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Research Article

Relative Almost Convergence and Approximation Theorems

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Abstract

In this paper, we introduce a new type of almost convergence and using this convergence, we give a Korovkin-type approximation theorem. Then, we construct an example such that our result is stronger than the results given before. Also, we present some consequences.

Keywords: Relative uniform almost convergence, Korovkin Theorem, Almost convergence, Statistical convergence

Relative Hemen Hemen Yakınsaklık ve Yaklaşım Teoremleri

Öz

Bu makalede, yeni bir hemen hemen yakınsaklık türü tanıtacağız ve bu yakınsaklığı kullanarak Korovkin tipi yaklaşım teoremi vereceğiz. Daha sonra bizim sonucumuzun önceden verilen sonuçlardan daha güçlü olduğunu gösteren bir örnek vereceğiz. Ayrıca, bazı sonuçlar sunacağız.

Anahtar Kelimeler: Relative düzgün hemen hemen yakınsaklık, Korovkin teorem, Hemen hemen yakınsaklık, İstatistiksel yakınsaklık

Introduction and Preliminaries

There are two well-known non-matrix regular summability methods, namely "almost convergence" and "statistical convergence". Let l_∞ and c respectively be the Banach spaces of all bounded and convergent sequences $x=(x_k)$ with the usual norm $\|x\|=\sup_k |x_k|$. A Banach limit L is defined on l_∞, as a continuous linear functional such that $(i)\, L(x)=L((x_k))\geq 0$ for $x_k\geq 0$ for each k $(ii)\, L(e)=1,\ e=(1,\ 1,\ 1,\ ...)$, and $L((x_k))=L((x_{k+1}))$ for all $x=(x_k)\in l_\infty$ [1]. In 1948, Lorentz [2] used this notion of a Banach limit to define a new type of convergence which known as the almost convergence. A sequence $x=(x_k)$ is said to be *almost convergent* to the number l if and only if all Banach limits of x are l. A bounded sequence $x=(x_k)$ is *almost convergent* to the number l if and only if

$$\lim_p \left| \frac{1}{p} \sum_{k=n}^{n+p-1} x_k - l \right| = 0, \text{ uniformly in } n.$$

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The concept of statistical convergence for sequences of real numbers was introduced by Fast [3] and Steinhaus [4] independently in the same year 1951 and since then several generalizations and applications of this notion have been investigated [5, 6]. It is known that statistical convergence and almost convergence are distinct concepts but their intersection is not empty.

E. H. Moore [7] introduced the notion of uniform convergence of a sequence of functions relative to a scale function. Then, E. W. Chittenden [8] gave the following definition of relatively uniform convergence equivalently to the definition given by Moore:

A sequence (f_k) of functions, defined on an interval $I \equiv (a \le x \le b)$, converges relatively uniformly to a limit function f if there exists a function $\sigma(x)$, called a scale function such that for every $\varepsilon > 0$ there is an integer k_ε such that for every $k > k_\varepsilon$ the inequality

$$|f_k(x) - f(x)| < \varepsilon\,|\sigma(x)|$$

holds uniformly in x on the interval I . The sequence (f_k) is said to converge *uniformly relative to the scale function* σ or more simply, *relatively uniformly*.

It will be observed that uniform convergence is the special case of relatively uniform convergence in which the scale function is a non-zero constant (for more properties and details, see also [8, 9, 10]).

In [11], Demirci and Orhan define a new type of statistical convergence by using the notions of the natural density and the relatively uniform convergence.

Let K be a subset of $\mathbb{N}$, the set of natural numbers, then the natural density of K, denoted by $\delta(K)$, is given by:

$$\delta(K) := \lim_n \frac{1}{n}\left|\{k \le n : k \in K\}\right|$$

whenever the limit exists, where $|B|$ denotes the cardinality of the set B [12].

Let f and f_k belong to $C(X)$, which is the space of all continuous real valued functions on a compact subset X of the real numbers and $\|f\|_{C(X)}$ denotes the usual supremum norm of f in $C(X)$.

