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A trapezoidal type-2 fuzzy multi-criteria decision making method based on TOPSIS for supplier selection: An application in textile sector

Tedarikçi seçimi için TOPSIS tabanlı ikizkenar yamuk tip-2 bulanık çok kriterli karar verme metodu: Tekstil sektöründe bir uygulama

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Abstract

Supplier evaluation and selection includes both qualitative and quantitative criteria and it is considered as a complex Multi Criteria Decision Making (MCDM) problem. Uncertainty and impreciseness of data is an integral part of decision making process for a real life application. The fuzzy set theory allows making decisions under uncertain environment. In this paper, a trapezoidal type 2 fuzzy multi-criteria decision making methods based on TOPSIS is proposed to select convenient supplier under vague information. The proposed method is applied to the supplier selection process of a textile firm in Turkey. In addition, the same problem is solved with type 1 fuzzy TOPSIS to confirm the findings of type 2 fuzzy TOPSIS. A sensitivity analysis is conducted to observe how the decision changes under different scenarios. Results show that the presented type 2 fuzzy TOPSIS method is more appropriate and effective to handle the supplier selection in uncertain environment.

Keywords: Type 2 fuzzy TOPSIS, Multi criteria decision making, Supplier selection

Öz

Tedarikçi değerlendirme ve seçimi, nitel ve nicel çok sayıda faktörün değerlendirilmesini gerektiren karmaşık birçok kriterli karar verme problemi olarak görülmektedir. Gerçek hayatta, belirsizlikler ve muğlaklık bir karar verme sürecinin ayrılmaz bir parçası olarak karşımıza çıkmaktadır. Bulanık küme teorisi, belirsizlik durumunda karar vermemize imkân sağlayan metotlardan bir tanesidir. Bu çalışmada, ikizkenar yamuk tip 2 bulanık TOPSIS yöntemi kısaca tanıtılmıştır. Tanıtılan yöntem, Türkiye'de bir tekstil firmasının tedarikçi seçimi problemine uygulanmıştır. Ayrıca, tip 2 bulanık TOPSIS yönteminin sonuçlarını desteklemek için aynı problem tip 1 bulanık TOPSIS ile de çözülmüştür. Duyarlılık analizi yapılarak önerilen çözümler farklı senaryolar altında incelenmiştir. Duyarlılık analizi sonuçlarına göre tip 2 bulanık TOPSIS daha efektif ve uygun çözümler üretmektedir.

Anahtar kelimeler: Tip 2 bulanık TOPSIS, Çok kriterli karar verme, Tedarikçi seçimi

1 Introduction

In recent decades, supply chain management (SCM) has taken remarkable attention in academic and business environment. The major aims of SCM are to maximize profit, improve customer relationship, reduce production costs and minimize inventory levels, and increase competitiveness. In competitive environment, supplier selection (SS) is very critical matter for firms which want to realize supply chain objectives such as competitive advantage. According to literature, the selection of the best supplier significantly decrease purchasing costs [1]. It is likely that the manufacturer allocates more than sixty percent of its total sales on raw materials, parts, and components [2]. Therefore, selecting the inappropriate suppliers increases operational and financial cost [3].

In the literature, SS has been addressed as a Multi Criteria Decision Making (MCDM) problem and a wide range of mathematical methods have been undertaken to provide more accurate and sufficient solutions [4]. Among them, we mention genetic algorithm, artificial neural networks, data envelopment analysis, linear programming, analytic hierarchy process, and grey system theory.

SS as a MCDM problem involves qualitative and quantitative criteria [4],[5]. Decision-making process is to determine the best one from a given alternative sets with respect to overall judgments [6],[7]. However, in many practical cases, the decision makers (DM) may be unable to assess precise numerical values to the supplier assessment in contrast to the traditional formulation of MCDM problems that human's judgments are symbolized as exact numbers. Because of the fact that some evaluation and selection criteria are qualitative and subjective in real life, it is difficult to represent preferences with numerical values for the DM [10]. Fuzzy methods are effective tools dealing with uncertainty resulting from subjective human judgments [11],[41]. In the classical set theory, an element cannot be in and out of a set at the same time. In contrast, fractional membership can be accepted in the fuzzy set theory [12]. The current fuzzy MCDM technics are based on conventional type-1 fuzzy sets (T1FS) [56]-[59]. In T1FSs, each element has a degree of membership which is described with a membership function (MF) valued in the interval [1].

Levels of uncertainty increase from numerical judgments to word and to perception, respectively [8]. In real life, DMs undertake decisions in uncertain environments and conventional modeling techniques are insufficient while taking into consideration these uncertainties [8].

Recently, number of studies using MCDM with type-2 fuzzy sets (T2FSs) is rapidly growing as T1FSs are unable to cope with high uncertainty and complexity. To solve the limitations of T1FSs theory, Zadeh (1975) developed T2FS theory in 1975 as an extension of ordinary fuzzy sets [9],[11]. Türkşen [34] argued that type-1 representation does not present a good approximation to verbal statements. Hence, T2FS may provide better approximation of uncertainty [8]. Handling more uncertainty means making less assumption and, thus, more realistic solutions to real problem. Due to these advantages, T2FSs have potential to go beyond T1FSs [32]. T2FSs are characterized by primary and secondary membership function. T2FSs can cope with uncertainty in complex systems more accurately than the T1FSs with the additional dimension of membership function. Although T2FSs are more difficult to apply than T1FSs, it is preferred by researchers to take into consideration uncertainty [12].

