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NEW DECOMPOSITIONS OF SOFT CONTINUITY IN SOFT TOPOLOGICAL SPACES

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Abstract. The aim of this paper is to produce new decompositions of soft continuity. For this reason, we first have introduced two new sets concepts named as soft AC-set with soft BC-set in soft topological spaces. Compairing with this two soft sets concepts and others soft set kinds, we have obtained two decompositions of soft open sets, supported by counterexamples. Utilizing soft AC-set and soft BC-set concepts, we have introduced two functions concepts named as soft AC-continuous with soft BC-continuous functions and we have obtained a new decomposition of soft continuity.

1. Introduction

The concept of soft sets was initiated by Molodtsov [11] as a new mathematical tool for dealing with uncertainty. In fact, a soft set is a parameterized family of subsets of a given universe set. The way of parameterization in problem solving makes soft set theory convenient and simple for application. Later Maji et al. [9] presented several operations in soft set theory. Shabir and Naz [12] introduced the soft topological spaces which are defined over an initial universe with a fixed set of parameters. They say that a soft topological space gives a parametrized family of topologies in the first universe, but the reverse is not true, i.e. if we are given some topology for each parameter, we cannot construct a soft topological space. As a result, we can say that soft topological spaces are more comprehensive and generalized than classical topological spaces. Many researchers [3, 6, 10, 17] studied some of basic concepts and properties of soft topological spaces. Recently, weak and strong forms of soft open sets were studied by many authors [1, 2, 4, 8, 13, 14, 15, 16]. The aim of this paper is to produce new decompositions of soft continuity. For this reason, we first give the concepts of soft AC-sets and soft BC-sets. We study the relationships between different types of soft sets in soft topological spaces. Later, we define soft AC-continuous and soft BC-continuous functions.

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2. Preliminaries

In this section, we present the basic definitions and results of soft set theory which may be found in earlier studies.

Definition 2.1. [11] Let X be an initial universe set and E be the set of all possible parameters with respect to X. Let $P(X)$ denote the power set of X. A pair (F, A) is called a soft set over X where $A \subseteq E$ and $F : A \to P(X)$ is a set valued mapping.

The set of all soft sets over X is denoted by $SS(X)_E$.

Definition 2.2. [9] A soft set (F, A) over X is said to be a null soft set denoted by Φ if for all $e \in A$, $F(e) = \emptyset$. A soft set (F, A) over X is said to be an absolute soft set denoted by $\widetilde{A}$ if for all $e \in A$, $F(e) = X$.

Definition 2.3. [12] Let Y be a nonempty subset of X, then $\widetilde{Y}$ denotes the soft set (Y, E) over X for which $Y(e) = Y$, for all $e \in E$. In particular, (X, E) will be denoted by $\widetilde{X}$.

Definition 2.4. [9] For two soft sets (F, A) and (G, B) over X, we say that (F, A) is a soft subset of (G, B) if $A \subseteq B$ and $F(e) \subseteq G(e)$ for all $e \in A$. We write $(F, A) \sqsubseteq (G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(G, B) \sqsubseteq (F, A)$. Then (F, A) and (G, B) are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.5. [9] The union of two soft sets (F, A) and (G, B) over X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$, $H(e) = F(e)$ if $e \in A \backslash B$, $H(e) = G(e)$ if $e \in B \backslash A$, $H(e) = F(e) \cup G(e)$ if $e \in A \cap B$. We write $(F, A) \sqcup (G, B) = (H, C)$.

Definition 2.6. [5] The intersection (H, C) of two soft sets (F, A) and (G, B) over X, denoted $(F, A) \sqcap (G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.7. [12] The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E) \backslash (G, E)$, is defined as $H(e) = F(e) \backslash G(e)$ for all $e \in E$.

Definition 2.8. [12] The relative complement of a soft set (F, E) is denoted by $(F, E)^c$ and is defined by $(F, E)^c = (F^c, E)$ where $F^c : E \longrightarrow P(X)$ is a mapping given by $F^c(e) = X \backslash F(e)$ for all $e \in E$.

