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IMPROVEMENT OF WOLF LEADER IN THE GREY WOLF OPTIMIZATION

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Highlights

- Developing optimization algorithms have advantages such as increasing performance, revenue, and efficiency in various fields, as well as reducing costs.
- In this study, the alpha wolf class, which is also called the wolf leader class in Gray Wolf Optimization (GWO), was improved with Whale Optimization Algorithm (WOA).
- This developed algorithm has been tested with 23 benchmark test functions and 10 CEC2019 test functions.
- After executing the suggested method 30 times, average fitness and standard deviation values were obtained and these findings were evaluated against the literature.
- It has been found that the proposed ILGWO algorithm is promising according to the literature and can be applied in various applications

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ABSTRACT: The development of optimization algorithms attracts the attention of many analysts as it has advantages such as increasing performance, revenue, and efficiency in various fields, and reducing cost. Swarm-based optimization algorithms, which are among the meta-heuristic methods, are more commonly preferred because they are generally successful. In this study, the alpha wolf class, also called the wolf leader class, in the Grey Wolf Optimization (GWO), has been improved with the Whale Optimization Algorithm (WOA). This improved method is called ILGWO. To evaluate the ILGWO, 23 benchmark test functions, and 10 CEC2019 test functions were used. After running 30 iterations of the suggested algorithm, average fitness and standard deviation values have been acquired; these findings have been compared to the literature. Based on the literature's comparisons of the algorithms, the ILGWO algorithm has achieved the most optimal result in 5 of 7 functions for unimodal benchmark functions, 3 of 6 functions for multimodal benchmark functions, 9 of 10 functions for fixed-dimension multimodal benchmark functions, and 8 of 10 functions for CEC2019 test functions. So the proposed algorithm is generally better than the literature results. It has been found that the suggested ILGWO is encouraging and may be used in a variety of implementations.

Keywords: Grey Wolf Optimization, Alpha Wolf, Whale Optimization Algorithm, Benchmark Test Functions

1. INTRODUCTION

Meta-heuristic optimization methods are used by many researchers because of their advantages, such as being used in various application areas, being easy to apply, and avoiding the local optimum as much as possible [1-4]. The purpose of using nature-inspired meta-heuristics is to aim for the successful solving of real-world and mathematical function problems [5]. For this purpose, new optimization algorithms are constantly being developed, or these enhanced algorithms are combined with existing algorithms to create new ones. In this way, performance and efficiency in applications are increased, and costs and energy consumption are reduced. It is possible to divide meta-heuristic methods into three main categories: swarm-based, physics-based, and evolutionary-based [1]. The literature contains examples of swarm-based optimizations, such as: Artificial Bee Colony (ABC) [3], Harris Hawks Optimization (HHO) [4], Particle Swarm Optimization (PSO) [6], Whale Optimization Algorithm (WOA) [1], Optimal Foraging Algorithm (OFA) [7], Grey Wolf Optimization (GWO) [2], Butterfly Optimization Algorithm (BOA) [8], Grasshopper Optimization Algorithm (GOA) [9], Optimal Foraging Algorithm (OFA) [7].

GWO and WOA, which are swarm-based methods inspired by nature, have been used in many areas of the literature in recent years. In [10], a hybrid approach, called HGWWO and based on Whale and Grey Wolf Optimization algorithms, is proposed to generate solutions for scheduling problems in cloud tasks. It was proposed in [11] to use a hybrid GWO and genetic WOA to analyze moving objects efficiently. They developed the OWC (Optimal Weighted Centroid) procedure for this hybrid method. A fusion of the WOA and GWO is used in the OWC procedure, which is a dynamic clustering technique for splitting up moving objects. To solve the coordination problem of directional overcurrent relays, a new algorithm that consists of hybrid metaheuristic optimization is presented in the [12]. By using a leadership hierarchy of

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the GWO, the proposed method enhances the exploitative phase of the WOA. An integrated landslide modeling framework is described in [13] to reduce the risk of fatalities and financial losses in Anyuan County, China by combining an ANFIS with the two optimization algorithms GWO and WOA. Multi-objective versions of GWO and WOA for resolving bi-objective next release problem (NRP) are presented in [14]. Moreover, four datasets were used to solve NRP problem instances using these two algorithms and three other evolutionary algorithms. In [15], an improved form of the ant-lion optimization algorithm (IALO), using a new boundary reduction procedure, is proposed to solve the image clustering problem. In order to obtain well-separated clusters, an objective function has been developed for image clustering in the literature. This is achieved by minimizing inter-cluster distances and minimizing inter-cluster similarities. Based on the sexual motivation and individual intelligence of chimps in their group hunting, [16]' paper proposes a novel metaheuristic algorithm called Chimp Optimization Algorithm (ChOA). ChOA has been evaluated using both tests and real-world issues. The findings show that ChOA performs better than the other benchmark optimization techniques. A feature selection approach called IWOAIKFS, which includes an improved whale optimization algorithm (IWOA) and improved k-nearest neighbors (IKNN), is proposed in [17]. The experimental findings demonstrate that IWOA not only performs better in optimization while solving benchmark functions of various dimensions but also when IKNN is utilized for feature selection, IWOAIKFS performs better in classification. In order to segment the brain's subregions, a hybrid method that combines the WOA and GWO is suggested in [18]. The suggested approach aids in the diagnosis of Alzheimer's disease. In [19], a new WOA with Gathering strategies (HWOAG) is suggested, and many innovative strategies are collected into WOA for high-dimensional function optimization issues and Fuzzy C-Means (FCM) optimization.

