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Investment decision support system using credibility analysis with fuzzy interest rate

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Abstract

Today, with a fluctuating course of the economy, it is inevitable that the interest method used by people for investment will also fluctuate. There may be serious inconsistency between the current interest rate and the interest rate at the time of the investment. Therefore, in order to eliminate these inconsistent situation, fuzzy set theory is used and the case where the interest rate parameter is fuzzy variable is examined. So, one of the important points to be overcome in real life investment models is the estimation of interest rate. In this way, it is provided that the uncertain interest rate is close to the real-life interest rate. In this study, we tried to obtain generalized closed formulas to calculate the future value of the capital investment although interest rate is fuzzy variable. Generalized closed formulas of the future value of the capital investment have been obtained by expected value function. And we applied credibility theory for determining the expected value function.

1. Introduction

The idea of investment is one of the issues that have been considered since the existence of humankind. Since the first days of human history, people have wanted for the valuation of their tangible assets for investment purposes and utilized interest methods in various ways. Today, due to the fluctuating course of national economies; components of investment decisions also include uncertainty, such as interest rate, investment life. Moreover, since these uncertainties are not predictable; components cannot be expressed as stochastic or deterministic variables. One of the important points to be overcome in real life investment models is the estimation of interest rate. In most of the studies in the current literature, it is assumed that the interest rate is fixed to make an appropriate approach regarding the interest rate (Jaimungal and Wang,2006; Hespos and Strassmann,1965; Mulvey and Vladimiriou,1989) However, due to the current economic fluctuations; even stochastic estimation of interest rates may not be possible. Therefore, in the evaluation of investment analyzes; the use of deterministic or stochastic data may not give very accurate results. In such cases, for the analysis of investment projects; it would make sense to use fuzzy set theory. In recent years, studies on investment decision have been made by using fuzzy set theory. So, we tried to obtain generalized closed formulas to calculate the future values of the capital investment although interest rate is fuzzy variable. Generalized closed formulas of the future value of the capital investment have been obtained by expected value function. And we applied credibility theory proposed for determining the expected value function.

Kim (2002), made of pricing performance under a number of interest rate. One choice pricing model under a special stochastic interest rate does not notably outperform another choice pricing model under an alternative stochastic interest rate.

Korn and Kraft (2001), considered investment problems where a financier can invest in an investments account, stocks, and bonds and tries to maximize her efficiency from terminal wealth. And they suppose the interest rate as a stochastic variable.

Zhao (2009), investigated the long-time performances of two groups of stochastic interest rate models.

Wang et al. (2017), proposes a Monte Carlo simulation-based methodology for measuring Value-at-Risk of a portfolio consisting of alternatives and bonds. A benefit of this study is that its implementation does not need knowledge of the joint distribution or other statistical characteristics of the related risk factors.

Kahraman and Kaya (2010) used fuzzy parameters in stochastic investment decision and then alternative investment analysis is studied by using fuzzy event probability.

Bi and Wang (2009), assumed that the net present value (NPV) of cash flow, the costs of investment and interest rate are fuzzy numbers and then suggested the fuzzy-real options evaluation model. After that, they demonstrated the validity of model using a numerical example.

Kahraman and Uçal (2009), suggested a hybrid model advised by Carlsson and Fuller before and they examined the discrete compounding. They applied this model to an oil field investment and in conclusion the loss of information caused by early-defuzzification has been determined.

Dimova et al. (2006), proposed a new method for generalization of aggregation schemes based on level-2 fuzzy sets. And theoretical consideration is proved by simple numerical examples.

Ustundağ et al. (2010), suggested an economic analysis for RFID investment. The elements of cost and benefits are determined to measure the value of an RFID investment. The expected net present value of investment is determined by using Monte-Carlo simulation.

Kahraman et al. (2006), developed the formulas for the analyses of project-investment analyses techniques on fuzzy environment and some numeric examples are demonstrated. After that, the cash flows are expanded to geometric and trigonometric cash flows. Finally, a fuzzy versus stochastic investment analysis is examined by using the probability of a fuzzy event.

Karsak and Tolga (2001), proposed a fuzzy decision algorithm to select the most suitable advanced manufacturing systems alternative. The fuzzy discounted cash flow analysis is used for the evaluating the economic aspects of the advanced manufacturing systems alternative. Also, they demonstrated the results of the analysis using comprehensive numerical example.

Some of the studies investigated in this subject are shown in table 1.