In particular, researchers have been applying interval T2FS theory to the field of MCDM problems. For example, Kahraman et al. [14] developed fuzzy MCDM approaches to select the most appropriate renewable energy alternatives. First they determine evaluation scores by using the analytic hierarchy process (AHP) and then they used method based on axiomatic design principles under fuzziness. The proposed methods were applied to select the most appropriate renewable energy alternative in Turkey. Chen and Lee [15] presented a new method to cope with fuzzy MCDM problems based on interval T2FSs. Chen et al. [16] proposed a novel fuzzy MCDM method based on interval T2FSs. Firstly, they proposed a novel method for ranking interval T2FSs. Then, they presented a novel technic for fuzzy MCDM based on the developed ranking method of interval T2FSs. Lou and Dong [17] developed a new methodology type-2 fuzzy neural networks. Paternain et al. [18] presented a construction method of Atanassov's intuitionistic fuzzy preference relations from the fuzzy preference relations given by experts. Wang et al. [19] addressed the MCDM problems under interval type-2 fuzzy environment, and presented an approach to cope with the situations in which the criteria values are represented by using interval T2FS.

Celik et al. [20] proposed an interval type-2 fuzzy (T2F) MCDM method based on TOPSIS and grey relationship analyzes to assess customer satisfaction at public transportation in Istanbul. Chen [21] presented a linear assignment method within the context of interval T2F numbers. The presented method is applied to the selection of a landfill site.

Chen et al. [22] developed an extended QUALIFLEX technic to solve MCDM problem in the interval T2FSs environment. The presented method was applied to a medical decision-making problem. Hu et al. [23] proposed a novel method based on possibility degree to figure out MCDM problem in the environment of interval T2FSs. The proposed method was applied to the overseas minerals investment for metals companies in China. Chen [24] developed an ELECTRE based MCDM within the environment of interval T2FSs.

Kahraman et al. [10] presented an interval T2F AHP method together with a novel ranking method for T2FSs. The presented method is applied to a SS problem. Temur et al. [12] presented T2F TOPSIS approach to determine the most appropriate reverse logistics facility location. The proposed method was applied to e-waste recycling industry. Kilic and Kaya [25] developed a new T2F AHP and T2F TOPSIS methods to evaluate investment projects for development agencies in Turkey. Qin and Liu [19] presented three novel average ranking value formulas related to the interval T2F information. They define interval T2F entropy with trigonometric sine function based on the aggregation and combinatorial optimization. Celik et al. [31] presented an effective method that combines T2FSs and AHP to determine importance weights of critical success factors in humanitarian relief logistics management and evaluate them. Abdullah and Najib [33] proposed a new fuzzy analytic hierarchy process characterized by interval T2FS for linguistic variables. The presented model is applied to work safety evaluation problem. Liao and Xu [35] proposed a hesitant fuzzy VIKOR method for MCDM problem using hesitant preference information. Zouggari and Benyoucef [42] presented a two-phase decision making approach for group multi-criteria supplier selection problem to integrate supplier selection process with order allocation. The first phase, suppliers are selected using fuzzy-AHP through four main criteria (Performance strategy, Quality of service, Innovation and Risk). In the second phase, via simulation based fuzzy TOPSIS; the criteria (price, quality and delivery) are evaluated for order allocation. Omurca [52] presented a hybrid method, which is consist of fuzzy c-means and rough set theory, for supplier selection, evaluation and development problem. Dogan and Aydin [53] developed the method that combines the Bayesian Networks and the Total Cost of Ownership methods for the supplier selection process. The proposed method is applied to automotive industry. Yue and Jia [54] proposed the TOPSIS method through using intuitionistic fuzzy information. Ayağ and Samanlioglu [55] developed analytic network process in the fuzzy environment.

The aim of this study is to present a trapezoidal type-1 fuzzy TOPSIS and T2F TOPSIS method for solving MCDM problem in vague information environment. The presented method is applied to a firm SS problem in which operates at textile sector in Turkey. The contribution of this paper is to present a trapezoidal type-1 fuzzy TOPSIS and type-2 fuzzy TOPSIS method for solving supplier selection problem in vague information environment in order to analyze the effect of the uncertainty level on solutions.

This paper is organized as follows. Section 2 briefly reviews the concepts of type-1 fuzzy TOPSIS, interval T2FSs and T2F TOPSIS. In Section 3, a real life application for SS problem in a textile firm is conducted by using T1F TOPSIS and T2F TOPSIS. Then sensitivity analysis is made to show solutions under different conditions. Finally, conclusions are presented and point out future research in Section 4.

2 Methodology

2.1 Type-1 fuzzy TOPSIS

The TOPSIS method was presented by Hwang and Yoon in 1981 [26]. Although it has been widely utilized for decision making process, TOPSIS method is not able to deal with uncertainties. Chen [7] presented Fuzzy TOPSIS method to solve MCDM problems under uncertain environment. Here, linguistic variables are utilized by the DMs D_r (r=1,..,k) to assess the weights of the criteria and the ratings of the alternatives. Thus, $\widetilde{W}_r^j$ denotes the weight of the *j*th criteria C_j (j=1,..,m), given by the *r*th DM. $\tilde{x}_{ij}^r$ denotes the rating of the *i*th alternative A_i (i=1,…,n), with respect to criteria j, given by the *r*th DM. The method comprises the following steps [7]:

1. The evaluation criteria for SS process are identified by decision-makers,
2. The importance of criteria and the alternatives' ratings with respect to each criteria are estimated using Eq. (1 and 2).

$$\tilde{w}_{ij} = \frac{1}{k}\left[\tilde{w}_j^1 + \tilde{w}_j^2 + \cdots + \tilde{w}_j^k\right] \quad (1)$$

$$\tilde{x}_{ij} = \frac{1}{k}\left[\tilde{x}_{ij}^1 + \tilde{x}_{ij}^2 + \cdots + \tilde{x}_{ij}^k\right] \quad (2)$$

Each criteria is evaluated by the DMs using linguistic variables depicted in Table 1 and alternatives are rated according to Table 2.