In this study, the development of the alpha wolf class, which is the most important class in the wolf hierarchy and takes important decisions such as hunting, was carried out using WOA in GWO. Since the alpha class in GWO is a class that contains the most important and valuable solutions, it contributes more to the algorithm than other classes. Therefore, it is aimed to improve this class in particular. In addition, since the alpha class is the leader of the wolf pack in nature, this developed method is called the ILGWO algorithm. F1-F23 benchmark test functions and CEC2019 functions are used to measure the performance of ILGWO. It is proved by the results that the ILGWO algorithm is further developed by the GWO. Furthermore, the ILGWO algorithm consistently outperformed other approaches in the literature when these findings were compared to those of other methods. Sections 2 and 3 present, respectively, the GWO and WOA. The ILGWO approach is described in Section 4. The ILGWO method's results are provided in Section 5, along with a comparison of those results to previous research. The ILGWO method's conclusions are provided in Section 6.

2. GREY WOLF OPTIMIZATION (GWO)

GWO is an optimization method improved by Mirjalili et al., emerging from the hunting strategies created by grey wolves according to the hierarchy among them [2]. As displayed in Figure 1, grey wolves are divided into 4 groups in terms of hierarchy: alpha(α), beta(β), delta(δ), and omega(ω). Alpha consists of a female and a male wolf, and they are the lead wolves who make important decisions such as hunting. Beta wolves are the second-ranked wolves who help implement the decisions made by the alpha wolves. Delta wolves are the third-placed wolves, obeying the decisions of the alpha and beta wolves. The bottom of the hierarchy is the omegas. So omegas are the last wolves to be allowed to eat. The three best solutions from the GWO represent alpha (α), beta (β) and delta (δ), respectively, while the remaining candidates' solutions represent omega (ω).

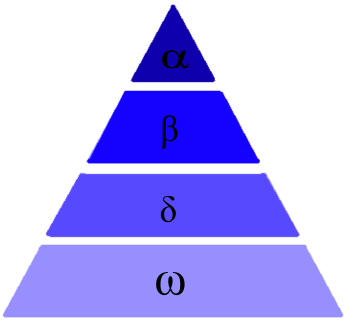


Figure 1. The Grey wolves’ hierarchy representation

The GWO method consists of 4 stages: Encircling prey, hunting, attacking prey (exploitation) and search for prey (exploration).

The grey wolves' encirclement of prey is expressed by Eqs. (1) and (2).

$$\vec{X}(t+1)=\vec{X}\,l(t)-\vec{A}.\vec{D} \tag{1}$$

$$\vec{D}=\left|\vec{C}.\vec{X}\,l(t)-\vec{X}(t)\right| \tag{2}$$

where $\vec{X}(t+1)$ is the location of a grey wolf. $\vec{X}\,l(t)$ indicates the prey's location at iteration *i*. *A* and *C* are the coefficient vectors derived from Eqs. (3) and (4).

$$\vec{A}=2\vec{a}.\vec{r}_1+\vec{a} \tag{3}$$

$$\vec{C}=2.\vec{r}_2 \tag{4}$$

where $\vec{r}_1$ and $\vec{r}_2$ are random numbers in the range of [0,1] and *a* value are linearly decreased from 2 to 0. By adjusting the *A* and *C* vectors, the best agent's nearby new positions can be checked.

After the encircling phase, the search for the best solution begins in the hunting phase. In this phase, alpha guides the hunt, while beta and delta occasionally participate in this process. Thus, grey wolves’ location is updated by using Eqs. (5), (6), and (7).

$$\vec{D}_\alpha=\left|\vec{C}_1.\vec{X}_\alpha-\vec{X}\right|,\quad \vec{D}_\beta=\left|\vec{C}_2.\vec{X}_\beta-\vec{X}\right|,\quad \vec{D}_\delta=\left|\vec{C}_3.\vec{X}_\delta-\vec{X}\right| \tag{5}$$

$$\vec{X}_1=\vec{X}_\alpha-\vec{A}_1.\vec{D}_\alpha,\quad \vec{X}_2=\vec{X}_\beta-\vec{A}_2.\vec{D}_\beta,\quad \vec{X}_3=\vec{X}_\delta-\vec{A}_3.\vec{D}_\delta \tag{6}$$

$$\vec{X}(t+1)=\frac{\vec{X}_1+\vec{X}_2+\vec{X}_3}{3} \tag{7}$$

After hunting phase, the phase of attacking the prey begins. When prey stops moving, the grey wolves attack the prey and finish the hunt. This phase is achieved by reducing the value of *a*. After attacking the prey phase, the phase of search for prey begins. According to the value of *A*, the search agent looks for better prey at this stage.

To summarize, in the GWO algorithm, the haphazard population is generated. Alpha, beta, and delta wolves forecast the likely prey's position. Then, distance of the candidate solution is updated. Then, *a* is reduced from 2 to 0 in order to emphasize exploration and exploitation, respectively. Next, If *A<1*, they go forward the prey. if *A>1*, they move away from attacking the prey. At long last, the GWO has arrived at an acceptable outcome and is ended [2]. The GWO pseudocode is presented in Figure 2.

```
Initialize the grey wolf population Xi (i = 1, 2, ..., n)
Initialize a, A, C
Calculate the fitness of each search agent
Xα= The best search agent
Xβ= The second best search agent
Xδ= The third best search agent
  while (t < Maximum iteration number)
     for each search agent
     Update the position of the current search agent using Eq (7)
     end for
   Update a, A, C
   Calculate the fitness of all search agent
   Update Xα, Xβ, Xδ
   t=t+1
  end while
return Xα
```

Figure 2. The GWO pseudocode [2]

3. WHALE OPTIMIZATION ALGORITHM (WOA)

Metaheuristic optimization algorithms have a wide range of use in recent years, as they can produce solutions to problems in different fields. Whale optimization algorithm, one of them, was proposed by Mirjalili and Lewis in 2016. WOA is inspired by the techniques used by humpback whales for bubbling and squeezing the prey into a bubble spiral [1].

Humpback whales collect prey, which usually consists of small fish communities, thanks to air bubbles, then rise to the surface and narrow the circle of target fish by making the movement of narrowing the bubble circle. The representation of hunting whales is given in Figure 3 [1]. WOA consists of the following stages: encircling prey, bubble-net attacking, and searching for prey.

