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INTERNATIONAL ADVANCED RESEARCHES and ENGINEERING JOURNAL

Journal homepage: www.dergipark.org.tr/en/pub/iarej

e-ISSN: 2618-575X

International Open Access

Volume 07
Issue 02

August, 2023

Research Article

Non-parametric analysis of maintenance data for Attitude Indicator of a commercial aircraft fleet

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ARTICLE INFO

Article history:
Received 04 October 2022
Accepted 30 May 2023
Published 15 August 2023

Keywords:
Aircraft
Mean cumulative function
Multiple repairable systems
Recurrence data
Recurrence rate

ABSTRACT

Analysis of maintenance data for a repairable system provides information about the failure behavior of the system. Such information is needed for determining preventive maintenance and retirement policy for the system. Parametric and non-parametric models can be used for analysis. Parametric models require more assumptions about the failure process of the systems under consideration compared to non-parametric models. To verify these assumptions statistical expertise needed. The purpose of this paper is to show that in practice non-parametric estimator of mean cumulative function can be utilized easily to model the failure behavior of a fleet. Mean cumulative function estimates the mean number of failures as function of operating hours. The method is exemplified on the attitude indicator units of a commercial aircraft fleet. Sampling uncertainty of the estimates is quantified by normal approximation confidence intervals.

1. Introduction

Many expensive and complex systems are repairable such as aircrafts, ships, nuclear power plants and production systems. Preventive maintenance is performed on a repairable system to keep it in the operating state and corrective maintenance is performed on a repairable system to return it to the operating state. Repeated maintenance actions produce data that composed of failure times, failure modes, preventive maintenance times, repair times etc. By analyzing this data, one gets valuable information about the failure behavior of the system under consideration, such as mean cumulative number of failures at a particular time, mean time between failures, and recurrence rate of failures. Then, this information is used for establishing a maintenance plan or evaluating the effectiveness of an existing plan.

Maintenance data is in the class of recurrence data since maintenance actions repeat in time. Methods used in the analysis of recurrence data can be categorized as parametric and non-parametric. Parametric models are defined based on the behavior of the recurrence rate of failures (ROCOF) and the distribution of inter failure times [1]. These models are called counting processes. There are different counting process models for different forms of ROCOFs. Determining a suitable counting process model is quite a challenge and needs expertise in statistics [2-6]. Counting process models can be classified as given below Rausand and Høyland [7]:

- Perfect Repair models: It is assumed that the system state after repair is just like a new system ("as good as new"). Therefore, the recurrence rate of the failures are constant in time. Homogeneous Poisson process (HPP) and Renewal process (PR) are in this class.
- Minimal Repair models: It is assumed that the system state after repair is same as just before the failure ("as bad as old"). Therefore, the recurrence rate of the failures is function of time. Non-homogeneous Poisson process (NHPP) is in this class.
- Imperfect repair models: It is assumed that the system state after repair is in between as good as new and as bad as old. Therefore, the recurrence rate of the failures is function of time. Generalized renewal process (GRP) and trend-renewal process (TRP) are in this class.

Each model has additional assumptions on probability distribution of times between failures. The detailed information about the counting process models can be find in Cook and Lawless [8]. Before using a specific

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DOI: 10.35860/iarej.1184070

parametric model, statistical tests have to be applied to show that problem at hand satisfy the assumptions of the model. If single system is modeled trend test are enough for choosing the appropriate model [9]. In the case of multiple repairable systems modeling, also homogeneity tests should be performed. If the systems are heterogeneous, they should be grouped into smaller homogeneous groups and each group should be analyzed separately. Garmabaki, Ahmadi [10] developed a comprehensive framework for deciding a suitable parametric model in the case of multiple repairable systems. More information on the trend tests and homogeneity tests can be found in Kvaløy and Lindqvist [11], Rigdon and Basu [12], Kvaløy and Lindqvist [13], Viertävä and Vaurio [14], Shen, Cassottana [15], Kvaløy and Lindqvist [16].

In practice, wrong models can be selected without probabilistic understanding of the models and overlooking the need for applying trend and homogeneity tests. Ascher and Hansen [17] discusses the common wrong use of HPP model. They showed that fitting a probability distribution to non-stationary times between failures (they are not identically distributed) leads wrong parametric model selection. Also, Trindade and Nathan [4] attention to the problem of using HPP without justifying the assumptions of the model. Ascher [18] emphasized the importance of the understanding of the statistical properties of the maintenance data. They exemplified the wrong model selection on three data sets. As another difficulty, Trindade and Nathan [6] stated that "parametric model estimation is computationally intensive and parametric models are not intuitive to the average person performing data analysis to address customer reliability concerns". They also observed that in practice the required statistical tests are not performed for model selection. Noticing this problem researchers developed non-parametric methods for analyzing recurrence data [3, 5, 6, 19-21].

