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AN EXTENSION OF THE LAST WORK OF OLEG A. TRETYAKOV: A NOVEL FORMAT OF THE WAVE EQUATION IN SI UNITS*

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ABSTRACT

Professor Oleg Alexandrovich Tretyakov is recognized in the field of radiophysics as founder of Evolutionary Approach to Electromagnetics (EAE). His analytical time domain approach which was acknowledged as an alternative to the method of complex amplitudes successfully applied to cavity and waveguide problems. In his final years, he proposed a novel format of Maxwell's equations in SI units where electric and magnetic field strength vectors have a common physical dimension of inverse meter. In this article, we pay special tribute to his exceptional career and contributions to science and extend his last work to derive a novel format of the wave equation in SI units.

Keywords: *Maxwell's Equations, Time-domain Electrodynamics, Electromagnetics, Evolutionary Approach, Wave Equation.*

OLEG A. TRETYAKOV'UN SON ÇALIŞMASININ BİR UZANTISI: SI BİRİM SİSTEMİNDE YENİ BİR DALGA DENKLEMİ FORMATI

ÖZ

Profesör Oleg Alexandrovich Tretyakov, radyofizik alanında Elektromanyetik Teoriye Evrimsel Yaklaşım'ın (ETEY) kurucusu olarak tanınmaktadır. Frekans domeni metodlarına bir alternatif olarak kabul edilen Tretyakov'un analitik zaman domeni metodu, kavite ve dalga kılavuzu problemlerine başarıyla uygulanmıştır. Son yıllarında, elektrik ve manyetik alan şiddet vektörlerinin bir bölü metre'lik ortak bir fiziksel boyuta sahip olduğu, SI birimler sisteminde yeni bir Maxwell denklemler formu önermiştir. Bu makalede, istisna kariyerine ve bilime yaptığı katkılara özel bir saygı sunarak önerdiği yeni formdaki Maxwell denklemler sisteminden faydalanarak SI birimler sisteminde yeni bir dalga denklemi formu önerilmiştir.

Anahtar Kelimeler: *Maxwell Denklemleri, Zaman Uzayı Elektrodinamiği, Elektromanyetik, Evrimsel Yaklaşım, Dalga Denklemi.*

1. INTRODUCTION

Professor Oleg Alexandrovich Tretyakov passed away on Tuesday, January 25, 2022, after a short battle with lung cancer. His contributions to the growth of electromagnetics research in Ukraine and Turkey are remarkable. He will be greatly missed by his friends and colleagues around the world.

As the Chairman of Commission B of the International Union of Radio Science (URSI) Committee of Ukraine since 1993, and a member of the Institute of Electrical and Electronics Engineers (IEEE), he was a distinguished reviewer of the November 2021 issue of the Journal of Naval Sciences and Engineering. Both URSI Radio Science Bulletin and IEEE Antennas and Propagation Magazine announced the sad news to the radiophysics and electromagnetic communities by publishing the *In Memoriam* letters reflecting his scientific career and main contributions (Erden, 2022a; Erden, 2022b).

In this article, we pay special tribute to his exceptional career and contributions to science and extend his final work to derive a novel format of the wave equation in SI units.

2. EVOLUTIONARY APPROACH

Professor Tretyakov is best known for the *Evolutionary Approach to Electromagnetics (EAE)* which was proposed by him for the analytical solution of cavity and waveguide problems directly in the time domain (Tretyakov, 1986; Tretyakov, 1993).

His approach was acknowledged as an alternative to *the classical time-harmonic field method*, namely *the method of complex amplitudes* (Tretyakov & Erden, 2012; Tretyakov, 2018). It was proved that the set of natural cavity modes originates a complete set of vector functions of coordinates in an appropriate class of quadratically integrable functions (i.e., it is a *modal basis*). Following the pioneering work by Kisunko (1949), Tretyakov (1989; 1994) pushed his approach into the waveguide problems.

Hence, by making use of this approach, the electromagnetic fields in cavities and waveguides were presented as *series in terms of their natural modes with their amplitudes sought for* where the cavity modal amplitudes depend on time *t*, and the waveguide modal amplitudes are functions of *t* and axial coordinate *z*. In the cavity case, *evolutionary ordinary* differential equations were derived for the modal amplitudes via projecting Maxwell's equations onto the modal basis. As for the waveguide modal amplitudes, *evolutionary partial* differential equations were obtained.