Definition 2.9. [12] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if

(1) Φ, $\widetilde{X} \in \tau$
(2) If (F, E), $(G, E) \in \tau$, then $(F, E) \sqcap (G, E) \in \tau$
(3) If $\{(F_i, E)\}_{i \in I} \in \tau$, $\forall i \in I$, then $\sqcup_{i \in I}(F_i, E) \in \tau$.

The triplet (X, τ, E) is called a soft topological space over X. Every member of τ is called a soft open set in X. A soft set (F, E) over X is called a soft closed set in X if its relative complement $(F, E)^c$ belongs to τ. We will denote the family of all soft open sets (resp., soft closed sets) of a soft topological space (X, τ, E) by SOS(X, τ, E) (resp., SCS(X, τ, E)).

Definition 2.10. Let (X, τ, E) be a soft topological space and (F, E) be a soft set over X.

(1) [12] The soft closure of (F, E) is the soft set $cl(F, E) = \sqcap\{(G, E) :\ (G, E)$ is soft closed and $(F, E) \sqsubseteq (G, E)\}$.

(2) [17] The soft interior of (F, E) is the soft set $int(F, E) = \sqcup\{(H, E) : (H, E)$ is soft open and $(H, E) \sqsubseteq (F, E)\}$.

Clearly, $cl(F, E)$ is the smallest soft closed set over X which contains (F, E) and $int(F, E)$ is the largest soft open set over X which is contained in (F, E).

Throughout the paper, the spaces X and Y (or (X, τ, E) and (Y, ν, K)) stand for soft topological spaces assumed unless stated otherwise.

Definition 2.11. *Let (X, τ, E) be a soft topological space.* A soft set (F, E) is called

(1) soft semi-open [4] in X if $(F, E) \sqsubseteq cl(int(F, E))$.
(2) soft α-open [1] in X if $(F, E) \sqsubseteq int(cl(int(F, E)))$.
(3) soft b-open [2] in X if $(F, E) \sqsubseteq int(cl(F, E)) \sqcup cl(int(F, E))$.

The relative complement of a soft semi-open (resp., soft α-open, soft b-open) set is called a soft semi-closed (resp., soft α-closed, soft b-closed) set.

Definition 2.12. *Let (X, τ, E) be a soft topological space. A soft set (F, E) is called*

(1) a soft regular open *(soft regular closed)* set [16] in X if $(F, E) = int(cl(F, E))$ $((F, E) = cl(int(F, E)))$.

(2) a soft A-set [14] in X if $(F, E) = (G, E)\backslash(H, E)$*,* where (G, E) is a soft open set and (H, E) is a soft regular open set in X.

(3) a soft t-set [14] in X if $int(cl(F, E)) = int(F, E)$.

(4) a soft B-set [14] in X if $(F, E) = (G, E) \sqcap (H, E)$, where (G, E) is a soft open set and (H, E) is a soft t-set in X.

(5) a soft α^*-set [13] in X if $int(cl(int(F, E))) = int(F, E)$.

(6) a soft C-set [13] in X if $(F, E) = (G, E) \sqcap (H, E)$, where (G, E) is a soft open set and (H, E) is a soft α^*-set in X.

(7) a soft semi-regular set [15] in X, if it is both soft semi-open and soft semi-closed.

(8) a soft AB-set [15] in X, if $(F, E) = (G, E) \sqcap (H, E)$, where (G, E) is a soft open set and (H, E) is a soft semi-regular set in X.

Definition 2.13. A soft set (F, E) is called a soft b-clopen set in a soft topological space X, if it is both soft b-open and soft b-closed.

Remark 2.14. In a soft topological space (X, τ, E);

(1) every soft open set is soft α-open [1],
(2) every soft regular open (closed) set is soft open (closed) [16],
(3) every soft open set is a soft A-set [14],
(4) every soft A-set is soft semi-open [14],
(5) every soft open set is a soft B-set [14],
(6) every soft A-set is a soft B-set [14].

