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Research Article

The relationship between multiple representations and thinking structures: example of the integral concept

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Abstract

This study, it was aimed to examine the effect of teaching the concept of integral with multiple representations on the concept definitions of teacher candidates. And also, the effect of teacher candidates' thinking structures on their use of multiple representations was investigated. In the experimental study process algebraic, graphical and numerical representations were used together in teaching the concept of integral at university level. Since the course content was designed by supporting multiple representations, a quasi - experimental research design was used in the research. The research was carried out in the 2021 - 2022 academic year with pre-service teachers studying in the primary school mathematics teaching department of a state university. Within the scope of the research, Mathematical Process and Integral Concept Test were used. The data obtained from these scales were analyzed quantitatively and qualitatively. Regardless of the thinking structures of the pre-service teachers, it has been determined that concept definitions include different representations depending on the teaching style supported by multiple representations in the course. It has been observed that the thinking structures of the pre-service teachers affect their representation preferences slightly, if not too much, while defining the concept. However, it has been determined that there are no sharp boundaries in the types of representation used by participants with different thinking structures. Even though the pre-service teachers had different thinking structures, they used multiple representations in their concept recognition. It can be concluded that this situation has a connection with the use of multiple representations in the lesson in addition to the thinking structures of the participants. It can be concluded that this situation has a connection with the use of multiple representations in the lesson in addition to the thinking structures of the participants. According to this result, the use of more than one representation in teaching a concept enables students to learn the concept in a versatile way. For this reason, it can be said that the use of multiple representations in teaching the concept of integral provides a higher level and deeper learning. This situation can be generalized to other mathematical concepts as well.

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Introduction

Mathematics is a course with different contents at different teaching levels and can be compared to a large building with many floors. Each floor of this building represents different levels of mathematical science, and rooms on these floors can also represent different mathematical concepts. One of these concepts is the concept of integral. The concept of integral is used for various calculations in many branches of mathematics, especially in the analysis course.

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The definite integral includes the calculation of area and volume and various geometric operations. In the numerical representation of the definite integral, Riemann sums are used. In this notation, cumulative sums, namely Riemann sums, are used to present the definite integral. In the graphical representation of the definite integral, the region bounded by the curves is calculated. In the algebraic representation of the definite integral, there is the Fundamental Theorem of Analysis and various algorithms (Thompson and Silverman, 2008). So, in general, the expression $\int f(x)dx$ is expressed with different meanings as cumulative sum, area, calculation, total change between two points on the graph. While the indefinite integral is expressed only with numerical calculations, the definite integral is expressed with geometric and applied solutions such as area and volume calculations (Edwards & Penney, 1994).

In general, according to the definitions used by students, it can be said that they generally define the integral as the inverse of the derivative or the area of a region. It is an understatement to say only "the inverse of the derivative" or just "the area of the region" for the integral. For this reason, the use of multiple representations is important in the teaching of the analysis course. Especially in teacher training faculties, the use of multiple representations becomes more important. Because mathematics teachers need to be trained in this subject in order to use multiple representations. As a general case, it is a common educational problem that students are unable to grasp mathematical thinking and relate multiple representations. For this reason, many studies in the literature suggest to deepen the meaning of the concept of integral by using multiple representations and presenting different contents (Rasslan & Tall, 2002).

In the studies carried out, the use of different representations in the teaching of the integral subject, as in other subjects of mathematics, is supported by many educators (Girard, 2002; Goldin, 2004). In a different study in the literature, it was seen that the majority of the answers of the students were wrong due to the difficulty of the integral subject and the fact that it was taught with single representations (Rasslan & Tall, 2002). Researchers who criticize that the definite integral is taught only at the operational level, stated that the use of various numerical and geometric-graphic approaches in problem solving would improve students' conceptual knowledge (Ostebee & Zorn, 1997). Studies in the literature draw attention to the fact that multi-representation-centered learning approaches can improve relational understanding by expressing the relevant conceptual structure in different ways (Keller & Hirsch, 1998; Kendal & Stacey, 2003). There are some studies showing that students who use only one type of representation or cannot convert between representations have insufficient conceptual understanding. And these studies argue that the level of conceptual understanding will only increase with the use of multiple representations and associative learning (Aspinwall & Shaw, 2002; Hallett, 1991). According to all these, the use of multiple representations makes a great contribution to meaningful and deep learning.