In encircling prey stage, the location of prey is determined by humpback whales and so they wrap prey. Eq. (8) and Eq. (9) are mathematically given this behavior.

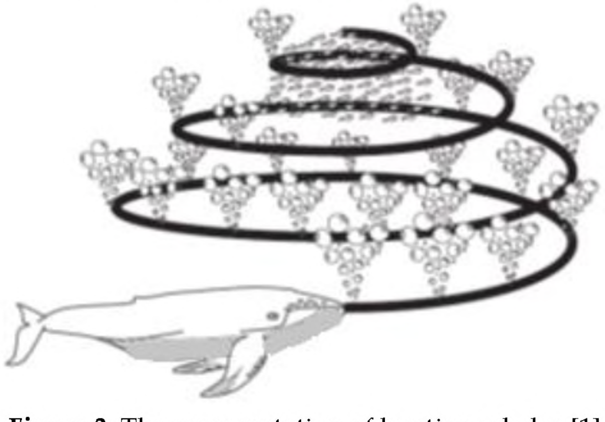


Figure 3. The representation of hunting whales [1]

$$\vec{D} = \left| \vec{C} . \vec{X}^*(t) - \vec{X}(t) \right| \tag{8}$$

$$\vec{X}(t+1) = \left| \vec{X}^*(t) - \vec{A} . \vec{D} \right| \tag{9}$$

where X stands for the position vector, X^* for the optimal solution, and t for the current iteration.

$$\vec{A} = 2\vec{a} . \vec{r} - \vec{a} \tag{10}$$

$$\vec{C} = 2 . \vec{r} \tag{11}$$

Using the variables r, a random vector between 0 and 1, and a, a linearly decreasing value between 2 and 0, the vectors A and C, which are coefficient vectors in Eqs. (10) and (11), are produced. Using the following formula, the a in Eq. (10) is produced:

$$a = 2 - t\frac{2}{MaxIter} \tag{12}$$

The number of iterations is given in Eq. (12) as t, and the greatest number of iterations denoted as $MaxIter$.

The bubble-net attacking stage consists of decreasing encircling movement and spiral movement. The decreasing encircling movement takes place by decreasing a in Eq.(10). The spiral movement occurs by Eq.(13).

$$\vec{X}(t+1) = D'.e^{bl}.\cos(2\pi l) + \vec{X}^*(t) \tag{13}$$

The $\vec{D}'$ is the distance between the whale and its prey in Eq. (13), and l is a random number between -1 and 1. The b indicates the spiral's shape [20].

Whales randomly choose one of these two shaped paths and this situation is symbolized in Eq.(14).

$$\vec{X}(t+1) = \begin{cases} Use\ Eq.9 & if\,(p < 0.5) \\ Use\ Eq.13 & if\,(p \geq 0.5) \end{cases} \tag{14}$$

During the hunt for prey, the prey is randomly searched by the whale. Hence, vector A, which produces random values, is employed. This mechanism is presented mathematically in Eqs. (15) and (16) [20].

$$\vec{D} = \left|\vec{C}.\vec{X}_{rand} - \vec{X}\right| \tag{15}$$

$$\vec{X}(t+1) = \vec{X}_{rand} - \vec{A}.\vec{D} \tag{16}$$

Eq.(9) is used in case of $|A|<1$ and Eq.(16) is used in case of $|A|>=1$. In Figure 4, the WOA's pseudocode is given.

```
Create Initial Population Xi (i = 1, 2, ..., n)
Compute the fitness value of each solution
X*=the best solution
while (t < Max_Iteration)
        for each solution
                Update a, A, C, l, and p
                If1 (p<0.5)
                        if2 (|A| < 1)
                                Calculate Eq. (9) to update the position of the current solution
                        else if2 (|A| ≥1)
                                Select a random solution (xrand)
                                Calculate Eq. (16)
                        end if2
                else if1 (p≥ 0.5)
                        Update the position of the current search by the Eq. (13)
                end if1
        end for
        Check if any solution goes beyond the search space and amend it
        Calculate the fitness of each solution If there is a better solution, update X*
        t=t+1
end while
return X*
```

Figure 4. Pseudocode of the WOA algorithm [1]

4. THE PROPOSED METHOD

GWO optimization may not produce the best solutions for each iteration until it reaches the maximum iteration. In order to improve these bad solutions, the wolf leader of the GWO method was improved with WOA. This improved method is called ILGWO and the flow diagram of the ILGWO method is given in Figure 5. In the ILGWO method, the alpha value of the GWO algorithm is aimed to be the best value to be obtained in the population. For this purpose, if the difference between the old fitness value and the new fitness value in the pure GWO algorithm is smaller than the value determined as a parameter, WOA is run using the same population members. The leader score obtained from WOA is compared with the alpha score obtained from GWO, and if a better WOA leader score is obtained than the alpha score of GWO, the population of WOA is assigned to GWO. Thus, more successful results were obtained.

In this method, the initial population is created first. The first population is given the GWO for the first iteration. In the GWO, the first best fitness value is assigned to the alpha score and its position to the alpha position, according to the fitness function obtained from the initial population. The second best fitness value is assigned to the beta score and its position to the beta position. The third best fitness value is assigned to the delta score and the position to the delta position.

In the GWO algorithm, if the difference between the old fitness value and the new fitness value is less than the value determined as a parameter, WOA is run using the same population members. In WOA, the fitness value is calculated for each search agent in the initial population, and then the processing steps for the WOA are run. Positions are updated according to these transaction steps and the best position is obtained. The fitness value of this best position is then assigned as the Leader score and the position as the Leader position.

Compares the Leader score from WOA with the Alpha score from GWO. If the Leader score is lower than the Alpha score, both the Alpha score will be replaced by the Leader score and the Alpha position will be replaced by the Leader position. To improve optimization in GWO, the current search agent position is updated with a high alpha coefficient and the fitness value of these positions is calculated. According to the updated positions, both the score and the position are updated for alpha, beta, delta. It is aimed to get the best result by performing this operation in each iteration.