Non-parametric models are based on the estimation of the mean cumulative function (MCF) for a population of systems. At a certain time point, each system in the population has usually different cumulative number of failures. The average of these cumulative numbers is the mean cumulative number of failures per system for that time point. The graph of the mean cumulative number of failures versus time is called as Mean Cumulative Function (MCF).

Before reviewing the literature on non-parametric MCF estimation, data types that encountered in reliability analyses have to be discussed. Each data type has its own statistical properties, so data type specific non-parametric estimators have to be developed. Reliability data types for repairable system can be categorized as follows. For detailed information on data types please see [6]:

- Complete data: Complete data occurs when we now the number of failures and their exact occurrence times.
- Left censored data: Left censored data occurs when we know that a failure occurs before a certain time but we do not know the exact failure time.
- Interval censored data: Interval censored data occurs when we know that a failure occurs in a specific interval but we do not know the exact failure time.
- Right censored data: Right censored data occurs when the item under observation is still in working condition at the end of the observation period.
- Left truncated data: Left truncated data occurs when we do not have any information regarding the number and times of failures before a certain time.
- Window observation data: Window observation data occurs when observation of the systems is done in time windows with possible gaps between the windows. During the gaps there is no observation so that number of systems at risk is zero for that time periods.

For the complete and right censored data, an unbiased nonparametric estimator of MCF is developed by Nelson [3]. For this estimator, a non-parametric approximate confidence interval was constructed by Nelson [22]. For left and interval censored data an unbiased non-parametric estimator of MCF is developed by Nelson [23]. In this study the author also discussed the required assumptions for the MCF estimate to be valid. In case of window observation data the regular non-parametric estimators of MCF would be inconsistent due to the periods without data. Zuo, Meeker [24] extended the non-parametric estimator of the MCF for window-observations. Sometimes observation of the systems could start much after their start of use resulting the left truncated data. Trindade and Nathan [6] developed the non-parametric estimate of MCF for this case. Jiang, Li [25] showed that Nelson's estimator is not robust after the observation period when units have different censoring times and developed a robust estimator for this case.

The purpose of this paper is to show that non-parametric estimate of the MCF can be utilized easily to model failure behavior of multiple systems on the example of the Attitude Indicator (AI) units for a commercial aircraft fleet. This estimate would provide the useful information for maintenance planning and evaluation.

2. MCF and its non-parametric estimate

For a single system, there is only one cumulative history function, $N(t)$ which represents the cumulative number of failures occurring by time t. When there is a population of systems, each one of them would has its own cumulative history function, $N_i(t)$, $i = 1,2,..,n$. At any age t, the cumulative number of failures will be usually different for each system so there will be a probability distribution of cumulative number of failures at each time point. The mean of these distributions as a function of time is called MCF which gives the mean number of failures occurring by a certain time for per system. Figure 1 depicts this idea. In Figure 1, as examples the distribution of cumulative number of failures at time points 3 and 5 is shown. Mean of these distributions for all time points construct the MCF which is the black curve in Figure 1 and named as *M(t)*.

Slope of MCF is called Rate of Occurrence of Failures (ROCOF). As a function of time ***t***, ROCOF gives the mean number of failures per system at time ***t***. Form of ROCOF gives information about the failure process. If ROCOF is increases by time, then times between failures are decrease and MCF has a convex shape. If ROCOF is decreases by time, then times between failures are increase and MCF has a concave shape. If ROCOF is constant in time, then times between failures have identical distribution and MCF has a linear shape. Figure 2 shows different shapes of MCF.

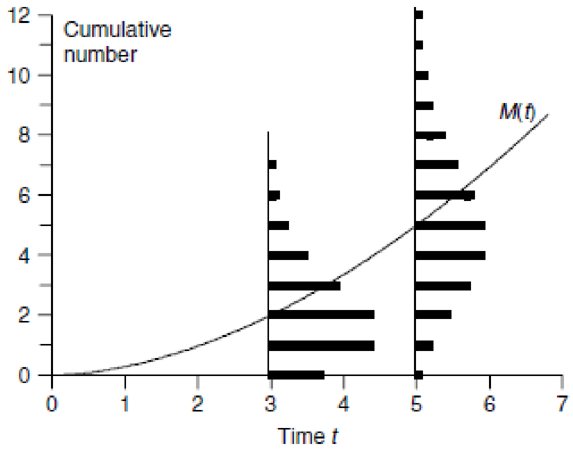

Figure 1. MCF and distributions of cumulative number of failures as a function of time [26].