Another structure related to different representation approaches in the learning process is students' thinking structures. Mathematical thinking profiles are divided into three groups (Krutetskii, 1976): Analytic, geometric and harmonic thinking (mixed - combination of visual and analytical thinking). Analytical thinkers tend to use symbolic representations in the problem-solving process. Those who are in the geometric thinking structure make use of visual representations rather than symbolic representations. Harmonic thinkers can use visual and symbolic elements together in cognitive processes. In other studies that group mathematical thinking profiles, it is stated that there are visual, non-visual (analytical) and mixed profiles (Clements, 1982). Although thinking structures are divided into groups according to different names, as a result, we can say that learners have different mental structures.

In researches on mathematics education, it is suggested that teaching processes should not be designed only for a single thinking structure, but should be designed in a holistic structure that includes multiple representations (Budak & Roy, 2013; Presmeg, 1986). Then, when different thinking structures are in question, we can talk about different learning processes and different course presentations. One of the tools that enables the same subject to be told in different ways is the use of multiple representations. Representation is the expression of a concept or situation in different and various ways. Mathematical representation is the verbal, numerical, visual or algebraic expression of mathematical concepts. Representations are generally divided into two categories as internal and external representations. External representations are in the student's responses, and internal representations are in the student's mind (Cobb, Yackel, & Wood, 1992).

In this study, student responses to the concept of integral, namely external representations, were examined. Although there are different representation models in the literature, representation classifications developed by Lesh, Behr and Post (1987) are based on this study, because these representations include all sub-definitions of the concept of integral. These representations used in the research are grouped into four groups as numerical, graphical, algebraic and spoken language. The multiple representations used in this study are divided into four groups. Verbal, algebraic, graphical and numerical representation. Spoken language (verbal) representation; Expressing the solution in words in the process of problem solving or thinking. Algebraic representation ; using mathematical symbols in problem solving or thinking process. Graphical representation; using picture number lines, diagrams or diagrams in problem solving and thinking processes. Numeric representation ; using tables or matrices in problem solving and thinking processes.

The use of a single type of representation can result in an incomplete teaching process because in such a case only one aspect of the concept is emphasized. It is stated that the use of multiple representations is more effective in conveying the various meanings of mathematical concepts. Based on these statements, it can be said that explaining the concept of integral with algebra, graphic and numerical representations will provide academic success to the students in the analysis course (Cuoco & Curcio, 2001). Considering previous research, in this study, the concept of integral is not defined incompletely as only the inverse of the derivative or only the area of a region. Instead, the concepts of definite integral and indefinite integral are defined using flexible, holistic and various representations. In this study, it was investigated how the simultaneous presentation of numerical, graphical and algebraic representations of the concept of integral affected students' definitions of the concept of integral. In addition, the differentiation of students' definitions of the concept of integral according to their thinking structures was examined. In general, in this study, it is aimed to investigate the effect of teaching the concept of integral with numerical, graphical and algebraic representations on the concept definitions of pre-service teachers and the effect of their thinking structures on the definitions of the integral concept.

Numerical, algebraic and graphical representations, which are multiple representation types for the concept of integral, were used in this study. This section is organized to show the expression of the integral concept in different types of representation. In this research, these three representation types of the integral were included in the teaching process of the integral. Detailed information on the experimental study is given in the method section.

Graphical Representation

The concept of integral can most simply be expressed as the area under the curve on a function graph.

