SUJE
ISSN: 2146-7455

Received : 02.11.2022
Published : 10.03.2023
Available Online : 30.04.2023

SAKARYA UNIVERSITY

Interpretations of Pre-service Elementary Mathematics Teachers on the Functions of Non-Textual Elements: Case Study on Algebra Learning Area

Murat GENÇ[a] Mustafa AKINCI[b]

a: 0000-0003-4525-7507 Zonguldak Bülent Ecevit University, Turkiye muratgenc@beun.edu.tr

b: 0000-0003-2096-7617 Zonguldak Bülent Ecevit University, Turkiye mustafa.akinci@beun.edu.tr

Abstract

The study aimed to investigate how pre-service elementary mathematics teachers perceive the intended use of non-textual elements in an algebra content area of an eighth-grade mathematics textbook. Non-textual elements in this qualitative exploratory case study refer to visual representations consisting of components that are not only verbal, numerical, or symbolic representations. Data were collected from thirty-one undergraduate students through a task-based written questionnaire including seven non-textual elements on the algebra learning domain. Data analysis was conducted using a content analysis approach to generate themes and uncover previously unspecified patterns. The results showed that pre-service teachers' interpretations of non-textual elements could be categorized into ten themes: (i) attractiveness, (ii) organizing, (iii) embodiment, (iv) informativeness, (v) reasoning, (vi) conciseness, (vii) essentiality, (viii) decorativeness, (ix) contextuality, and (x) connectivity. Pre-service teachers were found to have diverse but sometimes overlapping interpretations of the functions of each non-textual element. However, the functional diversity of non-textual elements may have differentiated their interpretations, as visual literacy skills and strategies are required to interpret the intended use of non-textual elements. Therefore, in order for pre-service mathematics teachers to better understand the functions of non-textual elements, various teaching approaches should be developed to support pre-service teachers' visual literacy, and these approaches to visual literacy should be incorporated into teacher education and professional development.

Keywords

Non-textual elements, mathematics textbook, algebra learning area, pre-service elementary mathematics teachers.

Ethics Committee Approval: Ethics committee permission for this study was obtained from Zonguldak Bülent Ecevit University Human Research Ethics Committee with the decision dated 05.10.2022 and numbered 221989/326.

Suggested Citation: Genç, M., & Akıncı, M. (2023). Interpretations of Pre-service Elementary Mathematics Teachers on the Functions of Non-Textual Elements: Case Study on Algebra Learning Area. *Sakarya University Journal of Education, 13*(1), 84-102. doi: https://doi.org/10.19126/suje.1200724

INTRODUCTION

Attracting students' visual attention is essential for teaching, communicating key concepts, and engaging students emotionally with course content (Araya, Farsani, & Hernández, 2016). Since visual representations can capture students' attention and have a positive impact on learning outcomes (Levin & Mayer 1993; Pettersson, 1990; Woodward, 1993), it can be seen that more images, illustrations, and diagrams are included in textbooks than in the past (Bazerman, 2006; Boling, Eccarius, Smith & Frick 2004). On the other hand, it has also been reported that visual images, particularly in science and reading books, can sometimes make it difficult for students to understand the content and even lead to confusion and misinterpretation of the text (Watkins, Miller & Brubaker, 2004). However, due to the abstract nature of mathematics, it is argued that representing or illustrating mathematical concepts or situations in different ways helps students develop their abstract thinking skills and contributes to mathematical understanding (National Council of Teachers of Mathematics [NCTM], 2014).

Visualization is not a new method for mathematicians, and they have long been aware of this method and have made great efforts to take advantage of it (Borwein & Jörgenson, 2001). Visualization makes it easy to understand how mathematical ideas are structured and how they relate to each other (Farmaki & Paschos, 2007). Since representations are directly related to both mathematical content and the learning process, they are considered to be effective in the formation of an individual's concept image, mathematical communication, and reasoning (Hershkowitz, Arcavi & Bruckheimer, 2001; Tall & Vinner 1981). Therefore, in addition to textual elements such as written texts, mathematical signs, and symbols, non-textual representations such as figures, diagrams, and various pictures are believed to play an important role in teaching mathematics (Arcavi, 2003; Brenner et al., 1997; Herman, 2007; Pape & Tchoshanov, 2001; Stylianou & Silver, 2004).

Algebra is a critical content area of focus in mathematics teaching as it provides important opportunities for the development of mathematical thinking, reasoning, and problem-solving skills (Van de Walle, Karp & Bay-Williams, 2012). NCTM's vision for school mathematics recognizes the importance of algebra and highlights why all students should learn algebra. First of all, algebra uses abstract structures and the principles of these structures in solving problems expressed with symbols. Besides, the ideas contained in the algebra standard are an important component of the school mathematics curriculum and help unify its content areas. For example, most of the symbolic and structural emphasis in algebra is built on students' knowledge of numbers. Algebra is also closely related to geometry and data analysis. In addition, algebraic competence is important in individuals' further education and later in their working lives. Moreover, algebra represents patterns in our daily lives and generalizes arithmetic. In other words, it is the language of generalization used to create a systematic representation of patterns and relationships between numbers and objects, to analyze change, and to model real-world events (NCTM, 2000, 2018).

The concepts of algebra can be studied and communicated through representations, enabling students to interpret relationships among quantities and make sense of symbols (Kieran, 2004). Being successful in algebra depends on algebraic thinking, defined as the individual's ability to generalize about mathematical operations and relationships/patterns, to make assumptions from these generalizations, and to discuss and express them (Kaput, 1999). Since algebraic thinking is expressed as the use of mathematical symbols and tools to represent verbally expressed mathematical knowledge with figures, tables, graphs, and equations by selecting the necessary information from the given problem situation (Herbert & Brown, 1997), there needs to be the effective use of multiple representations and relating these representations with each other in teaching algebra. Indeed, the use of representations in mathematics teaching is widely considered necessary, because abstract mathematical ideas, concepts, or relationships can only be accessed through representations and their effective use in teaching (Duval, 2006). On the other hand, a variety of representations appear to be readily available and widely used in curriculum materials, but research encourages educators to carefully examine both their benefits and limitations to support students' learning (Kamii, Lewis & Kirkland, 2001), rather than having a high expectation that the anticipated functions of these representations will somehow occur spontaneously (Ball, 1992). Mathematics curriculum materials include not only textual elements such as standard text, mathematical signs and symbols, but also various non-textual elements such as figures, tables, graphs, diagrams, pictures, images, and illustrations (Filloy, Rojano & Puig, 2008). Teachers are also expected to constantly interact with curriculum materials to assist and guide their teaching, including textbooks, teacher guides, student worksheets, and other types of resources (Stein, Remillard & Smith, 2007). However, if teachers interpret and apply representations inappropriately, incorrect messages may be transmitted and then basic mathematical concepts may be distorted, which can further confuse students (Bosse, Lynch-Davis, Adu-Gyamfi & Chandler, 2016). For this reason, teachers need to use various non-textual elements effectively for students to learn mathematics meaningfully. Despite the importance of non-textual elements in teaching and learning mathematics and the potential for misuse, their features and roles in mathematics curriculum materials are still elusive for many teachers, especially for prospective teachers (Lee & Ligocki, 2020). Therefore, this paper aims to explore pre-service teachers' interpretation of the functions of non-textual elements in a mathematics textbook. More specifically, the study attempts to investigate how pre-service elementary mathematics teachers perceive the characteristics and roles of non-textual elements selected from an algebra content area of an eighth-grade mathematics textbook.