5. RESULTS AND DISCUSSION

The F1-23 benchmark functions, which are made up of unimodal, multimodal, and fixed-dimension multimodal, were utilized to test the ILGWO approach. To compare with various methods described in the literature, these functions were used. To accurately compare this study to the literature, the same parameters were utilized as much as feasible, therefore it was attempted to locate papers that had the same parameters. The maximum iteration number, population number, and independent run number for the WOA, PSO, and GWO algorithms, respectively, were utilized in the literature as 500, 30, and 30 [1, 2]. The population number, the maximum number of iterations, and the number of independent runs for the ALO and IALO algorithms, respectively, were used in the literature to be 40, 500, and 30, respectively [15]. The suggested ILGWO algorithm ran each test function 30 times in order to compare with the literature. The independent run number and maximum iteration number were taken as 30 and 500, respectively. From these run data, mean and standard deviation results have been calculated. When the suggested ILGWO has been compared against the GWO, WOA, ALO, PSO, and IALO approaches, it has been discovered that the ILGWO generally produced better results. In Table 1, unimodal F1–F7 functions are listed. *Range* is the limit of the function's search space while *fmin* is the optimum value. A low border is *Lb*, while an upper boundary is *Ub*. *Dim* stands for the size of the function. F1 through F7 functions can evaluate the capacity and performance of the optimization method during the exploitation process.