Table 1. Interest rate studies

References	**Year**	**Interest rate variable**
Y.J. Kim	2002	stochastic
R.Korn and H. Kraft	2001	stochastic
Juan Zhao	2009	stochastic
X.Wang et al.	2017	stochastic
C. Kahraman and I. Kaya	2010	fuzzy
X. Bi and XF. Wang	2009	fuzzy
İ. Uçal and C. Kahraman	2009	fuzzy
L. Dimova et al.	2006	fuzzy
A. Üstündağ et al.	2010	fuzzy
C. Kahraman et al.	2006	fuzzy
E.E. Karsak and E. Tolga	2001	fuzzy

It can be seen that; the works on fuzzy interest rate are more limited than those in deterministic or stochastic ones. Therefore, we think that study, which will deal with fuzzy interest rate, may contribute to the literature. In this study, we tried to obtain generalized closed formulas in order to calculate the future value of the capital investment although interest rate is fuzzy variable. Generalized closed formulas of the future value of the capital investment have been obtained by expected value function defined by Xue et al (2008). And we applied credibility theory proposed by Liu et al. (2002), for determining the expected value function.

The rest of this paper is structured as follows. In the second part, definitions related to fuzzy set theory and credibility theory are mentioned. And the third part, we describe the present value of the capital, interest rate and the future value of the capital. Section 4 develops the formulas of the future value of the capital investment on fuzzy environment. And numerical examples of the proposed formulas in the section 4 are illustrated and sensitivity analysis has been obtained in the section 5. Then in the last section, we mentioned the conclusions and future studies.

2. Interest and the time value of money

As we all know, money has a time value. So, a hundred liras in yesterday are not a hundred pounds in today; because they have different purchase values. It is called as the time value of money. The factor that maintains this balance is interest. Thus, the interest rate is a function of capital, time, and interest rate. The function of the interest is:

$$Interest = f(Capital, Interest\ rate, Investment\ life)$$

The notations of the interest function are shown in Table 2:

Table 2. Interest function notations

F	Interest
P	Capital
i	Interest rate
n	Investment life

Interest is calculated in two ways: simple interest and compound interest.

Simple interest is calculated only on the capital amount and no interest is calculated on interest during the interest period. With the simple interest formula, interest is calculated as follow:

$$F = P * (1 + i * n) \quad (1)$$

Compound interest is calculated over the sum of the capital and interest in the previous period for each period. With the compound interest formula, interest is calculated as follow:

$$F = P * (1 + i)^n \quad (2)$$

In real life problems, compound interest is used in the calculation of interest, including time value.

3. Fuzzy set theory, credibility theory and expected value of a fuzzy variable function

In real life, there is not only randomness but also much uncertainty. For instances, a “large” house is not a clear in most cases. To identify such a situation, fuzzy set theory which was first proposed by Zadeh (1965), is used.

The characteristic function of a crisp set gives either 1 or 0 to each individual in the universal set, thereby belonging to or not belonging to the set. This function can be generalized as follows: a value which determines the degree of membership of the element is assigned to each element in a specific range in the universal set. This function is called as the membership function and the set is defined as a fuzzy set (Klir and Yuan, 1995).

A fuzzy set can be mathematically expressed as $\tilde{A} = (x, \mu_{\tilde{A}}(x)), \forall x \in X$ where X is the universal set and $\mu_{\tilde{A}}(x)$ is the membership function Zadeh (1965). Moreover, Zadeh proposed possibility measure to measure a fuzzy event. For a measure, the self-duality property is very important. But possibility measure is not self-dual. In order to define a measure which has self-dual property, Liu et al. (2002), suggested the credibility measure. Recently, credibility theory has been used in many fields (Huang,2010).

Definition 1: Let Θ be a nonempty set, and P(⊙) the power set of Θ, i.e., the largest σ-algebra over Θ. Each element in P(⊙) is called an event. The set function Cr is called a credibility measure if

a. (Normality) : $Cr(\odot) = 1$;
b. (Monotonicity): $Cr(A) \leq Cr(B)$ *whenever* $A \subset B$;
c. (Self-duality) : $Cr\{A\} + Cr\{A^C\} = 1$ *for any event* A;
d. (Maximality) : $Cr\{\cup_i A_i\} = sup_i\, Cr\{A_i\}$ *for any events* $\{A_i\}$ *with* $sup_i\, Cr\{A_i\} \leq 0{,}5$.

The value of $Cr\{A\}$ indicates the level that the event A will occur (Liu,2007).