Definition 1. [11]

A sequence (f_k) is said to be *statistically relatively uniform convergent* to f on X if there exists a function $\sigma(x)$ satisfying $|\sigma(x)| \neq 0$, called a scale function such that for every $\varepsilon > 0$,

$$\delta\left(\left\{k : \sup_{x\in X}\left|\frac{f_k(x) - f(x)}{\sigma(x)}\right| \ge \varepsilon\right\}\right) = 0 .$$

This limit is denoted by $(st) - f_k$ Ã $f\ (X;\sigma)$.

In the present paper, we introduce uniform almost convergence and also, the notion of almost convergence of a sequence of functions relative to a scale function.

Definition 2.

A sequence (f_k) is said to be *uniform almost convergent* to f on X if

$$\lim_p \sup_{x\in X}\left|\frac{1}{p}\sum_{k=n}^{n+p-1} f_k(x) - f(x)\right| = 0,$$

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uniformly in n.

Definition 3.

A sequence (f_k) is said to be *relatively uniform almost convergent* to f on X if there exists a function $\sigma(x)$ satisfying $|\sigma(x)| \neq 0$, called a scale function such that

$$\lim_p \sup_{x\in X} \left| \frac{1}{\sigma(x)} \left(\frac{1}{p} \sum_{k=n}^{n+p-1} f_k(x) - f(x) \right) \right| = 0,$$

uniformly in n.

For $n=1$, we will call this convergence as $(C,1)-$relatively uniform convergence.

It will be observed that uniform almost convergence is the special case of relative uniform almost convergence in which the scale function is a non-zero constant. Also, if $\sigma(x)$ is bounded, relative uniform almost convergence implies uniform almost convergence. However, relative uniform almost convergence does not imply uniform almost convergence, when $\sigma(x)$ is unbounded. This is illustrated by the following example.

Example 1.

For each $k \in \mathbb{N}$, define $h_k : [0,1] \to \mathbb{R}$ by

$$h_k(x) = \begin{cases} \dfrac{3}{2} + \dfrac{3kx}{1+k^2x^2}, & \text{if } k \text{ is odd}, \\ \dfrac{1}{2} + \dfrac{kx}{1+k^2x^2}, & \text{if } k \text{ is even}. \end{cases} \qquad (1)$$

Then observe that

$$\lim_p \sup_{x\in X} \left| \frac{1}{\sigma(x)} \left(\frac{1}{p} \sum_{k=n}^{n+p-1} h_k(x) - 1 \right) \right| = 0,$$

uniformly in n,

where $\sigma(x) = \begin{cases} \dfrac{1}{x}, & 0 < x \le 1 \\ 1, & x = 0 \end{cases}$, namely (h_k) is relatively uniform almost convergent to the function $f=1$ on the interval $[0,1]$. However, (h_k) is not uniform almost convergent to the function $f=1$ on the interval $[0,1]$. Also, we note that (h_k) is not statistically relatively uniform convergent.

A Korovkin-Type Approximation Theorem

The fundamental theorem of Korovkin [13] on approximation of continuous functions on a compact interval gives conditions in order to decide whether a sequence of positive linear operators converges to identity operator. This theorem has been extended in several directions[5, 14]. In 2011, Mohiuddine [15] gived the following version of the classical Korovkin approximation theorem.

Throughout the paper, we use the test functions e_i defined by $e_i(x) = x^i$ $(i = 0, 1, 2)$.

Theorem 1. [15]

Let (T_k) be a sequence of positive linear operators acting from $C(X)$ into itself and $D_{n,p}(f;x) := \dfrac{1}{p} \sum_{k=n}^{n+p-1} T_k(f;x)$.
Then, for all $f \in C(X)$,

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$\lim_p \sup_{x\in X}\left|D_{n,p}(f;x)-f(x)\right|=0$, uniformly in n,

if and only if

$\lim_p \sup_{x\in X}\left|D_{n,p}(e_i;x)-e_i(x)\right|=0$, uniformly in n, $i=0,\ 1,\ 2$.