Table 1: Linguistic variables for the importance of the criteria [16].

Linguistic terms	Type-1 fuzzy sets
Very Low -VL	(0.00,0.00,0.00,0.10;1,1)
Low -L	(0.00,0.10,0.10,0.30;1,1)
Medium Low -ML	(0.10,0.30,0.30,0.50;1,1)
Medium -M	(0.30,0.50,0.50,0.70;1,1)
Medium High -MH	(0.50,0.70,0.70,0.90;1,1)
High -H	(0.70,0.90,0.90,1.00;1,1)
Very High -VH	(0.90,1.00,1.00,1.00;1,1)

Table 2: Linguistic variables for the ratings [16].

Linguistic terms	Type-1 fuzzy sets
Very Poor -VP	(0,0,0,1;1,1)
Poor -L	(0,1,1,3;1,1)
Medium Poor -MP	(1,3,3,5;1,1)
Medium -M	(3,5,5,7;1,1)
Medium Good -MG	(5,7,7,9;1,1)
Good -G	(7,9,9,10;1,1)
Very Good -VG	(9,10,10,10;1,1)

3. Fuzzy MCDM problem which can be briefly depicted in matrix form as:

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & & \tilde{x}_{2n} \\ \vdots & & \cdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} \quad (3)$$

$$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \tilde{w}_3, \ldots . \tilde{w}_n] \quad (4)$$

4. Here, the linear scale transformation is utilized to transform the various criteria scales into a comparable scale so that the normalized fuzzy decision matrix is denoted as $\tilde{R}$:

$$\tilde{R} = [\tilde{r}_{ij}]_{m\times n} \quad (5)$$

where B denotes benefit criteria and C is the set of and cost criteria, respectively, and

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right), j \in B \quad (6)$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right), \; j \in C \quad (7)$$

$$c_j^* = \max_i c_{ij} \text{ if } j \in B \quad (8)$$

$$a_j^- = \min_i a_{ij} \text{ if } j \in C \quad (9)$$

5. Considering the different weight of each criteria, the weighted normalized fuzzy decision matrix is defined as:

$$\tilde{V} = [\tilde{v}_{ij}]_{m\times n} \quad \text{i=1,2,....,m and j=1,2,....,n} \quad (10)$$

$$\tilde{v}_{ij} = \tilde{x}_{ij} \times \tilde{w}_j \quad (11)$$

6. The fuzzy positive ideal solution (A^*), and fuzzy negative-ideal solution (A^-) are determined as:

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \ldots, \tilde{v}_n^*) \quad (12)$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-) \quad (13)$$

$$\tilde{v}_j^* = (1,1,1) \text{ ve } \tilde{v}_j^- = (0,0,0) \text{ j=1,2,....,n} \quad (14)$$

Distance of each alternative from positive ideal solution and negative ideal solution is calculated by using the following equations:

$$d_i^* = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^*), \quad \text{i=1,2,....,m} \quad (15)$$

$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-), \text{i=1,2,....,m} \quad (16)$$

where d(.,.) is difference between two fuzzy numbers.

7. Lastly, the closeness coefficient of each alternative is obtained as:

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}, \quad \text{i=1,2,....,m} \quad (17)$$

The ranking order of alternatives can be determined based on the closeness coefficient, CC_i. According to Chen et al. [27], using a linguistic variable to describe the current assessment status of each supplier according to its closeness coefficient may be more realistic approach. To describe the evaluation process of each supplier, the interval [0,1] is divided into five sub-intervals. Five linguistic variables for supplier assessment with respect to the sub-intervals are given in Table 3.

Table 3: Five linguistic variables for supplier assessment with respect to the sub-intervals [27].

CC_i	Evaluation results
[0,0.2]	Do not recommend
[0.2,0.4]	Recommend with high risk
[0.4,0.6]	Recommend with low risk
[0.6,0.8]	Approved
[0.8,1.0]	Approved and Preferred

2.2 Interval type-2 fuzzy sets

T1FSs cannot cope with uncertainty in data since its membership grades are crisp numbers. Thus, T2FSs are introduced as an extension of T1FSs with a third dimension. The additional dimension helps in handling more uncertainties than T1FSs [28],[29].

According to John and Coupland [37] imprecision levels increase numbers, words and perceptions, respectively. Zadeh [38] presented type-2 FSs and higher-types of FSs to deal with this issue. Appropriate techniques for corresponding levels of precision of data can be illustrated as Figure 1.

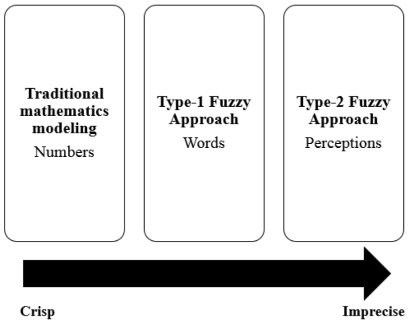

Figure 1: Suitable methods according to precision levels of data [29].

In this section, some basic definitions of T2FSs are presented [36],[15].

Definition 2.1: A T2FS $\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type-2 MF $\mu_{\tilde{\tilde{A}}}$, shown as follows:

$$\tilde{\tilde{A}}=\left\{\left((x,u),\mu_{\tilde{\tilde{A}}}(x,u)\right)|\forall x\in X,\forall u\in J_x\subseteq[0,1],0\le\mu_{\tilde{\tilde{A}}}(x,u)\le1\right\}$$

where J_x denotes an interval in [0,1]. Furthermore, the T2F set $\tilde{\tilde{A}}$ also can be represented as follows:

$$\tilde{\tilde{A}}=\int_{x\in X}\int_{u\in J_x}\mu_{\tilde{\tilde{A}}}(x,u)/(x,u)$$

$J_x\subseteq[0,1]$ and $\int\int$ shows union all acceptance u and x.

