



**RESEARCH ARTICLE**

**PERFORMANCE of MULTIUSER V-BLAST WITH LIMITED FEEDBACK for USER SCHEDULING over NAKAGAMI- $m$  FADING CHANNELS**

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**ABSTRACT**

In this paper, the performance of a downlink multiuser zero-forcing (ZF) Vertical Bell Laboratories Layered Space-Time (V-BLAST) with an ordering scheme over Nakagami- $m$  fading channels is presented. A system with two transmit antennas while each user is equipped with  $r$  ( $r \geq 2$ ) receiver antennas is considered. In every time slot, the base station schedules a single user out of  $U$  users and communicates with it by adopting a limited feedback (FB) technique for user scheduling. One user is chosen based on a set of bits each sent from a distinct user to the transmitter. The channel considered in this investigation is the fading channel with Nakagami- $m$  distribution which covers a wide range of multipath fading patterns. The adopted FB scheme requires a threshold value which is attained through a numerical optimization process. For a given shape parameter, the optimum threshold value approaches to zero as SNR increases. The results of the outage probability of the investigated systems with varying  $m$ -parameter value of Nakagami- $m$  fading channels are presented and compared with the no FB method. It is shown that the studied FB approach on multiuser ZF V-BLAST indeed outperforms the no FB technique. The shape parameter value clearly affects the performance of the FB system, whereas it seems have no significant effect on the random user scheduling (no FB) method.

**Keywords:** *Zero-Forcing, V-BLAST, User Scheduling, Limited Feedback, Multiuser System, Nakagami- $m$  Fading Channel*

**1. INTRODUCTION**

The transmission methods that rely on multiple antennas at both transmitter and receiver sides are reputable in significantly improving the channel capacity of wireless communication systems. Among of many techniques which have been suggested to employ the high spectral efficiency of multiple-input multiple-output (MIMO) channels, the Vertical Bell Laboratories Layered Space-Time (V-

BLAST) with zero-forcing (ZF) approach is capable of achieving high spectral efficiency. Moreover, it seems comparatively less complicated to implement. By using the V-BLAST technique, the input data streams are transmitted in parallel and simultaneously over the transmitting antennas without the requirement of channel state information (CSI) known at the transmitter side. The receiver applies sequential interference nulling and cancellation (SIC) processes to decode the received sub-streams, which are combined and overlapped by noise, one after one [1,2]. The SIC allows the symbol to be detected and decoded without inter symbol interference (ISI). The implementation of symbol detection and decoding at a specified SIC stage can be carried out by an ordering or non-ordering scheme. The ordering method is an approach to improve the drawback caused by the non-ordering scheme, i.e., error propagation. In fact that the detection error of the first symbol may incur more errors in the following symbol detections, the overall performance of the ZF V-BLAST with a non-ordering scheme is narrowed as the result. The ordering scheme based on the highest signal-to-noise ratio (SNR) level, offers a solution to mitigate this error propagation problem, i.e., the data sub-stream with the largest post-processing SNR is detected first [1].

In this paper, the multiuser ZF V-BLAST with ordering SIC scheme is combined with a feedback (FB) technique for user scheduling to improve performance. The FB technique that is exploited for user selection purposes practically lets every user to feed back the necessary information to the transmitter under the predetermined requirement. In general, the FB technique can be implemented by reporting the information in a scalar or limited form. Some studies exist regarding user scheduling in the multiuser scenario, such as opportunistic scheduling, threshold-based scheduling, and limited FB. The opportunistic scheduling technique implies the process based on the maximum SNR so that every user will send the information of its highest post-processing SNR to the transmitter [3-5]. While in the user selection that employs the threshold-based approach, one allows only the users satisfying the condition to report back their SNR values to the transmitter [6]. The threshold-based scheduling presents a solution for the disadvantage introduced by the opportunistic scheduling, i.e., the heavy FB load. However, the threshold-based scheduling also becomes not feasible as the number of users increases. Then, there is the combination between threshold-based scheduling and quantization of the FB values which has proposed a proper technique to improve the user selection process. This combination, which is also noted as the limited FB technique, enables users to feed back a bit 1 or 0 relying on compliance with the threshold requirement, then leads the reduction of the FB load [7]. The studies about the performance of multiuser V-BLAST with limited FB can be found in the papers [8-9]. The contributions by [8] assume Rayleigh channel fading condition through the work, whereas [9] utilizes transmit antenna selection and maximal-ratio combining (MRC) schemes at the transmitter and the receiver side, respectively. As for predetermining criteria to be implemented in the FB technique, various strategies can be applied for optimization purposes. Some studies have shown different approaches regarding the predetermining criteria for the FB requirement. For example, the study by [9] applies a threshold scheme to not only select a user but also choose the best transmit antenna for which the SNR at the scheduled user is maximized. The work in [10] employs several criteria for user scheduling of V-BLAST users.

