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Kesikli Chen Dağılımı için Bayes Tahmini

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Öz: Bu çalışmada, Kesikli Chen dağılımı için bayes tahmini incelenmiştir. Bayes tahmin edicilerini elde etmek zor olduğundan Bayes tahminlerinin yaklaşık değerleri Tierney-Kadane yaklaşımı kullanılarak elde edilmiştir. Tahmin edicilerin performansını gözlemlemek için simülasyon çalışmaları yapılmıştır. Ayrıca sayısal bir örnek verilmiştir.

Anahtar kelimeler: Bayes tahmini, Kesikli Chen dağılımı, Tierney Kadane yaklaşımı

Bayesian Estimation for Discrete Chen Distribution

Abstract: In this study, Bayesian estimation is discussed for the Discrete Chen Distribution. It is tedious to find the Bayesian estimation so we used Tierney-Kadane approximation to get the approximate Bayesian estimates. Simulation study is performed to monitor the performance of estimates. A numerical example is also provided.

Keywords: Bayes estimation, Discrete Chen Distribution, Tierney Kadane approximation.

1. Introduction

Sometimes in reliability (survival) analysis the reliability (survival) function is assumed to be a function of a count (discrete) random variable instead of being a function of continuous time random variable. There are several well-known discrete distributions such as binomial, poisson, negative-binomial and etc. Nowadays, the authors have introduced some new discrete distributions as alternative to well-known distribution. Please see Nakagawa and Osaki (1975), Stein and Dattero (1984), Khan et al. (1989), Roy (2003), (Roy, 2004), Krishna and Pundir (2007), Krishna and Pundir (2009), Jazi et al. (2009), Noughabi et al.(2013).

In this paper, the Bayes estimate of paremeters of discrete Chen distribution using Tierney-Kadene's (1986) approximation form is compared with their corresponding maximum likelihood estimate (MLE).

If the random variable X has a discrete Chen distribution (DCD) the the probability mass function (pmf) and cumulative distribution function (cdf) are given by

$$f(x)=q^{e^{x^{\beta}}-1}-q^{e^{(x+1)^{\beta}}-1},\ x=0,1,2,\ldots, \qquad (1)$$

and

$$F(x)=1-q^{e^{(x+1)^{\beta}}-1},\ x=0,1,2,\ldots(\text{jump point}) \qquad (2)$$

respectively, where $q=\exp(-\lambda)$ and $\beta>0$ (Noughabi et al., 2013). Note that X has continuous Chen distribution with paremeter λ and β then $[\![X]\!]$ will have DCD with parameter $q=\exp(-\lambda)$ and β.

2. Maximum Likelihood Estimation

$X_1, X_2, \ldots, X_n$ be the complete sample from DCD with parameter q and β. The likelihood and log-likelihood functions based on complete sample are

$$L(q,\beta)=\prod_{i=1}^{n}(q^{e^{x_i^{\beta}}-1}-q^{e^{(x_i+1)^{\beta}}-1}) \qquad (3)$$

$$\ell(q,\beta)=\sum_{i=1}^{n}\log(q^{e^{x_i^{\beta}}-1}-q^{e^{(x_i+1)^{\beta}}-1}) \qquad (4)$$

respectively.

Thus, likelihood equations are also given by

$$\frac{\partial \log(\ell(q,\beta))}{\partial\beta}=\sum_{i=1}^{n}\frac{x_i^{\beta}\log(x_i)\exp(x_i^{\beta})\log(q)q^{\exp(x_i^{\beta})-1}}{q^{e^{x_i^{\beta}}-1}-q^{e^{(x_i+1)^{\beta}}-1}} - \frac{(x_i+1)^{\beta}\log(x_i+1)\exp((x_i+1)^{\beta})\log(q)q^{\exp((x_i+1)^{\beta})-1}}{q^{e^{x_i^{\beta}}-1}-q^{e^{(x_i+1)^{\beta}}-1}} \qquad (5)$$

$$\frac{\partial \log(\ell(q,\beta))}{\partial q}=\sum_{i=1}^{n}\frac{q^{\exp(x_i^{\beta})-1}(\exp(x_i^{\beta})-1)}{q^{e^{x_i^{\beta}}-1}-q^{e^{(x_i+1)^{\beta}}-1}} + \frac{q^{\exp((x_i+1)^{\beta})-1}(1-\exp(x_i+1^{\beta}))}{q^{e^{x_i^{\beta}}-1}-q^{e^{(x_i+1)^{\beta}}-1}} \qquad (6)$$

The ML estimates of q and β must be derived numerically. Newton-Raphson algorithm is one of the standart methods to determine the ML estimates of the parameters**.**

3. Bayesian Estimation

For Bayesian infrence, we suppose that q has a Beta distributions with parameters a_1 and b_1 (denote as $Beta(a_1,b_1)$) and β has Gamma distribution with parameters a_2 and b_2 (denote as $Gamma(a_2, b_2)$) and q and β are independent. Then the prior pdf of q and β are given

$$\pi_1(q) = \frac{1}{B(a_1,b_1)} q^{a_1-1}(1-q)^{b_1-1}, p \in (0,1) \quad (7)$$

$$\pi_2(\beta) = \frac{b_2^{a_2}}{\Gamma(a_2)} \beta^{a_2-1} \exp(-b_2\beta), \beta > 0 \quad (8)$$

respectively, where $B(\cdot,\cdot)$ is the Beta function and $\Gamma(\cdot)$ is the well-known Gamma function.