Definition 2: Let Θ be a nonempty set, and P(⊙) the power set of Θ, and Cr a credibility measure. Then the triplet $(\odot, P(\odot), Cr)$ is called a credibility space (Liu,2007).

Definition 3: A fuzzy variable is defined as a function from a credibility space $(\odot, P(\odot), Cr)$ to the set of real numbers.

Definition 4: Let ε be a fuzzy variable with membership function μ. Then for any set A of real numbers, we have Liu et al. (2002);

$$Cr\{\varepsilon \in A\} = \frac{1}{2}\left(\sup_{t \in A}\mu(t) + 1 - \sup_{t \in A^c}\mu(t)\right) \tag{3}$$

Definition 5: Let ε be a fuzzy variable with membership function μ. Then it follows from equation (3) that the following equations hold (Huang,2010):

$$Cr\{\varepsilon = t\} = \frac{1}{2}\left(\mu(t) + 1 - \sup_{y \neq t}\mu(y)\right), \forall t \in \Re \tag{4}$$

$$Cr\{\varepsilon \leq t\} = \frac{1}{2}\left(\sup_{y \leq t}\mu(t) + 1 - \sup_{y > t}\mu(y)\right), \forall t \in \Re \tag{5}$$

$$Cr\{\varepsilon \geq t\} = \frac{1}{2}\left(\sup_{y \geq t}\mu(t) + 1 - \sup_{y < t}\mu(y)\right), \forall t \in \Re \tag{6}$$

Example: Let $\mu(x)$ be the membership function for the generalized L-R type fuzzy number expressed by equation (7); the credibility measure of this variable is shown in equation (8);

$$\mu_{\tilde{X}}(x) = \begin{Bmatrix} L(x) & a \leq x \leq b \\ 1 & b \leq x \leq c \\ R(x) & c \leq x \leq d \\ 0 & other \end{Bmatrix} \tag{7}$$

$$Cr\{\tilde{X} \leq x\} = \begin{cases} 0 & x \leq l \\ \frac{L(x)}{2} & l \leq x \leq m \\ \frac{1}{2} & m \leq x \leq n \\ 1 - \frac{R(x)}{2} & n \leq x \leq u \\ 1 & x \geq u \end{cases} \tag{8}$$

Definition 6: The expected value of a fuzzy number is defined as the center of the expected interval (Heilpern,1992). Moreover, most of the ranking methods used for fuzzy numbers are used to find the expected value of a fuzzy number (Yager,1981; Campos and Gonzalez,1989; Gonzalez,1990).

Let ξ be a fuzzy variable with the membership function $\mu(x)$ and r be a real number, the expected value of a fuzzy variable $E[\xi]$ can be calculated as in (9) (Liu et al. ,2002):

$$E[\xi] = \int_0^\infty Cr\{\xi \geq r\}dr - \int_{-\infty}^0 Cr\{\xi \leq r\}dr \tag{9}$$

Let ξ be a fuzzy variable with the membership function $\mu_\xi(x)$, and $f: \Re \to \Re$ is a strictly monotonic function. If the Lebesque integrals, $\int_0^\infty Cr\{\xi \geq r\}dr$ and $\int_{-\infty}^0 Cr\{\xi \leq r\}dr$ are finite, then the expected value of a function of a fuzzy variable $E[f(\xi)]$ can be calculated as in (10) (Dimova et al. , 2006);

$$E[f(\xi)] = \int_{-\infty}^{\infty} f(r)dCr\{\xi \leq r\} \quad (10)$$

Let ξ be a fuzzy variable whose support is $[a,b]$ and $f: \Re \to \Re$ is a strictly monotonic function. If the Lebesque integrals, $\int_0^{\infty} Cr\{\xi \geq r\}dr$ and $\int_{-\infty}^{0} Cr\{\xi \leq r\}dr$ are finite, then the expected value of a function of a fuzzy variable $E[f(\xi)]$ can be calculated as in (11) (Xue et al., 2008);

$$E[f(\xi)] = \int_a^b f(r)dCr\{\xi \leq r\} \quad (11)$$

4. The time value of money with fuzzy interest rate

The idea of investment is one of the issues that have been considered since the existence of humankind. Since the first days of human history; people have wanted for the valuation of their tangible assets for investment purposes and utilized interest methods in various ways. Today, due to the fluctuating course of national economies; components of investment decisions also include uncertainty; such as interest rate, investment life.