The Nakagami- $m$  distribution is regarded as one of the essential models because of its versatility in representing the fading envelope due to multipath fading in wireless communications [11]. The distribution in which the method of derivation and the fundamental characteristics were presented

initially based on field measurements is a model with two parameters, i.e., scale and shape parameter (or the fading parameter or  $m$ -parameter). This distribution, through its parameter  $m$  can be utilized to characterize fading channel conditions in the range from severe to moderate levels [12]; moreover also accommodates the Rayleigh distribution model as a particular case ( $m = 1$ ). Some studies have shown that the Nakagami- $m$  distribution is proper for data signals received in urban radio multipath channels [13-14]. The Nakagami- $m$  distribution is more suitable in land mobile and indoor propagation environments, so we are motivated to examine the limited FB concept by [10] over Nakagami- $m$  fading channels. To the best of our knowledge, the performance of multiuser V-BLAST with ordering scheme and 1-bit FB over Nakagami- $m$  fading channels has not yet been investigated in any literature.

The main points of the paper are summarized as follows. First, the average outage probability (AOP) of the investigated system at a certain number of  $m$ -parameter, SNR value, and  $U$  users is examined to a range value of threshold values. Consequently, the optimized threshold values will be attained for every  $m$ -parameter, SNR value, and  $U$  user of the studied systems. Furthermore, the AOP in a range of SNR values at a given  $m$ -parameter and  $U$  users is illustrated by applying the obtained threshold values from the threshold optimization process. Consequently, the gain of exploiting the limited FB scheme in V-BLAST for user scheduling over Nakagami- $m$  fading channels is represented by comparing its error performance with no FB method.

The rest of this paper is structured in the following way. Section 2 describes the V-BLAST system model used in the investigation. Section 3 covers the simulation results of the studied system. Finally, the conclusion is given in Section 4. Throughout the paper, the operators  $(\cdot)^H$  and  $\|\cdot\|$  represent the Hermitian transpose and Euclidean norm, respectively. Uppercase and lowercase bold letters denote the matrices and column vectors, respectively.

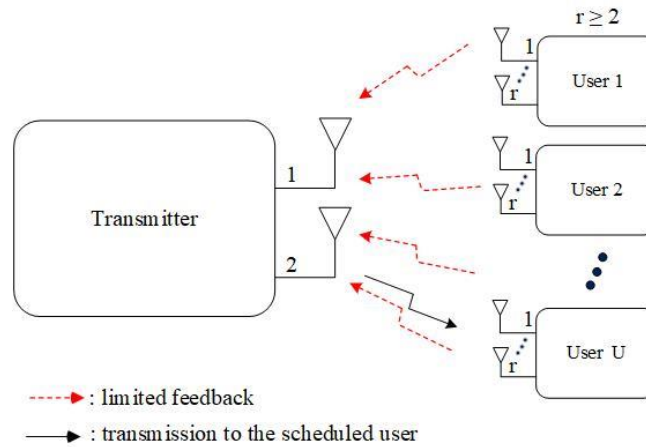
## 2. SYSTEM MODEL

We consider a MIMO downlink system that establishes communication between a base station equipped with two transmit antennas and  $U$  users, each with  $r$  ( $r \geq 2$ ) antennas as depicted in Figure 1. The complete knowledge of CSI is assumed to be available only at the receivers. Each user is required to report back to the transmitter in a form of a single bit which correlates with its sub-channel gain. Following this, the transmitter receives a set of FB bits from users and then opportunistically chooses only one user to communicate with in every training slot. Later, in each communication sequence, the transmitter dispatches two autonomously encoded streams through its antennas in a synchronous and parallel way to the scheduled user. At the receiver side of the chosen user, detecting and decoding the streams processes are performed by utilizing the ZF V-BLAST with the ordering scheme technique.