Let the function f be defined and continuous in the closed interval [a, b]. Also suppose that f (x) ≥0. The area bounded by the lines x = a, x = b, the y = f (x) and x-axis curve can be expressed as "the area under the curve f (x) from a to b" (Edwards & Penney, 1994)

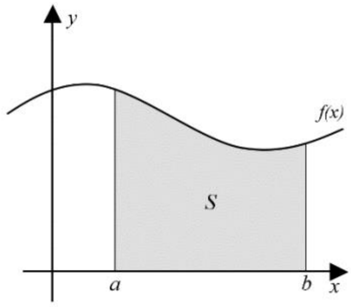

Figure 1. Limited Area

Algebraic Representation

A function F is an antiderivative of a function f if F'(x) = f(x).

We use the symbol $\int f(x)dx$ called the indefinite integral, to represent the family of all antiderivatives of f(x), and we write $\int f(x)dx = F(x) + C$ if F'(x) = f(x).

The representation of $\int_a^b f(x)dx$ is is called the definite integral of f from a to b. (Barnett, Ziegler & Byleen, 2005)

The situations in which algebraic representations are frequently used in the concept of integral are expressions such as "calculate the integral". The algebraic representation of the integral involves using the Fundamental Theorem of Analysis.

Numerical Representation

This method deals with Riemann sums. In this representation, the concept of integral is represented by the total amount of change in a given interval. This interval is divided into infinite parts and the limit calculation can be found (Thompson & Silverman, 2008).

By definition, the definite integral is the limit of the Riemann sum.

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} \; [f\,(x_{i^*})\,\Delta x_i]$$

$$\Delta x = \frac{b-a}{n} \text{ and } x_{i=} \text{ a} + \Delta x.\text{i}$$

Formula 1. Numerical formula of the integral

These three different representations of the concept of integra are related to each other. With the numerical representation of the integral concept, the total amount of change is expressed. In the graphical representation of the integral concept, area calculation is made. In the algebraic representation of the integral concept, there are algorithmic operations. As stated above, numerical, algebraic and graphical representations were used together in the teaching process of the concept of integral within the scope of the present study.

Problem of Study

Teaching the concept of integral only as the inverse of the derivative or just the area of a region is considered insufficient. For this reason, it is considered important to teach the concept of integral with holistic and multiple representations. In this study, the effect of simultaneous expression of the algebraic, graphical and numerical representations of the concept of integral on the integral definitions of pre-service teachers was examined and in addition, it was investigated whether these definitions differ according to their thinking structures. The sub-problems of this research are as follows:

- Which is the preferred form of representation for the definition of integral by the teacher candidates?
- Does the thinking structure of the teacher candidates have an effect on the representation they prefer for the definition of integral?

Method

Research Design

In this study, the effect of teaching the concept of integral with multiple representations on the concept definitions of mathematics teacher candidates was examined, and in addition, the effect of their thinking structures (geometric, analytical and harmonic thinking) on the concept definitions was investigated.

In this study, descriptive method was used. The aim of the research is to examine the effect of teaching the subject of integral with multiple representations on the definition of integral concept of teacher candidates, so a quasi-experimental research design was used. Within the scope of the quasi-experimental research, in the first lesson, in which the mathematics teacher candidates will be introduced to the concept of integral, the concept of integral is not presented with a single representation, but by supporting it with multiple representations.

Participants

In this research, a non-random, convenient sampling method was chosen to determine the study group. Within the scope of the research, first-year pre-service teachers who encountered the concept of integral for the first time at the university level were preferred. Because, first-year pre-service teachers were preferred in the study so that the cognitive skills they acquired at different grade levels do not affect the research results. The study group consisted of 37 pre-service mathematics teachers. The research was carried out in the 2021-2022 academic year in the primary school mathematics teaching department of a state university.