For the cavity problem, the time-dependent modal amplitudes were obtained explicitly when an impressed force is given as an arbitrary *integrable* function of time. For the waveguide problem, the evolutionary equations for the waveguide modes were obtained. Longitudinal components of the waveguide modes satisfy some partial differential equations, which are generalizations of the well-known Klein-Gordon equation. Modal amplitudes for transverse fields were found as *z*- and *t*- *derivatives* of the longitudinal components. Using his approach, exact explicit solutions for the cavity oscillations, and for waveforms propagating in waveguides are successfully studied over decades.

3. IMPLEMENTATION OF THE APPROACH

The method, first, was applied to free space, considered a regular structure, in a cylindrical coordinate system. Modes for this case had already been found in the frequency domain. Tretyakov and Dumin (2000) shifted this approach to the time domain for analysis of transient beam radiation and propagation using an expansion over Bessel modes with mode amplitudes being governed by the Klein–Gordon equation (KGE) formulated earlier. The main peculiarity, in this case, is that since the transverse domain is now unbounded, the mode spectrum is continuous which leads to integrals instead of sums in the mode expansion.

Butrym and Tretyakov (2002) introduced the next main improvement of the method for waveguides where in an inhomogeneous waveguide the permittivity and permeability functions of the filling are some functions of transverse coordinates (which goes into the boundary value problem for the modes) and a function of the longitudinal coordinate and time. The latter goes into the evolutionary waveguide equations as coefficients that govern the modes' amplitude transformation with propagation. In this case, the mode coupling was unavoidable.

A detailed implementation of the EAE for studying a time-variant hollow cavity system was published by Aksoy and Tretyakov (2002). Later, Aksoy and Tretyakov (2003a, 2003b) obtained explicit solutions for the cavity oscillations excited by a set of digital signals.

Within EAE, Butrym et al. (2004) solved the problems of short pulse propagation and diffraction from a permittivity step in a waveguide with a detailed time-domain energy flow analysis. Later, Butrym and Legenkiy (2009) considered the same problem for a conductive boundary with emphasis on the effect of transient charge displacement by a pulsed E-wave.

Aksoy et al. (2005) compared the calculation results of EAE with the Finite Difference Time Domain (FDTD) for the cavity filled with a dispersive medium. Antyufeyeva et al. (2009) considered a cavity with a double negative medium as an example. It was shown that due to the interaction of a spatial resonance in the cavity (due to waves bouncing between the walls) with local medium resonances (due to molecular oscillations), there are several possible frequency resonances for a single spatial mode distribution. Antyufeyeva and Tretyakov (2010) considered the problem of the transient (nonstationary) homogeneous medium in a cavity.

Erden and Tretyakov (2008) studied excitation of the electromagnetic fields by a wide-band current surge in a hollow cavity, and later, in a cavity filled with Debye and Lorentz kind dispersive medium (Tretyakov and Erden, 2008). The results were presented in several international conferences (Tretyakov & Erden, 2006; Erden & Tretyakov, 2011; Erden & Tretyakov, 2014; Erden, Tretyakov, & Bicer, 2016; Erden & Tretyakov, 2019). Therein, the Debye and Lorentz equations play the role of the dynamic constitutive relation between the polarization vector and the electric field. Tretyakov and Erden (2007) also presented the instantaneous and dynamic parts of polarization separately for the Debye medium.

Later, the interaction of the cavity fields with plasma is studied within EAE, making use of the motion equation for plasma (Erden, 2017a; Erden, 2017b; Erden, 2018; Erden, 2015; Erden & Bicer, 2016).

Regarding the waveguide problem, Aksoy and Tretyakov (2004) considered the excitation and propagation problem of digital signals in a hollow waveguide by making use of the method. Solving the KGE explicitly, a special case has been considered by Tretyakov and Akgun (2010) where the modal amplitudes of the waveguide fields are expressible explicitly via the Airy functions from mathematical physics. The complete set of TE- and TM- time-domain modal waves is established and studied in detail by Tretyakov and Kaya (2012; 2013). As a result of dealing with real-valued functions in the time domain, Eroglu, Aksoy, & Tretyakov (2012), Akgun and Tretyakov (2015) and Erden (2017) obtained explicit solutions for the energetic properties of the modal waveforms.

