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A method for calculation of lateral displacements of buildings under distributed loads

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ABSTRACT

Lateral displacement is a vary important parameter that we need to calculate when structures are subjected to lateral loads like earthquake and wind loads. In this study, a method is proposed for lateral displacement calculation of structures with different structural systems in different planes. This method is based on the continuum system calculation model. The method suggested in the literature for only top displacement in the case of uniform loading, is developed in this study for the calculation of displacements at each storey level and also in both uniform and triangular loading conditions. At the end of the study, twenty-eight storey building with shear wall-frame bearing system, which was taken from the literature, was solved with the presented method and Finite Element Method. The shear walls were modelled with three different element types for the analysis with the Finite Element Method with structural engineering program used. The results, obtained from the Continuum Method and Finite Element Method were presented in tables and by figures. Thus, the compatibility of the proposed method with the classical Finite Element Method was investigated. From the compared results of the method and the literature or finite element models it was seen that the method used for the case of uniform or triangular distributed loading gives very close results.

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1. Introduction

The desire of people to live in big cities for various reasons has increased the immigration to these cities. The increase in immigration has created the need for infrastructure and dwelling in cities. In order to meet these needs and to provide more economical use of residential areas, reinforced concrete or steel multi-storey structures were designed and built. In multi-storey buildings, the lateral loads on the structure increase with the increase in the height of the building. Therefore, lateral loads become more effective than vertical loads in the dimensioning of the bearing systems of such structures. At the same time, under the effect of increasing lateral loads, lateral displacements in the buildings reach high values and as a result, buildings can be damaged. Dynamic analysis is essential to determine this behavior of structures. However, equivalent static analysis methods are still used in low-rise and regular buildings, and the results obtained with dynamic analysis in all buildings are limited to the results obtained with the equivalent static analysis method.

Although static and dynamic calculations of buildings can be made quickly in parallel with the development of computer programs in recent years, approximate methods still maintain their importance both in the pre-sizing stage and in terms of controlling the program outputs. In the programs that analyze with the Finite Element Method, wrong results can be obtained as a result of the wrong element and model selection. At the same time, the structural behavior, which is difficult to understand in the inputs and outputs of complex finite element programs that make element-based analysis, is better understood by using approximate calculation methods and a few parameters. One of the methods used in the analysis of buildings is the Continuum Method. In this method, the multi-storey building is modeled as an equivalent cantilever beam.

The Continuum Method first emerged at the end of the 1940s. Chitty studied the behavior of parallel cantilever beams [1]. Rossmann performed the static analysis of shear walls under lateral loads using the Continuum Method [2]. Rutenberg and Heidebrecht used the Continuum Method in the analysis of asymmetrical buildings [3]. Murashev et al. conducted studies on the method and applied the method in housing estate projects [4]. The method has been studied by many researchers [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 1, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35] since then, and studies on the method are still ongoing. In Turkey, a number of studies have been carried out in the literature about the method used and developed by Bilyap [36] and his colleagues, called the Differential Equation Method. [37, 3, 39, 40, 41, 42, 43].

In this study, a method has been proposed for calculating lateral displacements of buildings with different structural systems in different planes. The method, which was suggested in the literature for only top displacement in the case of uniform loading, was developed in this study for the calculation of displacements at each storey level in both uniform and triangular loading conditions.

The following assumptions were made in the development of the method:

a) The material is linear elastic,
b) Second-order effects are negligible,
c) The axial and shear deformations in the beams are negligible,
d) The material, geometry and section properties of the building are constant throughout the height of the building.

2. Methods and materials

Each of the bearing systems (wall, frame) in different planes can be modeled as an equivalent Timoshenko beam as shown in Figure 1.

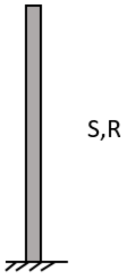

Figure 1 *Equivalent Timoshenko beam*

The total displacement (y) of the equivalent Timoshenko beam is equal to the sum of the bending displacement (y_e) and the shear displacement (y_k) as in Equation (1)

$$y = y_e + y_k \qquad (1)$$

In the equivalent Timoshenko beam, S denotes the bending stiffness and corresponds to the global bending stiffness created by the axial displacements in the frames and is calculated with the following equation.

$$S = D = E\sum_{i=1}^{m}\left(A_i t_i^2\right) \qquad (2)$$

In this equation; Ai is the cross-sectional area of the i^{th} column element, t^i is the distance of the i^{th} column element to the center of gravity of the column system, m is the number of elements.