Therefore, the joint prior of (q,β) can be expressed as

$$\pi(q,\beta) = \pi_1(q)\pi_2(\beta)$$
$$= \frac{b_2^{a_2}}{B(a_1,b_1)\Gamma(a_2)} q^{a_1-1}(1-q)^{b_1-1} \beta^{a_2-1} \exp(-b_2\beta) \quad (9)$$

Under the squared error loss function the Bayes estimates of the q and β can be expressed as

$$\hat{q} = E(q/x) = \frac{\int_0^\infty \int_0^1 qL(q,\beta|x)\pi(q,\beta)dq d\beta}{\int_0^\infty \int_0^1 L(q,\beta|x)\pi(q,\beta)dq d\beta} \quad (10)$$

and

$$\hat{\beta} = E(\beta/x) = \frac{\int_0^\infty \int_0^1 \beta L(q,\beta|x)\pi(q,\beta)dq d\beta}{\int_0^\infty \int_0^1 L(q,\beta|x)\pi(q,\beta)dq d\beta} \quad (11)$$

respectively.

These Bayes estimates cannot be obtained explicitly but they can be approximately obtained by using Tierney and Kadene (1986)'s method. This method is discussed in Section 4.

3.1. Tierney-Kadene Approximation

The ratio of two integrals given by Eq.10 and Eq.11 cannot be obtained in a closed form. We used here Tierney-Kadene approximation to obtained approximate Bayes estimates of q and β. According to this approach, the approximate Bayes estimate of any function of the parameters q, β and $\phi(q,\beta)$ is expressed by

$$\hat{\phi} = \sqrt{\frac{|\Sigma^*|}{|\Sigma|}} \frac{\phi(\hat{q}^*, \hat{\beta}^*) p(\hat{q}^*, \hat{\beta}^*|x)}{p(\hat{q}, \hat{\beta}|x)} \quad (12)$$

where $p(q,\beta/x)$ is the posterior distribution of q β, $\hat{q}^*$ and $\hat{\beta}^*$ maximize

$$G(q,\beta) = \frac{1}{n} p \log(q,\beta/x)$$

and

$$G^*(q,\beta) = G(q,\beta) + \frac{1}{n}\log(\phi(p,\beta))$$

respectively, $\sum^*$ and $\sum$ are the negatives of the inverses of the matrices of second derivatives of $G^*(q,\beta)$ and $G(q,\beta)$ at $(\hat{q}^*,\hat{\beta}^*)$ and $(\hat{q},\hat{\beta})$ respectively. For details see Tierney and Kadene (1986). This method used by a lot of paper such as Singh et al. (2014), Danish and Aslam (2013).

4. Simulation Study and Conclusion

In this section, simulation study (based on 5000 repitations) is performed to investigate the performance of the ML and Bayes estimators in that their estimated risks.

Table 1. Estimated risks for prior parameters $a_1 = 2, b_1 = 3, a_2 = 2, b_2 = 3$

$a_1 = 2, b_1 = 3, a_2 = 2, b_2 = 3$				
n	**MLE**		**Bayes Est.**	
	q	β	q	β
5	0,0483	0,9075	0,0187	0,4915
10	0,0237	0,6046	0,0121	0,3983
20	0,0117	0,4271	0,0075	0,3030
50	0,0042	0,2521	0,0033	0,1890
100	0,0020	0,1655	0,0018	0,1345

Different sample size and prior parameters are considered. Obtained average estimated risks of ML and Bayes estimators for two different choices of priors are given in Table 1 and Table 2.

Table 2. Estimated risks for prior parameters $a_1 = 5, b_1 = 3, a_2 = 2, b_2 = 3$.

$a_1 = 5, b_1 = 3, a_2 = 2, b_2 = 3$				
n	**MLE**		**Bayes Est.**	
	q	β	q	β
5	0,0395	1,0044	0,0136	0,3814
10	0,0185	0,4085	0,0089	0,2458
20	0,0085	0,1931	0,0054	0,1448
50	0,0030	0,0738	0,0025	0,0635
100	0,0014	0,0370	0,0013	0,0340

In this article, we studied Bayes estimate of paremeters of discrete Chen Distribution using Thirney-Kadene's (1986) approximation. Also, compared risks with their corresponding maximum likelihood estimate. Finally, we obtained results to use Monte Carlo Simulation in simulation study. According to simulation study, In Table 1, when n increases estmated risks of ML and Bayes estimators of q and β parameters decrease as expected. In addition estimated risks of Bayes estimators of q and β are smaller than those of ML estimators. The differences between estimated risks of ML and Bayes estimators are approaching to zero when n increases. Similar results can be seen from Table 2.

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