Definition 2.2: Let $\tilde{\tilde{A}}$ be a T2FS in the universe discourse X represented by the type-2 MF $\mu_{\tilde{\tilde{A}}}$. If all $\mu_{\tilde{\tilde{A}}}(x,u)=1$, then $\tilde{\tilde{A}}$ is called an interval T2FS. An interval T2FS $\tilde{\tilde{A}}$ can be considered as a special case of a T2FS, given as following:

$$\tilde{\tilde{A}}=\int_{x\in X}\int_{u\in J_x}1/(x,u)$$

where $J_x\subseteq[0,1]$.

Definition 2.3: The upper and the lower MF of an interval T2FS are type-1 MFs. The reference points in the universe of discourse and the heights of the upper and the lower MFs of interval T2FSs are utilized to characterize interval T2FSs. As it can be seen in Figure 1, a trapezoidal interval T2FS

$\tilde{\tilde{A}}_i=(\tilde{A}_i^U,\tilde{A}_i^L)=\left(a_{i1}^U,a_{i2}^U,a_{i3}^U,a_{i4}^U;H_1(\tilde{A}_i^U),H_2(\tilde{A}_i^U)\right)$,

$\left(a_{i1}^L,a_{i2}^L,a_{i3}^L,a_{i4}^L;H_1(\tilde{A}_i^L),\ H_2(\tilde{A}_i^L)\right)$ where $H_j(\tilde{A}_i^U)$ shows the membership value of the element $a_{i(j+1)}^U$ in the upper trapezoidal membership function

$\tilde{A}_i^U,1\le j\le2,\ \ as\ Interval\ Type-2\ Fuzzy\ sets\ H_j(\tilde{A}_i^L)$

shows the membership value of the element $a_{i(j+1)}^L$ in the lower trapezoidal MF $\tilde{A}_i^L,1\le j\le2,H_j(\tilde{A}_i^U)\in[0,1],H_1(\tilde{A}_i^L)\in[0,1],\ H_2(\tilde{A}_i^L)\in[0,1]$, and $1\le j\le n$.

Definition 2.4: The addition operation between the trapezoidal interval T2FSs.

$\tilde{\tilde{A}}_1=(\tilde{A}_1^U,\tilde{A}_1^L)=\left(a_{11}^U,a_{12}^U,a_{13}^U,a_{14}^U;H_1(\tilde{A}_1^U),H_2(\tilde{A}_1^U)\right)$,

$\left(a_{11}^L,a_{12}^L,a_{13}^L,a_{14}^L;H_1(\tilde{A}_1^L),\ H_2(\tilde{A}_1^L)\right)$

$\tilde{\tilde{A}}_2=(\tilde{A}_2^U,\tilde{A}_2^L)=\left(a_{21}^U,a_{22}^U,a_{23}^U,a_{24}^U;H_1(\tilde{A}_2^U),H_2(\tilde{A}_2^U)\right)$,

$\left(a_{21}^L,a_{22}^L,a_{23}^L,a_{24}^L;H_1(\tilde{A}_2^L),\ H_2(\tilde{A}_2^L)\right)$

$\tilde{\tilde{A}}_1\oplus\tilde{\tilde{A}}_2=(\tilde{A}_1^U,\tilde{A}_1^L)\oplus(\tilde{A}_2^U,\tilde{A}_2^L)=[a_{11}^U+a_{21}^U,a_{12}^U+a_{22}^U,a_{13}^U+a_{23}^U,a_{14}^U+a_{24}^U;$

$\min\left(H_1(\tilde{A}_1^U),H_1(\tilde{A}_2^U)\right),\min\left(H_2(\tilde{A}_1^U),H_2(\tilde{A}_2^U)\right)]$,

$\left(a_{11}^L+a_{21}^L,a_{12}^L+a_{22}^L,a_{13}^L+a_{23}^L,a_{14}^L+a_{24}^L;\min\left(H_1(\tilde{A}_1^L),H_1(\tilde{A}_2^L)\right),\min\left(H_2(\tilde{A}_1^L),H_2(\tilde{A}_2^L)\right)\right)$

Definition 2.5: The subtraction operation between the trapezoidal interval T2FSs $\tilde{\tilde{A}}_1=(\tilde{A}_1^U,\tilde{A}_1^L)=\left(a_{11}^U,a_{12}^U,a_{13}^U,a_{14}^U;H_1(\tilde{A}_1^U),H_2(\tilde{A}_1^U)\right)$,

$\left(a_{11}^L,a_{12}^L,a_{13}^L,a_{14}^L;H_1(\tilde{A}_1^L),\ H_2(\tilde{A}_1^L)\right)$

$\tilde{\tilde{A}}_2=(\tilde{A}_2^U,\tilde{A}_2^L)=\left(a_{21}^U,a_{22}^U,a_{23}^U,a_{24}^U;H_1(\tilde{A}_2^U),H_2(\tilde{A}_2^U)\right)$,

$\left(a_{21}^L,a_{22}^L,a_{23}^L,a_{24}^L;H_1(\tilde{A}_2^L),\ H_2(\tilde{A}_2^L)\right)$

$\tilde{\tilde{A}}_1\ominus\tilde{\tilde{A}}_2=(\tilde{A}_1^U,\tilde{A}_1^L)\ominus(\tilde{A}_2^U,\tilde{A}_2^L)=[a_{11}^U-a_{21}^U,a_{12}^U-a_{22}^U,a_{13}^U-a_{23}^U,a_{14}^U-a_{24}^U;$

$\min\left(H_1(\tilde{A}_1^U),H_1(\tilde{A}_2^U)\right),\min\left(H_2(\tilde{A}_1^U),H_2(\tilde{A}_2^U)\right)]$,

$[a_{11}^L-a_{21}^L,a_{12}^L-a_{22}^L,a_{13}^L-a_{23}^L,a_{14}^L-a_{24}^L;$

$\min\left(H_1(\tilde{A}_1^L),H_1(\tilde{A}_2^L)\right),\min\left(H_2(\tilde{A}_1^L),H_2(\tilde{A}_2^L)\right)]$

Definition 2.6: The multiplication operation between the trapezoidal interval T2FSs (see Figure 2).