Remark 2.15. [15] Since every soft regular closed set is soft semi-regular and since every soft semi-regular set is soft semi closed, then the following implications are obvious.

$$\text{soft A-set} \implies \text{soft AB-set} \implies \text{soft B-set}$$

Remark 2.16. [13, 14] Since every soft closed set is a soft t-set and since every soft t-set is a soft α^*-set, then the following implications are obvious.

$$\text{soft A-set} \implies \text{soft B-set} \implies \text{soft C-set}$$

Definition 2.17. [7] Let $SS(X)_E$ and $SS(Y)_K$ be families of soft sets, $u : X \longrightarrow Y$ and $p : E \longrightarrow K$ be mappings. Then the mapping $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ is defined as:

(1) Let $(F, E) \in SS(X)_E$. The image of (F, E) under f_{pu}, written as $f_{pu}(F, E) = (f_{pu}(F), p(E))$, is a soft set in $SS(Y)_K$ such that

$$f_{pu}(F)(y) = \begin{cases} \cup_{x \in p^{-1}(y) \cap A} u(F(x)) & , p^{-1}(y) \cap A \neq \emptyset \\ \emptyset & , otherwise \end{cases}$$

for all $y \in K$.

(2) Let $(G, K) \in SS(Y)_K$. The inverse image of (G, K) under f_{pu}, written as $f_{pu}^{-1}(G, K) = (f_{pu}^{-1}(G), p^{-1}(K))$, is a soft set in $SS(X)_E$ such that

$$f_{pu}^{-1}(G)(x) = \begin{cases} u^{-1}(G(p(x))) & , p(x) \in K \\ \emptyset & , otherwise \end{cases}$$

for all $x \in E$.

Definition 2.18. [17] Let (X, τ, E) and (Y, υ, K) be soft topological spaces and $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then f_{pu} is called a soft continuous function if for each $(G, K) \in \upsilon$ we have $f_{pu}^{-1}(G, K) \in \tau$.

Definition 2.19. Let (X, τ, E) and (Y, υ, K) be soft topological spaces and $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then f_{pu} is called

(1) a soft semi-continuous function [8] if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft semi-open set in X.

(2) a soft α-continuous function [1] if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft α-open set in X.

(3) a soft A-continuous function [14] if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft A-set in X.

(4) a soft B-continuous function [14] if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft B-set in X.

(5) a soft C-continuous function [13] if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft C-set in X.

(6) a soft AB-continuous function [15] if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft AB-set in X.

Remark 2.20. Let (X, τ, E) and (Y, υ, K) be soft topological spaces and $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then,

(1) every soft continuous function is soft α-continuous [1].
(2) every soft continuous function is soft A-continuous [14].
(3) every soft A-continuous function is soft semi-continuous [14].
(4) every soft continuous function is soft B-continuous [14].
(5) every soft A-continuous function is soft B-continuous [14].
(6) every soft B-continuous function is soft C-continuous [13].

Theorem 2.21. [15] *Let* (X, τ, E) *and* (Y, υ, K) *be soft topological spaces and* $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ *be a function. Then,*

(1) every soft A-continuous function is soft AB-continuous,
(2) every soft AB-continuous function is soft B-continuous.

3. Soft AC-sets and Soft BC-sets

Definition 3.1. A soft set (F, E) is called a soft BC-set in a soft topological space X if $(F, E) = (G, E) \sqcap (H, E)$ where (G, E) is soft open and (H, E) is soft b-closed.

Definition 3.2. A soft set (F, E) is called a soft AC-set in a soft topological space X if $(F, E) = (G, E) \sqcap (H, E)$ where (G, E) is soft open and (H, E) is soft b-clopen.

The family of all soft BC-sets (soft AC-sets) of a soft topological space (X, τ, E) is denoted by $SBCS(X)$ $(SACS(X))$.

Remark 3.3. The following implications hold and none of these implications is reversible as shown by examples given below.

$$\begin{array}{ccccccc} \text{soft } A\text{-set} & \Longrightarrow & \text{soft } AB\text{-set} & \Longrightarrow & \text{soft } AC\text{-set} & & \\ & & \Downarrow & & \Downarrow & & \\ & & \text{soft } B\text{-set} & \Longrightarrow & \text{soft } BC\text{-set} & \Longrightarrow & \text{soft } C\text{-set} \end{array}$$

Example 3.4. [15] Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where $\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)$ are soft sets over X, defined as follows:

$$\begin{aligned} (F_1, E) &= \{(e_1, \{x_2\}),\ (e_2, \{x_2\})\}, \\ (F_2, E) &= \{(e_1, \{x_3\}),\ (e_2, \{x_3\})\}, \\ (F_3, E) &= \{(e_1, \{x_2, x_3\}),\ (e_2, \{x_2, x_3\})\}. \end{aligned}$$

Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X .Then, $(H, E) = \{(e_1, \{x_2\}),\ (e_2, \{x_1, x_2\})\}$ is a soft AB-set in X but not a soft A-set. Also $(G, E) = \{(e_1, \{x_1\}),\ (e_2, \{x_1\})\}$ is a soft B-set in X but not a soft AB-set.