Let consider an investment of P is made. This amount of P will not be equivalent to the amount after n years. Therefore, interest is applied to the capital at the rate of i for the n year. In real life problems; compound interest is used in the calculation of interest, because of including time value.
We consider the interest rate(i) as a trapezoidal fuzzy variable. A trapezoidal fuzzy number is represented as $\tilde{\imath} = (k,l,m,n)$ where $k < l < m < n$. Here $k\ and\ l$ are the most possible values of the fuzzy interest rate and k and n are the minimum and maximum values. The membership function of a trapezoidal fuzzy number is illustrated as shown in Equation (7).

Because the interest rate (i) is a fuzzy variable; the function of the interest is also become a fuzzy function since it contains fuzzy variable. And we used the analytical method based on credibility theory. Xue et al. (2008), has developed this theory to find the expected value of a function of a fuzzy variable. Credibility theory is based on credibility function which has the property of self-duality. Therefore, the credibility function shows characteristics like those of probability measure.

The fuzzy compound interest is calculated as follow:

$$\tilde{F} = P * (1+\tilde{\imath})^n$$

By using equation (11); the expected values of the fuzzy compound interest rate represented as, $E[\tilde{F}]$ can be formulated as following;

$$E[\tilde{F}] = \int_a^i P * (1+r)^n\, d\, Cr\{i \leq r\} + \int_i^d P * (1+r)^n\, d\, Cr\{i \leq r\} \quad (12)$$

For the computational convenience; the defuzzification of interest rate (i) can be calculated in this stage and then for the value of the defuzzified value of it; we can calculate the interest value using Equation (12). Different defuzzification methods can be applied for changing fuzzy numbers to the crisp numbers. In this study we used GMIR (Graded Mean Integration Representation) defuzzification method (see Yager,1981; Campos and Gonzalez,1989; Gonzalez,1990) for numerical analysis.The defuzzified value using GMIR defuzzification method for a trapezodial fuzzy number $\tilde{X} = (a;b;c;d)$ can be calculated as;

$$X_{GMIR} = \frac{a+2b+2c+d}{6} \quad (13)$$

Here there will be three cases to be analyzed for the value of i discussing the credibility value of interest rate;

<u>Situation 1</u> $\boldsymbol{(a < i < b)}$

$$E[\tilde{F}(i)] = P * \left[\int_a^i (1+r)^n\, d\, Cr\{i \leq r\} + \int_i^b (1+r)^n\, d\, Cr\{i \leq r\} + \int_c^d (1+r)^n\, d\, Cr\{i \leq r\}\right] = P * \left[\frac{b^{n+1} - a^{n+1}}{2*(b-a)*(n+1)} + \frac{d^{n+1} - c^{n+1}}{2*(d-c)*(n+1)}\right] \quad (14)$$

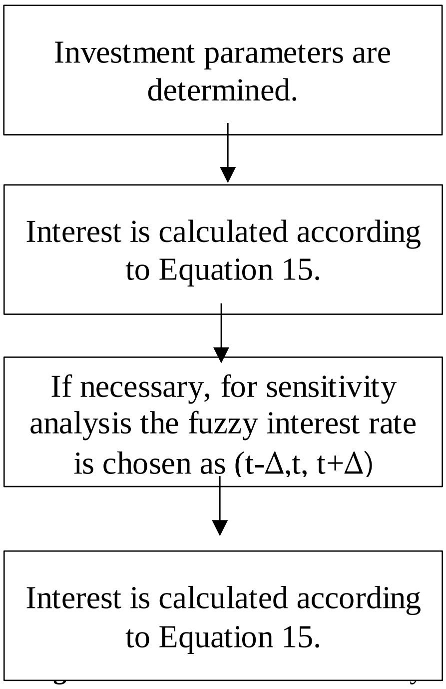
Situation 2($b < i < c$):

$$E[\tilde{F}(i)] = P * \left[\int_a^b (1+r)^n \, d\, Cr\{i \le r\} + \int_b^i (1+r)^n \, d\, Cr\{i \le r\} + \int_i^c (1+r)^n \, d\, Cr\{i \le r\} + \int_c^d (1+r)^n \, d\, Cr\{i \le r\} \right]$$
$$= P * \left[\frac{c^{n+1} - a^{n+1}}{2*(b-a)*(n+1)} + \frac{d^{n+1} - c^{n+1}}{2*(d-c)*(n+1)} \right] \quad (15)$$

Situation 3($c < i < d$):

$$E[\tilde{F}(i)] = P * \left[\int_a^b (1+r)^n \, d\, Cr\{i \le r\} + \int_c^i (1+r)^n \, d\, Cr\{i \le r\} + \int_i^d (1+r)^n \, d\, Cr\{i \le r\} \right]$$
$$= P * \left[\frac{b^{n+1} - a^{n+1}}{2*(b-a)*(n+1)} + \frac{d^{n+1} - c^{n+1}}{2*(d-c)*(n+1)} \right] \quad (16)$$

The workflow of the study can be shown as shown in figure 1:



5. Numerical illustrations

In this section; numerical examples are given for the demonstration of the proposed models. Firstly; we examined the proposed model in the previous section. After that, we focus on the fuzziness aspect of the interest rate.

