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Statistical Convergence of Spliced Sequences in Terms of Power Series on Topological Spaces

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Abstract
In the present paper, $P-$distributional convergence which is defined by power series method has been introduced. We give equivalent expressions for $P-$distributional convergence of spliced sequences. Moreover, convergence of a bounded ∞-spliced sequence via power series method is represented in terms of Bochner integral in Banach spaces.

Keywords: *P-distributional convergence; P-density; power series; P-statistical convergence; spliced sequences.*

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1. Preliminaries

Results of the ordinary summability theory in topological setting can not be obtained in the lack of addition operator. Hence summability theory has been studied in topological spaces under some assumptions. In this context some convergences such as Abel-statistical convergence and Abel distributional convergence have been introduced in topological spaces. From another perspective Osikiewicz [1] has introduced the concept of spliced sequences. And then this concept has been studied by Ünver [2] and Ünver and et al. [3] in topological spaces. Also Yurdakadim and et al. [4] have generalized this concept by using bounded sequences instead of convergent sequences. On the other hand spliced sequences have been studied from a different perspective in [5]. In the present paper, we introduce P-distributional convergence which generalizes Abel distributional convergence in the Hausdorff topological space. We also give equivalent expressions for P-distributional convergence of spliced sequences. Furthermore, we show that convergence of a bounded ∞-spliced sequence via power series method can be represented in terms of Bochner integral in Banach spaces.

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Definition 1.1. If the limit

$$\delta(G) := \lim_{n\to\infty} \frac{1}{n+1} |\{j \le n : j \in G\}|$$

exists then it is said to be the natural density of the subset $G \subset \mathbb{N}_0 = \{0\} \cap \mathbb{N}$. Here by $|.|$, we denote the cardinality of elements of enclosed set. If for every $\epsilon > 0$, $\delta(G_\epsilon) = 0$ where $G_\epsilon = \{j \in \mathbb{N}_0 : |x_j - L| \ge \epsilon\}$ then it is said that $x = (x_j)$ converges statistically to L [6–8].

Definition 1.2. Let (p_j) be a real sequence such that $p_0 > 0$, $p_1, p_2, ... \ge 0$ and $p(t) := \sum_{j=0}^{\infty} p_j t^j$ has radius of convergence R with $0 < R \le \infty$,

$$C_P := \left\{ x = (x_j) \mid P_x(t) := \sum_{j=0}^{\infty} p_j t^j x_j \text{ has radius of convergence } \ge R \text{ and } P_x \in C_p \right\}.$$

The functional $P - \lim : C_P \to \mathbb{R}$ defined by

$$P - \lim x = \lim_{0<t\to R^-} \frac{1}{p(t)} \sum_{j=0}^{\infty} p_j t^j x_j$$

is called a power series method and x is said to be P-convergent [9], where

$$C_p := \left\{ f : (-R, R) \to \mathbb{R} \mid \lim_{0<t\to R^-} \frac{1}{p(t)} f(t) \text{ exists} \right\}.$$

Now consider $x = (1, -1, 1, -1, ...)$, $R = \infty$, $p(t) = e^t$ and for $j \ge 0$, $p_j = \frac{1}{j!}$. Then we immediately see that

$$\lim_{t\to\infty} \frac{1}{e^t} \sum_{j=0}^{\infty} \frac{x_j t^j}{j!} = \lim_{t\to\infty} \frac{1}{e^t} \sum_{j=0}^{\infty} \frac{(-1)^j t^j}{j!} = \lim_{t\to\infty} \frac{1}{e^t} e^{-t} = 0.$$

Hence while the sequence $x = (x_j)$ is P-convergent to 0, it does not converge in the ordinary sense. This illustrates us that ordinary convergence is not as useful as power series method.

If $\lim x = l$ implies $P - \lim x = l$, then it is said that the method P is regular. A power series method P is regular if and only if for any $j \in \mathbb{N}_0$

$$\lim_{0<t\to R^-} \frac{p_j t^j}{p(t)} = 0$$

holds [9].