$\tilde{\tilde{A}}_1=(\tilde{A}_1^U,\tilde{A}_1^L)=\left(a_{11}^U,a_{12}^U,a_{13}^U,a_{14}^U;H_1(\tilde{A}_1^U),H_2(\tilde{A}_1^U)\right)$,

$\left(a_{11}^L,a_{12}^L,a_{13}^L,a_{14}^L;H_1(\tilde{A}_1^L),\ H_2(\tilde{A}_1^L)\right)$

$\tilde{\tilde{A}}_2=(\tilde{A}_2^U,\tilde{A}_2^L)=\left(a_{21}^U,a_{22}^U,a_{23}^U,a_{24}^U;H_1(\tilde{A}_2^U),H_2(\tilde{A}_2^U)\right)$,

$\left(a_{21}^L,a_{22}^L,a_{23}^L,a_{24}^L;H_1(\tilde{A}_2^L),\ H_2(\tilde{A}_2^L)\right)$

$\tilde{\tilde{A}}_1\otimes\tilde{\tilde{A}}_2=(\tilde{A}_1^U,\tilde{A}_1^L)\otimes(\tilde{A}_2^U,\tilde{A}_2^L)$

$=[a_{11}^U\times a_{21}^U,a_{12}^U\times a_{22}^U,a_{13}^U\times a_{23}^U,a_{14}^U\times a_{24}^U;$

$\min\left(H_1(\tilde{A}_1^U),H_1(\tilde{A}_2^U)\right),\min\left(H_2(\tilde{A}_1^U),H_2(\tilde{A}_2^U)\right)]$,

$[a_{11}^L\times a_{21}^L,a_{12}^L\times a_{22}^L,a_{13}^L\times a_{23}^L,a_{14}^L\times a_{24}^L;$

$\min\left(H_1(\tilde{A}_1^L),H_1(\tilde{A}_2^L)\right),\min\left(H_2(\tilde{A}_1^L),H_2(\tilde{A}_2^L)\right)]$

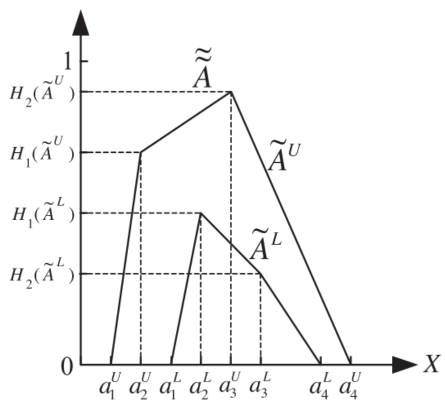

Figure 2: The upper trapezoidal MF $\tilde{A}_i^U$ and the lower trapezoidal MF $\tilde{A}_i^L$ of the interval T2F set $\tilde{\tilde{A}}_i$ [30].

Definition 2.7: The arithmetic operations between the trapezoidal interval T2FSs.

$\tilde{\tilde{A}}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = \left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)\right),$

$\left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(\tilde{A}_1^L),\ H_2(\tilde{A}_1^L)\right)$

$k\tilde{\tilde{A}}_1 = \left(k \times a_{11}^U, k \times a_{12}^U, k \times a_{13}^U, k \times a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)\right),$

$[k \times a_{11}^L, k \times a_{12}^L, k \times a_{13}^L, k \times a_{14}^L;$

$H_1(\tilde{A}_1^L),\ H_2(\tilde{A}_1^L)]$

$\frac{\tilde{\tilde{A}}_1}{k} = \left(\frac{a_{11}^U}{k}, \frac{a_{12}^U}{k}, \frac{a_{13}^U}{k}, \frac{a_{14}^U}{k}; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)\right),$

$\left(a_{11}^L/k, a_{12}^L/k, a_{13}^L/k, a_{14}^L/k; H_1(\tilde{A}_1^L),\ H_2(\tilde{A}_1^L)\right)$

Definition 2.8: The ranking value Rank $\left(\tilde{\tilde{A}}_i\right)$ of the trapezoidal interval T2FSs $\tilde{\tilde{A}}_i$ is defined as follows [20]:

$$Rank\left(\tilde{\tilde{A}}_i\right) = M_1(\tilde{A}_i^U) + M_1(\tilde{A}_i^L) + M_2(\tilde{A}_i^U) + M_2(\tilde{A}_i^L) + M_3(\tilde{A}_i^U) + M_3(\tilde{A}_i^L) - \frac{1}{4}\left(S_1(\tilde{A}_i^U) + S_1(\tilde{A}_i^L) + S_2(\tilde{A}_i^U) + S_2(\tilde{A}_i^L) + S_3(\tilde{A}_i^U) + S_3(\tilde{A}_i^L) + S_4(\tilde{A}_i^U) + S_4(\tilde{A}_i^L)\right) + H_1(\tilde{A}_i^U) + H_1(\tilde{A}_i^L) + H_2(\tilde{A}_i^U) + H_2(\tilde{A}_i^L)$$