Example 3.5. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where $\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)$ are soft sets over X, defined as follows:

$$\begin{aligned} (F_1, E) &= \{(e_1, \{x_2\}),\ (e_2, \{x_2\})\}, \\ (F_2, E) &= \{(e_1, \{x_1, x_2\}),\ (e_2, \{x_1, x_2\})\}, \\ (F_3, E) &= \{(e_1, \{x_2, x_3\}),\ (e_2, \{x_2, x_3\})\}. \end{aligned}$$

Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X. Clearly, $(G, E) = \{(e_1, \{x_2, x_3\}),\ (e_2, \{x_1, x_3\})\}$ is a soft AC-set in X but not a soft AB-set.

Example 3.6. Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$.Let us take the soft topology τ on X and the soft set $(G, E) = \{(e_1, \{x_1\}),\ (e_2, \{x_1\})\}$ in Example 3.4. Then, (G, E) is a soft BC-set in X but not a soft AC-set.

Example 3.7. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where $\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)$ are soft sets over X, defined as follows:

$$
\begin{aligned}
(F_1, E) &= \{(e_1, \{x_1\}),\ (e_2, \{x_1\})\},\\
(F_2, E) &= \{(e_1, \{x_2\}),\ (e_2, \{x_2\})\},\\
(F_3, E) &= \{(e_1, \{x_1, x_2\}),\ (e_2, \{x_1, x_2\})\}.
\end{aligned}
$$

Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X. Clearly, $(G, E) = \{(e_1, \{x_3\}),\ (e_2, \{x_1, x_3\})\}$ is a soft BC-set in X but not a soft B-set.

Example 3.8. Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where $\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)$ are soft sets over X, defined as follows:

$$
\begin{aligned}
(F_1, E) &= \{(e_1, \{x_4\}),\ (e_2, \{x_1, x_2\})\},\\
(F_2, E) &= \{(e_1, \{x_1, x_2\}),\ (e_2, \{x_4\})\},\\
(F_3, E) &= \{(e_1, \{x_1, x_2, x_4\}),\ (e_2, \{x_1, x_2, x_4\})\}.
\end{aligned}
$$

Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X. Clearly, $(G, E) = \{(e_1, \{x_2, x_3, x_4\}),\ (e_2, \{x_1, x_2, x_4\})\}$ is a soft C-set in X but not a soft BC-set.

Theorem 3.9. *In a soft topological space* (X, τ, E)*,*

(1) Every soft open set is a soft AC-set.
(2) Every soft b-clopen set is a soft AC-set.

Proof. (1) Let (F, E) be a soft open set in X. Then $(F, E) = (F, E) \sqcap \widetilde{X}$ such that $\widetilde{X}$ is soft b-clopen and hence it is a soft AC-set.
(2) It is shown in a similar way. □

Example 3.10. Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology τ on X and the soft set $(G, E) = \{(e_1, \{x_2, x_3\}),\ (e_2, \{x_1, x_3\})\}$ in Example 3.5. Then, (G, E) is a soft AC-set in X but not soft open.