Definition 1.3. Let P be regular and $G \subset \mathbb{N}_0$. If the limit

$$\delta_P(G) := \lim_{0<t\to R^-} \frac{1}{p(t)} \sum_{j\in G} p_j t^j$$

exists then it is said to be the P-density of G [10].

Definition 1.4. The sequence $x = (x_j)$ of real numbers P-statistically converges to L if for every $\epsilon > 0$, $\delta_P(G_\epsilon) = 0$ that is for any $\epsilon > 0$,

$$\lim_{0<t\to R^-} \frac{1}{p(t)} \sum_{j\in G_\epsilon} p_j t^j = 0$$

is satisfied [10].

A criteria has been given for P-statistical convergence in [11].

Now, we introduce P-statistical convergence in a Hausdorff topological space (X, τ).

Definition 1.5. Consider a Hausdorff topological space (X, τ). The sequence $x = (x_j)$ in X is P-statistically convergent to $\alpha \in X$ if for any open set H that contains α

$$\lim_{0<t\to R^-} \frac{1}{p(t)} \sum_{x_j \notin H} p_j t^j = 0$$

holds.

Definition 1.6. Consider a Hausdorff topological space (X, τ) and the Borel sigma field $\sigma(\tau)$ on (X, τ). Let $F : \sigma(\tau) \to [0, 1]$ be a set function such that $F(X) = 1$ and if $H_0, H_1, ...$ are pairwise disjoint sets in $\sigma(\tau)$ then

$$F(\bigcup_{j=0}^{\infty} H_j) = \sum_{j=0}^{\infty} F(H_j)$$

holds. Then F is said to be a distribution on $\sigma(\tau)$ [2].

Definition 1.7. Consider a distribution F on $\sigma(\tau)$ and a nonnegative summability matrix $A = (a_{nj})$ such that whose each row adds up to one. Let $x = (x_j)$ be a sequence in X and ∂H be the boundary of H. Then the sequence x is said to be A-distributionally convergent to F if for all $H \in \sigma(\tau)$ with $F(\partial H) = 0$ we have ([2])

$$\lim_{n\to\infty} \sum_{x_j \in H} a_{nj} = F(H).$$

The next definition is power series version of the above one and Abel distributional convergence [2].

Definition 1.8. Consider a distribution F on $\sigma(\tau)$ and a sequence $x = (x_j)$ in X. Let ∂H be the boundary of H. Then the sequence x is said to be P-*distributionally convergent* to F if for any $H \in \sigma(\tau)$ with $F(\partial H) = 0$, we have

$$\lim_{0<t\to R^-} \frac{1}{p(t)} \sum_{x_j \in H} p_j t^j = F(H)$$

where ∂H is the boundary of H.

Definition 1.9. Let T be a fixed positive integer. A $T-$partition of $\mathbb{N}_0$ consists of infinite sets $E_i = \{v_i(j)\}$ for $i = 0, 1, ..., T-1$ such that $\bigcup_{i=0}^{T-1} E_i = \mathbb{N}_0$ and $E_i \cap E_j = \varnothing$ for all $i \neq j$. An $\infty-$partition of $\mathbb{N}_0$ consists of a countably infinite number of infinite sets $E_i = \{v_i(j)\}$ for $i \in \mathbb{N}_0$ such that $\bigcup_{i=0}^{\infty} E_i = \mathbb{N}_0$ and $E_i \cap E_j = \varnothing$ for all $i \neq j$.

Definition 1.10. Let $\{E_i : i = 0, 1, ..., T-1\}$ be a fixed $T-$partition of $\mathbb{N}_0$ and $x^{(i)} = (x_j^{(i)})$ be a sequence in X with for $i = 0, 1, ..., T-1$, $\lim_{j\to\infty} x_j^{(i)} = \alpha_i$. If $k \in E_i$, then $k = v_i(j)$ for some j. Define $x = (x_k)$ by $x_k = x_{v_i(j)} = x_j^{(i)}$. Then x is called a $T-$ *splice* on $\{E_i : i = 0, 1, ..., T-1\}$ with limit points $\alpha_0, \alpha_1, ..., \alpha_{T-1}$ [1].