where $M_p(\tilde{A}_i^j)$ denotes the average of the elements a_{ip}^j and $a_{i(p+1)}^j$, $M_p(\tilde{A}_i^j) = \frac{(a_{ip}^j + a_{i(p+1)}^j)}{2}$, $1 \le p \le 3$ denotes the standard deviation of the elements a_{ip}^j and $a_{i(p+1)}^j$, $S_p(\tilde{A}_i^j) = \sqrt{\frac{1}{2}\sum_{k=q}^{q+1}\left(a_{ik}^j - \frac{1}{2}\sum_{k=q}^{q+1} a_{ik}^j\right)^2}$ $1 \le q \le 3$, denotes the standard deviation of the elements $a_{i1}^j, a_{i2}^j, a_{i3}^j, a_{i4}^j$, $S_4(\tilde{A}_i^j) = \sqrt{\frac{1}{4}\sum_{k=1}^{4}\left(a_{ik}^j - \frac{1}{4}\sum_{k=1}^{4} a_{ik}^j\right)^2}$ $H_p(\tilde{A}_i^j)$ denotes the membership value of the element $a_{i(p+1)}^j$ in the trapezoidal MF $\tilde{A}_i^j$, $1 \le p \le 3$,$j \in \{U, L\}$, and$1 \le i \le n$.

2.3 Type-2 fuzzy TOPSIS

In the most of multi-criteria decision-making problems, crisp numbers and fuzzy sets should be utilized simultaneously [25].

It is assumed that there are X alternatives, where X ={$x_1, x_2, \ldots, x_n$} and Y criteria, where Y={$y_1, y_2, \ldots, y_n$}. There are k DMs $D_1, D_2, \ldots$, and D_k. The set Y of criteria can be divided into two sets Y1 and Y2, where they denote set of benefit, and cost attributes, respectively, Y1 ∩ Y2=∅ and Y1 ∪ Y2=Y. The details of the method is presented as follows [13],[12]:

Step 1: Using linguistic terms and interval T2FSs (Table 4), establish the decision matrix Dk of the kth decision-maker and construct the average decision matrix D̄, respectively, shown as follows:

Table 4: Linguistic terms and their corresponding interval T2F sets [12].

Linguistic Terms	Interval Type-2 Fuzzy Sets
Very Low (VL)	((0,0,0.1;1,1),(0,0,0,0.05;0.9,0.9))
Low (L)	((0,0.1,0.1,0.3;1,1),(0.05,0.1,0.1,0.2;0.9,0.9))
Medium Low(ML)	((0.1,0.3,0.3,0.5;1,1),(0.2,0.3,0.3,0.4;0.9,0.9))
Medium (M)	((0.3,0.5,0.5,0.7;1,1),(0.4,0.5,0.5,0.6;0.9,0.9))
Medium High (MH)	((0.5,0.7,0.7,0.9;1,1),(0.6,0.7,0.7,0.8;0.9,0.9))
High (H)	((0.7,0.9,0.9,1;1,1),(0.8,0.9,0.9,0.95;0.9,0.9))
Very High (VH)	((0.9,1,1,1;1,1),(0.95,1,1,1;0.9,0.9))

$$Y_k = (\tilde{\tilde{y}}_{ij}^k)_{m\times n} = \begin{array}{c} \\ y_1 \\ y_2 \\ \vdots \\ y_m \end{array}\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{y}}_{11}^k & \tilde{\tilde{y}}_{12}^k & \ldots & \tilde{\tilde{y}}_{1n}^k \\ \tilde{\tilde{y}}_{21}^k & \tilde{\tilde{y}}_{22}^k & \ldots & \tilde{\tilde{y}}_{2n}^k \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{y}}_{m1}^k & \tilde{\tilde{y}}_{m2}^k & \ldots & \tilde{\tilde{y}}_{mn}^k \end{bmatrix}\end{array} \tag{18}$$

$$\bar{Y} = (\tilde{\tilde{y}}_{ij})_{m\times n} \tag{19}$$

where $\tilde{\tilde{y}}_{ij} = \left(\frac{\tilde{\tilde{y}}_{ij}^1 \otimes \tilde{\tilde{y}}_{ij}^2 \otimes \tilde{\tilde{y}}_{ij}^3 \otimes \tilde{\tilde{y}}_{ij}^4}{k}\right)$, $\tilde{\tilde{y}}_{ij}$ is an interval T2F set, $1 \le i \le m, 1 \le j \le n, 1 \le p \le k$ and k denotes the number of decision-makers.

Step 2: Obtain the weighting matrix Wk of the criteria of the kth DMs and find the average weighting matrix W̄:

$$W_k = (\tilde{\tilde{w}}_i^k)_{1\times n} = \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & \cdots & y_n \end{array} \\ [\tilde{\tilde{w}}_1^k & \tilde{\tilde{w}}_2^k & \ldots & \tilde{\tilde{w}}_m^k] \end{array} \tag{20}$$

$$\bar{W} = (\tilde{\tilde{w}}_i)_{1\times m} \tag{21}$$

where $\tilde{\tilde{w}} = \left(\frac{\tilde{\tilde{w}}_i^1 \otimes \tilde{\tilde{w}}_i^2 \otimes \tilde{\tilde{w}}_i^3 \otimes \tilde{\tilde{w}}_i^4}{k}\right)$, $\tilde{\tilde{w}}_i$ is an interval T2F set, $1 \le i \le m, 1 \le j \le n, 1 \le p \le k$ and k denotes the number of decision-makers.