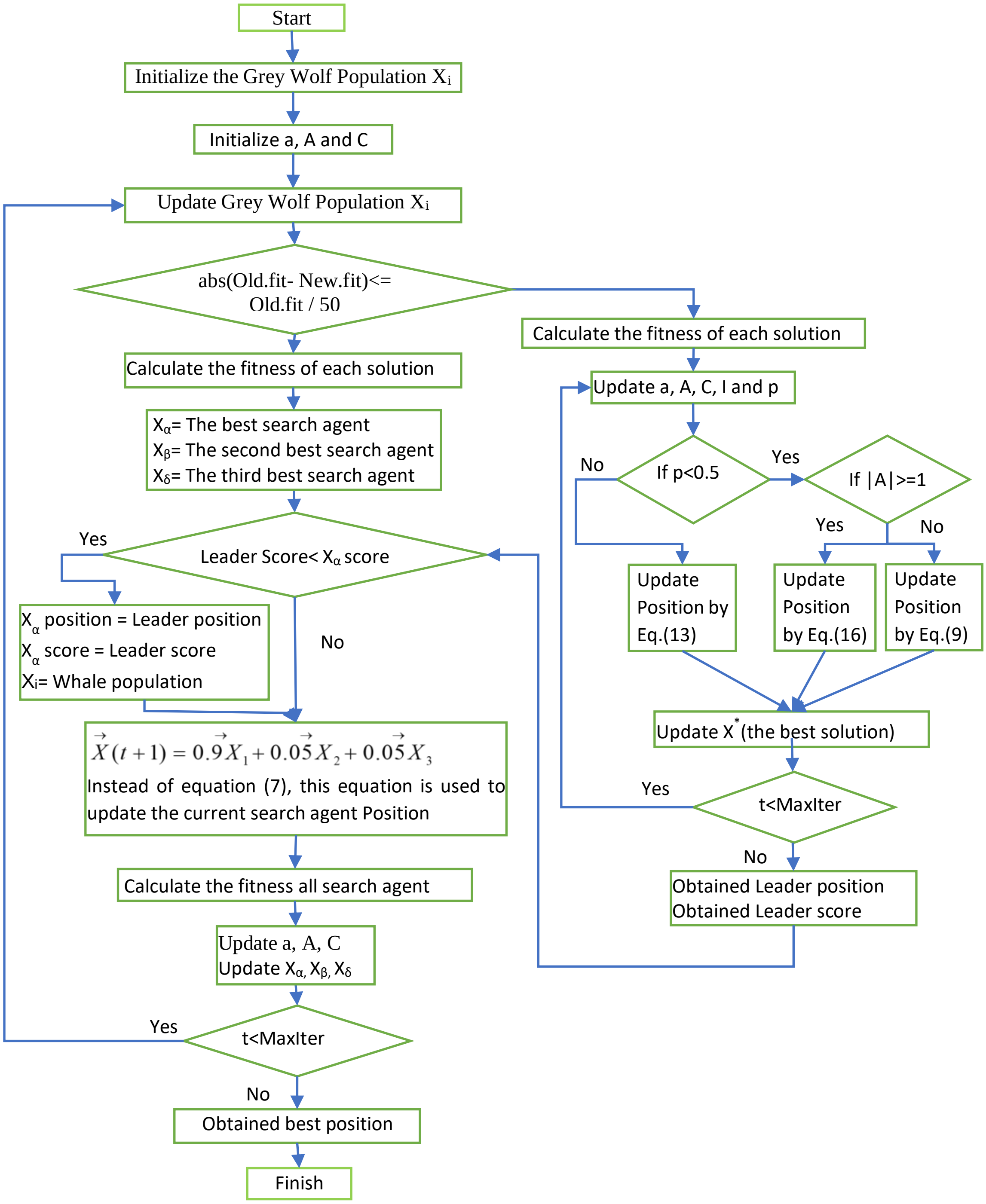


Figure 5. Flowchart of the proposed ILGWO

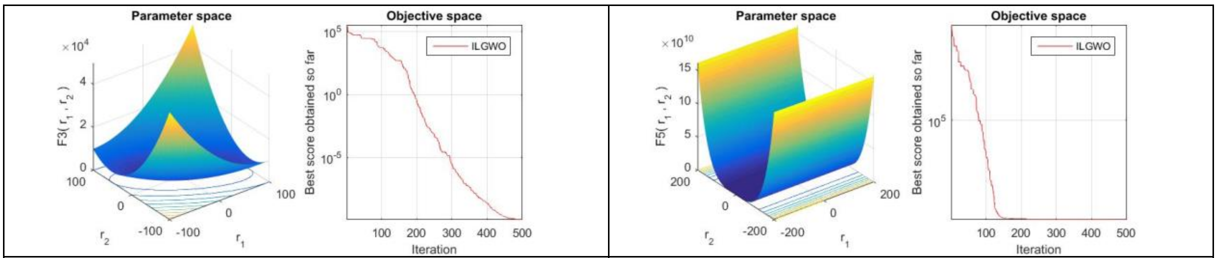
Table 1. Unimodal functions

Func. Num	Function	f_{min}	Range Lb	Range Ub	*Dim*
$F_1(r)$	$\sum_{i=1}^{n} r_i^2$	0	-100	100	30
$F_2(r)$	$\sum_{i=1}^{n} \lvert r_i \rvert + \prod_{i=1}^{n} \lvert r_i \rvert$	0	-10	10	30
$F_3(r)$	$\sum_{i=1}^{n} (\sum_{j-1}^{i} r_j)^2$	0	-100	100	30
$F_4(r)$	$\max_i \{\lvert r_i \rvert, 1 \le i \le n\}$	0	-100	100	30
$F_5(r)$	$\sum_{i=1}^{n-1} [100(r_{i+1} - r_i^2)^2 + (r_i - 1)^2]$	0	-30	30	30
$F_6(r)$	$\sum_{i=1}^{n} ([r_i + 0.5])^2$	0	-100	100	30
$F_7(r)$	$\sum_{i=1}^{n} i r_i^4 + random[0,1)$	0	-1.28	1.28	30

Unimodal functions' results are presented in Table 2. In 5 of 7 functions for unimodal functions, the ILGWO algorithm produced the best outcome. As a result of the findings, it is evident that the GWO algorithm has been developed. Moreover, ILGWO has been found to be better than the other comparison algorithms except for the F2 and F6 results. WOA produced the best results for F2, while IALO produced the best results for F6. The best examples for F1–F7 functions in search and objective space are given in Figure 6.

Table 2. Unimodal functions' results

F	GWO[2]		WOA[1]		ALO [15]		PSO[1]		IALO [15]		ILGWO	
	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.
F1	6.5900E-28	6.3400E-05	1.4100E-30	4.9100E-30	4.3800E-09	1.8100E-09	1.3600E-04	2.0200E-04	3.6800E-11	1.1400E-10	8.4249E-32	5.5081E-32
F2	7.1800E-17	2.9014E-02	1.0600E-21	2.3900E-21	5.5354E-01	1.3245E+00	4.2144E-02	4.5421E-02	3.4600E-04	7.7400E-04	7.0959E-21	3.0817E-21
F3	3.2900E-06	7.9150E+01	5.3900E-07	2.9300E-06	6.5900E-04	8.3500E-04	7.0126E+01	2.2119E+01	5.7663E-01	6.2807E-01	3.3126E-10	2.6852E-10
F4	5.6100E-07	1.3151E+00	7.2581E-02	3.9747E-01	8.5600E-04	1.1980E-03	1.0865E+00	3.1704E-01	2.7898E-02	9.2275E-02	9.6518E-08	3.6654E-08
F5	2.6813E+01	6.9905E+01	2.7866E+01	7.6363E-01	2.7842E+01	6.2201E+01	9.6718E+01	6.0116E+01	3.4984E+02	7.4489E+02	2.5789E+01	2.9517E-01
F6	8.1658E-01	1.2600E-04	3.1163E+00	5.3243E-01	4.6200E-09	2.2200E-09	1.0200E-04	8.2800E-05	4.5400E-11	1.7600E-10	7.6812E-01	2.1301E-01
F7	2.2130E-03	1.0029E-01	1.4250E-03	1.1490E-03	1.5767E-02	9.8230E-03	1.2285E-01	4.4957E-02	1.3740E-02	9.3790E-03	4.04897E-04	1.4293E-04



(F3) (F5)

Figure 6. The best examples for F1–F7 functions in search and objective space

Table 3 lists multimodal F8-F13 functions. F8-F13 functions can assess an optimization algorithm's strength in terms of its ability to avoid local optima since in order to attain the global optimum, the optimization algorithm must attempt to avoid all local optimums.