The fading channel vector between the  $s$ th transmit antenna and the receiver antennas at the user  $u$  is denoted by  $\mathbf{h}_s^u$  for  $u \in \{1, 2, \dots, U\}$  and  $s \in \{1, 2\}$ . Hence, the  $k$ th entry of  $\mathbf{h}_s^u$  represents the fading coefficient between the  $s$ th transmit antenna and the  $k$ th receive antenna at the user  $u$ . Nakagami- $m$  fading scenario is considered in this study, and the random channel matrices for the users are assumed to be not correlated with each other. Each channel of any user remains fixed in the course of one

transmission cycle and switches independently from one to another transmission process. The norms of entries of  $\mathbf{h}_s^u$  are independent and identically distributed Nakagami- $m$  random variables, each with a certain  $m$ -parameter ( $m \geq 0.5$ ) and a scale parameter of unity. Furthermore, we assume that a capacity-achieving code applied on both sub-streams.

The transmitter decides one user out of  $U$  users based on the FB reported back from all users in a limited format. The limited FB utilized in this work as the FB scheme is a technique that allows users to send a single bit (bit 1 or 0) based on the decided standard. For this reason, this approach is also noted as the 1-bit FB technique. The  $u$ th user sends back bit 1 to the transmitter if its highest outcome of sub-channel gain on the first layer  $\gamma_{u1}$  is greater or equal to a threshold value  $\gamma_{th}$ , and bit 0 any other way. Let a set of users feed back bit 1 to the transmitter symbolized by  $B$ , and then transmitter performs the selection for user scheduling by observing the cardinality of  $B$  denoted as  $\bar{B}$ . If  $\bar{B}$  is unity, the user in  $B$  is scheduled by the transmitter. In case  $B$ 's number of elements exceeds one, the transmitter randomly selects and schedules one user from  $B$ . Otherwise (when  $B$  equals empty set), which means that all the users feed back bit 0, the transmitter chooses one user out of  $U$  users in a random fashion.



**Figure 1.** The system's block diagram.

Assume that  $u'$  ( $u' \in \{1, 2, \dots, U\}$ ) represents the index of the scheduled user to communicate with the transmitter. Then, the received complex signal at the selected user is given by

$$\mathbf{y}^{u'} = \mathbf{h}_1^{u'} x_1 + \mathbf{h}_2^{u'} x_2 + \mathbf{n} \quad (1)$$

where  $x_s$  and  $\mathbf{n}$  respectively denote the transmitted modulated symbol from the  $s$ th transmit antenna and the additive white Gaussian noise (AWGN) at the receiver. The ZF V-BLAST with an optimum ordering approach is applied to decode the symbols at the receiver. The decoding processes start with a sub-stream with the highest post-processing sub-channel gain, hence the outage probability is

minimized. First, the received signal is multiplied with a particular matrix obtained employing QR factorization, thereby the interference at a given sub-stream induced by other sub-streams is eliminated. Later, that specific sub-stream is decoded without any interference. Subsequently, the interference caused by the previously decoded sub-stream is subtracted from the total signal, and the other symbol is decoded. The first and second squared sub-channel gains produced by the ZF V-BLAST with the optimum ordering scheme are expressed as  $\gamma_{u1} = \max \left\{ \left( \mathbf{h}_1^{u'} \right)^H \mathbf{P}_{\mathbf{h}_2^{u'}}^\perp \mathbf{h}_1^{u'} \right\}, \left( \mathbf{h}_2^{u'} \right)^H \mathbf{P}_{\mathbf{h}_1^{u'}}^\perp \mathbf{h}_2^{u'} \right\}$  and  $\gamma_{u2} = \min \{ \|\mathbf{h}_1^{u'}\|^2, \|\mathbf{h}_2^{u'}\|^2 \}$ , respectively [2]. Here,  $\mathbf{P}_{\mathbf{h}_s^u}^\perp$  represents the projection matrix onto the null space of the vector  $\mathbf{h}_s^u$ . The result of  $\arg \max \left\{ \left( \mathbf{h}_1^{u'} \right)^H \mathbf{P}_{\mathbf{h}_2^{u'}}^\perp \mathbf{h}_1^{u'} \right\}, \left( \mathbf{h}_2^{u'} \right)^H \mathbf{P}_{\mathbf{h}_1^{u'}}^\perp \mathbf{h}_2^{u'} \right\}$  indicates the index of the transmitted symbol which is best to be decoded first [2].