In numerical analysis; we consider that a firm wants to make an investment according to interest rate which is a fuzzy variable.

Table 3. Investment parameter values

Parameters		Values
P	Capital	$10.000
i	Interest rate	(0.1;0.6;0.65;0.7)
n	Investment life	5

The interest is calculated as in Equation (15); the interest value is obtained as $781,6.

One of the important points in investment decision is to obtain the interest rate value. When there is not enough historical data, interest rate variant can be shown using fuzzy set theory. Therefore, in this section; we focus on the fuzziness aspect of the interest rate uncertainty. As a numerical example, we suppose that the fuzzy interest rate is a trapezoidal fuzzy variable. The interest rate per year of the investment is determined with trapezoidal fuzzy number as $(\tilde{\imath} = 0.6 - \Delta, 0.6, 0.6 + \beta, 0.6 + \alpha)$ with $\Delta \geq 0$, $\beta \geq 0$ and $\alpha \geq 0$ and also $\alpha \geq \beta$. And the parameter values are the same as the parameter values used in the previous analysis.
The interest values obtained by changing the interest rate variance are shown in Table 4.

Table 4. The Interest Values Obtained by Changing the Interest Rate Variance

Interval	a	b	c	d	F
1	0.1	0.6	0.65	0.7	782
	0.1	0.6	0.64	0.7	757
	0.1	0.6	0.63	0.7	734
	0.1	0.6	0.62	0.7	712
2	0.1	0.6	0.65	0.8	1115
	0.1	0.6	0.64	0.8	1085
	0.1	0.6	0.63	0.8	1056
	0.1	0.6	0.62	0.8	1028
3	0	0.6	0.65	0.7	769
	0	0.6	0.64	0.7	744
	0	0.6	0.63	0.7	721
	0	0.6	0.62	0.7	699
4	0	0.6	0.65	0.8	1102
	0	0.6	0.64	0.8	1072
	0	0.6	0.63	0.8	1043
	0	0.6	0.62	0.8	1015

It is seen from the tables that in the case of diminishing interest rate fuzziness, the value of interest is also decreasing. Because the interest rate is uncertain; people also experience uncertainty in their investment decisions. Interest values in the second interval are higher than interest values in the third interval because the interest rate in this interval is also higher.

6. Conclusions, discussions, and future studies

The idea of investment is one of the issues that have been considered since the existence of humankind. Since the first days of human history; people have wanted for the valuation of their tangible assets for investment purposes and utilized interest methods in various ways. Today, due to the fluctuating course of national economies; components of investment decisions also include uncertainty; such as interest rate, investment life. Moreover, since these uncertainties are not predictable; components cannot be expressed as stochastic or deterministic variables. One of the important points to be overcome in real life investment models is the estimation of interest rate.

It can be seen from the literature; the works on fuzzy interest rate are more limited than those in deterministic or stochastic ones. Therefore, we think that study, which will deal with fuzzy interest rate, may contribute to the literature. In this study, we tried to obtain generalized closed formulas in order to calculate the future value of the capital investment although interest rate is fuzzy variable. Generalized closed formulas of the future value of the capital investment have been obtained by expected value function. And we applied credibility theory for determining the expected value function.

Numerical examples are given for the demonstration of the proposed models. Firstly; we examined the proposed model. After that; we focus on the fuzziness aspect of the interest rate.

It is seen from the study that in the case of diminishing interest rate fuzziness, the value of interest is also decreasing. Because the interest rate is uncertain; people also experience uncertainty in their investment decisions. Interest values in the second interval are higher than interest values in the third interval because the interest rate in this interval is also higher.

For the future studies; not only the interest rate is fuzzy variable; but also, the other parameters of the interest calculation formula can be fuzzy variables. Also, the valuation methods of the investment decision can be examined in the fuzzy environment.

Contribution of researchers

Authors have equal contribution in all the sections.

Conflict of interest

The authors declared that there is no conflict of interest.

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