Example 3.11. Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E), (F_{10}, E), (F_{11}, E)\}$ where $\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E), (F_{10}, E), (F_{11}, E)$ are soft sets over X, defined as follows:

$$
\begin{aligned}
(F_1,E) &= \{(e_1,\{x_1\}),\ (e_2,\{x_1\})\},\\
(F_2,E) &= \{(e_1,\{x_2\}),\ (e_2,\{x_2\})\},\\
(F_3,E) &= \{(e_1,\{x_1,x_2\}),\ (e_2,\{x_1,x_2\})\},\\
(F_4,E) &= \{(e_1,\{x_1,x_2,x_3\}),\ (e_2,\{x_1,x_3\})\},\\
(F_5,E) &= \{(e_1,\{x_1,x_2,x_4\}),\ (e_2,\{x_1,x_2,x_3\})\},\\
(F_6,E) &= \{(e_1,\{x_2\}),\ (e_2,\emptyset)\},\\
(F_7,E) &= \{(e_1,\{x_1,x_2\}),\ (e_2,\{x_1\})\},\\
(F_8,E) &= \{(e_1,\{x_1,x_2,x_3\}),\ (e_2,\{x_1,x_2,x_3\})\},\\
(F_9,E) &= \{(e_1,X),\ (e_2,\{x_1,x_2,x_3\})\},\\
(F_{10},E) &= \{(e_1,\{x_1,x_2\}),\ (e_2,\{x_1,x_2,x_3\})\},\\
(F_{11},E) &= \{(e_1,\{x_1,x_2\}),\ (e_2,\{x_1,x_3\})\}.
\end{aligned}
$$

Then τ defines a soft topology on X and thus (X,τ,E) is a soft topological space over X [16]. Then, $(G,E) = \{(e_1,\{x_2,x_3\}),\ (e_2,\{x_3\})\}$ is a soft AC-set in X but not a soft b-clopen set.

Theorem 3.12. [2] *Let (X,τ,E) be a soft topological space. If (F,E) is soft open and (G,E) is soft b-open then $(F,E)\sqcap(G,E)$ is a soft b-open set in X.*

Theorem 3.13. *In a soft topological space (X,τ,E), every soft AC-set is a soft b-open set.*

Proof. Let (F,E) be a soft AC-set in X. Then $(F,E) = (G,E)\sqcap(K,E)$ where (G,E) is soft open and (K,E) is soft b-clopen. □

Theorem 3.14. *In a soft topological space (X,τ,E),*

(1) Every soft open set is a soft BC-set.
(2) Every soft b-closed set is a soft BC-set.

Proof. The proofs are similar with Theorem 3.9. □

Example 3.15. Let $X = \{x_1,x_2,x_3\}$ and $E = \{e_1,e_2\}$. Let us take the soft topology τ on X and the soft set $(G,E) = \{(e_1,\{x_1\}),\ (e_2,\{x_1\})\}$ in Example 3.6. Then, (G,E) is a soft BC-set in X but not soft open.

Example 3.16. Let $X = \{x_1,x_2,x_3,x_4\}$ and $E = \{e_1,e_2\}$. Let us take the soft topology τ on X and the soft set $(G,E) = \{(e_1,\{x_2,x_3\}),\ (e_2,\{x_3\})\}$ in Example 3.11. Then, (G,E) is a soft BC-set in X but not a soft b-closed set.

Definition 3.17. [2] Let (X,τ,E) be a soft topological space and (F,E) be a soft set over X. Then soft b-closure of (F,E) ($sbcl(F,E)$) is the soft set $sbcl(F,E) = \sqcap\{(F,E)\sqsubseteq(G,E) : (G,E)$ is a soft b-closed set of $X\}$.

Theorem 3.18. *Let (X,τ,E) be a soft topological space. A soft set (F,E) over X is a soft BC-set if and only if $(F,E) = (G,E)\sqcap sbcl(F,E)$ for some soft open set (G,E).*

Proof. $\Longrightarrow$: Let (F,E) be a soft BC-set. We get $(F,E) = (G,E)\sqcap(H,E)$ where (G,E) is soft open and (H,E) is soft b-closed. Since $(F,E)\sqsubseteq(H,E)$, $sbcl(F,E)\sqsubseteq$

$sbcl(H,E) = (H,E)$. Thus $(G,E) \sqcap sbcl(F,E) \sqsubseteq (G,E) \sqcap (H,E) = (F,E) \sqsubseteq (G,E) \sqcap sbcl(F,E)$ and hence $(F,E) = (G,E) \sqcap sbcl(F,E)$.