Literature Review and Conceptual Background

In this study, we are particularly concerned with the intended use and overall quality of non-textual elements referring to visual representations consisting of components that are not purely verbal, numerical, or mathematical symbolic representations (Kim, 2009). For example, the equation $a^2 + b^2 = c^2$ used in the Pythagorean Theorem is not a non-textual element, as it consists only of symbolic representations. However, if it is illustrated with a right triangle picture in which the necessary symbols and signs are used, the picture is a non-textual element because it is not a purely symbolic representation despite some symbols and signs it contains. Kim (2012) emphasizes that research on visual representations in mathematics textbooks usually focuses on mathematical representations such as formulae, numbers, tables, graphs, charts, diagrams, symbolic equations, and the like. On the other hand, pictorial representations such as pictures, drawings, photos, and illustrations are often considered decorations or a part of the visual design of a textbook. To provide a more systematic understanding of non-textual elements in mathematics textbooks, Kim (2009, 2012) has developed a conceptual framework, each conceptual component of which is based on various studies on mathematics education, semiotics, metaphor theory, visual rhetoric, and information design. In this conceptual framework, the important aspects that constitute the quality of non-textual elements in mathematics textbooks are identified as accuracy, connectivity, contextuality, conciseness, and aesthetics. Accuracy refers to the mathematical clarity and precision of non-textual elements according to the definition of a mathematical concept. Connectivity signifies how closely the non-textual elements are related to the mathematical content contained in the texts. Contextuality indicates the presentation of mathematical expressions in a realistic context. Conciseness means mathematical simplicity in a non-textual element. Aesthetics implies the visual appeal of non-textual elements to facilitate and motivate learning.

In addition, seven functions of explicative illustrations distinguished by Duchastel and Waller (1979) are stated as descriptive, expressive, constructional, functional, logico-mathematical, algorithmic, and data-display. Descriptive denotes the function of a visual element to provide information about what a described object actually looks like. Expressive refers to a function that aims to make an impact on the learner beyond simple explanation. Constructional indicates the function of describing how the various components of an object fit together to form the whole. Functional represents the function that aims the learner to understand how a process or system works. Logico-mathematical is a function of displaying diagrams, figures, drawings, and graphs used to explain mathematical relationships. Algorithmic illustrates the function that provides a holistic picture of the various possibilities for an action plan. Data-display refers to the function that provides a quick visual comparison and easy access to data.

Moreover, Carney and Levin (2002) underline that pictures fulfill five traditional functions in text processing: decorative, representational, organizational, interpretive, and transformational. Decorative pictures simply decorate the page with little or no relation to the text content. Representative pictures reflect some or all of the text content and are decisively the most widely used type of illustration. Organizational pictures show qualitative relationships between different elements, allowing a useful structural framework for text content. Interpretive pictures help clarify difficult texts by providing the function of interpretation and reflection thanks to their explanatory aspect. Transformative pictures contain systematic mnemonic (memory-enhancing) components designed to enhance the reader's recall by re-encoding text information to make it more tangible and then relating it through a meaningful, interactive illustration. Similarly, Elia and Philippou (2004) propose four functions (categories) of pictures in mathematical problem-solving: decorative, representational, organizational, and informational. Decorative pictures do not provide any significant information about the solution to the problem. Representative pictures illustrate all or part of the problem's content. Organizational pictures specify guidelines for drawing or written work that support the solution procedure. Finally, informative pictures provide the information necessary to solve the problem; in other words, the solution to the problem cannot be done without the picture.

It can be seen that there are both similarities and differences between the functions of non-textual elements identified by different researchers (Carney & Levin, 2002; Duchastel & Waller, 1979; Elia & Philippou, 2004; Kim, 2009, 2012). For example, the functions of pictorial illustrations by Duchastel and Waller (1979) and Carney and Levin (2002) are useful in identifying the role of non-textual elements in textbooks, especially in reading and science textbooks. Therefore, they may not be sufficient to understand the functions of non-textual elements in mathematics textbooks. In contrast to the typical non-textual elements in science, non-textual elements in mathematics are used not only as informational tools, but also as tools for reasoning, argumentation, and reflection (Cuoco & Curcio, 2001). Based on this argument, Kim (2009, 2012) discussed the functions of non-textual elements in mathematics textbooks under the headings of accuracy, connectivity, contextuality, conciseness, and aesthetics. Although there are partial differences between all these functions of non-textual elements mentioned by these researchers, it can be said that functions such as aesthetic and decorative, descriptive, expressive, informative and interpretive, as well as logico-mathematical and connectivity have many aspects that support and complement each other. Moreover, it is found that some functions of non-textual elements such as decorative, representational, and organizational functions are underlined by both Carney and Levin (2002) and Elia and Philippou (2004). Accordingly, the conceptual framework of the study addresses all these highlighted similarities and differences in the functions of non-textual elements together. Despite the significance of non-textual elements in mathematics education, their function in the curriculum is still unclear to many teachers, especially pre-service teachers. Thus, the purpose of this article is to examine how prospective teachers identify the role of non-textual elements in a mathematics textbook. More clearly, it seeks to explore how pre-service elementary mathematics teachers interpret the functions of non-textual elements from the algebra learning domain in an eighth-grade mathematics textbook.

METHOD

Research Design

Qualitative studies are preferred in the research process, using unique methods to comprehensively and in detail capture the phenomena under investigation (Creswell, 2017). An exploratory case study, which is one of the qualitative research designs, deeply probes how individuals see themselves based on their experiences, perceptions, and feelings depending on the context, and the reasons behind them (Yin, 2014). Accordingly, this research study lends itself well to the use of a qualitative exploratory case study that focuses on pre-service teachers' interpretation of the non-textual elements in mathematics textbooks for their intended use.

Participants

The study was conducted with a total of thirty-one pre-service elementary mathematics teachers (twenty females and eleven males) enrolled in the third-year mathematics course "Analysis of Mathematics Textbooks" at a public university in Turkey. This course is designed to provide students with an overview of the pedagogical, structural, and organizational components of mathematics textbooks including didactic and graphic visual design features, language standards, contribution to meaningful learning, ease of use in the classroom, suitability for student-level, consistency with study objectives, etc. The participants of the study also volunteered based on the convenience sampling technique of the purposive (or purposeful) sampling method in qualitative research (Patton, 2014).

Data Collection

Data were collected through a task-based written questionnaire developed by the researchers and administered to the pre-service teachers as an individual assignment to be completed outside of the classroom. Evaluation done in this way not only gave the researchers flexibility in terms of time but also provided more systematic and comparable data from the participants. This questionnaire includes seven non-textual elements related to the learning outcomes in the algebra content area of the eighth-grade mathematics textbook (Middle School and Imam hatip Middle School 8 Textbook by Böge and Akıllı (2019)) published by the Ministry of National Education in Turkey. Pre-service teachers were required to interpret the functions of twenty-three non-textual elements from the algebra content area of this textbook. Among these elements, seven non-textual elements with various functions were selected to be appropriate for the study. Another point considered in selecting these seven non-textual elements is that they serve different outcomes of the algebra learning area in the mathematics curriculum. Accordingly, one non-textual element for the learning outcome "Perform multiplication of algebraic expressions" (Ministry of National Education [MoE], 2018), and two non-textual elements for each of the learning outcomes "Explain identities with models" (MoE, 2018), "Solve first-degree inequalities in one variable" (MoE, 2018), and "Explain the slope of a line using models, relate linear equations and their graphs to slope" (MoE, 2018) were selected for investigation (see in Appendix 1). Participants were asked to write down their thoughts about the usefulness of these non-textual elements compared to symbols and textual information, what kind of roles or functions they have in learning and teaching mathematics, and in which aspects they are more remarkable. Prior to data collection, a task-based questionnaire was also piloted with four students not involved in the actual study to assess the extent to which the questionnaire could elicit responses to address the research question. Expert opinions were also taken into consideration. Based on the feedback received, necessary amendments were made to arrive at the final version of the task-based written questionnaire. In this respect, two questions were removed from the task-based questionnaire, which initially consisted of five questions. One of them was excluded from the questionnaire because it was a general statement covering other questions. The other one was excluded because it was thought that pre-service teachers did not have sufficient expertise in textual and visual literacy about the lack of features that non-textual elements should have. Therefore, it was not evaluated within the scope of this study.