Definition 1.11. Let $\{E_i : i \in \mathbb{N}_0\}$ be a fixed ∞ $-$partition of $\mathbb{N}_0$, consider a sequence $x^{(i)} = (x_j^{(i)})$ in X with $\lim_{j\to\infty} x_j^{(i)} = \alpha_i$, $i \in \mathbb{N}_0$. If $k \in E_i$, then $k = v_i(j)$ for some j. Define $x = (x_k)$ by $x_k = x_{v_i(j)} = x_j^{(i)}$. Then it is said that x is an $\infty-$ *splice* on $\{E_i : i \in \mathbb{N}_0\}$ with limit points $\alpha_0, \alpha_1, ..., \alpha_T, ...$ [1].

From [3], it is useful to recall the following

Proposition 1.1. *Consider a Banach space $(X, \|.\|)$ and a nonnegative regular summability matrix $A = (a_{nj})$ such that each row adds up to one and an $\infty-$partition of $\mathbb{N}$, $\{E_i = \{v_i(j)\} : i \in \mathbb{N}\}$. If $\delta_A(E_i)$ exists for $i \in \mathbb{N}$ and $\sum_{i=1}^{\infty} \delta_A(E_i) = 1$ then for every bounded $\infty-$ spliced sequence $x = (x_j)$ on $\{E_i : i \in \mathbb{N}\}$*

$$\lim_{n\to\infty} \sum_{j=1}^{\infty} a_{nj} x_j = \int_X t dF, \tag{1.1}$$

where the integral in (1.1) is the Bochner integral and

$$F(H)=\sum_{\alpha_i\in H}\delta_A(E_i),\, H\in\sigma(\tau)$$

is a distribution.

2. Statistical convergence of spliced sequences in terms of power series

In this section, we aim to characterize P-statistical convergence in topological spaces and also obtain equivalent expressions of P-distributional convergence of spliced sequences. The convergence of a bounded ∞-spliced sequence via power series method is also represented in terms of Bochner integral in Banach spaces.

The next theorem characterizes P-statistical convergence in topological spaces.

Theorem 2.1. *Consider X being a Hausdorff topological space and a sequence $x=(x_j)$ in X. The following assessments are equivalent:*

- *(i) x is P-statistically convergent to $\alpha\in X$.*
- *(ii) x is P-distributionally convergent to $F:\sigma(\tau)\to[0,1]$ defined by*

$$F(H)=\begin{cases}0 & , & \alpha\notin H\\ 1 & , & \alpha\in H\end{cases}.$$

Proof. Assume that x is P-statistically convergent to α. So for every open set H which contains α we get

$$\lim_{0<t\to R^-}\frac{1}{p(t)}\sum_{x_j\notin H}p_jt^j=0.$$

First, let us give the following notations that we will need to complete the proof and use in the remainder of the paper.

Consider $\mathbf{D}=\{(z_n)\subset\mathbb{R}:\forall n\in\mathbb{N}_0, 0<z_n<R \text{ and } \lim_{n\to\infty}z_n=R\}$ and let

$$\mathbf{C}=\left\{C(z)=(c_{nk}):c_{nk}=\frac{1}{p(z_n)}p_jz_n^j\right\},$$

where

$$p(t):=\sum_{j=0}^{\infty}p_jt^j.$$

Hence for any $z\in\mathbf{D}$ we get

$$\lim_{n\to\infty}\frac{1}{p(z_n)}\sum_{x_j\notin H}p_jz_n^j=0.$$

This limit shows that for any $C(z)\in\mathbf{C}$, the sequence x is $C(z)$-statistically convergent to α. According to Proposition 1 of [3], for any $C(z)\in\mathbf{C}$, x is $C(z)$-distributionally convergent to F. This completes the proof.

