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Analytical Solutions of Coupled Boiti-Leon-Pempinelli Equation with Fractional Derivative

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Abstract. In this study, the sub-equation method is used as a tool for finding the analytical solutions of Coupled Boiti-Leon-Pempinelli (CBLP) equation where the derivatives are in conformable form with the fractional term. In the introduction section, the advantages of the conformable derivative are expressed. By using the fractional wave transform and chain rule for conformable derivative, the nonlinear fractional partial differential equation turns into a nonlinear integer order differential equation. This translation gives us a great advantage in obtaining analytical solutions and interpreting the physical behavior of the acquired solutions. In the rest of article, the sub-equation method is applied to Coupled Boiti-Leon-Pempinelli equation, and the analytical results are derived successfully. This means that our method is effective and powerful for constructing exact and explicit analytic solutions to nonlinear PDEs with the fractional term. While this process, symbolic computation such as Mathematica is used. It is shown that, with the help of symbolic computation, sub-equation method ensures a powerful and straightforward mathematical tool for solving nonlinear partial differential equations.

1. Introduction

Nonlinear phenomena draws great attraction in the last decades. Understanding the physical nature of the nonlinear mathematical models allures scientists because the only way for interpreting natural events arises as a result of this curiosity [3–6, 16–18]. For this aim, many methods are developed such as homotopy analysis method [7], differential transform method [8], exp-function method [9], Jacobi elliptic function expansion method [10] and etc. As we see, both numerical methods and analytical methods are applied to get the results. But analytical solutions of very few of the differential equations that arise as a result of mathematical models of events encountered in nature can be obtained. This makes the analytical method valuable. Because numerical methods give the approximate value for the expected solution and give us a restricted chance to understand the physical nature of the solution. In spite of that the solutions which are obtained as a result of analytical methods give us extensive perspective for explaining the behavior of the solution.

In the beginning, integer order derivative and integral are used for modeling the natural event. But by the time it is understood that Newtonian type derivative and integral fall short of modeling the event that arises in the nonlinear nature. So the survey for fractional differentiation and integration is started. By the time

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it is understood that fractional calculus has a clearer physical meaning and a simpler statement compared with the integer order models while describing complicated physical mechanics problems. This motivation helped fractional calculus to be improved faster. First of all, scientists need to give an efficient and applicable definition of fractional differentiation and integration. Riemann-Liouville, Caputo and Grünwald-Letnikov definitions were the popular definition. But there were some deficiencies while describing the mathematical model. For instance, the Riemann-Liouville fractional derivative of a constant is not zero. In addition to this fractional initial/boundary conditions of problems which are described as mathematical models of different physical, chemical, or engineering problems must be expressed in fractional form. More than these, basic properties such as a derivative of the quotient of two functions, derivative of the product of two functions, chain rule and etc. are not satisfied by Riemann-Liouville, Caputo, and Grünwald-Letnikov definitions. As a result, Khalil et al. expressed a new definition that obeys the basic properties.

Definition 1.1. *α^{th} order conformable derivative(CFD) of a function f can be expressed as*

$$T_\alpha(f)(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - (f)(t)}{\varepsilon},$$

for $f : [0, \infty) \to R$ and for all $t > 0$, $\alpha \in (0, 1)$.

Definition 1.2. *Let f function is defined with n variables $x_1, ..., x_n$. The fractional partial derivatives of f of order $\alpha \in (0, 1]$ in conformable sense with respect to x_i is given by [19]*

$$\frac{d^\alpha}{dx_i^\alpha} f(x_1, ..., x_n) = \lim_{\varepsilon \to 0} \frac{f(x_1, ..., x_{i-1}, x_i + \varepsilon x_i^{1-\alpha}, ..., x_n) - f(x_1, ..., x_n)}{\varepsilon}.$$

Definition 1.3. *Let $\alpha \geqslant 0$. Then conformable fractional integral of a function is defined as [13]*

$$I_\alpha^a(f)(s) = \int_a^s \frac{f(t)}{t^{1-\alpha}} dt.$$

In this article, the analytical solutions of Coupled Boiti-Leon-Pempinelli equation where the derivatives are in the conformable sense with the fractional term are obtained. To the best of our knowledge, these solutions are seen firstly in the literature. Also, some graphical representations of the solutions are given to understand the physical behavior of the solutions.