Now we present the following main result.

Theorem 2.

Let (T_k) be a sequence of positive linear operators acting from $C(X)$ into itself and $D_{n,p}(f;x):=\frac{1}{p}\sum_{k=n}^{n+p-1}T_k(f;x)$. Then, for all $f\in C(X)$,

$$\lim_p \sup_{x\in X}\left|\frac{D_{n,p}(f;x)-f(x)}{\sigma(x)}\right|=0, \qquad (2)$$

uniformly in n,

if and only if

$$\lim_p \sup_{x\in X}\left|\frac{D_{n,p}(e_i;x)-e_i(x)}{\sigma_i(x)}\right|=0, \qquad (3)$$

uniformly in n, $i=0,\ 1,\ 2$,

where $\sigma(x)=\max\left\{\left|\sigma_i(x)\right|;\ i=0,\ 1,\ 2\right\}$, $\left|\sigma_i(x)\right|\neq 0$ and $\sigma_i(x)$ is unbounded, $i=0,\ 1,\ 2$.

Proof.

Condition (3) follows immediately from condition (2), since each of the functions $1,\ x,\ x^2$ belongs to $C(X)$. Let $f\in C(X)$. Then, reasoning exactly as in the proof of Theorem 2.1 in [15], we arrive at,

$$\begin{aligned}
&D_{n,p}(f;x)-f(x)\\
&\leq \varepsilon\left[D_{n,p}(e_0;x)-e_0(x)\right]+\varepsilon\\
&\quad+\frac{2\|f\|_{C(X)}}{\delta^2}\Big\{\left[D_{n,p}(e_2;x)-e_2(x)\right]\\
&\quad-2x\left[D_{n,p}(e_1;x)-e_1(x)\right]\\
&\quad+x^2\left[D_{n,p}(e_0;x)-e_0(x)\right]\Big\}\\
&\quad+f(x)\left[D_{n,p}(e_0;x)-e_0(x)\right],
\end{aligned}$$

therefore

$$\begin{aligned}
&\left|D_{n,p}(f;x)-f(x)\right|\leq \varepsilon+\Big(\varepsilon+\|f\|_{C(X)}\\
&\quad+\frac{2\|f\|_{C(X)}\|e_2\|_{C(X)}}{\delta^2}\Big)\left|D_{n,p}(e_0;x)-e_0(x)\right|\\
&\quad+\frac{4\|f\|_{C(X)}\|e_1\|_{C(X)}}{\delta^2}\left|D_{n,p}(e_1;x)-e_1(x)\right|\\
&\quad+\frac{2\|f\|_{C(X)}}{\delta^2}\left|D_{n,p}(e_2;x)-e_2(x)\right|\\
&\leq \varepsilon+M\Big\{\left|D_{n,p}(e_0;x)-e_0(x)\right|\\
&\quad+\left|D_{n,p}(e_1;x)-e_1(x)\right|+\left|D_{n,p}(e_2;x)-e_2(x)\right|\Big\}
\end{aligned}$$

where

$$M=\varepsilon+\|f\|_{C(X)}+\frac{2\|f\|_{C(X)}}{\delta^2}\left(\|e_2\|_{C(X)}+2\|e_1\|_{C(X)}+1\right).$$

We get

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$$\sup_{x\in X}\left|\frac{D_{n,p}(f;x)-f(x)}{\sigma(x)}\right|$$

$$\leq \sup_{x\in X}\frac{\varepsilon}{|\sigma(x)|}+M\left\{\sup_{x\in X}\left|\frac{D_{n,p}(e_0;x)-e_0(x)}{\sigma_0(x)}\right|\right.$$

$$+\sup_{x\in X}\left|\frac{D_{n,p}(e_1;x)-e_1(x)}{\sigma_1(x)}\right|$$

$$\left.+\sup_{x\in X}\left|\frac{D_{n,p}(e_2;x)-e_2(x)}{\sigma_2(x)}\right|\right\}$$

where $\sigma(x)=\max\{|\sigma_i(x)|;\ i=0,\ 1,\ 2\}$. Then using the hypothesis (3) and supposing that $p\to\infty$, we get

$$\lim_p \sup_{x\in X}\left|\frac{D_{n,p}(f;x)-f(x)}{\sigma(x)}\right|=0 \quad \text{uniformly}$$

in n.