Data Analysis

Data analysis was performed using the content analysis method, which requires an in-depth analysis of the collected data and allows to uncover of previously unspecified themes and dimensions (Corbin & Strauss, 2015). All participants were coded from P1 to P31 to protect their confidentiality. Their responses to the task-based questionnaire were received in writing. Qualitative data analysis software NVivo (QSR International, 2012) was also used to assist in coding and categorizing data to identify common themes and patterns. Using this software allowed us to eliminate potential conflicts in the interpretation of data and ensure the accuracy of the research work by comparing it to the participants' responses. Taking into account both the relevant literature (Carney & Levin, 2002; Duchastel and Waller, 1979; Elia & Philippou, 2004; Kim, 2009, 2012) and the data itself, an initial list of codes was developed by assigning a code to each piece of information received from pre-service teachers. For example, the preliminary code list for pre-service teachers' interpretations of the image of the fourth problem (see Figure 4 in Appendix 1), in which the algebraic identity $a^2 - b^2 = (a-b)(a+b)$ is shown by the area models, was created as attention, clarifying, embodying, explanatory, facilitating, helping solution, informative, instructive, interpretative, logical, modeling, rational, representative, simple, supportive, understandable, and useful. After the coding phase was completed, similar codes were grouped into categories, which were further merged to form main themes. For instance, codes such as embodying, modeling, and representative were included under the theme of embodiment while codes such as facilitating, helping solutions, supportive, understandable and useful were placed under the theme of organizing. Similarly, codes such as clarifying, informative, and instructive were classified into the theme of informativeness while codes such as interpretative, logical, and rational were grouped into the theme of reasoning. After the creation of themes identifying the functions of the non-textual elements was completed, their frequencies were also calculated. Participants' responses were evaluated separately for each theme created, and the ones that best reflected everyone's views were quoted directly. Since credibility and transferability were important to ensure the validity of the research, direct quotations with the views of the participants were often included while presenting the findings, and the results were interpreted based on those views. Through member checks, respondents were also asked to clarify or disagree with something in the transcripts to identify and minimize potential bias. The aim was therefore to disclose the research results accurately and impartially so that the emerging themes could form a coherent and meaningful whole as much as possible (Merriam & Tisdell, 2016). Moreover, to facilitate inter-coder reliability (Miles & Huberman, 1994), a mathematics education researcher was asked to act as an external rater. Accordingly, in the last step of the analysis, in addition to the expert review, this mathematics education researcher and the authors double-checked the codes in all transcripts and reexamined the resulting categories. The comparison of the two codings resulted in an overall agreement of 88 percent across all categories. The raters resolved any disagreements and revised the codes until full agreement was reached for the categories to ensure the confirmability and dependability of the data collected.

Ethical Principles

Ethics committee permission for this study was obtained from Zonguldak Bülent Ecevit University Human Research Ethics Committee with the decision dated 05.10.2022 and numbered 221989/326.

FINDINGS

A cross-case analysis of the data from thirty-one pre-service teachers revealed ten themes that captured their views on the functions of seven images selected from the eighth-grade mathematics textbook in the algebra content area focusing on algebraic expressions and identities, linear equations, and inequalities: *(i) attractiveness, (ii) organizing, (iii) embodiment, (iv) informativeness, (v) reasoning, (vi) conciseness, (vii) essentiality, (viii) decorativeness, (ix) contextuality, and (x) connectivity.* Pre-service teachers' views on the functions of the non-textual elements were thematized and the frequencies of each theme are given in Table 1. The following sections provide the findings concerning these emergent themes.

Table 1

Frequencies and functions attributed by pre-service teachers to non-textual elements

	Number of Participants						
Functions	*(Non-textual Element-1)*	*(Non-textual Element-2)*	*(Non-textual Element-3)*	*(Non-textual Element-4)*	*(Non-textual Element-5)*	*(Non-textual Element-6)*	*(Non-textual Element-7)*
Attractiveness	22 (71%)	20 (65%)	21 (68%)	20 (65%)	9 (29%)	18 (58%)	8 (26%)
Organizing	9 (29%)	21 (68%)	23 (74%)	22 (71%)	14 (45%)	27 (87%)	15 (48%)
Embodiment	17 (55%)	17 (55%)	20 (65%)	22 (71%)	8 (26%)	22 (71%)	12 (39%)
Informativeness	0 (0%)	8 (26%)	10 (32%)	8 (26%)	6 (19%)	19 (61%)	7 (23%)
Reasoning	4 (13%)	1 (3%)	3 (10%)	6 (19%)	8 (26%)	8 (26%)	1 (3%)
Conciseness	0 (0%)	0 (0%)	9 (29%)	5 (16%)	2 (6%)	0 (0%)	3 (10%)
Essentiality	0 (0%)	11 (35%)	1 (3%)	0 (0%)	9 (29%)	12 (39%)	8 (26%)
Decorativeness	21 (68%)	1 (3%)	0 (0%)	0 (0%)	1 (3%)	2 (6%)	0 (0%)
Contextuality	0 (0%)	1 (3%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)
Connectivity	0 (0%)	0 (0%)	0 (0%)	0 (0%)	1 (3%)	0 (0%)	0 (0%)

Attractiveness

Most of the pre-service teachers in this study mentioned the attractiveness of non-textual elements. For example, twenty-two participants indicated that the image of the first problem about daily life (see

Figure 1 in Appendix 1), which requires writing a mathematical expression with first-degree inequalities in one variable, can draw students' attention to the problem. One of the pre-service teachers put it as follows:

… with the picture given in a text, it was aimed to draw students' attention to the problem by enabling them to visualize the problem situation in their minds (P13).

Similarly, twenty-one participants stated that the image belonging to the third problem (see Figure 3 in Appendix 1), where the multiplication of algebraic expressions is illustrated using area models, has the function of attractiveness and that it can arouse students' curiosity and positively affect the attempts to solve the problem. As one of the participants expressed:

... I think that it is a visual that will stimulate the curiosity and attention of the students and will capture their interest in solving the problem (P27).

In addition, twenty participants pointed out that the attractive function of the image in the second problem (see Figure 2 in Appendix 1), dealing with transforming the real-life problem into a mathematical statement, can increase students' motivation to solve the problem. As stated by one of the participants:

…This picture has a role in raising the motivation of students...The fact that the visual is colorful and the tools used in real life may attract students' attention (P9).

Organizing

Organizing is a function emphasized by most of the pre-service teachers in all non-textual elements. For instance, twenty-seven participants referred to the organizing function of the image in the sixth problem (see Figure 6 in Appendix 1), where the slope is explained with models from everyday life. They mentioned that this image supports textual information and makes it useful for solving the problem. One of the participants expressed this as follows:

…I think textual information about the problem is supported by this picture and it is quite useful for solving the problem. … It is a visual that helps students understand the problem and therefore reinforces the topic being covered (P28).

Twenty-two participants also considered the purpose of using the image belonging to the fourth problem (see Figure 4 in Appendix 1), in which identities are explained with area models, as organizing because the image facilitates the solution of the given problem by organizing given information logically and coherently to produce mathematical ideas. As stated by one pre-service teacher:

…this image organizes information given in the problem and facilitates its solution in a way that encourages students to form their own ideas (P10).

Similarly, twenty-one participants mentioned the organizing function of the image used for the second problem (see Figure 2 in Appendix 1), which is about expressing the real-life problem of inequalities mathematically, as it provides students with information about the problem and helps them plan its solution. As one of the participants remarked:

… equal arm weighing scales in the figure are used to inform students, support their understanding, and facilitate the planning of the solution for the problem (P15).

Embodiment

Embodiment is also a function revealed by most of the pre-service teachers in all non-textual elements. For example, twenty-two participants expressed their thoughts about the role of the image of the fourth problem (see Figure 4 in Appendix 1), in which identities are illustrated with area models. Pre-service teachers mentioned that the given image allows students to discover the algebraic identity $a^2 - b^2 = (a-b)(a+b)$ by embodying the learning process. They stated that mathematical principles that are difficult to visualize in a way that makes sense in the mind are embodied using visual representations or images. That is:

…thanks to this visual, the identity $a^2 - b^2 = (a-b)(a+b)$, which is an abstract fact for students, is modeled using areas, and thus it is proved by embodied (P24).