For shear walls, S represents the bending stiffness of the wall and can be calculated by Equation (3).

$$S = EI = \sum_{i=1}^{n}\left(EI_i\right) \qquad (3)$$

R, seen in Figure 1, shows the shear stiffness of the equivalent Timoshenko beam and is calculated with the following equation for frames.

$$R = \frac{12}{h\left(\frac{1}{r} + \frac{1}{s}\right)} \qquad (4)$$

In this equation; r is the sum of all beams stiffness factors, s is the sum of all column stiffness factors at the storey, and h is the storey height. r and s are calculated by Equation (5) and Equation (6), respectively.

$$r = \sum_{i=1}^{n-1}\left(\frac{EI_{ki}}{l_i}\right) \qquad (5)$$

$$s = \sum_{i=1}^{n}\left(\frac{EI_{ci}}{h_i}\right) \qquad (6)$$

In the above equations; n is the total number of columns, EI_{ki} is the total bending stiffness of the beams, EI_{ci} is the total bending stiffness of the columns, l_i is the length of i^{th} beam, h_i represents the height of each column at the storey.

For shear wall elements, R can be calculated by Equation (7).

$$R = \frac{E}{k*2*(1+\mu)}*A \qquad (7)$$

Here, A is the cross-sectional area of the shear wall, k is the shear shape factor, and μ is the poisson's ratio. k is defined as 1.2 for rectangular sections.

2.1 Uniformly Distributed Load Case

In the case of a uniformly distributed load (Figure 2), the displacement (y_e) caused by bending is calculated by Equation (8).

$$y_e(z) = \frac{q}{S}\left[\frac{z^4}{24} - \frac{Hz^3}{6} + \frac{H^2z^2}{4}\right] \tag{8}$$

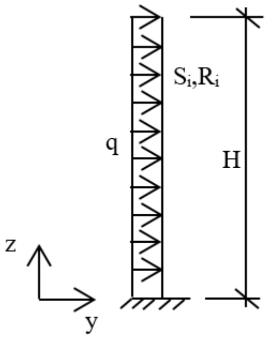

Figure 2 *Equivalent Timoshenko beam under uniformly distributed load*

The displacement caused by shear is calculated by Equation (9).

$$y_k = \frac{qH^2}{2R}\left[\frac{z}{H} - \frac{z^2}{2H^2}\right] \tag{9}$$

2.2 Triangle Distributed Load Case

In case of triangular distributed load shown in Figure 3, the displacement (y_e) caused by bending is calculated by Equation (10).

$$y_e(z) = \frac{q}{S}\left[\frac{z^5}{120H} - \frac{Hz^3}{12} + \frac{H^2z^2}{6}\right] \tag{10}$$

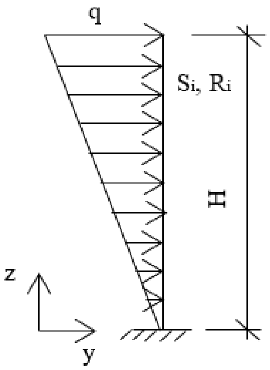

Figure 3 *Equivalent Timoshenko beam under triangular distributed load*

The displacement (y_k) caused by shear is calculated by Equation (11).