Conversely assume that x is P-distributionally convergent to F. So for any $C(z)\in\mathbf{C}$, x is $C(z)$-distributionally convergent to F. Again according to Proposition 1 of [3], x is $C(z)$-statistically convergent to α for any $C(z)\in\mathbf{C}$. This implies x is P-statistically convergent to α. □

The next theorem deals with P-distributional convergence of finite spliced sequences.

Theorem 2.2. *Consider X being a Hausdorff topological space and a $T-$partition of $\mathbb{N}_0$, $\{E_i:i=0,1,...,T-1\}$. Then the following assessments are equivalent:*

- *(a) For each $i=0,1,...,T-1$, $\delta_P(E_i)$ exists.*

- *(b) There exist* $s_0, s_1, ..., s_{T-1} \in [0,1]$ *such that* $\sum_{i=0}^{T-1} s_i = 1$ *and any* $T-spliced$ *sequence on* $\{E_i : i = 0,1,...,T-1\}$ *with limit points* $\alpha_0, \alpha_1, ..., \alpha_{T-1}$ *is P-distributionally convergent to the distribution* $F : \sigma(\tau) \to [0,1]$ *where*
$$F(H) = \sum_{\substack{0 \le i \le T-1 \\ \alpha_i \in H}} s_i, \text{for all } H \in \sigma(\tau).$$

- *(c) There exist* $s_0, s_1, ..., s_{T-1} \in [0,1]$ *such that* $\sum_{i=0}^{T-1} s_i = 1$ *and the* $T-$*splice of* $x^{(0)}, x^{(1)}, ..., x^{(T-1)}$ *on* $\{E_i : i = 0,1,...,T-1\}$ *where* $x^{(i)} = (\alpha_i, \alpha_i, ...)$ *is a constant sequence and is* P*-distributionally convergent to the distribution* $F : \sigma(\tau) \to [0,1]$ *where*
$$F(H) = \sum_{\substack{0 \le i \le T-1 \\ \alpha_i \in H}} s_i, \text{ for all } H \in \sigma(\tau).$$

Proof. $(a) \implies (b)$: Assume that $\delta_P(E_i)$ exists for any $i = 0,1,...,T-1$ and set $s_i = \delta_P(E_i)$ for $i = 0,1,...,T-1$. Since $\{E_i : i = 0,1,...,T-1\}$ is a $T-$partition of $\mathbb{N}_0$, it is obvious that
$$1 = \sum_{i=0}^{T-1} \delta_P(E_i) = \sum_{i=0}^{T-1} p_i.$$
Let $F : \sigma(\tau) \to [0,1]$ be defined as follows
$$F(H) = \sum_{\substack{0 \le i \le T-1 \\ \alpha_i \in H}} p_i, \text{ for all } H \in \sigma(\tau).$$
Obviously one can see that F is a distribution on $\sigma(\tau)$.

For each $i = 0,1,...,T-1$ by the existence of $\delta_P(E_i)$, we observe that for any $C(z) \in \mathbf{C}$ and any $i = 0,1,...,T-1$, $\delta_{C(z)}(E_i)$ exists and equals to $\delta_P(E_i)$. Since for each $i = 0,1,...,T-1$, $s_i = \delta_P(E_i) = \delta_{C(z)}(E_i)$ and for any $C(z) \in \mathbf{C}$ from Theorem 1 of [3] , x is obviously $C(z)$-distributionally convergent to F i.e. for any $z \in \mathbf{D}$ and for all $H \in \sigma(\tau)$ with $F(\partial H) = 0$
$$\lim_{n\to\infty} \frac{1}{p(z_n)} \sum_{x_j \in H} p_j z_n^j = F(H)$$
holds and implies
$$\lim_{0<t\to R^-} \frac{1}{p(t)} \sum_{x_j \in H} p_j t^j = F(H).$$
Then the sequence x is P-distributionally convergent to F.

$(b) \implies (c)$: Since for each $i = 0,1,...,T-1$, $x^{(i)} = (\alpha_i, \alpha_i, ...)$ is convergent, the proof can be obtained immediately.