Likewise, twenty participants argued that the image belonging to the third problem (see Figure 3 in Appendix 1), which is about performing multiplication with algebraic expressions with area models, embodies the given problem and makes its solution understandable. As stated by one of the participants:

…the geometric shapes in the picture are visual elements used to make algebraic expressions meaningful. Explaining this subject only with algebraic expressions remains abstract for students, but the shapes used to represent the problem embody its solution for students (P13).

Also, twelve participants explained that the image of the seventh problem (see Figure 7 in Appendix 1), discovering the slopes of parallel and perpendicular lines, embodies abstract concepts in the problem that are difficult for students to understand. One of the participants put it simply:

…If only textual information is included, the problem may be difficult to understand and the problem may remain abstract. The inclusion of the image embodies the solution in the mind (P23).

Informativeness

Informativeness is another function that is noted by the pre-service teachers in all images except the image of the first problem. For example, nineteen participants mentioned the informative function of the image on the sixth problem (see Figure 6 in Appendix 1), illustrating that the slope is the ratio of vertical length to horizontal length in a model related to daily life. As pointed out by one participant:

…The three shapes with different slopes given in the image make it easier for students to understand that as the height increases, the slope will increase and as the length increases, it will decrease. For this reason, the images provided are informative to understand the subject and contribute to learning (P17).

Eight participants also supported that the image (see Figure 4 in Appendix 1), in which the algebraic identity $a^2 - b^2 = (a-b)(a+b)$ is proved by the area models, has an informative function as it makes the learning more effective and permanent by explaining the relationships between concepts. One of the pre-service teachers expressed that:

…demonstrating the identity with an area model instead of just showing it as a long mathematical expression makes learning more permanent. Even if students forget the algebraic identity, they can remember this model and prove it themselves (P5).

Similarly, seven participants emphasized the informative function of the image belonging to the seventh problem (see Figure 7 in Appendix 1), examining the relationship between parallel and perpendicular lines and the concept of slope. As one participant stressed:

…this visual is used to explain the relationship between the slopes of two lines that are parallel or perpendicular… In place of writing a long mathematical text, it is used to summarize the text and highlight the important information students need to know (P10).

Reasoning

The other function of the non-textual elements stated by the pre-service teachers is reasoning. For example, eight participants expressed that allowing the comparison of the slopes given in the figure on the sixth problem (see Figure 6 in Appendix 1) contributes to the interpretation of factors affecting the magnitude of the slope, thus revealing the reasoning function. As put by one of the participants:

…thanks to the shapes in the given figure, it can be easily interpreted in which cases the slope increases and in which cases it decreases (P12).

Similarly, six participants mentioned that the image of the fourth problem (see Figure 4 in Appendix 1), in which the algebraic identity $a^2 - b^2 = (a-b)(a+b)$ is shown by the area models, allows the interpretation of the problem situation and reasoning by thinking deeply about the problem situation. In other words, they stated that the visual can encourage students to think mathematically, as it helps to see how the given problem can be solved. In the words of a pre-service teacher:

…instead of memorizing the given identity, the students try to think and interpret the reason for it with the help of the given visual...if only textual information is included, it will be more abstract and difficult to explain and interpret the identity (P28).

Conciseness

Conciseness is another function underlined by the pre-service teachers in all non-textual elements except the first, second, and sixth non-textual elements. For instance, nine participants stated that the image presented in the third problem (see Figure 3 in Appendix 1) about performing the multiplication of algebraic expressions can simply and concisely convey the meaning of the multiplication operation on algebraic expressions by modeling the equation $x(2x+1) = 2x^2 + x$ using areas. As one of the participants noted:

…I think it is very helpful compared to textual information because what is meant to be explained in the text is modeled with a given visual more simply and understandably (P26).

In a like manner, three participants stated that the image of the seventh problem (see Figure 7 in Appendix 1), aiming to relate the concept of slope to parallel and perpendicular lines, shows the relationship between these concepts clearly. As posited by one of the participants:

…It is a useful visual as it clearly shows what is meant to be given in the problem (P11).

Essentiality

Essentiality was indicated as another function of non-textual elements in all visuals except the first and fourth ones. For example, eleven participants pointed to the essentiality function by stating that the mathematical expression required in the problem could not be written without the image given in the second problem (see Figure 2 in Appendix 1), focusing on expressing mathematical statements of real-life problems related to inequalities. As remarked by one of the participants:

…the visual is necessary for solving the problem. Otherwise, the text describing the question is useless without the image provided (P4).

Similarly, nine participants stated that the image of the fifth problem (see Figure 5 in Appendix 1), in which the algebraic identity $(a+b)^2 = a^2 + 2ab + b^2$ is shown, is essential for understanding the related problem. In other words,

…the given visual is part of the problem and is necessary for the solution of the problem (P1).

Decorativeness

Most of the pre-service teachers emphasized the decorative function of the given non-textual elements. For instance, twenty-one participants pointed to the decorativeness of the visual given with the first problem (see Figure 1 in Appendix 1), which is about writing a mathematical expression suitable for daily life situations involving first-order inequalities with one unknown. They also commented on whether the given problem can be solved without its visuals. One of the participants put it as follows:

…this visual does not contain any information about the solution to the problem, it is only used for decorative purposes. Even if the visual is not used here, this problem can be solved (P10).

Contextuality

Contextuality was highlighted by one pre-service teacher in the image of the second problem (see Figure 2 in Appendix 1), which requires transforming a real-life problem based on inequalities into mathematical expressions. It was assumed that with this image, students can establish a relationship between everyday life and mathematics. As stated by one of the pre-service teachers:

…given a weighing instrument used in everyday life, this visual helps students to relate the subject of inequality to daily life (P3).

Connectivity

Connectivity emerged as another function of non-textual elements, which was pointed out by only one participant in the image of the fifth problem (see Figure 5 in Appendix 1), where the algebraic identity $(a+b)^2 = a^2 + 2ab + b^2$ is proved. A pre-service teacher asserted that since the given image is related to the content of the problem there is a connectivity function.

…I think that the visual chosen in connection with the textual information of the problem helps students embody what is described in the text and visualize the problem in their minds (P1).

DISCUSSIONS AND CONCLUSIONS

In this study, pre-service elementary mathematics teachers emphasized various functions of seven non-textual elements that are part of algebra learning outcomes in an eighth-grade mathematics textbook. In this regard, ten different functions were identified in total: *(i) attractiveness, (ii) organizing, (iii) embodiment, (iv) informativeness, (v) reasoning, (vi) conciseness, (vii) essentiality, (viii) decorativeness, (ix) contextuality, and (x) connectivity*. It was seen that pre-service teachers attributed at least five different functions to each of the non-textual elements (see Table 1). Therefore, as stated in the literature, it has been shown that non-textual elements can have more than one function, or more than one meaning can be attributed to a non-textual element (Carney & Levin, 2002; Elia & Philippou, 2004; Kim, 2009, 2012; Lee & Ligocki, 2020).

It has often been emphasized that each non-textual element used in this study has the functions of attractiveness, organizing, and embodiment. Similarly, some studies asserted that most of the pictures used in mathematics textbooks were attractive because pictures did contribute to the attractiveness of the learning material and the enjoyment of reading (Biron, 2006; Peeck, 1993). Besides, in the studies analyzing the functions of the pictures in the problem-solving process (Elia & Philippou, 2004), the roles of pictorial illustrations in mathematics textbooks (Carney & Levin, 2002), and the way prospective teachers interpret and use non-textual elements in mathematics curriculum materials (Lee & Ligocki, 2020), visuals were found to organize the problem-solving process. In addition, visualization is essential for mathematical generalization and abstraction because visual representations can embody a concept in various ways and make it comprehensible (Demircioğlu & Polat, 2015; Dufour-Janvier, Bednarz & Belanger, 1987; Yilmaz & Argun, 2018). Therefore, it can be argued that these three functions of visuals (attractiveness, organizing, and embodiment) will have a critical role in the effective internalization of the abstract content of algebra in mathematics textbooks.