$$y_k = -\frac{qz^3}{6HR} + \frac{qH}{2R}z \tag{11}$$

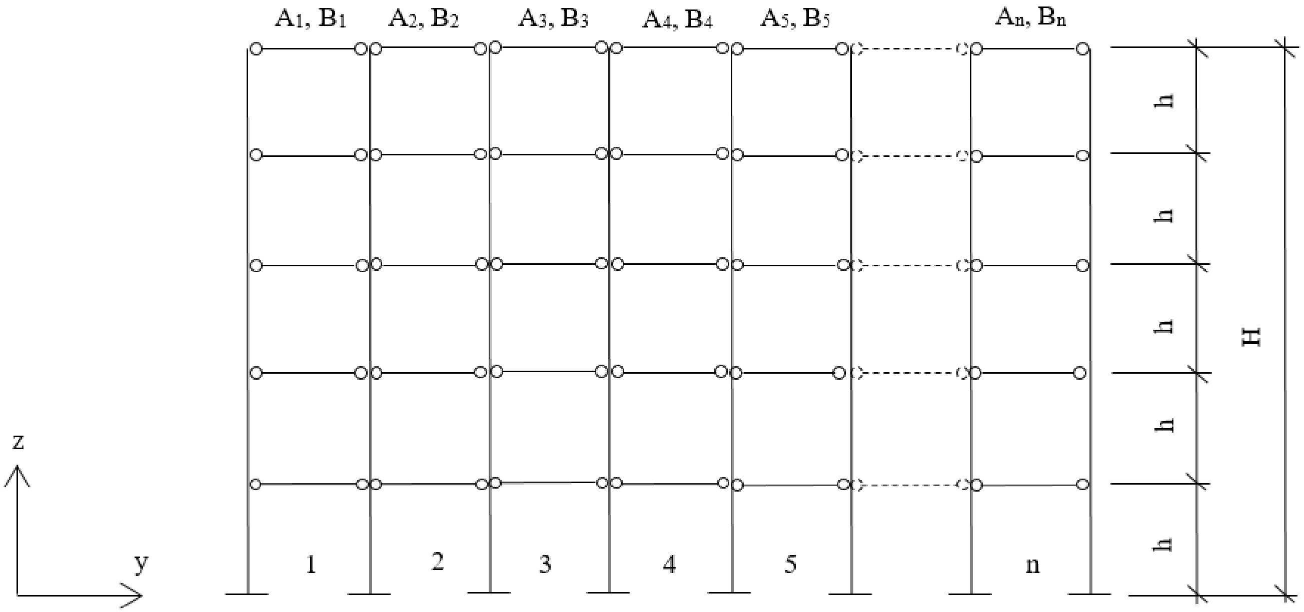

Figure 4. *Equivalent Timoshenko beam models representing the different bearing systems in different planes*

For element jth in Figure 4, the relative storey drift at ith storey is calculated by Equation (12).

$$\Delta_i^j = y_{i+1} - y_i \qquad (12)$$

In this equation, y_{i+1}, y_i represent the total displacements at i+1 and ith stories, respectively.

With the shear beam assumption, the lateral shear stiffness of the jth bearing system at ith storey is calculated by Equation (13).

$$k_i^j = \frac{V_i}{\Delta_i^j} \qquad (13)$$

In this equation, k_i^j represents the lateral stiffness of the jth element, V_i represents the shear force and Δ_i^j represents the relative storey drifts of the jth element at ith storey. The total stiffness of each storey is calculated with Equation (14).

$$K_i = \sum_{j=1}^{n} k_i^j \qquad (14)$$

The drift of the ith storey of the building is calculated with the following equation.

$$\Delta_i = \frac{V_i}{K_i} \qquad (15)$$

3. Methods and materials

3.1 Example:

In the example taken from the literature [35], 28-storey reinforced concrete structure whose layout is shown in Figure 5 and whose cross-sectional features are shown in Table 1 was investigated. In this structure; the height of each storey was 3.00 m, the Modulus Elasticity was $25*10^6$ kN/m^2 and the poisson ratio was 0.2.