This completes the proof of the theorem.

We now show that our result Theorem 2 is stronger than Theorem 1.

Example 2.

Let $X=[0,1]$ and consider the classical Bernstein polynomials $B_k(f;x)$ on $C[0,1]$.Using these polynomials, we introduce the following positive linear operators on $C[0,1]$:

$$Q_k(f;x)=h_k(x)B_k(f;x), \tag{4}$$

$x\in[0,1]$ and $f\in C[0,1]$,

where $h_k(x)$ is given by (1).

If we choose $z_k=\begin{cases}1, & \text{if } k \text{ is odd}\\ 0, & \text{if } k \text{ is even}\end{cases}$ and $g_k(x)=\frac{2kx}{1+k^2x^2}$, $x\in[0,1]$, then we can write $h_k(x)=\left(\frac{1}{2}+z_k\right)(1+g_k(x))$ and

$$Q_k(f;x)=\left(\frac{1}{2}+z_k\right)(1+g_k(x))B_k(f;x)$$

$x\in[0,1]$ and $f\in C[0,1]$.

Observe that

$$Q_k(e_0;x)=h_k(x)e_0(x),$$

$$Q_k(e_1;x)=h_k(x)e_1(x),$$

$$Q_k(e_2;x)=h_k(x)\left[e_2(x)+\frac{x(1-x)}{k}\right],$$

and the sequence (Q_k) satisfies the condition (3). Hence, by Theorem 2

$$\lim_p \sup_{x\in X}\left|\frac{D_{n,p}(f;x)-f(x)}{\sigma(x)}\right|=0 \quad \text{uniformly}$$

in n,

where $D_{n,p}(f;x):=\frac{1}{p}\sum_{k=n}^{n+p-1}Q_k(f;x)$ and

$$\sigma(x)=\begin{cases}\frac{1}{x}, & 0<x\leq 1\\ 1, & x=0\end{cases}.$$

On the other hand for each $k\in\mathbb{N}$,

$$\sup_{x\in[0,1]}|D_{n,p}(1;x)-1|\geq\left|D_{n,p}\left(1;\frac{1}{k}\right)-1\right|$$

$$=\frac{2}{\mathrm{p}}\sum_{k=n}^{n+p-1}z_k.$$

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Since $\lim_p \sup_n \frac{2}{\mathrm{p}} \sum_{k=n}^{n+p-1} z_k = 1$, we have

$$\lim_p \sup_n \sup_{x\in[0,1]} \left| D_{n,p}(1;x) - 1 \right| \neq 0$$

and we can say that Theorem 1 does not work for our operators defined by (4).

Some Consequences

It is easy to see that relatively uniform convergence implies relatively uniform almost convergence and relatively uniform almost convergence implies $(C,1)-$relatively uniform convergence, but in this section, we prove the converse of these cases under an additional condition.

Theorem 3.

Let (T_n) be a sequence of positive linear operators on $C(\mathrm{X})$ such that

$$\lim_n \sup_{x\in X} \left| \frac{T_{n+1}(f;x) - T_n(f;x)}{\sigma(x)} \right| = 0, \qquad (5)$$

and

$$\lim_p \sup_{x\in X} \left| \frac{1}{\sigma_i(x)} \left(\frac{1}{p} \sum_{k=n}^{n+p-1} T_k(e_i;x) - e_i(x) \right) \right| = 0, (6)$$

uniformly in n, $i = 0,\ 1,\ 2$.