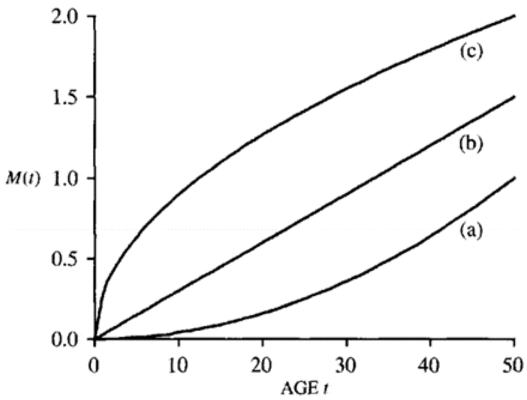

Figure 2. MCFs with (a) increasing, (b) constant, and (c) decreasing recurrent rate [26].

MCF provides the following important information about the failure behavior of multiple systems:

- Mean number of failures occurring as a function of time for per system in the fleet
- Trend of failure behavior
- Slope of the MCF gives the ROCOF.

Maintenance data can have different details. For example, failure times can be known exactly (complete data), known to be in an interval (interval censored data), known to be less than a certain time (left censored data) and known to be greater than a certain time (right censored data). Since a system can fail in different ways, failures modes also can be recorded as a part of maintenance data. This situation is called as mix events in counting process terminology. Non-parametric estimators of MCF should take into consideration the different details of maintenance data. Nelson [27] provides unbiased non-parametric estimates of MCF for different data details.

In our example, maintenance data of AI units are complete and right censored and failure modes are not recorded so there is only one kind of event that is failure. For this settings of the data, William and Escobar [28] gives an algorithm for the calculation of Nelson's unbiased point wise non-parametric estimate of MCF as follows:

1. Order the unique recurrence times t_{ij} among all of the *n* systems. Let *m* denote the number of unique times. These ordered unique times are denoted by $t_l < \dots < t_m$.
2. Compute $d_i(t_k)$, the total number of recurrences for system *i* at t_k.
3. Let $\delta_i(t_k) = 1$ if system *i* is still being observed at time t_k and $\delta_i(t_k) = 0$ otherwise.
4. Compute

$$\hat{\mu}(t_j) = \sum_{k=1}^{j}\left[\frac{\sum_{i=1}^{n}\delta_i(t_k)d_i(t_k)}{\sum_{i=1}^{n}\delta_i(t_k)}\right] = \sum_{k=1}^{j}\frac{d.(t_k)}{\delta.(t_k)} = \sum_{k=1}^{j}\bar{d}(t_k) \quad (1)$$

for $j = 1, \dots, m$, where $d.(t_k) = \sum_{i=1}^{n}\delta_i(t_k)d_i(t_k)$, $\delta.(t_k) = \sum_{i=1}^{n}\delta_i(t_k)$, and $\bar{d}(t_k) = d.(t_k)/\delta.(t_k)$, where $\hat{\mu}(t_j)$ is the estimate of MCF at j^{th} failure time, $d.(t_k)$ is the total number of system recurrences at time t_k, $\delta.(t_k)$ is the number of units observed at t_k, and $\bar{d}(t_k)$ is the average number of recurrences per system at t_k. Point wise estimate of MCF will be a step function, jumping at failure times. Normal approximation confidence intervals for the estimate are given in (2). Details of the calculation of Confidence Interval (CI) are given in William and Escobar [28].

$$\frac{\hat{\mu}(t_j)}{ekl\left[\frac{\left(z_{1-\frac{\alpha}{2}}\right)s\hat{e}_{\hat{\mu}(t)}}{\hat{\mu}(t)}\right]} < \hat{\mu}(t) < \hat{\mu}(t_j)e^{\left[\left(z_{1-\frac{\alpha}{2}}\right)s\hat{e}_{\hat{\mu}(t)}\hat{\mu}(t)\right]} \quad (2)$$

$s\hat{e}_{\hat{\mu}(t)}$ is the estimated standard error of MCF and $(Z_{1-\alpha/2})$ is the value of a normal distribution at significance level α.