Another important finding is that reasoning, which pre-service teachers believe plays a crucial role in the implementation of all mathematical skills, is a function expressed in all non-textual elements used in the study. It has long been known that using certain types of representations (visual, concrete, etc.) allows students to develop mathematical skills such as reasoning and helps them acquire advanced problem-solving skills (Presmeg, 2020). Therefore, considering the fact that non-textual elements used in teaching mathematics facilitate students' reasoning processes (Alsina & Nelseni, 2006), it is important to include such non-textual elements in the algebra content area of mathematics textbooks.

It was also found that pre-service teachers attributed an informative function to six non-textual elements and an essentiality function to five non-textual elements in the study. Seffah (2017) indicated that in most textbooks, visual representations, which are an important tool that can contribute to a comprehensive understanding of the concept of the series, are rarely used, and they have the function of essentiality as in this study. Karakaya (2011) noted that most of the visual representations of functions used in mathematics textbooks have an informative function. Therefore, it is important for the teaching of algebraic concepts that the non-textual elements in the textbooks have informative and essentiality functions, as they initiate mathematical thinking by explaining the text and providing the necessary information for solving the problem.

Moreover, conciseness, interpreted as mathematical simplicity by pre-service teachers, was a function that emerged in the four non-textual elements used in the study. Pettersson (2001) contended that too much detail or complexity reduces interest in visual content, while too little detail or complexity makes it impossible to understand the picture. In other words, since mathematics is such a precise subject, the ambiguity of complex visual representations can prevent students' understanding of the concept (Goldin & Shteingold, 2001). Simple or concise non-textual elements facilitate understanding of the mathematical concept and making connections between the concept and other related mathematical concepts (Kim, 2012). Because concise non-textual elements can clearly and effectively convey the meaning and idea when teaching a new concept, such elements can also help students better understand algebraic concepts.

Another function expressed in four non-textual elements is decorativeness, referred to as the aesthetics or visual appeal of the images. Sinclair (2006) argued that aesthetics should be considered an important step for success in mathematics because it is closely related to students' understanding and learning styles of mathematics. Goldin (2000) emphasized that aesthetics is important not only to help students discover beauty in mathematics, but also because it affects students' affective characteristics such as emotions, beliefs, and attitudes toward mathematics. In this respect, one can better understand that the decorative function of the visuals in the textbooks is more valuable for teaching algebra.

On the other hand, contextuality, mentioned as the representation of mathematical expressions in a realistic context, was a function emphasized by the pre-service teacher only in one non-textual element. Wiggins (1993) asserted that it is inappropriate to evaluate information out of its context. Ferratti and Okolo (1996) showed that students' thinking skills and attitudes improve as they solve problems in realistic contexts. Non-textual elements with a contextual function can facilitate students' understanding and learning through connections between mathematics and real life (Kim, 2012). Therefore, it should be kept in mind that such non-textual elements in textbooks can enable students to understand and deepen algebraic concepts by encouraging them to think in their own context. In addition, connectivity, interpreted by the pre-service teacher as the non-textual elements being closely related to the mathematical content in the texts, was a function specified only in one non-textual element, as in the contextuality function. Since visual representations can serve as models or problem-solving tools to show students what they cannot see in texts and symbols (Arcavi, 2003), it is important to establish close connections between mathematical texts and visual representations in teaching algebra. Because textual literacy may not be at the same level as visual literacy for every student, students who have difficulty reading and comprehending mathematical texts can learn more and understand better through visuals (Kim, 2012). Therefore, visual representations can provide some students with opportunities to learn mathematics that they cannot understand only from algebraic textual expressions.

Overall, this study provides an overview of pre-service elementary mathematics teachers' interpretations of the functions of non-textual elements from the content area of algebra in a mathematics textbook. Pre-service teachers were found to have diverse but sometimes overlapping interpretations of the functions of each non-textual element. However, the functional diversity of non-textual elements may have challenged their interpretations, as visual processing skills and strategies are required to interpret the intended use of non-textual elements. Some pre-service teachers had difficulty seeing the connections between the information in the pictures and the mathematical texts containing algebraic expressions. Understanding the intended message of the non-textual elements in mathematics textbooks seems to be something that some prospective teachers are unlikely to do unless they have carefully planned, intentional training on the matter. Otherwise, despite increasing informational texts and visual content in textbooks, these prospective teachers will hardly be able to teach their students to read visual elements as well (Metros, 2008). Therefore, in order to enhance pre-service teachers' understanding of the functions of non-textual elements, not only should researchers develop various instructional approaches that promote pre-service teachers' visual literacy, but these instructional approaches to visual literacy should also be incorporated into teacher education and professional development. Moreover, while our study will guide future research by documenting the functions of non-textual elements encountered in an algebra content area of a mathematics textbook, it would be valuable to extend this work to other possible functions of non-textual elements by considering different learning domains and various mathematics textbooks.

REFERENCES

Alsina, C., & Nelsen, R. B. (2006). *Math made visual: Creating images for understanding mathematics*. Washington D. C.: Mathematical Association of America. https://doi.org/10.5948/UPO9781614441007

Araya, R., Farsani, D., & Hernández, J. (2016). How to attract students' visual attention. In K. Verbert, M. Sharples, T. Klobučar (Eds.), *Adaptive and adaptable learning. EC-TEL 2016. Lecture notes in computer science* (pp. 843–908). Springer, Cham. https://doi.org/10.1007/978-3-319-45153-4_3

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, *52*(3), 215–241. https://doi.org/10.1023/A:1024312321077

Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator*, *16*(2), 14–18. https://eric.ed.gov/?id=EJ449539

Bazerman, C. (2006). Analyzing the multidimensionality of texts in education. In J. L. Green, G. Camilli, & P. B. Elmore (Eds.), *Handbook of complementary methods in education research* (pp.77–94). New York, NY: Routledge. https://doi.org/10.4324/9780203874769

Biron, D. (2006). Fonctions de l'image dans les manuels scolaires de mathématiques au primaire. In M. Lebrun (Dir.) *Le manuel scolaire, un outil à multiples facettes* [The textbook, a multifaceted tool]. (pp. 191–208). Quebec: Presses Université du Québec.

Boling, E., Eccarius, M., Smith, K., & Frick, T. (2004). Instructional illustrations: Intended meanings and learner interpretations. *Journal of Visual Literacy*, *24*(2), 185–204. https://doi.org/10.1080/23796529.2004.11674612

Borwein, P., & Jörgenson, L. (2001). Visible structures in number theory. *The American Mathematical Monthly*, *108*(10), 897–910. https://doi.org/10.1080/00029890.2001.11919824

Bosse, M. J., Lynch-Davis, K., Adu-Gyamfi, K., & Chandler, K. (2016). Using integer manipulatives: Representational determinism. *International Journal for Mathematics Teaching and Learning*, *17*(3), 1–20. https://www.cimt.org.uk/ijmtl/index.php/IJMTL/article/view/37/0

Böge, H. & Akıllı, R. (2019). *Ortaokul ve imamhatip ortaokulu matematik 8 ders kitabı* [Middle school and imamhatip middle school 8th-grade mathematics textbook]. Ankara: Milli Eğitim Bakanlığı Yayınları.

Brenner, M. E., Mayer, R. E., Moseley, B., Brar, T., Durán, R., Reed, B. S., & Webb, D. (1997). Learning by understanding: The role of multiple representations in learning algebra. *American Educational Research Journal*, *34*(4), 663–689. https://doi.org/10.3102/00028312034004663

Carney, R. N., & Levin, J. R. (2002). Pictorial illustrations still improve students' learning from text. *Educational psychology review*, *14*(1), 5–26. https://doi.org/10.1023/A:1013176309260

Corbin, J. M., & Strauss, A. L. (2015). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (4th ed.) Thousand Oaks, CA: SAGE. https://doi.org/10.4135/9781452230153

Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches*. Sage Publications.

Cuoco, A. A., & Curcio, F. R. (Eds.). (2001). *The roles of representation in school mathematics*. *(NCTM 2001 Yearbook)*. National Council of Teachers of Mathematics.