The velocity of transportation of the modal field energy, defined by the power flow and energy densities, is derived, and presented as a function of time and axial coordinate (Erden, Cosan, & Tretyakov, 2016; Tretyakov & Erden, 2016; Erden & Tretyakov, 2017; Cosan, Erden, & Tretyakov, 2019). Recently, analytical solutions for the causal oscillations in a cavity with lossy walls presented through the upgraded version of the EAE (Erden, 2021a; Erden, 2021b; Erden et al, 2021; Tretyakov et al., 2021).

Lately, the electromagnetic inertia and the mechanical momentum of the electromagnetic fields for any closed cylindrical waveguide are obtained and presented to the radiophysics community (Tretyakov, 2017; Erden & Tretyakov, 2017; Erden, Tretyakov, & Cosan, 2018; Erden, Cosan, & Tretyakov, 2017; Erden & Tretyakov, 2018; Erden, Cosan, & Tretyakov, 2018; Tretyakov & Erden, 2020; Tretyakov & Erden, 2021).

Ongoing research within the framework of the evolutionary approach by Professor Tretyakov's former students and colleagues is listed in detail in the recent book chapter (Tretyakov, Butrym, & Erden, 2021).

4. A NOVEL FORMAT OF MAXWELL'S EQUATIONS

In his final years, Professor Tretyakov presented a novel format of Maxwell's equations in SI units (Tretyakov & Erden, 2020; Tretyakov & Erden, 2021, Tretyakov, Butrym, & Erden, 2021). Quantities occurring in Maxwell's equations are rescaled so that the various relationships appear in a dimensionally simpler format.

This perspective is implemented in the evolutionary approach to develop an adequate time-domain theory of cavities and waveguides. As the new format of Maxwell's equations is rather simpler than the standard one, the upgraded version of the EAE was found more convenient for practical applications (Kobayashi & Smith, 2021, p. 16).

Utilizing the novel format of Maxwell's equations proposed by Professor Tretyakov, energetic and mechanical field characteristics are obtained as functions of time, and the concept of modes in the time-harmonic setting for oscillations in cavities and waves in waveguides has its counterpart in time-dependent modal amplitudes.

The central point in rearranging Maxwell's equations to a new format in SI units is based on the *novel definition* of the free-space constants and charge density as

$$\varepsilon_0^V = \sqrt{\frac{1N}{\varepsilon_0}} \left\lfloor V = \frac{Nm}{As} \right\rfloor, \quad \mu_0^A = \sqrt{\frac{1N}{\mu_0}} \lfloor A \rfloor, \quad \varrho = \frac{\rho}{\sqrt{N\varepsilon_0}} \lfloor m^{-2} \rfloor \tag{1}$$

where N is a force of one *newton*. ε_0^V, with its numerical value of 3.361×10^5, and μ_0^A, with its numerical value of 8.921×10^2, are used as the *scaling coefficients* for the standard electric, $\mathcal{E}$, and magnetic, $\mathcal{H}$, fields to divide the physical dimensions of $\lfloor V/m \rfloor$ and $\lfloor A/m \rfloor$ as

$$\left.\begin{array}{l}
\underbrace{\boldsymbol{\mathcal{E}}(\mathbf{r},t)}_{\lfloor V/m \rfloor} = \underbrace{\varepsilon_0^V}_{\lfloor V \rfloor} \underbrace{\mathbb{E}(\mathbf{r},t)}_{\lfloor 1/m \rfloor} = \underbrace{3.361\times10^5}_{\lfloor V \rfloor} \times \underbrace{\mathbb{E}(\mathbf{r},t)}_{\lfloor 1/m \rfloor} \\
\underbrace{\boldsymbol{\mathcal{H}}(\mathbf{r},t)}_{\lfloor A/m \rfloor} = \underbrace{\mu_0^A}_{\lfloor A \rfloor} \underbrace{\mathbb{H}(\mathbf{r},t)}_{\lfloor 1/m \rfloor} = \underbrace{8.921\times10^2}_{\lfloor A \rfloor} \times \underbrace{\mathbb{H}(\mathbf{r},t)}_{\lfloor 1/m \rfloor} \\
\underbrace{\boldsymbol{\mathcal{J}}(\mathbf{r},t)}_{\lfloor A/m^2 \rfloor} = \underbrace{\mu_0^A}_{\lfloor A \rfloor} \underbrace{\mathbb{J}(\mathbf{r},t)}_{\lfloor 1/m^2 \rfloor} = \underbrace{8.921\times10^2}_{\lfloor A \rfloor} \times \underbrace{\mathbb{J}(\mathbf{r},t)}_{\lfloor 1/m^2 \rfloor}.
\end{array}\right\} \tag{2}$$