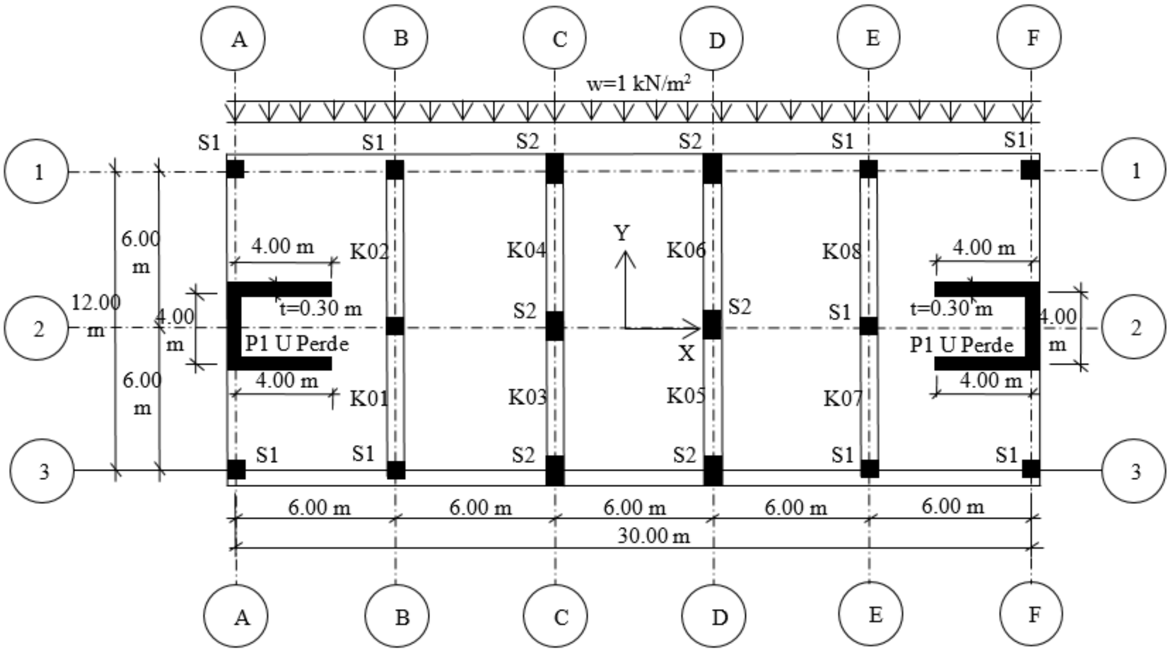

Figure 5. *Layout of the example [35]*

Table 1. *Cross-sections of the building elements*

Beam / Column	Dimensions (m)
K01, K02, K03, K04, K05, K06, K07, K08	0.40/0.40
S1	0.40/0.40
S2	0.40/0.70

In the example, the weight of the building was neglected and the lateral displacements of the building under uniformly distributed load representing the wind loads in the (–y) direction, were calculated analytically by the proposed method and by using the SAP2000 program to analyze with the Finite Element Method.

The lateral displacement of the top floor of the building under uniformly distributed load representing the wind load in the (-y) direction in the same example, was calculated by Zalka [35] as 0.184 m.

In this study, the lateral displacement values at each storey were calculated and shown in the table. In addition, 3 models were used to idealize the structure in the SAP2000 package program for Finite Element Analysis.

In the first model, the centers of gravity of the U-section wall elements were calculated. Then, the strength properties of these shear walls were calculated and these properties were assigned to the frame elements.

In the second model, each rectangular wall element constituting the U-section wall elements was formed as a frame element and assigned to the center of gravity of these elements. Then, these frame elements were connected to each other with infinitely rigid frame elements.

In the third model, each wall element constituting the U-section wall elements was considered as a shell element and divided into 20x20 cm in size square elements.

In all three models, the slabs were assumed to be infinitely rigid and at each storey rigid diaphragms were assigned.

The lateral displacement of each storey in the y-axis direction was calculated by analytical and Finite Element Methods, then shown in Table 2 and in the graph in Figure 6, comparatively.

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Table 2. *Comparison of lateral storey displacements under uniformly distributed load*

	Lateral Displacements in the y axis line (m)			
Number of Storey	**Model 1**	**Model 2**	**Model 3**	**Analytical**
Ground Floor	-0.0008	-0.0009	-0.0009	-0.0009
1.	-0.0027	-0.0029	-0.0029	-0.0032
2.	-0.0056	-0.0059	-0.0058	-0.0067
3.	-0.0092	-0.0096	-0.0096	-0.0112
4.	-0.0137	-0.0141	-0.0141	-0.0166
5.	-0.0188	-0.0193	-0.0192	-0.0226
6.	-0.0244	-0.025	-0.0249	-0.0292
7.	-0.0306	-0.0313	-0.0311	-0.0363
8.	-0.0372	-0.0380	-0.0378	-0.0437
9.	-0.0442	-0.0450	-0.0448	-0.0515
10.	-0.0515	-0.0524	-0.0520	-0.0595
11.	-0.0591	-0.0600	-0.0596	-0.0676
12.	-0.0668	-0.0678	-0.0673	-0.0759
13.	-0.0748	-0.0757	-0.0752	-0.0842
14.	-0.0828	-0.0838	-0.0833	-0.0925
15.	-0.0910	-0.0920	-0.0914	-0.1007
16.	-0.0992	-0.1002	-0.0995	-0.1088
17.	-0.1074	-0.1085	-0.1077	-0.1168
18.	-0.1156	-0.1167	-0.1159	-0.1247
19.	-0.1238	-0.1249	-0.1240	-0.1323
20.	-0.1320	-0.1331	-0.1321	-0.1396
21.	-0.1401	-0.1413	-0.1402	-0.1467
22.	-0.1482	-0.1494	-0.1482	-0.1534
23.	-0.1563	-0.1574	-0.1562	-0.1598
24.	-0.1642	-0.1654	-0.1641	-0.1658
25.	-0.1722	-0.1733	-0.1720	-0.1713
26.	-0.1800	-0.1812	-0.1798	-0.1763
27.	-0.1879	-0.1890	-0.1875	-0.1807