Data Collection Tools

Two different measurement tools were used in this study. Firstly, the Mathematical Process Scale was used to determine the thinking structures of participants, and secondly, the Integral Concept Test was used to determine their integral concept definitions. Within the scope of the quasi-experimental study in the research, the concept of integral is presented by supporting it with multiple representations. After teaching the teacher candidates with multiple representations, the scales used in the research were applied sequentially. Detailed information about the two scales can be found in the section below.

Integral Concept Test

The concept definitions of the participants were evaluated with the test developed by Rasslan and Tall (2002). The Integral Concept Test is a test applied to determine the cognitive diagrams for the students' integral concept. In this test, there are two different questions. First, there is the statement "please define the concept of ....." to determine the formal definition of the concept, and then the question "What does the concept of ... mean to you" is included to determine the informal definition of the concept. The other questions in the Integral Concept Test were prepared to examine the solution processes of the students' problems related to the integral subject, but the problem solving processes were not included in this research because the concept definitions of the integral were emphasized in this research. This test is a test adapted to Turkish and expert opinion was sought for the linguistic equivalence of the test.

The Mathematical Process Scale

The Mathematical Process Scale is a measurement tool consisting of three parts. This scale was prepared by Krutetskii (1976) in order to measure the individual's thinking structures. It serves to determine the preferences of both teachers and students towards visual and non-visual methods in the solution processes of non-routine problems. The Mathematical Process Scale consists of three parts and these parts are named as A, B and C. The target audience of these three sections differs. Parts A and B are for students, parts B and C are for teachers. That is, the part that is common to both teachers and students is part B. Within the scope of this study, only certain parts of the scale were used, because the study was carried out with pre-service teachers, that is, students. There are 12 problems in section B, and 6 problems in sections A and C. Since parts B and C were used in this study, a scale with 18 questions in total was applied to the participants.

After the scale questions were applied to the students, a list of possible solutions was given to these questions. And students were asked to mark solutions similar to their own. In cases where the student could not find a solution close to his own solution, they were told to mark the "other" option.

Data Analysis

In the Integral Concept Test, participant responses were analyzed according to various categories using a descriptive method. In the process of determining the categories, the common themes in the answers of the pre-service teachers were determined and these common themes were expressed as frequency and percentage. For the analysis of this test, the evaluation instruction prepared by Rasslan and Tall (2002) was used. Here, the answers given by the pre-service teachers about the concept of integral were evaluated in three categories: Area, inverse of the derivative, and formula-specific answers. These response categories are also associated with graphical, numerical and algebraic representations. The expression of the concept of integral as the opposite of the concept of derivative has been evaluated in relation to algebraic notation. Expressing the integral as area calculation is under the theme of graphical representation. The numerical representation is represented as the calculation of the change between two points. The expressions in Table 1 below were used for all these categorical classifications. The answers given by the pre-service teachers were evaluated according to these categories and the frequency and percentage values were calculated. In addition, the answers of the participants were divided into categories as completely correct answers, partially correct answers, incorrect answers and blank answers.

Table 1. Response Categories According to Multiple Representation Types of Integral Concept Definition

Concept	Response Categories	Assessment Criteria	Multiple Representation Types
Integral	Completely Correct Answers	Exactly correct answers to the concept of integral	Graphical Representation
			Algebraic Representation
			Numerical Representation
	Partially Correct Answers	Partially correct or incomplete answers to the concept of integral	Graphical Representation
			Algebraic Representation
			Numerical Representation
	Incorrect Answers	Incorrect answers to the concept of integral	Graphical Representation
			Algebraic Representation
			Numerical Representation
	Blank Answers	No answer to the question	Graphical Representation
			Algebraic Representation
			Numerical Representation

The Mathematical Process Tool consists of two parts and there are eighteen questions in total. In the Mathematical Process Tool, there is a scale with students' own solutions and a list of different solutions for each problem. Participants are asked to mark one or more solutions that are similar to their own solution from this list of problem solutions. All questions in this scale can be solved by visual or non-visual methods. In the scoring stage of the scale, 2 points are given to visual solutions and 0 points to non-visual solutions. Solutions whose type is not specific are given 1 point. While scoring, it is not taken into account whether the answer is correct or incorrect, and points are given only according to whether it is visual or not. According to this scoring, the maximum score a participant can get from this test is 36 and the minimum score is 0.