2. Brief Description of Considered Method

2.1. Sub-Equation Method

In this section sub-equation method [11] which established on the Riccati equation

$$\varphi'(\xi) = \sigma + (\varphi(\xi))^2 . \tag{1}$$

is going to be described. Regarding the general form of nonlinear time fractional partial differential equation as

$$P\left(u, D_t^\alpha u, D_x u, D_x^2 u, ...\right) = 0 \tag{2}$$

where $D_t^\alpha u$ indicates fractional order differentiation in conformable sense. The fractional wave transformation [12] could be expressed

$$u(x, y, t) = U(\xi),\ \xi = kx + wy + c\frac{t^\alpha}{\alpha} \tag{3}$$

where c is a constant to be calculated later and by the help of the chain rule [14], Eq. (2) can turn into an nonlinear differential equation with integer order

$$G(U(\xi), U'(\xi), U''(\xi), ...) = 0. \tag{4}$$

Suppose that the solution of the reduced Eq. (4) can be expressed as

$$U(\xi) = \sum_{i=0}^{N} a_i \varphi^i(\xi),\ a_N \neq 0, \tag{5}$$

where a_i $(0 \leq i \leq N)$ are constant coefficients to be calculated and positive integer N is going to be obtained by using balancing principle [15] in equation (4) and $\varphi(\xi)$ is the solution of Riccati equation (1). Some solutions of equation (1) is given as follows.

$$\varphi(\xi) = \begin{cases} -\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}\xi\right), & \sigma < 0 \\ -\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}\xi\right), & \sigma < 0 \\ \sqrt{\sigma}\tan\left(\sqrt{\sigma}\xi\right), & \sigma > 0 \\ -\sqrt{\sigma}\cot\left(\sqrt{\sigma}\xi\right), & \sigma > 0 \\ -\frac{1}{\xi+\varpi},\ \varpi \text{ is a cons.}, & \sigma = 0 \end{cases} \tag{6}$$

After the all solutions procedure we get a polynomial due to $\varphi(\xi)$. Equating zero to all the coefficients of $\varphi^i(\xi)$ $(i = 0, 1, ..., N)$ ends with a nonlinear algebraic equation system depending on c, a_i $(i = 0, 1, ..., N)$. By solving this algebraic equation system we have the values of c, a_i $(i = 0, 1, ..., N)$. Substituting all the results in the formulas (6) we get the exact solutions for equation (2).

3. Solutions of the Equation

Consider (CBLP) equation in conformable sense as

$$\begin{aligned} &D_y D_t^\alpha u - D_x D_y(u^2 - D_x u) - 2D_x^3 v = 0, \\ &D_t^\alpha v - D_x^2 v + 2u D_x v = 0. \end{aligned} \tag{7}$$

Using the following wave transformation

$$u(x, y, t) = U(\xi), v(x, y, t) = V(\xi),\ \xi = kx + wy + c\frac{t^\alpha}{\alpha} \tag{8}$$

and chain rule [14] (7) turns into nonlinear differential equation system

$$\begin{aligned} &cwU'' - kw(U^2 - kU')'' - 2k^3 V''' = 0, \\ &cV' - k^2 V'' - 2kUV' = 0 \end{aligned} \tag{9}$$

where the derivatives described in integer order. Now integrating twice the first equation in (7), we have

$$V' = \frac{cw}{2k^3}U - \frac{w}{2k^2}(U^2 - kU'). \tag{10}$$

Using this result in the second equation of Eq. (7) we have the following equation.