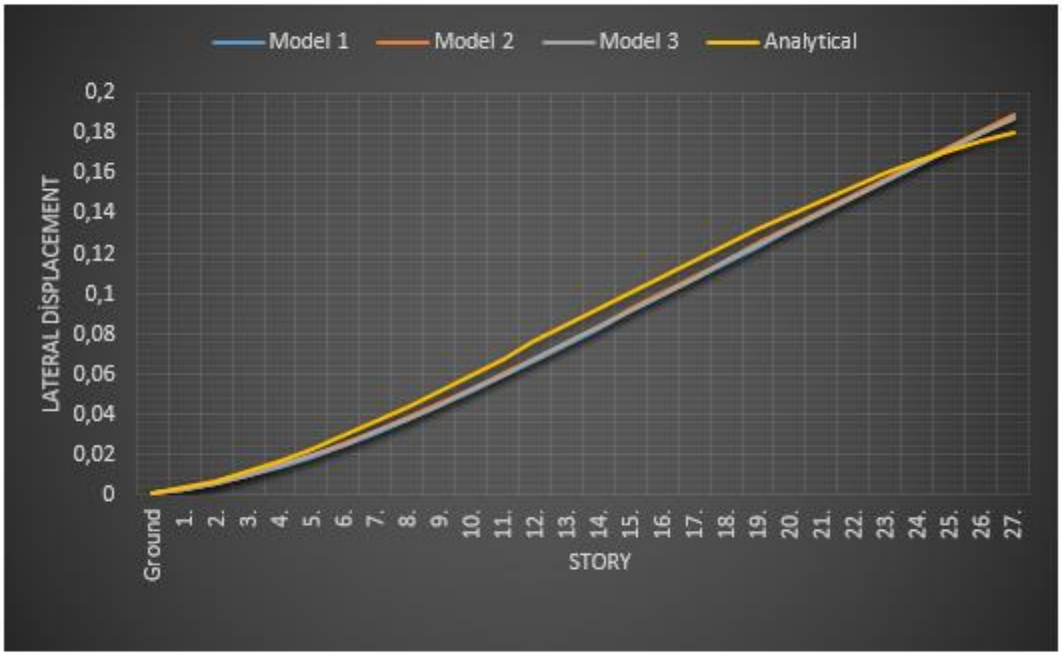

Figure 6. *Graphical comparison of lateral storey displacements under uniformly distributed load*

For the same example, the lateral displacement of each storey of the building under the triangular distributed load representing the earthquake loads in the (–y) direction shown in Figure 7 was calculated both analytically and using the SAP2000 program, which analyzes with the Finite Element Method.

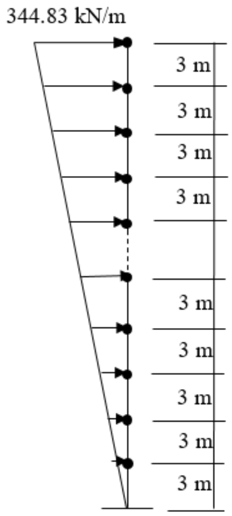

Figure 7. *Triangular distributed load*

The results obtained by the calculation of the displacements in the (–y) direction of each storey using analytical and Finite Element Method are shown in Table 3 and in the graph in Figure 8, comparatively.

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Table 3. *Comparison of lateral storey displacements under triangular distributed load*

	Lateral Displacements in the y axis line (m)			
Number of Storey	**Model 1**	**Model 2**	**Model 3**	**Analytical**
Ground Floor	-0.0020	-0.0022	-0.0022	-0.0023
1.	-0.0068	-0.0073	-0.0072	-0.0084
2.	-0.0142	-0.0149	-0.0148	-0.0175
3.	-0.0240	-0.0248	-0.0247	-0.0294
4.	-0.0359	-0.0369	-0.0368	-0.0435
5.	-0.0498	-0.0510	-0.0507	-0.0597
6.	-0.0655	-0.0668	-0.0664	-0.0775
7.	-0.0827	-0.0842	-0.0837	-0.0969
8.	-0.1013	-0.1030	-0.1024	-0.1175
9.	-0.1212	-0.1230	-0.1223	-0.1391
10.	-0.1422	-0.1442	-0.1432	-0.1617
11.	-0.1642	-0.1662	-0.1651	-0.1849
12.	-0.1869	-0.1891	-0.1878	-0.2087
13.	-0.2103	-0.2126	-0.2111	-0.2328
14.	-0.2343	-0.2367	-0.2350	-0.2572
15.	-0.2587	-0.2612	-0.2593	-0.2817
16.	-0.2834	-0.2860	-0.2839	-0.3062
17.	-0.3084	-0.3110	-0.3088	-0.3304
18.	-0.3335	-0.3362	-0.3337	-0.3543