### 3. SIMULATION RESULTS

In this section, we present the numerical results based on the Monte Carlo simulations regarding the AOP. First, separately assuming three distinct values of the shape parameter, i.e.,  $m = 1$ ,  $m = 2$ , and  $m = 5$ , each scenario at a specific number of users is examined independently within a range of estimated threshold values and varying SNR. The threshold value  $\gamma_{th}$  is used by all the users to feed back either bit 1 or bit 0. For the optimization, we numerically evaluate AOP for a range of threshold values and the one minimizing AOP is adopted as  $\gamma_{th}$ . This procedure is followed for all the inspected scenarios. Following this, the proposed scheme of multiuser ZF V-BLAST is simulated using all the obtained  $\gamma_{th}$  corresponding to each SNR value. The comparison between the FB and the no FB technique with various fading parameter values is presented as well. The simulation is implemented under the condition of  $t = r = 2$  for  $U = 5$  and  $U = 10$ . In addition, each sub-channel is assumed to have a target spectral efficiency of  $R = 2$  bits/s/Hz.

In this work, we utilize Nakagami- $m$  distribution, whose probability density function is stated below.

$$f_{|\mathbf{h}_{s,r}|}(z) = \frac{2m^m z^{2m-1} \exp(-mz^2)}{\Gamma(m)} \quad (2)$$

for  $z \geq 0$ , where  $\mathbf{h}_{s,r}$  denotes the multipath channel coefficients between transmit and receive antennas which are independent complex variables,  $m$  represents the fading figure or shape parameter,  $\exp(\cdot)$  denotes the exponential function, and  $\Gamma(\cdot)$  is the Gamma function. All multipath channel coefficients are assumed to have the same  $m$  value. Moreover, the average received power here is considered by  $E \left\{ |\mathbf{h}_{s,r}|^2 \right\} = 1$ , i.e., all the channel coefficients have identical average power.

As mentioned before, the optimization processes are performed to obtain the best threshold values for varying numbers of users,  $m$ -parameter, and SNR values such that AOP is minimized. Outage probability is defined as the probability that the attained (achievable) information rate is less than the required information rate. In such a case the receiver cannot decode the transmitted message signal and an outage occurs. The acquired  $\gamma_{th}$  with respect to SNR values for  $U = 5$  and  $U = 10$  are shown in

Table 1 and 2, respectively. Figure 2 and Figure 3 respectively show the suitable values  $\gamma_{th}$  for  $U = 5$  (with  $m = 2$ ) and  $U = 10$  (with  $m = 5$ ) in terms of AOP. Based on the optimization results, it is shown that for every scenario of the  $m$ -parameter, the optimum threshold value  $\gamma_{th}$  reaches zero when SNR escalates.