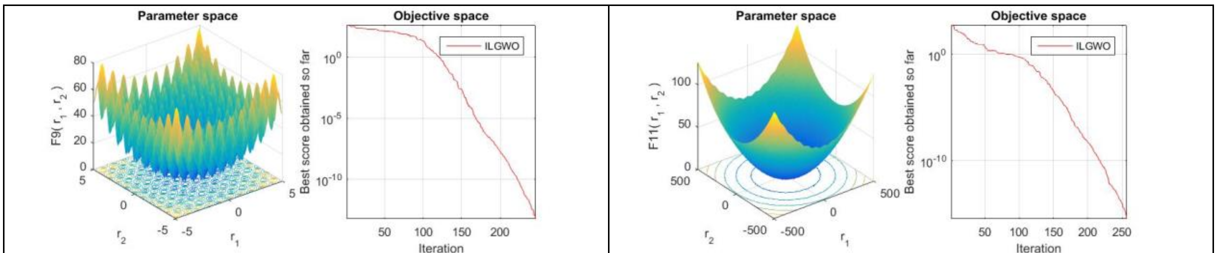
Table 3. Multimodal functions

Func. Num	Function	f_{min}	Range Lb	Range Ub	Dim
$F_8(r)$	$\sum_{i=1}^{n} -r_i \sin(\sqrt{\lvert r_i \rvert})$	-418.9829x5	-500	500	30
$F_9(r)$	$\sum_{i=1}^{n}\left[r_i^2 - 10\cos(2\pi r_i + 10)\right]$	0	-5.12	5.12	30
$F_{10}(r)$	$-20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} r_i^2}) - \exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi r_i)) + 20 + e$	0	-32	32	30
$F_{11}(r)$	$\frac{1}{4000}\sum_{i=1}^{n} r_i^2 - \prod_{i=1}^{n}\cos(\frac{r_i}{\sqrt{i}}) + 1$	0	-600	600	30
$F_{12}(r)$	$\frac{\pi}{n}\left\{10\sin(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{i+1})\right] + (y_n - 1)^2\right\} + \sum_{i=1}^{n} u(r_i, 10, 100, 4)$ $y_i = 1 + \frac{r_i + 1}{4} \quad u(r_i, a, k, m) = \begin{cases} k(r_i - a)^m & r_i > a \\ 0 & -a < r_i < a \\ k(-r_i - a)^m & r_i < -a \end{cases}$	0	-50	50	30
$F_{13}(r)$	$0.1\left\{\sin^2(3\pi r_1) + \sum_{i=1}^{n}(r_i - 1)^2\left[1 + \sin^2(3\pi r_i + 1)\right] + (r_n - 1)^2\left[1 + \sin^2(2\pi r_n)\right]\right\} + \sum_{i=1}^{n} u(r_i, 5, 100, 4)$	0	-50	50	30

Table 4 displays the outcomes of the multimodal functions. In 3 of the 6 multimodal comparison functions, the ILGWO algorithm produced the best result. According to the results of F8-F13, half of the results are improved. F9-F11 has been found to be better than the other comparison algorithms. ALO produced the best results for F8 and F13, while PSO produced the best results for F12. The best examples for multimodal functions in search and objective space are given in Figure 7.

Table 4. Multimodal functions' results

F	GWO[2]		WOA[1]		ALO [15]		PSO[1]		IALO [15]		ILGWO	
	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.
F8	-6.1231E+03	-4.0874E+03	-5.0808E+03	6.9580E+02	-2.4391E+03	4.4985E+02	-4.8413E+03	1.1528E+03	-2.8191E+03	3.1346E+02	-7,1924+03	2,4306+02
F9	3.1052E-01	4.7356E+01	0	0	1.9402E+01	1.1247E+01	4.6704E+01	1.1629E+01	1.4725E+01	5.0693E+00	0	0
F10	1.0600E-13	7.7835E-02	7.4043E+00	9.8976E+00	2.9240E-01	6.1341E-01	2.7602E-01	5.0901E-01	7.8411E-01	1.0061E+00	2,81256E-14	3,14091E-15
F11	4.4850E-03	6.6590E-03	2.8900E-04	1.5860E-03	2.2125E-01	1.0754E-01	9.2150E-03	7.7240E-03	2.0489E-01	1.0017E-01	0	0
F12	5.3438E-02	2.0734E-02	3.3968E-01	2.1486E-01	1.4850E+00	1.7889E+00	6.9170E-03	2.6301E-02	1.1932E-01	2.3377E-01	4,4980E-02	7,2107E-03
F13	6.5446E-01	4.4740E-03	1.8890E+00	2.6609E-01	7.0100E-04	3.8380E-03	6.6750E-03	8.9070E-03	2.1990E-03	4.4720E-03	8,2727 E-01	1,2069 E-01



(F9) (F11)

Figure 7. The best examples for F8–F13 functions in search and objective space

Table 5 presents fixed-dimension multimodal F14-F23 functions. Since optimization algorithms must keep the balance between the exploitation and exploration stages of the search process, F14–F23 functions can evaluate this balance.

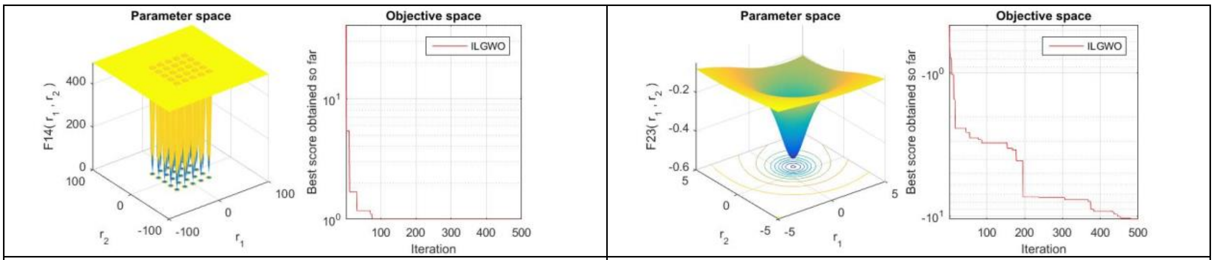
Table 5. Fixed-dimension multimodal F14-F23 functions

Func. Num	Function	f_{min}	Range Lb	Range Ub	Dim
$F_{14}(r)$	$\left(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(r_i-a_{ij})^6}\right)^{-1}$	1	-65	65	2
$F_{15}(r)$	$\sum_{i=1}^{11}\left[a_i-\frac{r_1(b_i^2+b_i r_2)}{b_i^2+b_i r_3+r_4}\right]^2$	0.00030	-5	5	4
$F_{16}(r)$	$4r_1^2-2.1r_1^4+\frac{1}{3}r_1^6+r_1r_2-4r_2^2+4r_2^4$	-1.0316	-5	5	2
$F_{17}(r)$	$(r_2-\frac{5.1}{4\pi^2}r_1^2+\frac{5}{\pi}r_1-6)^2+10(1-\frac{1}{8\pi})\cos r_1+10$	0.398	-5	5	2
$F_{18}(r)$	$\left[1+(r_1+r_2+1)^2(19-14r_1+3r_1^2-14r_2+6r_1r_2+3r_2^2)\right]\times \left[30+(2r_1-3r_2)^2\times(18-32r_1+12r_1^2+48r_2-36r_1r_2+27r_2^2)\right]$	3	-2	2	2
$F_{19}(r)$	$-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{3}a_{ij}(r_j-p_{ij})^2)$	-3.86	1	3	3
$F_{20}(r)$	$-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{6}a_{ij}(r_j-p_{ij})^2)$	-3.32	0	1	6
$F_{21}(r)$	$-\sum_{i=1}^{5}\left[(R-a_i)(R-a_i)^T+c_i\right]^{-1}$	-10.1532	0	10	4
$F_{22}(r)$	$-\sum_{i=1}^{7}\left[(R-a_i)(R-a_i)^T+c_i\right]^{-1}$	-10.4028	0	10	4
$F_{23}(r)$	$-\sum_{i=1}^{10}\left[(R-a_i)(R-a_i)^T+c_i\right]^{-1}$	-10.5363	0	10	4