In next section, MCF for AI units calculated using the above algorithm. Also, to quantify the sampling uncertainty normal approximation confidence intervals are calculated using Equation (2).

3. Estimate of the MCF for AI units

An AI unit shows the aircraft's orientation relative to Earth's horizon and gives on immediate indication of the smallest orientation change. Therefore, it is very important that an AI unite works without failure for safety of a flight.

In this paper, maintenance data for 27 AI units is analyzed to evaluate the failure behavior of the units. These AI units are used in a commercial aircraft fleet. They are subjected to both preventive and corrective maintenance. According to the preventive maintenance plan, an AI unit goes through a benchmark at every 3000 flight hours and an overhaul at every 6000 flight hours.

Analyzed maintenance data consists of the corrective maintenance dates of the AI units. An AI unit is given a unique identification number (ID) and traced in the maintenance system by its ID. Table 1 summarizes the corrective maintenance data. According to table AI 16, AI 23 and AI 27 have the least number of failures which is one failure. AI 5 has the maximum number of failures which is 12. Based on Table 1, the total number of failures for the whole sample of AI units is 123 failures.

Event plots of recurrence data provides first information about trend behavior of the occurrences of events. For our case events are failures of the AIs. Event plots of the AIs are given in Figure 3. Cross marks represent the failure points of an AI. From Figure one, it seems that AIs are failed more often as flight hours increases. Therefore, we expect to see a convex MCF.

Table 1. The AI units that have a certain number of failures.

Number of Failures	1	2	3	4	5	6	7	8	9	10	11	12
Number of AIs	3	4	3	7	4	0	2	1	0	1	1	1
AIs' ID	16; 23; 27	3; 13; 19; 24	9; 18; 21	1; 6; 10; 14; 20; 25; 26	7; 12; 15; 22	-	2; 17	4	-	11	8	5

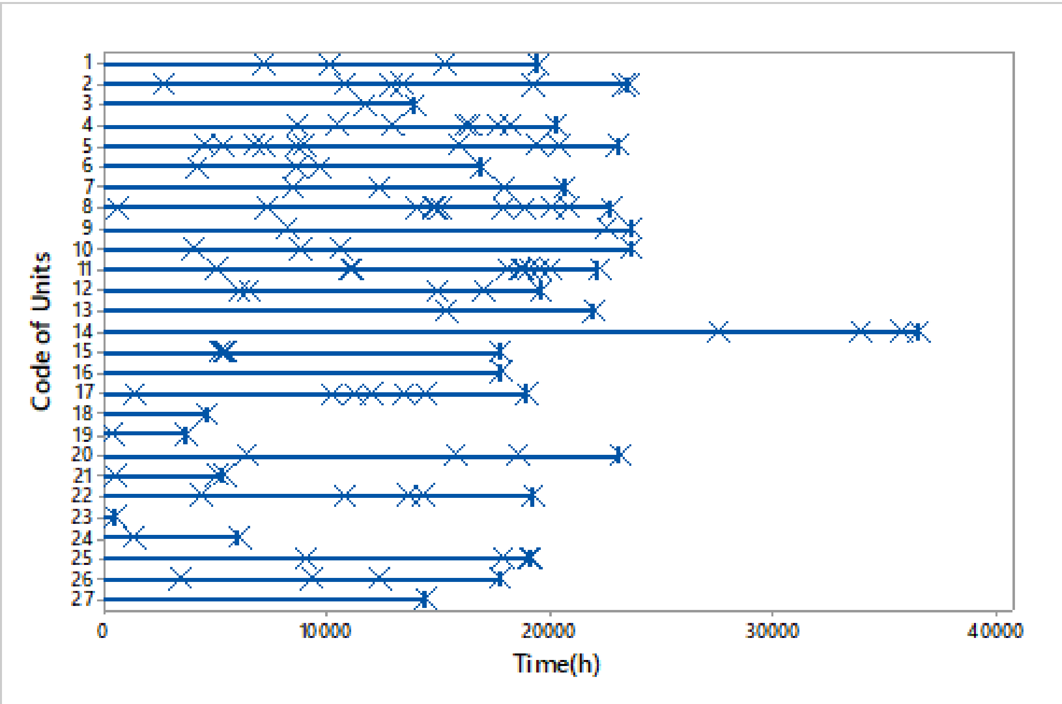

Figure 3. Event plot for units of AI.