19.	-0.3587	-0.3615	-0.3588	-0.3778
20.	-0.3839	-0.3867	-0.3838	-0.4007
21.	-0.4091	-0.4120	-0.4088	-0.4228
22.	-0.4342	-0.4371	-0.4337	-0.4439
23.	-0.4592	-0.4621	-0.4585	-0.4641
24.	-0.4840	-0.4869	-0.4832	-0.4829
25.	-0.5087	-0.5116	-0.5077	-0.5002
26.	-0.5333	-0.5362	-0.5320	-0.5158
27.	-0.5578	-0.5606	-0.5562	-0.5293

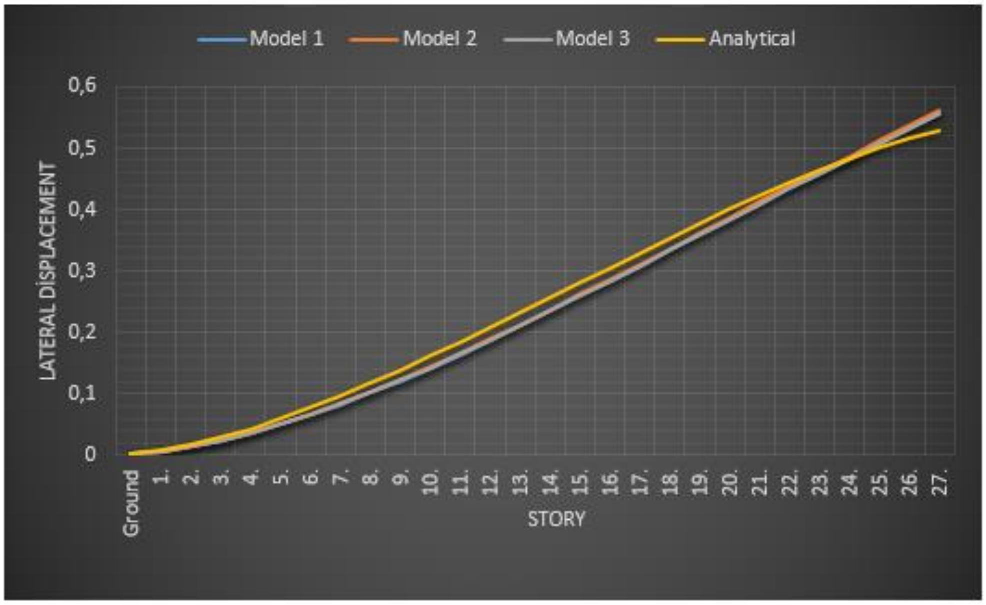

Figure 8. *Graphical comparison of lateral storey displacements under triangular distributed load*

4. Conclusions

In this study, a practical method was proposed to determine the lateral displacements at each storey level of the buildings. The method which was suggested in the literature for only top displacement in the case of uniform loading, was developed in this study for the calculation of displacements at each storey level. In addition to the uniform loading presenting the wind loads, triangular loading presenting the earthquake loads was developed in this study.

At the end of the study, example taken from the literature was solved with the presented method. In addition, for the same example, the lateral displacement at each storey was calculated using different models created by the Finite Element Method. The lateral displacement of the top floor of the building under uniformly distributed load was calculated as 0.184 m in the literature, which was calculated as 0.1807 m by the proposed method. Also for the different three models created by the Finite Element Method the lateral displacement of the top floor was calculated as 0.1879m, 0.1890 m and 0.1875 m. For the convergence of the presented method with respect to the Finite Element Method, it was seen from tables that in the case of uniform and triangular distributed loads, close results were obtained. Thus, the presented method can be used especially in the pre-sizing stage and controlling the package programs.

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