Demircioğlu, H., & Polat, K. (2015). Ortaöğretim matematik öğretmen adaylarının "sözsüz ispat" yöntemine yönelik görüşleri [Secondary mathematics pre-service teachers' opinions about "proof without words"]. *The Journal of Academic Social Science Studies*, *41*, 233–254. https://web.archive.org/web/20201214055604id_/https://jasstudies.com/files/jass_makaleler/1607767651_15-Yrd.%20Do%C3%A7.%20Dr.%20Handan%20DEM%C4%B0RC%C4%B0O%C4%9ELU.pdf

Duchastel, P., & Waller, R. (1979). Pictorial illustration in instructional texts. *Educational Technology*, *19*(11), 20–25. https://www.jstor.org/stable/44421421

Dufour-Janvier, B., Bednarz, N., & Belanger, M. (1987). Pedagogical considerations concerning the problem of representation. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 109–122). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, *61*(1–2), 103–131. https://doi.org/10.1007/s10649-006-0400-z

Elia, I., & Philippou, G. (2004). The functions of pictures in problem solving. In: M.J. Hoines, & A.B. Fuglestad (Eds.), *Proceedings of the 28th conference of the international group for the psychology of mathematics education*. (Vol. 2, pp. 327–334). University College.

Farmaki, V., & Paschos, T. (2007). The interaction between intuitive and formal mathematical thinking: a case study. *International Journal of Mathematical Education in Science and Technology*, *38*(3), 353–365. https://doi.org/10.1080/00207390601035302

Ferratti, R. P., & Okolo, C. M. (1996). Authenticity in learning: multimedia design projects in the social studies for students with disabilities. *Journal of Learning Disability*, *29*(5), 450–460. https://doi.org/10.1177/002221949602900501

Filloy, E., Rojano, T., & Puig, L. (2008). *Educational algebra: A theoretical and empirical approach*. New York, NY: Springer. https://doi.org/10.1007/978-0-387-71254-3

Goldin, G. A. (2000). Affective pathways and representation in mathematical problem solving. *Mathematical Thinking and Learning*, *2*(3), 209–219. https://doi.org/10.1207/S15327833MTL0203_3

Goldin, G. A., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics*. Reston, VA: NCTM.

Herbert, K., & Brown, R. H. (1997). Patterns as tools for algebraic reasoning. *Teaching Children Mathematics*, 3(6), 340–344. https://doi.org/10.5951/TCM.3.6.0340

Herman, M. (2007). What students choose to do and have to say about use of multiple representations in college algebra. *Journal of Computers in Mathematics and Science Teaching*, *26*(1), 27–54. https://www.learntechlib.org/primary/p/21086/

Hershkowitz, R., Arcavi, A., & Bruckheimer, M. (2001). Reflections on the status and nature of visual reasoning-the case of the matches. *International Journal of Mathematical Education in Science and Technology*, *32*(2), 255–265. https://doi.org/10.1080/00207390010010917

Kamii, C., Lewis, B., & Kirkland, L. (2001). Manipulatives: When are they useful? *Journal of Mathematical Behavior*, *20*(1), 21–31. https://doi.org/10.1016/S0732-3123(01)00059-1

Kaput, J. J. (1999). Teaching and learning a new algebra. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 133–155). Mahwah, NJ: Lawrence Erlbaum Associates.

Karakaya, İ. (2011). *Dokuzuncu sınıf matematik ders kitaplarındaki fonksiyon kavramıyla ilgili görsel objelerin incelenmesi* [Investigating visual objects related to the function concept in high school mathematics textbooks] [Unpublished master's thesis]. Marmara University.

Kieran, C. (2004). Algebraic thinking in the early grades: What is it. *The mathematics educator*, *8*(1), 139–151. https://www.researchgate.net/profile/Carolyn-Kieran-2/publication/228526202_Algebraic_thinking_in_the_early_grades_What_is_it/links/55895b8408ae2affe714d428/Algebraic-thinking-in-the-early-grades-What-is-it.pdf

Kim, R. Y. (2009). *Text + book = textbook? Development of a conceptual framework for non-textual elements in middle school mathematics textbooks*. [Unpublished doctoral dissertation]. Michigan State University, USA.

Kim, R. Y. (2012). The quality of non-textual elements in mathematics textbooks: An exploratory comparison between South Korea and the United States. *ZDM—The International Journal on Mathematics Education*, *44*(2), 175–187. https://doi.org/10.1007/s11858-012-0399-9

Lee, J. E., & Ligocki, D. (2020). Analysis of pre-service teachers' interpretation and utilization of non-textual elements in mathematics curriculum materials. *Research in Mathematical Education*, *23*(4), 181–217. https://koreascience.kr/article/JAKO202009651118533.page

Levin, J. R., & Mayer, R. E. (1993). Understanding illustrations in text. In B. K. Britton, A. Woodward, & M. R. Binkley (Eds.), *Learning from textbooks: Theory and practice* (pp. 95–111). Hillsdale, NJ: Lawrence Erlbaum Associates.

Milli Eğitim Bakanlığı [Ministry of National Education]. (2018). *Matematik dersi öğretim programı (İlkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. sınıflar) [Mathematics curriculum: Elementary and middle school grades 1, 2, 3, 4, 5, 6, 7 and 8]*. Ankara: MEB.

Merriam, S. B., & Tisdell, E. J. (2016). *Qualitative research: A guide to design and Implementation* (4th ed.). San Francisco, CA: Jossey Bass.

Metros, S. E. (2008). The educator's role in preparing visually literate learners. *Theory into Practice*, *47*(2), 102–109. https://doi.org/10.1080/00405840801992264

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis* (2nd ed.). Thousand Oaks, CA: Sage.

National Council of Teachers of Mathematics (NCTM). (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: NCTM.

Pape, S. J., & Tchoshanov, M. A. (2001). The role of representation(s) in developing mathematical understanding. *Theory into Practice*, *40*(2), 118–127. https://doi.org/10.1207/s15430421tip4002_6

Patton, M. Q. (2014). *Qualitative research & evaluation methods: Integrating theory and practice.* Sage Publications.

Peeck, J. (1993). Increasing picture effects in learning from illustrated text. *Learning and Instruction*, 3(3), 227–238. https://doi.org/10.1016/0959-4752(93)90006-L

Pettersson, R. (1990). Teachers, students and visuals. *Journal of Visual Literacy*, *10*(1), 45–62. https://doi.org/10.1080/23796529.1990.11674450

Pettersson, R. (2001). Assessing image contents. In R. E. Griffin, V. S. Williams & J. Lee (Eds.), *Exploring the visual future: Art design, science, and technology* (pp. 233–246). Arizona: International Visual Literacy Association.

Presmeg, N. (2020). Visualization and learning in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education.* Cham: Springer. https://doi.org/10.1007/978-3-030-15789-0_161

Q. S. R. International (2012). *NVivo qualitative data analysis software* (Version 10) [Computer software]. Victoria, Australia: QSR International Pty Ltd.

Seffah, R. (2017). *Le concept de série dans les manuels au niveau collégial: Registres de représentation et activités cognitives* [The concept of series in college textbooks: Registers of representation and cognitive activities]. [Unpublished masters' thesis] Université de Montréal.

Sinclair, N. (2006). *Mathematics and beauty: Aesthetic approaches to teaching children*. New York: Teachers College Press.

Stein, M. K., Remillard, J., & Smith, M. (2007). How curriculum influences student learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 319–369). Greenwich, CT: Information Age.

Stylianou, D. A., & Silver, E. A. (2004). The role of visual representations in advanced mathematical problem solving: An examination of expert-novice similarities and differences. *Mathematical Thinking and Learning*, *6*(4), 353–387. https://doi.org/10.1207/s15327833mtl0604_1

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*(2), 151–169. https://doi.org/10.1007/BF00305619

Van de Walle, J., Karp, K., & Bay-Williams, J. M. (2019). *Elementary and middle school mathematics: Teaching developmentally* (10th edition). New York, NY: Pearson Education, Inc.

Watkins, J. K., Miller, E., & Brubaker, D. (2004). The role of the visual image: What are students really learning from pictorial representations? *Journal of Visual Literacy*, *24*(1), 23–40. https://doi.org/10.1080/23796529.2004.11674601

Wiggins, G. (1993). Assessment: Authenticity, context, and validity. *Phi Delta Kappan*, *75*(3), 200–214.