Then for any function $f \in C(\mathrm{X})$, we have

$$\lim_n \sup_{x\in X} \left| \frac{T_n(f;x) - f(x)}{\sigma(x)} \right| = 0,$$

where $\sigma(x) = \max\left\{ \left|\sigma_i(x)\right|;\ i = 0,\ 1,\ 2 \right\}$, $\left|\sigma_i(x)\right| \neq 0$ and $\sigma_i(x)$ is unbounded for each $i = 0,\ 1,\ 2$.

Proof.

By Theorem 2, condition (6) implies that

$$\lim_p \sup_{x\in X} \left| \frac{1}{\sigma(x)} \left(\frac{1}{p} \sum_{k=n}^{n+p-1} T_k(f;x) - f(x) \right) \right| = 0, \quad (7)$$

uniformly in n.

We can get

$$\sup_{x\in X} \left| \frac{T_n(f;x) - f(x)}{\sigma(x)} \right|$$

$$\leq \sup_{x\in X} \left| \frac{1}{\sigma(x)} \left(\frac{1}{p} \sum_{k=n}^{n+p-1} T_k(f;x) - f(x) \right) \right| + \frac{1}{p} \sum_{k=n+1}^{n+p-1} \left(\sum_{m=n+1}^{k} \sup_{x\in X} \left| \frac{T_m(f;x) - T_{m-1}(f;x)}{\sigma(x)} \right| \right)$$

$$\leq \sup_{x\in X} \left| \frac{1}{\sigma(x)} \left(\frac{1}{p} \sum_{k=n}^{n+p-1} T_k(f;x) - f(x) \right) \right| + \frac{p-1}{2} \left\{ \sup_{k\geq n} \left(\sup_{x\in X} \left| \frac{T_k(f;x) - T_{k-1}(f;x)}{\sigma(x)} \right| \right) \right\}.$$

Therefore using (5) and (7), we get

$$\lim_n \sup_{x\in X} \left| \frac{T_n(f;x) - f(x)}{\sigma(x)} \right| = 0.$$

This completes the proof of the theorem.

Theorem 4.

Let (T_n) be a sequence of positive linear operators on $C(\mathrm{X})$ such that

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$$\lim_p \sup_{x\in X}\left|\frac{1}{\sigma_i(x)}\left(\frac{1}{p}\sum_{k=1}^{p}T_k(e_i;x)-e_i(x)\right)\right|=0, \quad (8)$$

$i=0,\ 1,\ 2$

and

$$\lim_p\left|\sup_{n\ge p}\frac{n}{p}\left(\sup_{x\in X}\left|\frac{\varphi_{n+p-1}(f;x)-\varphi_{n-1}(f;x)}{\sigma(x)}\right|\right)\right|=0, \quad (9)$$

where $\varphi_n(f;x)=\frac{1}{n+1}\sum_{k=0}^{n}T_k(f;x)$. Then for any function $f\in C(X)$, we have

$$\lim_p \sup_{x\in X}\left|\frac{1}{\sigma(x)}\left(\frac{1}{p}\sum_{k=n}^{n+p-1}T_k(f;x)-f(x)\right)\right|=0,$$

uniformly in n,

where

$\sigma(x)=\max\{|\sigma_i(x)|;\ i=0,\ 1,\ 2\}$,

$|\sigma_i(x)|\neq 0$ and $\sigma_i(x)$ is unbounded for each $i=0,\ 1,\ 2$.

Proof.