According to any test result, it is not possible to distinguish individuals with sharp boundaries according to analytical, harmonic and geometric thinking structures. However, in this test, the thinking structures of teacher candidates are determined according to the lower and upper score limits of each thinking type. In the evaluation of this test, it is accepted that the group with the highest score has the geometric thinking structure, and the group with the lowest score has the analytical thinking structure. Participants with medium scores are evaluated in the harmonic thinking structure (Presmeg, 1986).

To determine the upper and lower limits of test scores, the difference between the highest score and the lowest score is calculated. Thus, the dispersion range is obtained. To obtain the class range, the distribution range is divided by the number of groups. Since there were three groups in total in this study as harmonic thinking structure, geometric thinking structure and analytical thinking structure, the distribution range was divided into three in this study. Thus, the minimum and maximum scores of the thinking structures, namely the boundaries, are determined. Within the scope of the current research, the lowest score that the pre-service teachers got from this test was 5 and the highest score was 29. Accordingly, the distribution range was calculated as 24. When this value is divided by 3, which is the number of groups, 8 is obtained. The class interval in this study was calculated as 8. According to this class range, it was accepted within the scope of this study that the participants who scored between 4-12 had an analytical thinking structure, those who scored between 13-21 had a harmonic thinking structure, and those who scored between 22-28 had a geometrical thinking structure. In this study, a dynamic structure was preferred in terms of scoring. Because this is a quasi-experimental study, only the participants in this study were considered.

Results

In this section, the findings obtained from the research are given. The basic statistical data regarding the scores of the participants from the Mathematical Process Scale are given below.

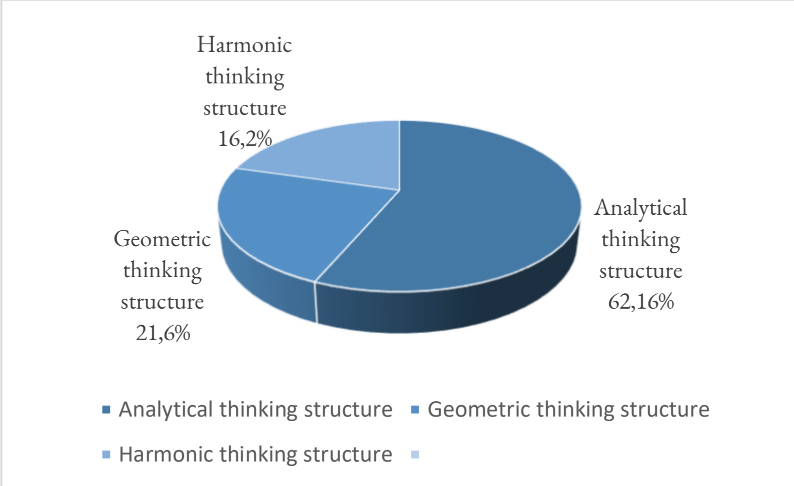

Figure 1. Pre-service Teachers' Thinking Structures Data

In this study, the lowest score that pre-service teachers got from the Mathematical Process Scale was 5 and the highest score was 29. While the maximum score that can be obtained from this test is 36, the average score of the participants in this study was calculated as 17,96. According to this average score, it can be said that the participants in this study preferred analytical processes instead of visual processes in mathematical problems.

In this study, which examined the types of concept definition responses according to the thinking structures of the participants. It was determined that 23 (62.1%) of the participants had analytical thinking structure and 8 (21.6%) had geometric thinking structure. It was determined that 6 of the teacher candidates (16.2%) had a harmonic thinking structure.