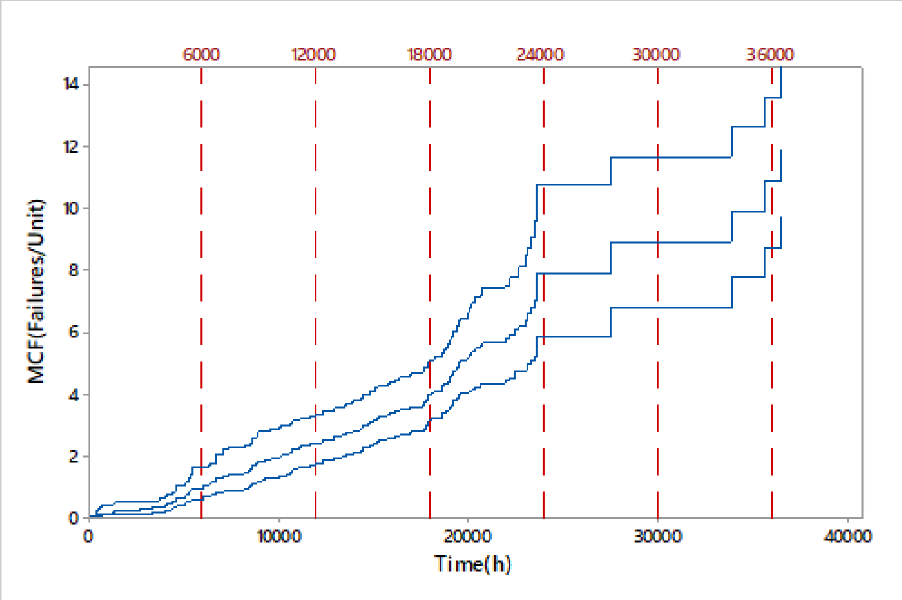

Figure 4. MCF for AI units.

Table 2. Value of MCF for each 3000 flight-hour interval.

Flight Hours (h)	*MCF*
3000	0,285
6000	1,003
9000	1,799
12000	2,413
15000	3,150
18000	4,014
21000	5,679
24000	7,978
27000	8,748
30000	9,272
33000	9,743
36000	11,282

The estimate of MCF and 95 % CI are given in Figure 4. Doted vertical lines in the figure show the approximate overhaul times. Fort the first 6000 flight hours MCF has a convex shape indicating an increasing recurrence rate. According to the existing maintenance plan first, second and third overhauls is performed on approximately at 6000, 12000 and 18000 flight hours. Between 6000-18000 flight hours MCF is approximately linear indicating a constant recurrence rate. This shows the effectiveness of the overhauls for that period. Between 18000-20600 flight hours it is linear with bigger slope. Since the overhauls are performed on the same quality, this might be the result of the starting wear out effect of the AI units. After 20600 flight hours MCF is convex again indicating increasing recurrence rate. This shows that the wear out of AI units sets in. Only one system is observed after 23626 flight hours. Looking at this analysis of the graph of MCF overhauls has positive effect on the reliability of the AI units and they are effective. But when the wear out sets in, it decreases the effectiveness of overhauls.

Estimated values of MCF at every 3000 flight hours per aircraft are given in Table 2. Estimate of mean cumulative number of failures for the fleet of 40 aircrafts can be calculated multiplying the estimates given in Table 2 by 40. For example, up to and including 6000 flight hours it is expected to observe approximately 40 failures for the whole fleet.

4. Conclusions

Non-parametric analysis of the maintenance data easy and quick to apply and provides valuable information about the failure process for planning the maintenance actions. As a population model, MCF gives the estimate of cumulative number of failures as a function of time. Shape of MCF gives information about the trend of failures. For the AI units ROCOF is increasing in the early part of the life pointing that overhaul is necessary. First and second overhauls showed the effect of linear ROCOF. Starting wear out resulted an approximately linear MCF with bigger slope between third and fourth interval. After the fourth overhaul MCF becomes convex as wear out sets in. In this example we used non-parametric estimate of MCF for evaluating the existed preventive maintenance plan.

Declaration

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article. The authors also declared that this article is original, was prepared in accordance with international publication and research ethics, and ethical committee permission or any special permission is not required.

Author Contributions

S.K. Ulusoy and M.S. Sasmazturk. contributed to the publication with the modelling and writing of the article. They contributed to study, evaluation of methods and discussion of results and review of the article.

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