The SI dimensions of volt $\lfloor V \rfloor$ and of ampere $\lfloor A \rfloor$ are assigned to the factors ε_0^V and μ_0^A, in the new definition. So, instead of the standard format of Maxwell's equations in SI units presented as

$$\begin{gathered}
\nabla \times \boldsymbol{\mathcal{H}}(\mathbf{r},t) = \varepsilon_0 \frac{\partial}{\partial t} \boldsymbol{\mathcal{E}}(\mathbf{r},t) + \boldsymbol{\mathcal{J}}(\mathbf{r},t) \\
\nabla \times \boldsymbol{\mathcal{E}}(\mathbf{r},t) = -\mu_0 \frac{\partial}{\partial t} \boldsymbol{\mathcal{H}}(\mathbf{r},t) \\
\nabla \cdot \boldsymbol{\mathcal{H}}(\mathbf{r},t) = 0 \\
\nabla \cdot \boldsymbol{\mathcal{E}}(\mathbf{r},t) = \frac{1}{\varepsilon_0} \rho(\mathbf{r},t)
\end{gathered} \tag{3}$$

the *new* SI *format* of the set of Maxwell's equations was proposed as

$$\begin{gathered}
\nabla \times \mathbb{H}(\mathbf{r},t) = \frac{1}{c} \frac{\partial}{\partial t} \mathbb{E}(\mathbf{r},t) + \mathbb{J}(\mathbf{r},t) \\
\nabla \times \mathbb{E}(\mathbf{r},t) = -\frac{1}{c} \frac{\partial}{\partial t} \mathbb{H}(\mathbf{r},t) \\
\nabla \cdot \mathbb{H}(\mathbf{r},t) = 0 \\
\nabla \cdot \mathbb{E}(\mathbf{r},t) = \varrho(\mathbf{r},t)
\end{gathered} \tag{4}$$

where c is the speed of light. The convenience of the *new format* of Maxwell's equations (4) to upgrade the EAE for solving some practical problems was exhibited in late studies (Erden, Tretyakov, & Cosan, 2018; Erden & Tretyakov, 2017; Tretyakov, 2018; Tretyakov, Butrym, & Erden, 2021; Tretyakov & Erden, 2021). One can easily obtain the standard format (3) by making use of the novel format of Maxwell's equations (4) together with equation (2). The novel field vectors in (4), $\mathbb{E}$ and $\mathbb{H}$, have the inverse meter $\lfloor 1/m \rfloor$ physical dimension, whereas the standard field vectors in (3), $\boldsymbol{\mathcal{E}}(\mathbf{r},t)$ and $\boldsymbol{\mathcal{H}}(\mathbf{r},t)$, have physical dimension of $\lfloor V/m \rfloor$ and $\lfloor A/m \rfloor$, respectively.

5. DERIVATION OF A NOVEL FORMAT OF WAVE EQUATION

Tretyakov and Erden (2021) exhibited Maxwell's equations in the novel format for the fields in free space, plasma and dielectrics. The mathematical format of (4) one-to-one coincides with the Heaviside-Lorentz form of Maxwell's equations (Carron, 2015; Rothwell & Cloud, 2001) albeit the latter have been derived within the framework of the CGS metric system. There is one more coincidence between (4) and Heaviside-Lorentz form, namely: the electric and magnetic field vectors have their common dimensions. In Heaviside-Lorentz form, this dimension is in centimeter-gram-second as $\left\lfloor g^{\frac{1}{2}} cm^{-\frac{1}{2}} s^{-1} \right\rfloor$. But in (4); $\mathbb{E}$ and $\mathbb{H}$, have the common SI dimension meter of $\lfloor 1/m \rfloor$ (Tretyakov & Erden, 2021).