The results of multiuser ZF V-BLAST with FB technique are shown in Figure 4 and Figure 5 for  $U = 5$  and  $U = 10$ , respectively, by applying the attained optimal  $\gamma_{th}$  values (at which AOP is minimized), which correspond to a given shape parameter and SNR value. The comparisons with the random user scheduling ZF V-BLAST (no FB technique) can be observed as well from the graphs. The FB technique outperforms the random user scheduling technique in all shape parameters of Nakagami- $m$  fading channels. Applying the limited FB has shown its advantage on the performance of multiuser ZF V-BLAST over Nakagami- $m$  channels, i.e the acquired AOP performances of FB method are better than those of no FB, which share the same values of  $U$ ,  $m$ -parameter and SNR. For  $U = 5$ , an AOP of  $10^{-4}$  is reached by the FB method at SNR values around of 27.5 dB, 18 dB and 15 dB for  $m = 1$ ,  $m = 2$  and  $m = 5$ , respectively. On the other hand, the no FB method for all  $m$ -parameter values cannot reach an AOP of  $10^{-4}$ , even though the SNR value already reaches 28 dB. In the scenario of  $U = 10$ , the 1-bit FB approach again surpasses the no FB technique in every  $m$ -parameter value. For an AOP of  $10^{-4}$ , the attained SNR gains of FB technique are about 27, 18, and 12 dB for  $m = 1$ ,  $m = 2$  and  $m = 5$ , respectively. The graphs also exhibit that the performances for the no FB scheme are almost the same regardless of the shape parameter values. Conversely, the performances improve as the  $m$ -parameter increases for the FB method.

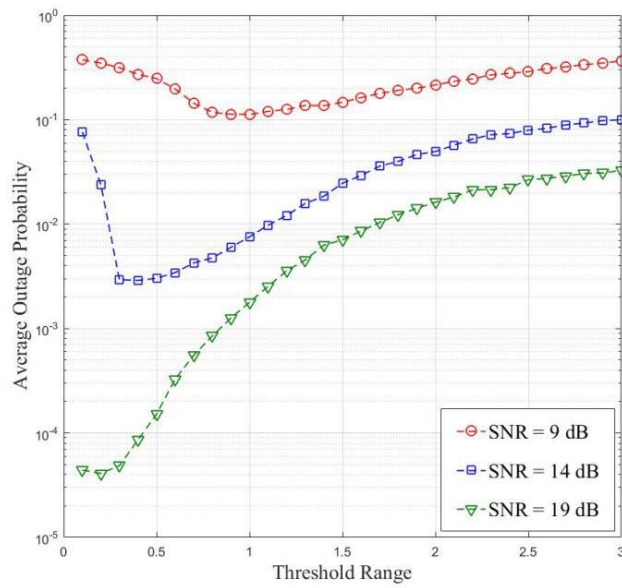
**Table 1.** Threshold values with respect to SNR values for  $U = 5$ .

$m$	Threshold values $\gamma_{th}$		
	SNR (10 dB)	SNR (19 dB)	SNR (28 dB)
1	1.1	0.5	0.4
2	SNR (9 dB)	SNR (14 dB)	SNR (19 dB)
	1	0.4	0.2
5	SNR (10 dB)	SNR (13 dB)	SNR (16 dB)
	0.6	0.4	0.2

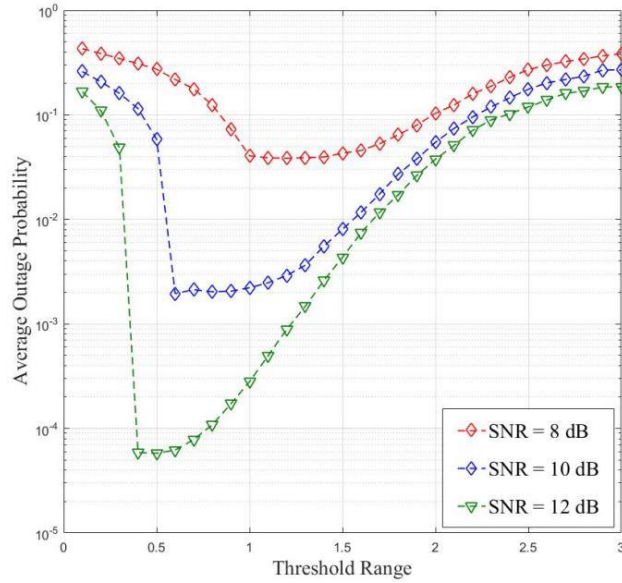
**Table 2.** Threshold values with respect to SNR values for  $U = 10$ .