Woodward, A. (1993). Do illustrations serve an instructional purpose in US textbooks? In B. K. Britton, A. Woodward & M. Binkley (Eds.), *Learning from textbooks: Theory and practice* (pp. 115–134). Hillsdale, NJ: Lawrence Erlbaum Associates.

Yilmaz, R., & Argun, Z. (2018). Role of visualization in mathematical abstraction: The case of congruence concept. *International Journal of Education in Mathematics, Science and Technology*, *6*(1), 41–57.

Yin, R. K. (2014). *Case study research: Design and methods* (5th ed.). Thousand Oaks, CA: Sage Publications.

APPENDIX

Appendix 1. Non-textual elements given in the task-based written questionnaire

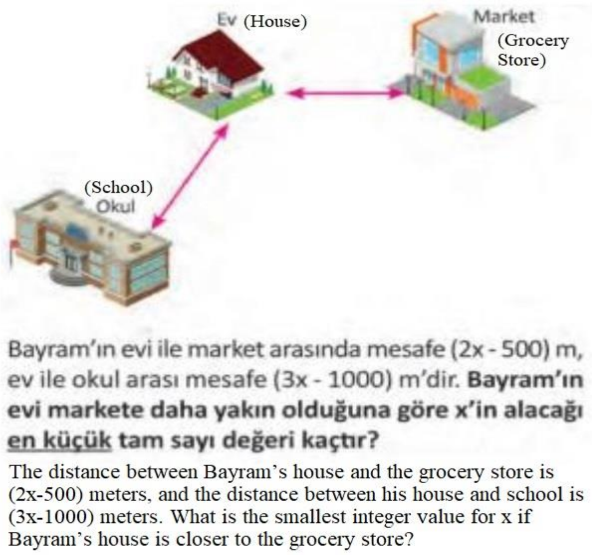
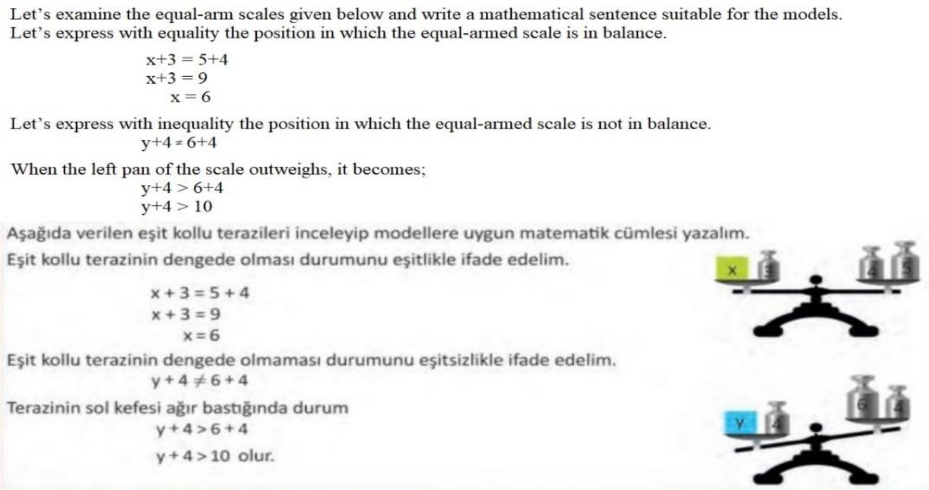


Figure 1. Non-textual element of the learning outcome "*Solve first-degree inequalities in one variable*" (MoE, 2018). Source: Middle school and imamhatip middle school 8th-grade mathematics textbook (Böge & Akıllı, 2019, p. 143) [Ortaokul ve imamhatip ortaokulu matematik 8 ders kitabı (Böge & Akıllı, 2019, ss. 143)].


Let's examine the equal-arm scales given below and write a mathematical sentence suitable for the models.
Let's express with equality the position in which the equal-armed scale is in balance.

$x+3 = 5+4$
$x+3 = 9$
$x = 6$

Let's express with inequality the position in which the equal-armed scale is not in balance.
$y+4 \neq 6+4$

When the left pan of the scale outweighs, it becomes;
$y+4 > 6+4$
$y+4 > 10$

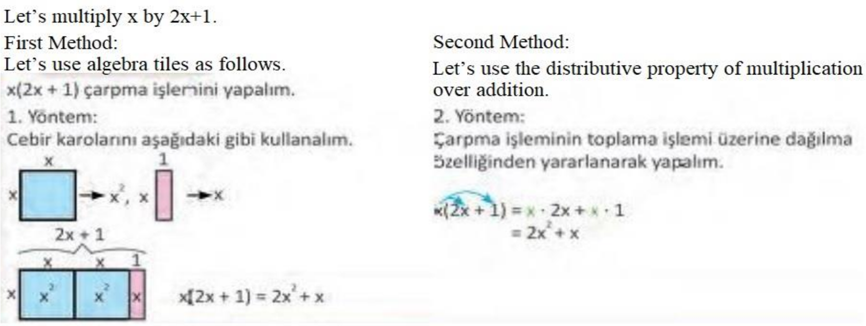
Aşağıda verilen eşit kollu terazileri inceleyip modellere uygun matematik cümlesi yazalım.
Eşit kollu terazinin dengede olması durumunu eşitlikle ifade edelim.

$x + 3 = 5 + 4$
$x + 3 = 9$
$x = 6$

Eşit kollu terazinin dengede olmaması durumunu eşitsizlikle ifade edelim.
$y + 4 \neq 6 + 4$

Terazinin sol kefesi ağır bastığında durum
$y + 4 > 6 + 4$
$y + 4 > 10$ olur.

Figure 2. Non-textual element of the learning outcome "*Solve first-degree inequalities in one variable*" (MoE, 2018). Source: Middle school and imamhatip middle school 8th-grade mathematics textbook (Böge & Akıllı, 2019, p. 134) [Ortaokul ve imamhatip ortaokulu matematik 8 ders kitabı (Böge & Akıllı, 2019, ss. 134)].


Let's multiply x by 2x+1.

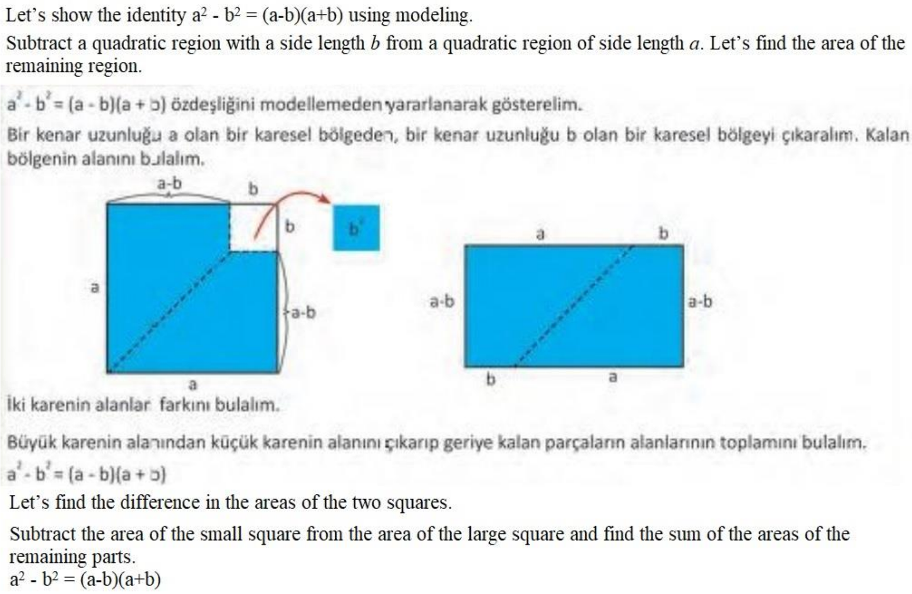
First Method:
Let's use algebra tiles as follows.
$x(2x + 1)$ çarpma işlemini yapalım.
1. Yöntem:
Cebir karolarını aşağıdaki gibi kullanalım.