Derivation of the Lorentz force law with new field vectors was presented in (Tretyakov & Erden, 2021, p. 88275). Energetic characteristics (i.e., field energy, Poynting vector, the velocity of transportation of energy), and mechanical equivalents of the energetic field characteristics were also derived with the novel field vectors $\mathbb{E}$ and $\mathbb{H}$ in (Tretyakov, Butrym, & Erden, 2021; Tretyakov & Erden, 2021).

Herein, we aim to derive an *uncoupled second-order partial differential equation*, that is wave equation, from the novel format of Maxwell's equations, which is a set of *coupled first-order partial differential equations*.

The first two equations of the novel format of Maxwell's equations in differential form, as given by (4), are first-order coupled differential equations; that is, both unknown fields ($\mathbb{E}$ and $\mathbb{H}$) appear in each equation. Usually, it is very desirable to uncouple these equations. This can be accomplished at the expense of increasing the order of the differential equations to the second order (Balanis, 2012).

Taking the curl of both sides of the first two equations of (4) and assuming a homogeneous medium, we can write that

$$
\begin{aligned}
\nabla\times\nabla\times\mathbb{H}(\mathbf{r},t) &= \frac{1}{c}\frac{\partial}{\partial t}\nabla\times\mathbb{E}(\mathbf{r},t)+\nabla\times\mathbb{J}(\mathbf{r},t) \\
\nabla\times\nabla\times\mathbb{E}(\mathbf{r},t) &= -\frac{1}{c}\frac{\partial}{\partial t}\nabla\times\mathbb{H}(\mathbf{r},t)
\end{aligned}
\tag{5}
$$

Using the vector identity

$$
\nabla\times\nabla\times\mathbb{F}=\nabla(\nabla\cdot\mathbb{F})-\nabla^2\mathbb{F} \tag{6}
$$

into the left side of (5), we can rewrite it as

$$
\begin{aligned}
\nabla\left(\nabla\cdot\mathbb{H}(\mathbf{r},t)\right)-\nabla^2\mathbb{H}(\mathbf{r},t) &= \frac{1}{c}\frac{\partial}{\partial t}\nabla\times\mathbb{E}(\mathbf{r},t)+\nabla\times\mathbb{J}(\mathbf{r},t) \\
\nabla\left(\nabla\cdot\mathbb{E}(\mathbf{r},t)\right)-\nabla^2\mathbb{E}(\mathbf{r},t) &= -\frac{1}{c}\frac{\partial}{\partial t}\nabla\times\mathbb{H}(\mathbf{r},t).
\end{aligned}
\tag{7}
$$

Substituting; (*i*) the first equation of (4) into the right side of the second equation of (7), (*ii*) the second equation of (4) into the right side of the first equation of (7), (*iii*) the third equation of (4) into the left side of the first equation of (7), (*iv*) the fourth equation of (4) into the left side of the second equation of (7) as

$$
\begin{aligned}
\nabla(0)-\nabla^2\mathbb{H}(\mathbf{r},t) &= \frac{1}{c}\frac{\partial}{\partial t}\left(-\frac{1}{c}\frac{\partial}{\partial t}\mathbb{H}(\mathbf{r},t)\right)+\nabla\times\mathbb{J}(\mathbf{r},t) \\
\nabla\left(\varrho(\mathbf{r},t)\right)-\nabla^2\mathbb{E}(\mathbf{r},t) &= -\frac{1}{c}\frac{\partial}{\partial t}\left(\mathbb{J}(\mathbf{r},t)+\frac{1}{c}\frac{\partial}{\partial t}\mathbb{E}(\mathbf{r},t)\right)
\end{aligned}
\tag{8}
$$

and rearranging its terms, we have that

$$
\begin{aligned}
\nabla^2\mathbb{H}(\mathbf{r},t)-\frac{1}{c^2}\frac{\partial^2\mathbb{H}(\mathbf{r},t)}{\partial t^2} &= -\nabla\times\mathbb{J}(\mathbf{r},t) \\
\nabla^2\mathbb{E}(\mathbf{r},t)-\frac{1}{c^2}\frac{\partial^2\mathbb{E}(\mathbf{r},t)}{\partial t^2} &= \frac{1}{c}\frac{\partial\mathbb{J}(\mathbf{r},t)}{\partial t}+\nabla\varrho(\mathbf{r},t)
\end{aligned}
\tag{9}
$$

uncoupled second-order differential equations for the new field vectors $\mathbb{E}$ and $\mathbb{H}$. Equations in (9) can be referred to as the *vector wave equation for the novel field vectors* $\mathbb{E}$ *and* $\mathbb{H}$.