$(c) \implies (a)$: Consider the sequence x which is the T-spliced of $x^{(0)}, x^{(1)}, ..., x^{(T-1)}$ on $\{E_i : i = 0,1,...,T-1\}$ where $x^{(i)} = (\alpha_i, \alpha_i, ...)$ is a constant sequence and is P-distributionally convergent to the distribution F and let $s_0, s_1, ..., s_{T-1} \in [0,1]$ such that $\sum_{i=0}^{T-1} s_i = 1$. Then for any $C(z) \in \mathbf{C}$, x is $C(z)$-distributionally convergent to F. According to Theorem 1 of [3] for any $i = 0,1,...,T-1$ and for every $C(z) \in \mathbf{C}$, $\delta_{C(z)}(E_i)$ exists and equals to s_i which implies
$$\lim_{n\to\infty} \frac{1}{p(z_n)} \sum_{x_j \in E_i} p_j z_n^j = s_i.$$
Thus for each $i = 0,1,...,T-1$, we have
$$\delta_P(E_i) = \lim_{0<t\to R^-} \frac{1}{p(t)} \sum_{x_j \in E_i} p_j t^j = s_i.$$
This completes the proof. □

The following result deals with P-distributional convergence of $\infty-$spliced sequences.

Theorem 2.3. *Consider a Hausdorff topological space X and an $\infty-$partition of $\mathbb{N}_0$, $\{E_i=\{v_i(j)\}: i\in\mathbb{N}_0\}$. Then $\delta_P(E_i)$ exists for all $i\in\mathbb{N}_0$ and $\sum_{i=0}^{\infty}\delta_P(E_i)=1$ if and only if there exist $s_i\in[0,1]$ for $i\in\mathbb{N}_0$ such that $\sum_{i=0}^{\infty}s_i=1$ and any $\infty-$splice sequence on $\{E_i: i\in\mathbb{N}_0\}$ with limit points $\alpha_0,\alpha_1,\alpha_2,...$ is P-distributionally convergent to the distribution $F:\sigma(\tau)\to[0,1]$ where for all $H\in\sigma(\tau)$*

$$F(H)=\sum_{\alpha_i\in H}s_i.$$

Proof. Assume that $\delta_P(E_i)$ exists for any $i\in\mathbb{N}_0$. Hence for any $C(z)\in\mathbf{C}$ and for all $i\in\mathbb{N}_0$, $\delta_{C(z)}(E_i)$ exists and equals to $\delta_P(E_i)$. Hence for any $C(z)\in\mathbf{C}$ and for each $i\in\mathbb{N}_0$, we get

$$\sum_{i=0}^{\infty}\delta_{C(z)}(E_i)=1.$$

According to Theorem 2 of [3], for any $C(z)\in\mathbf{C}$ we get that every $\infty-spliced$ sequence $x=(x_k)$ on $\{E_i: i\in\mathbb{N}_0\}$ with limit points $\alpha_0,\alpha_1,\alpha_2,...$ is $C(z)$-distributionally convergent to the distribution $F:\sigma(\tau)\to[0,1]$ where for all $H\in\sigma(\tau)$

$$F(H)=\sum_{\alpha_i\in H}\delta_{C(z)}(E_i)=\sum_{\alpha_i\in H}\delta_P(E_i),$$

i.e.

$$\lim_{n\to\infty}\frac{1}{p(z_n)}\sum_{x_j\in H}p_jz_n^j=F(H)$$

holds for every $H\in\sigma(\tau)$ with $F(\partial H)=0$ and for any $z\in\mathbf{D}$. We have

$$\lim_{0<t\to R^-}\frac{1}{p(t)}\sum_{x_j\in H}p_jt^j=F(H)$$

which means that x is P-distributionally convergent to F.