Table 6 presents the results of the F14-F23 functions. In 9 of 10 functions for fixed-dimension multimodal comparison functions, the ILGWO algorithm produced the best result. As can be seen from the results obtained, the GWO algorithm has been developed. Moreover, ILGWO has been found to be better than the other comparison algorithms except for the F17 results. For the F17, the best performance has been attained by WOA. The best examples for F14–F23 functions in search and objective space are given in Figure 8.

Table 6 F14-F23 functions' results

F	GWO[2]		WOA[1]		ALO [15]		PSO[1]		IALO [15]		ILGWO	
	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.
F14	4.0425E+00	4.2528E+00	2.1120E+00	2.4986E+00	2.7076E+00	2.3599E+00	3.6272E+00	2.5608E+00	1.2295E+00	6.2052E-01	9,9800E-01	7,9583E-12
F15	3.3700E-04	6.2500E-04	5.7200E-04	3.2400E-04	2.8430E-03	5.9450E-03	5.7700E-04	2.2200E-04	2.1910E-03	4.9450E-03	3,0773E-04	2,59658E-07
F16	-1.0316E+00	-1.0316E+00	-1.0316E+00	4.2000E-07	-1.0316E+00	9.8600E-14	-1.0316E+00	6.2500E-16	-1.0316E+00	5.7600E-16	-1.0316E+00	7,78488E-10
F17	3.9789E-01	3.9789E-01	3.9791E-01	2.7000E-05	3.9789E-01	5.5900E-14	3.9789E-01	0.0000E+00	3.9789E-01	0.0000E+00	3,9788E-01	3,15429E-08
F18	3.0000E+00	3.0000E+00	3.0000E+00	4.2200E-15	3.0000E+00	3.3100E-13	3.0000E+00	1.3300E-15	3.0000E+00	4.9300E-15	3.0000E+00	9,81502E-08
F19	-3.8626E+00	-3.8628E+00	-3.8562E+00	2.7060E-03	-3.8628E+00	2.3000E-13	-3.8628E+00	2.5800E-15	-3.8628E+00	2.8900E-12	-3,8603E+00	2,3151E-03
F20	-3.2865E+00	-3.2506E+00	-2.9811E+00	3.7665E-01	-3.2624E+00	6.0657E-02	-3.2663E+00	6.0516E-02	-3.2775E+00	5.9564E-02	-3,3219E+00	3,74551E-06
F21	-1.0151E+01	-9.1402E+00	-7.0492E+00	3.6296E+00	-6.3766E+00	3.2796E+00	-6.8651E+00	3.0196E+00	-7.2848E+00	2.7947E+00	-1,0151E+01	5,0788E-04
F22	-1.0402E+01	-8.5844E+00	-8.1818E+00	3.8292E+00	-7.1015E+00	3.4428E+00	-8.4565E+00	3.0871E+00	-8.3333E+00	3.2587E+00	-1.0402E+01	5,6599E-04
F23	-1.0534E+01	-8.5590E+00	-9.3424E+00	2.4147E+00	-8.2471E+00	3.3601E+00	-9.9529E+00	1.7828E+00	-8.2543E+00	3.3636E+00	-1,0535E+01	3,1834E-04



(F14)	(F23)

Figure 8. The best examples for F14–F23 functions in search and objective space

There are 10 functions in this contemporary benchmark collection that were created for the CEC conference [21]. For an annual optimization challenge known as "The 100-Digit Challenge," these ten functions were developed. All of these functions have scalable dimensionalities despite having various dimensionalities. Dimensionalities vary between functions CEC01, CEC02 and CEC03 [22]. The dimensionality of the other functions is [-100,100]. Functions for CEC2019 are listed in Table 7 [21, 23].

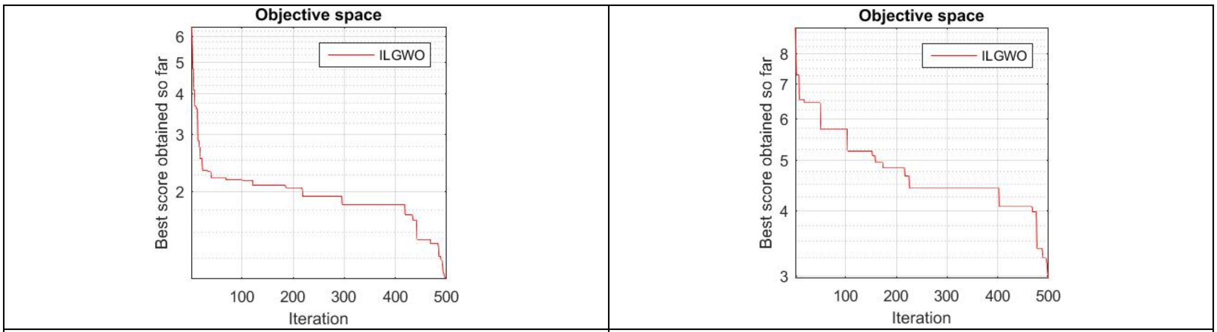
Table 7. CEC2019 Benchmark Functions

No.	*Functions*	$F_i^* = F_i(x^*)$	*D*	*Search Range*
1	*Storn's Chebyshev Polynomial Fitting Problem*	1	9	[−8192, 8192]
2	*Inverse Hilbert Matrix Problem*	1	16	[−16,384, 16,384]
3	*Lennard-Joes Minimum Energy Cluster*	1	18	[−4, 4]
4	*Rastrigin's Function*	1	10	[−100, 100]
5	*Griewangk's Function*	1	10	[−100, 100]
6	*Weierstrass Function*	1	10	[−100, 100]
7	*Modified Schwefel's Function*	1	10	[−100, 100]
8	*Expand Schaffer's F6 function*	1	10	[−100, 100]
9	*Happy Cat Function*	1	10	[−100, 100]
10	*Ackley Function*	1	10	[−100, 100]