Within the scope of this research, the integral concept representations of the participants were examined according to their thinking structures. Table 2 shows the data of mathematics teacher candidates' preferences for integral concept representation according to their thinking structures.

Table2. Representations Used According to Thinking Structures

Concept	Thinking structure	Response categories	Algebraic Representation		Graphical Representation		Numerical Representation	
			f	%	f	%	f	%
Integral	Analytical thinking structure	Completely Correct Answers	8	34,7	6	26,0	4	17,3
		Partially Correct Answers	4	17,3	7	30,4	5	21,7
		Incorrect answers / Blank answers	11	47,8	10	43,4	14	60,8
	Geometric thinking structure	Completely Correct Answers	2	25,0	4	50	1	12,5
		Partially Correct Answers	3	37,5	2	25,0	2	25,0
		Incorrect answers / Blank answers	3	37,5	2	25,0	5	62.5
	Harmonic thinking structure	Completely Correct Answers	1	16,6	1	16,6	1	16,6
		Partially Correct Answers	2	33,3	2	33,3	3	50,0
		Incorrect answers / Blank answers	3	50,0	3	50,0	2	33,0

In Table 2, it was seen that the participants gave answers to the concept of integral in all types of representation, including algebraic, graphical and numerical representation. In the table, each type of thinking structure was examined in its own category and frequency and percentage values were written accordingly. For this reason, when the values in each thinking structure are collected separately, 100% has been reached.

First of all, the table was explained according to the correct answers given by the teacher candidates.
According to the correct answers of the participants to the concept of integral, it was found that 34.7% of the participants with analytical thinking preferred algebraic representation, 26.0% preferred graphical representation and 17.3% preferred numerical representation.

According to the correct answers given by the participants with geometrical thinking, 50% of them preferred graphical representation, 25% of them preferred algebraic representation, 12.5% of them preferred numerical representation. According to the number of correct answers of the pre-service teachers with harmonic thinking, it was seen that there was a rate of 16.6% in each type of representation. It was found that the most correct answers of the integral concept definitions of the pre-service teachers with analytical thinking were in the algebraic representation, while the pre-service teachers with the geometrical thinking structure had the most correct answers in the graphical representation.

Within the scope of this research, the relationship between the thinking structures of the participants and the number of multiple representations they used was also examined. In Table 3 below, the data of the pre-service teachers who used two or more representations for the concept of integral were given. In this table, the number of representations used by pre-service teachers was also examined according to their thinking structures.

Table3. Pre-service Teachers' Thinking Structures and Representations

Thinking Structure	Dual Representation (Algebraic-Graphical)		Dual Representation (Algebraic-Numeric)		Dual Representation (Graphical-Numeric)		Triple Representation (Algebraic Graphical-Numeric)	
	F	%	f	%	F	%	f	%
Analytical thinking structure	6	26,0	4	17,39	3	13,04	1	4,3
Geometric thinking structure	2	25,0	1	12,5	2	25,0	1	12,5
Harmonic thinking structure	1	16,6	1	16,6	1	16,6	1	16,6

According to the number of representation usage in Table 3, it was determined that the most frequently used binary representation (f=9) by all participants with different thinking structures was graphical-algebraic representation. Algebraic-graphical representation was the most preferred binary representation by participants with analytical thinking. Participants with geometric thinking structure preferred algebraic-graphical and algebraic-numerical representations equally. And finally, participants with harmonic thinking structure preferred an equal number of binary representations among different representation types. In addition, whether the participants used all representations together in their integral definitions was also examined. Considering the participant groups in analytical, geometric and harmonic thinking structures, only one participant from each group showed the concept of integral with triple representation.

Conclusion

In this study, algebraic, graphical and numerical representations were used together in teaching the concept of integral at university level. In this study, which was carried out with the use of multiple representations, firstly, the effect of teaching the concept of integral with multiple representations on the definitions of teacher candidates was examined. Secondly, the effect of teacher candidates' thinking structures on concept definitions was examined.