Step 3: Calculate the weighted decision matrix $\bar{Y}_w$,

$$\bar{Y}_w = (\tilde{\tilde{v}}_{ij})_{m\times n} = \begin{array}{c} \\ y_1 \\ y_2 \\ \vdots \\ y_m \end{array}\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{v}}_{11} & \tilde{\tilde{v}}_{12} & \ldots & \tilde{\tilde{v}}_{1n} \\ \tilde{\tilde{v}}_{21} & \tilde{\tilde{v}}_{22} & \ldots & \tilde{\tilde{v}}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{v}}_{m1} & \tilde{\tilde{v}}_{m2} & \ldots & \tilde{\tilde{v}}_{mn} \end{bmatrix}\end{array} \tag{22}$$

Step 4: Calculate Rank($\tilde{\tilde{v}}_{ij}$) of the interval T2F set $\tilde{\tilde{v}}_{ij}$ where $1 \le j \le n$. Obtain the ranking weighted decision matrix $\bar{Y}_w^*$:

$$\begin{aligned} Rank\left(\tilde{\tilde{A}}_i\right) = M_1(\tilde{A}_i^U) + M_1(\tilde{A}_i^L) + M_2(\tilde{A}_i^U) \\ + M_2(\tilde{A}_i^L) + M_3(\tilde{A}_i^U) + M_3(\tilde{A}_i^L) \\ - \frac{1}{4}\left(S_1(\tilde{A}_i^U) + S_1(\tilde{A}_i^L) + S_2(\tilde{A}_i^U)\right. \\ + S_2(\tilde{A}_i^L) + S_3(\tilde{A}_i^U) + S_3(\tilde{A}_i^L) \\ \left. + S_4(\tilde{A}_i^U) + S_4(\tilde{A}_i^L)\right) + H_1(\tilde{A}_i^U) \\ + H_1(\tilde{A}_i^L) + H_2(\tilde{A}_i^U) + H_2(\tilde{A}_i^L) \end{aligned} \tag{23}$$

$$\bar{Y}_w^* = \text{Rank}(\tilde{\tilde{v}}_{ij})_{m\times n} \tag{24}$$

where $1 \le i \le m, 1 \le j \le n$.

Step 5: Find the positive ideal solution $x^+ = (v_1^+, v_1^+, \ldots, v_m^+)$ and the negative ideal solution $x^- = (v_1^-, v_1^-, \ldots, v_m^-)$, where

$$v_i^+ = \begin{cases} max\{\text{Rank}(\tilde{\tilde{v}}_{ij}), \text{if } y_i \in Y_1 \\ min\{\text{Rank}(\tilde{\tilde{v}}_{ij}), \text{if } y_i \in Y_2 \end{cases} \quad 1 \le j \le n \tag{25}$$

$$v_i^- = \begin{cases} min\{\text{Rank}(\tilde{\tilde{v}}_{ij}), \text{if } y_i \in Y_1 \\ max\{\text{Rank}(\tilde{\tilde{v}}_{ij}), \text{if } y_i \in Y_2 \end{cases} \quad 1 \le j \le n \tag{26}$$

Y_1 denotes the set of benefit criteria, Y_2 denotes the set of cost criteria, and $1 \le i \le m$.

Step 6: Calculate the distances positive ideal solution and the negative ideal solution and find the relative degree of closeness C(xj) using the equations below:

$$d^{+}(x_j) = \sqrt{\sum_{i:1}^{m} \left(\text{Rank}(\tilde{v}_{ij}) - v_i^{+}\right)^2}, \tag{27}$$

$$d^{-}(x_j) = \sqrt{\sum_{i:1}^{m} \left(\text{Rank}(\tilde{v}_{ij}) - v_i^{-}\right)^2}, \tag{28}$$

$$C(x_j) = \frac{d^{-}(x_j)}{d^{+}(x_j) + d^{-}(x_j)} \tag{29}$$

Step 7: Finally, rank the closeness scores $C(x_j)$ in a descending order. Select the alternative with the highest $C(x_j)$.

3 A case study

The presented method is applied to SS problem in textile industry. There are three potential suppliers S_i(i=1,2,3) to be evaluated with seven criteria given in Figure 3; C1: Quality (Araz and Ozkaran [44]; Amid et al. [45]; Ha and Krishnan [46]; Weber et al. [47]; Dickson [43]), C2: Purchasing Cost (Kumar et al. [48]; Bevilacqua et al. [49]; Amid et al. [45]; Weber et al. [47]; Dickson [43]), C3: Delivery Performance (Araz and Özkaran [44]; Ha and Krishnan [46]; Weber et al. [47]; Dickson [43]), C4: Customer Relationships (Dickson [43]), C5: Payment Options (Dickson [43]), C6: Technical Capability (Dickson [43]; Liu and Hai [50]; Chen et al. [51]), and C7: References (Dickson [43]). DMs group consists of three experts DM_k (k=1,2,3).

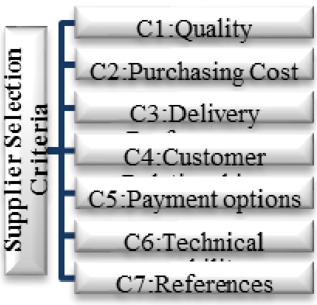

Figure 3: Criteria for SS problem.

3.1 Type-1 fuzzy TOPSIS solutions

The computational procedure for type-1 fuzzy TOPSIS is summarized as follows:

Step 1: The DMs (DM1, DM2, DM3) determine the evaluation criteria in order to evaluate suppliers. The related criteria is given in Figure 3.

Table 5: DMs' evaluations of importance of the criteria.

	DM1	DM2	DM3
C1:Quality	VH	VH	VH
C2:Purchasing Cost	M	VH	VH
C3:Delivery Performance	ML	ML	M
C4:Customer Relationships	VH	H	H
C5:Payment options	H	H	H
C6:Technical capability	H	H	H
C7:References	H	H	ML

The DMs use the linguistic weighting variables given in Table 2. The obtained subjective evaluations of each DM are given in Table 5.

Step 2: The DMs use the linguistic rating variables (given in Table 2) to assess the rating of alternative textile suppliers S_i (i=1, 2, 3) with respect to each criterion shown in Table 6.