The CEC2019 functions' results are presented in Table 8. The proposed ILGWO is compared with ChOA, WOA, and GWO. The maximum iteration number, the population number, and the independent run number for this table were used as 500, 30, and 30, respectively. The ILGWO algorithm achieved optimum results in 8 of 10 functions for CEC2019 functions. The best of objective space samples for CEC2019 functions are given in Figure 9.

Table 8. Results of CEC2019 functions

CEC	ChOA[23]		WOA[22]		GWO		ILGWO	
	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.	Ave.	Stan. Dev.
1	4.24E+09	9.67E+09	411E+08	542E+08	7.8299E+08	1.0886E+09	1.2690879E+07	1.6934569E+07
2	1.8408E+01	1.858E-02	1.7349E+01	4.5E-03	1.9847E+01	3.6134E-15	1.984762E+01	3.6134E-15
3	1.37024E+01	7.11E-06	1.37024E+01	0	1.3702E+01	1.9515E-04	1.3702E+01	1.4108E-09
4	5.93262E+03	2.8552E+03	3.9467E+02	2.4856E+02	2.1180E+02	5.5601E+02	4.1676E+01	6.5916E+00
5	4.2094E+00	8.873E-01	2.7342E+00	2.917E-01	2.3266E+00	2.4688E-01	2.1862E+00	3.1306E-02
6	1.2154E+01	6.826E-01	1.0708E+01	1.0325E+00	1.1918E+01	7.9262E-01	1.0102E+01	7.3297E-01
7	1.0071E+03	1.7901E+02	4.9068E+02	1.9483E+02	4.4050E+02	3.1599E+02	1.4273E+01	6.4592E+01
8	6.7846E+00	1.562E-01	6.909E+00	4.269E-01	5.2205E+00	9.2289E-01	3.6725E+00	3.0476E-01
9	4.4927E+02	2.4549E+02	5.9371E+00	1.6566E+00	5.3354E+00	6.9542E-01	4.5957E+00	1.7125E-01
10	2.14985E+01	7.195E-02	2.1276E+01	1.111E-01	2.1501E+01	1.0239E-01	2.0586E+01	3.0153E+00



(CEC05)	(CEC08)

Figure 9. The best of objective space samples for CEC2019 functions

Wilcoxon signed-rank test (p value) statistical comparison of the results obtained by running ILGWO and GWO 30 times is given in Table 9. According to the p-value results of this test, all of the 23 classical functions except F6 and all of the functions of CEC2019 except F2 are less than 0.05, which shows that the proposed ILGWO method is successful in GWO.

Table 9. Wilcoxon signed-rank test (p value) statistical comparison of the ILGWO and GWO

Classical Benchmark	ILGWO vs. GWO	Classical Benchmark	ILGWO vs. GWO	CEC2019 Benchmark	ILGWO vs. GWO
1	1,73E-06	13	2,6E-05	1	1,73E-06
2	1,73E-06	14	1,73E-06	2	1
3	1,73E-06	15	1,73E-06	3	1,73E-06
4	1,73E-06	16	1,73E-06	4	1,92E-06
5	1,73E-06	17	1,73E-06	5	5,32E-03
6	9,915E-01	18	1,73E-06	6	1,73E-06
7	1,73E-06	19	3,41E-05	7	1,73E-06
8	1,73E-06	20	5,22E-06	8	1,73E-06
9	1,6E-06	21	2,6E-06	9	5,22E-06
10	1,63E-06	22	1,73E-06	10	1,73E-06
11	1,56E-02	23	5,75E-06		
12	3,85E-03				

6. CONCLUSION

In this study, the alpha class, which is the top class of grey wolves and called leader wolves, was improved by using the WOA algorithm. Here, because the leader wolf was improved in the GWO, this method was named ILGWO. Since the basis of optimization is to increase efficiency and reduce costs, these objectives were targeted in the development of this ILGWO algorithm. For testing the efficiency of this algorithm, 23 mathematical functions were used. The proposed method has been run 30 times. Later, the average fitness and standard deviation values were compared with those in the literature. The ILGWO was compared with the WOA, GWO, ALO, PSO, and IALO algorithms. It was found that ILGWO achieved the best results in 5 of 7 tests for unimodal benchmarks, 3 of 6 tests for multimodal benchmarks, and 9 of 10 tests for fixed-dimension multimodal benchmarks. In addition, 10 CEC2019 test functions were used to evaluate the ILGWO, and it has been compared with the WOA, GWO, and ChOA methods. It has been found that ILGWO achieved the best results in 8 of the 10 tests for the CEC 2019 benchmarks. As a consequence, the suggested method performs better than the results from the literature in general. Also, objective space sample representations of the test functions are given.

According to the results, the proposed ILGWO method appears to be promising. In addition, it has been revealed that it can be used in engineering applications. It is planned to test the proposed method in more test functions and engineering implementations. We also want to use this strategy to develop various hybrid approaches.

Declaration of Ethical Standards

Authors declare to comply with all ethical guidelines, including authorship, citation, data reporting, and original research publication.

Credit Authorship Contribution Statement

OI and MSU conducted the literature review of the manuscript and the design of the proposed method together. It also contributed equally to obtaining the results of the proposed method and interpreting the results. MSU and OI read and approved the final manuscript.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data Availability

Research data has not been made available in a repository.

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