$\Longleftarrow$: Suppose that $(F,E) = (G,E) \sqcap sbcl(F,E)$ for some soft open set (G,E). Since $sbcl(F,E)$ is soft b-closed, (F,E) is a soft BC-set in X. □

Theorem 3.19. *Let (X,τ,E) be a soft topological space. For a soft set (F,E) over X, the following are equivalent:*

(1) (F,E) is soft open in X,
(2) (F,E) is soft α-open and a soft AC-set in X,
(3) (F,E) is soft α-open and a soft BC-set in X,
(4) (F,E) is soft α-open and a soft C-set in X.

Proof. (1) $\Longrightarrow$ (2) : Since every soft open set is both soft α-open and a soft AC-set, the proof is completed.

(2) $\Longrightarrow$ (3) : The proof is obvious, since every soft AC-set is a soft BC-set.
(3) $\Longrightarrow$ (4) : Since every soft BC-set is a soft C-set, it is clear.
(4) $\Longrightarrow$ (1) : Follows from Proposition 3.8 of [13]. □

Theorem 3.20. *Let (X,τ,E) be a soft topological space and (F,E) be a soft set over X. If (F,E) is a soft BC-set, then*

(1) $sbcl(F,E) \setminus (F,E)$ is soft b-closed.
(2) $(F,E) \sqcup (sbcl(F,E))^c$ is soft b-open.

Proof. (1) Let (F,E) be a soft BC-set. From Theorem 3.18, $(F,E) = (G,E) \sqcap sbcl(F,E)$ for some soft open set (G,E). Thus $sbcl(F,E) \setminus (F,E) = sbcl(F,E) \setminus ((G,E) \sqcap sbcl(F,E)) = sbcl(F,E) \sqcap ((G,E) \sqcap sbcl(F,E))^c = sbcl(F,E) \sqcap ((G,E)^c \sqcup (sbcl(F,E))^c) = (sbcl(F,E) \sqcap (G,E)^c) \sqcup (sbcl(F,E) \sqcap (sbcl(F,E))^c)$
$= (sbcl(F,E) \sqcap (G,E)^c) \sqcup \Phi = sbcl(F,E) \sqcap (G,E)^c$. Hence we obtain $sbcl(F,E) \setminus (F,E)$ is soft b-closed.

(2) Since $sbcl(F,E) \setminus (F,E)$ is soft b-closed, $(sbcl(F,E) \setminus (F,E))^c$ is soft b-open. Thus $(sbcl(F,E) \setminus (F,E))^c = (sbcl(F,E) \sqcap (F,E)^c)^c = (sbcl(F,E))^c \sqcup (F,E)$. Hence we get $(F,E) \sqcup (sbcl(F,E))^c$ is soft b-open. □

4. Decompositions of Soft Continuity

Definition 4.1. Let (X,τ,E) and (Y,υ,K) be soft topological spaces. Let $u : X \longrightarrow Y$ and $p : E \longrightarrow K$ be mappings. Let $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then the function f_{pu}

(1) is called a soft AC-continuous if for each $(G,K) \in SOS(Y)$, $f_{pu}^{-1}(G,K)$ is a soft AC-set in X.

(2) is called a soft BC-continuous if for each $(G,K) \in SOS(Y)$, $f_{pu}^{-1}(G,K)$ is a soft BC-set in X.

Remark 4.2. We have the following diagram for f_{pu}:

soft A-continuous	$\Longrightarrow$	soft AB-continuous	$\Longrightarrow$	soft AC-continuous		
		$\Downarrow$		$\Downarrow$		
		soft B-continuous	$\Longrightarrow$	soft BC-continuous	$\Longrightarrow$	soft C-continuous

Theorem 4.3. *Let (X,τ,E) and (Y,υ,K) be soft topological spaces and $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then the following are equivalent:*

(1) f_{pu} is soft continuous,
(2) f_{pu} is soft α-continuous and soft AC-continuous,
(3) f_{pu} is soft α-continuous and soft BC-continuous,
(4) f_{pu} is soft α-continuous and soft C-continuous.

Proof. The proof is an immediate consequence of Theorem 3.19. □

5. Conclusion

In this study, we have presented soft AC-sets and soft BC-sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. We have introduced their some basic properties with the help of counterexamples. Also, we have studied soft AC-continuos and soft BC-continuous functions and we have obtained the new decompositions of soft continuity. We hope that results in this paper will be heplfull for future studies in soft topological spaces.

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