With a holistic approach, in this study, definitions of the integral concept are presented with algebraic, graphical and numerical representations instead of a single representation. And teaching with multiple representations was also reflected in the answers of the pre-service teachers, they used different representations while defining the concept of integral. Another situation examined in the study was the thinking structures of the participants. It has been observed that the thinking structures of the pre-service teachers affect their representation preferences slightly, if not too much, while defining the concept. However, it has been determined that there are no sharp boundaries in the types of representation used by participants with different thinking structures. Even though the pre-service teachers had different thinking structures, they used multiple representations in their concept recognition. It can be concluded that this situation has a connection with the use of multiple representations in the lesson in addition to the thinking structures of the participants.

In this study, students' explanations of definitions with different representations and most of their answers being in the category of fully correct and partially correct revealed the importance of teaching the integral subject simultaneously with multiple representations. There are also different studies supporting the results obtained from the first sub-problem of this research. Similar research results also supported the use of multiple representations and showed that the conceptual understanding levels of students who adhere to a single representation type or who do not have the ability to transform between representations may not develop sufficiently (Girard, 2022; Goldin, 200). In another study, it was seen that teaching the integral subject with a single representation caused incomplete and wrong learning (Rasslan & Tall, 2002). Criticizing that the meanings attributed to definite integrals remain at the operational level, researchers stated that the use of multiple representations in problem solving would improve students' conceptual knowledge (Ostebee & Zorn, 1997).

According to the second sub-problem result of the study, it can be mentioned that there is a low level of connection between students' use of multiple representations and their thinking structures. Considering the number of correct answers of the participants, it was seen that those with analytical thinking used algebraic representation, while those with geometric thinking used graphical representation. Krutetskii (1976) also emphasized the existence of different mathematical thinking profiles to ensure a successful performance in mathematics. However, regardless of whether the answers of the participants are right or wrong, it can be said that the different thinking structures of the pre-service teachers do not have a significant effect when viewed only in terms of representation types. In another study, it was concluded that students' solution methods were not related to their spatial-visual and verbal-logical reasoning skills (Hacıömeroğlu, Chicken & Dixon, 2013). This result is similar to the result of the present study. This showed that regardless of the thinking structures of the students, their dominant experiences in the learning process affect their representation preferences in the solution processes.

Considering the results of the current study and other studies in the literature, if the concept is suitable to be presented with multiple representations, it has been seen that using multiple representations contributes to the learning process, regardless of the thinking structures of the learners. Accordingly, teaching the concept of integral with multiple representations has also positively affected the cognitive development of students. The aim of a course should be to explain a concept not with a single representation, but also with multiple representations. And this purpose is also a priority for the analysis course. According to both the views suggested by the educators and the results of this research, teachers should use multiple representations in explaining a concept or explaining a subject if the concept has a structure that can be explained with more than one representation.

Recommendations

As a result, it can be suggested to use multiple representations and to establish a relationship between representations not only in the teaching of the concept of integral but also in the teaching of different contents. In order to gain more comprehensive information about both multiple representations and thinking structures, it is recommended to conduct research on different working groups and different subjects.

Recommendations for Further Research

- This research was carried out with 37 pre-service teachers. Similar studies with different sample groups may be recommended.
- Similar studies can be carried out within the scope of different disciplines.

Limitations of Study

This study is limited to the teaching the concept of integral with multiple representations on the concept definitions of teacher candidates. In terms of the study group, it is limited to 37 pre-service teachers who voluntarily participated in the research in the 2021-2022 academic year.

Conflicts of Interest

I wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

Statements of Publication Ethics

This research was reviewed by the Izmir Demokrasi University Social and Humanities Ethics Committee and it was decided that the research was ethically appropriate. Date and ethical decision number: 08/04/2022- 2022/04-02

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