$m$	Threshold values $\gamma_{th}$		
	SNR (10 dB)	SNR (19 dB)	SNR (28 dB)
1	1.9	1	0.7
2	SNR (9 dB)	SNR (14 dB)	SNR (19 dB)
	1.3	1.2	0.6
	SNR (8 dB)	SNR (10 dB)	SNR (12 dB)

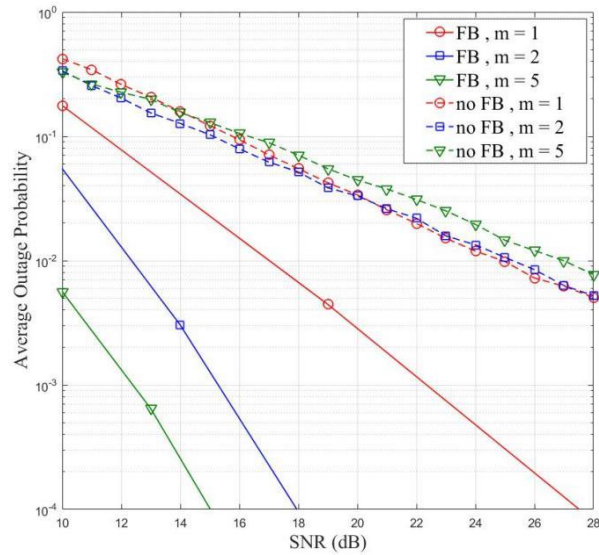
5 1.2 0.6 0.5



**Figure 2.** Average outage probability vs threshold range for  $U = 5$  and  $m = 2$  with varying SNR values.

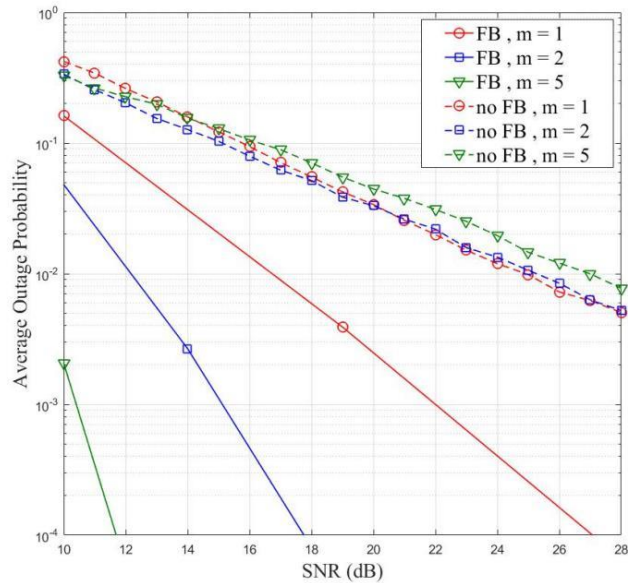


**Figure 3.** Average outage probability vs threshold range for  $U = 10$  and  $m = 5$  with varying SNR values.



**Figure 4.** Average outage probability comparison between the limited FB and no FB (random user scheduling) of ZF V-BLAST for  $U = 5$  with varying SNR values.





**Figure 5.** Average outage probability comparison between the limited FB and no FB (random user scheduling) of ZF V-BLAST for  $U = 10$  with varying SNR values.

#### 4. CONCLUSION

The performance of the ZF V-BLAST with the optimum ordering and limited FB method has been presented in a  $U$ -user downlink network system where the transmitter has two antennas and every user is equipped with  $r$  ( $r \geq 2$ ) antennas. Each user reports back a single bit FB (either bit 1 or 0) regarding its sub-channel gain to the transmitter. In each training cycle, the transmitter only selects and schedules one user to communicate with based on the FB data. The user scheduling is carried out by applying a threshold value. The limited FB strategy is implemented in different scenarios, including the number of users, the fading parameter of Nakagami- $m$  channels, the SNR values, and the threshold values. The numerical optimization process is implemented to obtain the optimum threshold values and the results show that as SNR increases, the attained threshold gets closer to zero value for every shape parameter. The simulation results of the investigated FB system on multiuser ZF V-BLAST have shown that the performance of the adopted limited FB method exceeds that of the no FB technique. Furthermore, for the multiuser ZF V-BLAST, the AOP outcomes of the random user scheduling (no FB) technique show almost similar results regardless of the value of the  $m$ -parameter while the FB scheme yields better performance as the shape parameter value gets higher.

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