For sufficiency, let $s_i\in[0,1]$ for $i\in\mathbb{N}_0$ such that $\sum_{i=0}^{\infty}s_i=1$ and consider every $\infty-spliced$ sequence on $\{E_i: i\in\mathbb{N}_0\}$ with limit points $\alpha_0,\alpha_1,\alpha_2,...$ is P-distributionally convergent to F. Then

$$\lim_{0<t\to R^-}\frac{1}{p(t)}\sum_{x_j\in H}p_jt^j=F(H)$$

holds for all $H\in\sigma(\tau)$ with $F(\partial H)=0$. So for any $H\in\sigma(\tau)$ with $F(\partial H)=0$ and for each $z\in\mathbf{D}$, we have

$$\lim_{n\to\infty}\frac{1}{p(z_n)}\sum_{x_j\in H}p_jz_n^j=F(H). \tag{2.1}$$

Then from Theorem 2 of [3] and by (2.1) for each $C(z)\in\mathbf{C}$ and for all $i\in\mathbb{N}_0$, $\delta_{C(z)}(E_i)$ exists and equals to s_i with $\sum_{i=0}^{\infty}s_i=1$. Therefore for any $z\in\mathbf{D}$ and for each $i\in\mathbb{N}_0$

$$\lim_{n\to\infty}\frac{1}{p(z_n)}\sum_{j\in E_i}p_jz_n^j=s_i$$

which implies

$$\delta_P(E_i)=\lim_{0<t\to R^-}\frac{1}{p(t)}\sum_{j\in E_i}p_jt^j=s_i$$

for all $i\in\mathbb{N}_0$. Hence $\delta_P(E_i)$ exists for all $i\in\mathbb{N}_0$ and $\sum_{i=0}^{\infty}\delta_P(E_i)=1$. □

In the next theorem the convergence of a bounded ∞-spliced sequence via power series method is represented by Bochner integral in Banach spaces.

Theorem 2.4. *Consider a Banach space* $(X, \|.\|)$ *and an* $\infty-$*partition of* $\mathbb{N}_0$, $\{E_i = \{v_i(j)\} : i \in \mathbb{N}_0\}$. *If* $\delta_P(E_i)$ *exists for each* $i \in \mathbb{N}_0$ *and* $\sum_{i=0}^{\infty} \delta_P(E_i) = 1$ *then for every bounded* $\infty-$*spliced sequence* $x = (x_j)$ *on* $\{E_i = \{v_i(j)\} : i \in \mathbb{N}_0\}$

$$\lim_{0<t\to R^-} \frac{1}{p\,(t)} \sum_{j=0}^{\infty} p_j t^j x_j = \int_X t dF \tag{2.2}$$

where the integral in (2.2) is the Bochner integral and

$$F(H) = \sum_{\alpha_i \in H} \delta_P(E_i), \text{for every } H \in \sigma(\tau)$$

is a distribution.

Proof. Let $\delta_P(E_i)$ exists for all $i \in \mathbb{N}_0$ and $\sum_{i=0}^{\infty} \delta_P(E_i) = 1$. Hence for any $C(z) \in \mathbf{C}$ and for each $i \in \mathbb{N}_0$, $\delta_{C(z)}(E_i)$ exists and equals to $\delta_P(E_i)$ with $\sum_{i=0}^{\infty} \delta_{C(z)}(E_i) = 1$. Then from Proposition 1.1, we obtain for every bounded $\infty - spliced$ sequence $x = (x_j)$ on $\{E_i = \{v_i(j)\} : i \in \mathbb{N}_0\}$, for each $C(z) \in \mathbf{C}$

$$\lim_{n\to\infty} \frac{1}{p(z_n)} \sum_{j=0}^{\infty} p_j z_n^j x_j = \int_X t dF \tag{2.3}$$

where F is defined by

$$F(H) = \sum_{\alpha_i \in H} \delta_{C(z)}(E_i) = \sum_{\alpha_i \in H} \delta_P(E_i). \tag{2.4}$$

Hence from (2.3) and (2.4), we obtain

$$\lim_{0<t\to R^-} \frac{1}{p(t)} \sum_{j=0}^{\infty} p_j t^j x_j = \int_X t dF.$$

This completes the proof. □

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