For source-free ($\mathbb{J}=0,\ \varrho=0$) and lossless ($\sigma=0$) media, equations in (9) reduce to the simplest form of the vector wave equation for the novel field vectors

$$\begin{aligned}\nabla^2\mathbb{H}(\mathbf{r},t)-\frac{1}{c^2}\frac{\partial^2\mathbb{H}(\mathbf{r},t)}{\partial t^2}=0\\ \nabla^2\mathbb{E}(\mathbf{r},t)-\frac{1}{c^2}\frac{\partial^2\mathbb{E}(\mathbf{r},t)}{\partial t^2}=0.\end{aligned} \qquad \textbf{(10)}$$

For time-harmonic fields (time variations of the form $e^{j\omega t}$), replacing $\partial/\partial t \equiv j\omega$, $\partial^2/\partial t^2 \equiv (j\omega)^2 = -\omega^2$, and the instantaneous new field vectors $\mathbb{E}$ and $\mathbb{H}$, respectively, with the complex field vectors E and H, in (9) results in

$$\begin{aligned}\nabla^2\mathrm{H}(\mathbf{r})+\frac{\omega^2}{c^2}\mathrm{H}(\mathbf{r})=-\nabla\times\mathrm{J}(\mathbf{r})\\ \nabla^2\mathrm{E}(\mathbf{r})+\frac{\omega^2}{c^2}\mathrm{E}(\mathbf{r})=\frac{j\omega}{c}\mathrm{J}(\mathbf{r})+\nabla\varrho(\mathbf{r})\end{aligned} \qquad \textbf{(11)}$$

where $\frac{\omega^2}{c^2}=\omega^2\mu_0\varepsilon_0=k^2$, and k represents the wave number.

For source-free ($\mathbb{J}=0,\ \varrho=0$) and lossless ($\sigma=0$) media, equations in (11) reduce to

$$\begin{aligned}\nabla^2\mathrm{H}(\mathbf{r})+k^2\mathrm{H}(\mathbf{r})=0\\ \nabla^2\mathrm{E}(\mathbf{r})+k^2\mathrm{E}(\mathbf{r})=0\end{aligned} \qquad \textbf{(12)}$$

which are homogeneous vector Helmholtz's equations. Analogously, it is possible to derive the wave equation for the novel vector potential $\mathbb{A}$, and the wave equation for the novel scalar potential V.

7. CONCLUSION

The analytical time domain approach presented to the literature by Professor Oleg A. Tretyakov, i.e., Evolutionary Approach to Electromagnetics, allows including the well-developed mathematical theory of evolutionary differential equations to powerful tools of electromagnetics. In the general case of *EAE*, the evolutionary equations may be obtained as linear or nonlinear: as the constitutive relations for a medium under consideration dictate it. A series of young scientists are attracted by his idea and pursue their research to advance the method.

The upgraded version of the EAE with the novel format of Maxwell's equations, proposed lately by Tretyakov, will be more convenient for practical applications since the solutions can be found in the class of the real-valued functions. Obtaining the energetic and mechanical field characteristics as the functions of time opens a way to study the energetic wave processes that should inevitably accompany the oscillations in cavities and waves in waveguides.

Nowadays, multiscale and multiphysics disciplines are appealing for the development of modern techniques and technologies, i.e., Quantum Electromagnetics. Possibly, applying a factorization of the physical dimensions, as Tretyakov presented in the novel format of Maxwell's equations, may be useful also for further development of these modern topics.

Professor Tretyakov showed a commitment to radiophysics and electrodynamics throughout his career. The breadth and depth of his technical knowledge made him an outstanding professor, respected and sought out by both students and the radiophysics community. As students, colleagues, and friends, we not only lost a prominent scientist and mentor, but also a remarkable person. He will be sorely missed.

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