$$c^2U - 3kcU^2 + 2k^2U^3 - k^4U'' = 0. \tag{11}$$

Let the solution of Eq. (11) is given in terms of $\varphi(\xi)$ as

$$U(\xi) = \sum_{i=0}^{N} a_i \varphi^i(\xi),\ a_N \neq 0. \tag{12}$$

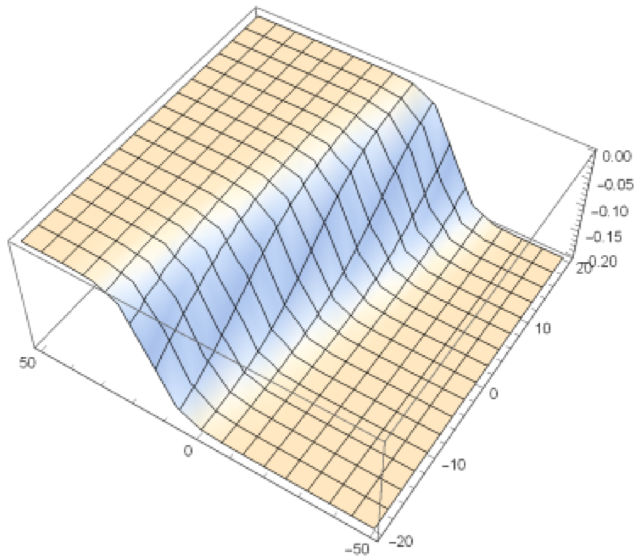

Figure 1: Surface plot of the exact solution $u_1(x, y, t)$ for $w = 0.1, a_0 = -0.1, \sigma = -2, \alpha = 0.75, t = 0.1$

Using the balancing principle [15], we calculate $N = 1$. Collecting all the obtained results in Eq. (11), an algebraic equation system come to exist with respect to w, k, c, a_0, a_1. Solving this obtained system led to following solution set

$$a_1 = \frac{a_0}{\sqrt{-\sigma}}, c = -\frac{2a_0^2}{\sqrt{-\sigma}}, k = \frac{-a_0}{\sqrt{-\sigma}} \tag{13}$$

where $\sigma < 0$ and a_0 and w are free constants. Using (6) and (3) the traveling wave solutions of Eq. (7) can be deducted

$$\begin{aligned} u_1(x, y, t) &= a_0 - a_0 \tanh\left(\xi \sqrt{-\sigma}\right), \\ v_1(x, y, t) &= -\frac{w\sigma \tanh\left(\xi \sqrt{-\sigma}\right)}{\sqrt{-\sigma}}, \\ u_2(x, y, t) &= a_0 - a_0 \coth\left(\xi \sqrt{-\sigma}\right), \\ v_2(x, y, t) &= -\frac{w\sigma \coth\left(\xi \sqrt{-\sigma}\right)}{\sqrt{-\sigma}} \end{aligned}$$

where $\xi = wy - \frac{a_0 x}{\sqrt{-\sigma}} - \frac{2a_0^2 t^\alpha}{\alpha \sqrt{-\sigma}}$. Some graphical representations of the obtained results are given in Figure 1 and Figure 2.

4. Conclusion

In this article, it is obtained that sub-equation method shows great performance while obtaining the exact solutions of the nonlinear partial differential equations where the derivatives are in conformable sense with fractional term. While obtaining the solution symbolic computer software called Mathematica is used. Also some graphical simulations of the obtained solutions are given. This article may give an insight to the researchers who study on obtaining the analytical solutions of nonlinear fractional partial differential equations.

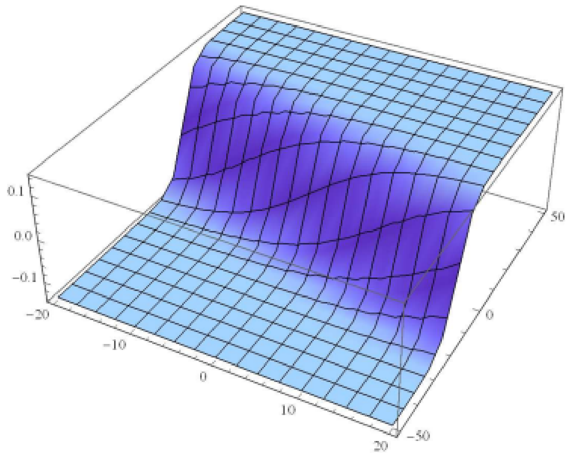

Figure 2: Surface plot of the exact solution $v_1(x, y, t)$ for $w = 0.1, a_0 = -0.1, \sigma = -2, \alpha = 0.75, t = 0.1$

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