For $n\ge p\ge 1$, it is easy to see that

$$\frac{1}{p}\sum_{k=n}^{n+p-1}T_k(f;x)=\varphi_{n+p-1}(f;x)+\frac{n}{p}\left(\varphi_{n+p-1}(f;x)-\varphi_{n-1}(f;x)\right)$$

which implies

$$\sup_{n\ge p}\sup_{x\in X}\left|\frac{1}{\sigma(x)}\left(\frac{1}{p}\sum_{k=n}^{n+p-1}T_k(f;x)-\varphi_{n+p-1}(f;x)\right)\right| =\sup_{n\ge p}\frac{n}{p}\left(\sup_{x\in X}\left|\frac{\varphi_{n+p-1}(f;x)-\varphi_{n-1}(f;x)}{\sigma(x)}\right|\right). \quad (10)$$

From Theorem 2, we have that if (8) holds, then

$$\lim_p \sup_{x\in X}\left|\frac{1}{\sigma(x)}\left(\frac{1}{p}\sum_{k=1}^{p}T_k(f;x)-f(x)\right)\right|=0. \quad (11)$$

Using $(8)-(11)$ and because of relatively uniform almost convergence implies $(C,1)-$relatively uniform convergence, we conclude that

$$\lim_p \sup_{x\in X}\left|\frac{1}{\sigma(x)}\left(\frac{1}{p}\sum_{k=n}^{n+p-1}T_k(f;x)-f(x)\right)\right|=0,$$

uniformly in n

and the proof is complete.

Theorem 5.

Let (T_p) be a sequence of positive linear operators on $C(\mathrm{X})$ such that

$$\lim_p \sup_n \frac{1}{p}\sum_{k=n}^{n+p-1}\sup_{x\in X}\left|\frac{T_p(f;x)-T_k(f;x)}{\sigma(x)}\right|=0$$

and

$$\lim_p \sup_{x\in X}\left|\frac{1}{\sigma_i(x)}\left(\frac{1}{p}\sum_{k=n}^{n+p-1}T_k(\mathrm{e}_i;x)-e_i(x)\right)\right|=0 \quad (12)$$

uniformly in n, $i=0,\ 1,\ 2$.

Then for any function $f\in C(\mathrm{X})$, we have

$$\lim_p \sup_{x\in X}\left|\frac{T_p(f;x)-f(x)}{\sigma(x)}\right|=0,$$

where $\sigma(x)=\max\{|\sigma_i(x)|;\ i=0,\ 1,\ 2\}$, $|\sigma_i(x)|\neq 0$ and $\sigma_i(x)$ is unbounded for each $i=0,\ 1,\ 2$.

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Proof.

By Theorem 2, condition (12) implies that

$$\lim_p \sup_{x\in X}\left|\frac{1}{\sigma(x)}\left(\frac{1}{p}\sum_{k=n}^{n+p-1}T_k(f;x)-f(x)\right)\right|=0, \quad (13)$$

uniformly in n.

Because of

$$\sup_n \sup_{x\in X}\left|\frac{1}{\sigma(x)}\left(T_p(f;x)-\frac{1}{p}\sum_{k=n}^{n+p-1}T_k(f;x)\right)\right|$$

$$\leq \sup_n \frac{1}{p}\sum_{k=n}^{n+p-1}\sup_{x\in X}\left|\frac{T_p(f;x)-T_k(f;x)}{\sigma(x)}\right|,$$

we have

$$\lim_p \sup_n \sup_{x\in X}\left|\frac{1}{\sigma(x)}\left(T_p(f;x)-\frac{1}{p}\sum_{k=n}^{n+p-1}T_k(f;x)\right)\right|=0. \quad (14)$$

By the triangle inequality, we get

$$\left|\frac{T_p(f;x)-f(x)}{\sigma(x)}\right|$$

$$\leq\left|\frac{1}{\sigma(x)}\left(T_p(f;x)-\frac{1}{p}\sum_{k=n}^{n+p-1}T_k(f;x)\right)\right|$$

$$+\left|\frac{1}{\sigma(x)}\left(\frac{1}{p}\sum_{k=n}^{n+p-1}T_k(f;x)-f(x)\right)\right|$$

and hence from (13) and (14), we have

$$\lim_p \sup_{x\in X}\left|\frac{T_p(f;x)-f(x)}{\sigma(x)}\right|=0.$$

This completes the proof of the theorem.

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