$x(2x + 1) = 2x^2 + x$

Second Method:
Let's use the distributive property of multiplication over addition.
2. Yöntem:
Çarpma işleminin toplama işlemi üzerine dağılma özelliğinden yararlanarak yapalım.

$x(2x + 1) = x \cdot 2x + x \cdot 1$
$= 2x^2 + x$

Figure 3. An example of the learning outcome "*Perform multiplication of algebraic expressions*" (MoE, 2018). Source: Middle school and imamhatip middle school 8th-grade mathematics textbook (Böge & Akıllı, 2019, p. 92) [Ortaokul ve imamhatip ortaokulu matematik 8 ders kitabı (Böge & Akıllı, 2019, ss. 92)].


Let's show the identity $a^2 - b^2 = (a-b)(a+b)$ using modeling.
Subtract a quadratic region with a side length *b* from a quadratic region of side length *a*. Let's find the area of the remaining region.

$a^2 - b^2 = (a - b)(a + b)$ özdeşliğini modellemeden yararlanarak gösterelim.
Bir kenar uzunluğu a olan bir karesel bölgeden, bir kenar uzunluğu b olan bir karesel bölgeyi çıkaralım. Kalan bölgenin alanını bulalım.

İki karenin alanları farkını bulalım.
Büyük karenin alanından küçük karenin alanını çıkarıp geriye kalan parçaların alanlarının toplamını bulalım.
$a^2 - b^2 = (a - b)(a + b)$

Let's find the difference in the areas of the two squares.
Subtract the area of the small square from the area of the large square and find the sum of the areas of the remaining parts.
$a^2 - b^2 = (a-b)(a+b)$

Figure 4. Non-textual element of the learning outcome "*Explain identities with models*" (MoE, 2018). Source: Middle school and imamhatip middle school 8th-grade mathematics textbook (Böge & Akıllı, 2019, p. 99) [Ortaokul ve imamhatip ortaokulu matematik 8 ders kitabı (Böge & Akıllı, 2019, ss. 99)].

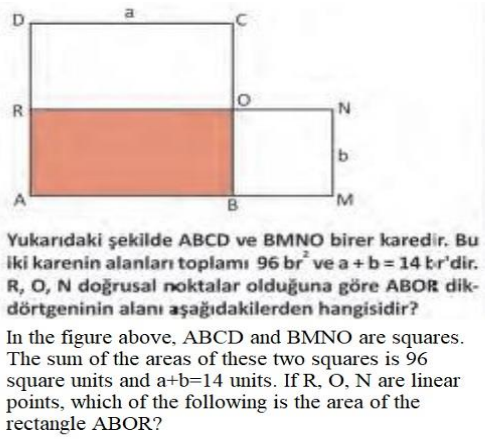
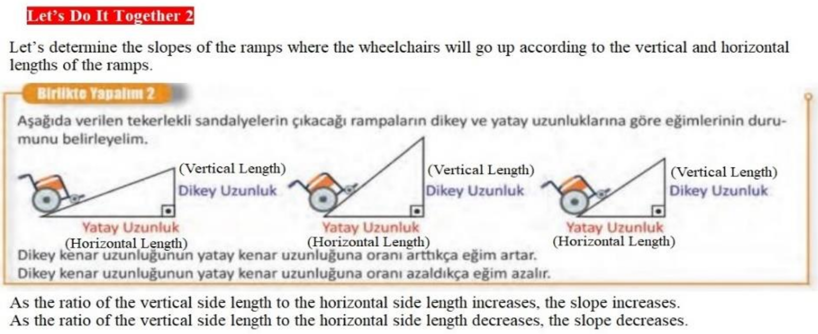


Figure 5. Non-textual element of the learning outcome "*Explain identities with models*" (MoE, 2018). Source: Middle school and imamhatip middle school 8th-grade mathematics textbook (Böge & Akıllı, 2019, p. 108) [Ortaokul ve imamhatip ortaokulu matematik 8 ders kitabı (Böge & Akıllı, 2019, ss. 108)].



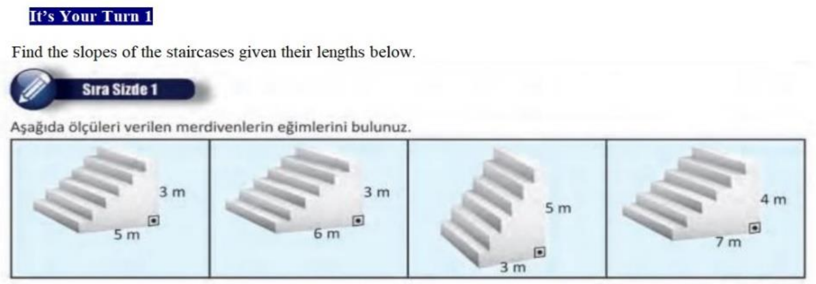


Figure 6. Non-textual element of the learning outcome "*Explain the slope of a line using models, relate linear equations and their graphs to slope*" (MoE, 2018). Source: Middle school and imamhatip middle school 8th-grade mathematics textbook (Böge & Akıllı, 2019, p. 131) [Ortaokul ve imamhatip ortaokulu matematik 8 ders kitabı (Böge & Akıllı, 2019, ss. 131)].

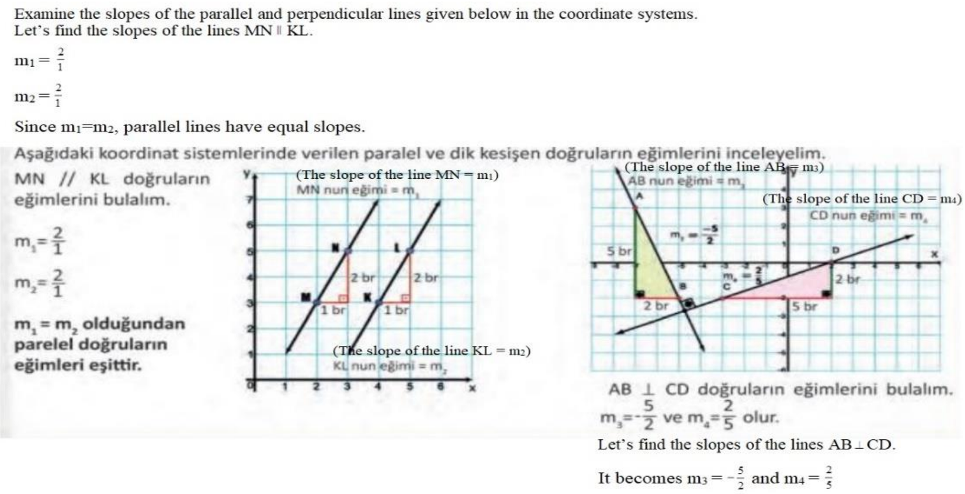


Figure 7. Non-textual element of the learning outcome "*Explain the slope of a line using models, relate linear equations and their graphs to slope*" (MoE, 2018). Source: Middle school and imamhatip middle school 8th-grade mathematics textbook (Böge & Akıllı, 2019, p. 133) [Ortaokul ve imamhatip ortaokulu matematik 8 ders kitabı (Böge & Akıllı, 2019, ss. 133)].

Author Contributions

The authors contributed equally for writing the article, conceptualization of the article, data collection, analysis and discussion.

Conflict of Interest

No potential conflict of interest was declared by the author.

Supporting Individuals or Organizations

No grants were received from any public, private or non-profit organizations for this research.

Ethical Approval and Participant Consent

Ethics committee permission for this study was obtained from Zonguldak Bülent Ecevit University Human Research Ethics Committee with the decision dated 05.10.2022 and numbered 221989/326.

Plagiarism Statement

Similarity rates of this article was scanned by the iThenticate software. No plagiarism detected.

Availability of Data and Materials

Not applicable.

Acknowledgements

This manuscript was presented at the 14th National Congress of Science and Mathematics Education (Online) at Burdur Mehmet Akif Ersoy University, May 19-21, 2021.