Table 6: Evaluation for supplier with respect to each criterion.

DM1	S1	H	M	M	VH	MH	H	H
	S2	H	M	H	MH	H	M	H
	S3	H	M	H	H	MH	L	H
DM2	S1	M	H	H	H	VH	M	H
	S2	H	MH	VH	H	M	MH	VH
	S3	VH	MH	M	H	ML	VH	VH
DM3	S1	MH	MH	ML	H	H	MH	MH
	S2	MH	MH	MH	M	H	H	MH
	S3	M	M	H	MH	H	ML	MH

Step 3: Linguistic terms are transformed into trapezoidal fuzzy numbers and the fuzzy weight of each criterion is determined as Table 7. Table 8 gives aggregated fuzzy decision matrix.

Table 7: Fuzzy decision matrix for textile product.

		DM1	DM2	DM3
C1	S1	(7,9,9,10)	(3,5,5,7)	(5,7,7,9)
	S2	(7,9,9,10)	(7,9,9,10)	(5,7,7,9)
	S3	(7,9,9,10)	(9,10,10,10)	(3,5,5,7)
C2	S1	(3,5,5,7)	(7,9,9,10)	(5,7,7,9)
	S2	(3,5,5,7)	(5,7,7,9)	(5,7,7,9)
	S3	(3,5,5,7)	(5,7,7,9)	(3,5,5,7)
C3	S1	(3,5,5,7)	(7,9,9,10)	(1,3,3,5)
	S2	(7,9,9,10)	(9,10,10,10)	(5,7,7,9)
	S3	(7,9,9,10)	(3,5,5,7)	(7,9,9,10)
C4	S1	(9,10,10,10)	(7,9,9,10)	(1,3,3,5)
	S2	(5,7,7,9)	(7,9,9,10)	(3,5,5,7)
	S3	(7,9,9,10)	(7,9,9,10)	(5,7,7,9)
C5	S1	(5,7,7,9)	(9,10,10)	(7,9,9,10)
	S2	(7,9,9,10)	(3,5,5,7)	(7,9,9,10)
	S3	(5,7,7,9)	(1,3,3,5)	(7,9,9,10)
C6	S1	(7,9,9,10)	(3,5,5,7)	(5,7,7,9)
	S2	(3,5,5,7)	(5,7,7,9)	(7,9,9,10)
	S3	(0,1,1,3)	(9,10,10,10)	(1,3,3,5)
C7	S1	(7,9,9,10)	(7,9,9,10)	(5,7,7,9)
	S2	(7,9,9,10)	(9,10,10,10)	(5,7,7,9)
	S3	(7,9,9,10)	(9,10,10,10)	(5,7,7,9)

Table 8: Aggregation Fuzzy decision matrix.

	S1	S2	S3
C1	(5.00,7.00,7.00,8.67)	(6.33,8.33,8.33,9.67)	(6.33,8.00,8.00,9.00)
C2	(5.00,7.00,7.00,8.67)	(4.33,6.33,6.33,8.33)	(3.67,5.67,5.67,7.67)
C3	(3.67,5.67,5.67,7.33)	(7.00,8.67,8.67,9.67)	(5.67,7.67,7.67,9.00)
C4	(7.67,9.33,9.33,10.0)	(5.00,7.00,7.00,8.67)	(6.33,8.33,8.33,9.67)
C5	(7.00,8.67,8.67,9.67)	(5.67,7.67,7.67,9.00)	(4.33,6.33,6.33,8.00)
C6	(5.00,7.00,7.00,8.67)	(5.00,7.00,7.00,8.67)	(3.33,4.67,4.67,6.00)
C7	(6.33,8.33,8.33,9.67)	(7.00,8.67,8.67,9.67)	(7.00,8.67,8.67,9.67)

Step 4: Normalization is performed as seen in Table 9.

Step 5: Using Table 9 and the weights of criteria in Table 10, the weighted normalized fuzzy decision matrix is obtained as Table 11.

Table 9: Fuzzy normalized decision matrix for textile product.

	S1	S2	S3
C1	(0.52,0.72,0.72,0.90)	(0.66,0.86,0.86,1.00)	(0.66,0.86,0.86,1.00)
C2	(0.58,0.81,0.81,1.00)	(0.50,0.73,0.73,0.96)	(0.50,0.73,0.73,0.96)
C3	(0.38,0.59,0.59,0.76)	(0.72,0.90,0.90,1.00)	(0.72,0.90,0.90,1.00)
C4	(0.77,0.93,0.93,1.00)	(0.50,0.70,0.70,0.87)	(0.50,0.70,0.70,0.87)
C5	(0.72,0.90,0.90,1.00)	(0.59,0.79,0.79,0.93)	(0.59,0.79,0.79,0.93)
C6	(0.58,0.81,0.81,1.00)	(0.58,0.81,0.81,1.00)	(0.58,0.81,0.81,1.00)
C7	(0.66,0.86,0.86,1.00)	(0.72,0.90,0.90,1.00)	(0.72,0.90,0.90,1.00)

Table 10: The weights of criteria.

Criteria	Linguistic Weight	Weight
C1	MH	(0.5,0.7,0.7,0.9)
C2	VH	(0.9,1.0,1.0,1.0)
C3	M	(0.3,0.5,0.5,0.7)
C4	M	(0.3,0.5,0.5,0.7)
C5	H	(0.7,0.9,0.9,1.0)
C6	M	(0.3,0.5,0.5,0.7)
C7	M	(0.3,0.5,0.5,0.7)

Table 11: The weighted normalize fuzzy decision matrix for textile product.

	S1	S2	S3
C1	(0.26,0.51,0.51,0.81)	(0.33,0.60,0.60,0.90)	(0.33,0.58,0.58,0.84)
C2	(0.52,0.81,0.81,1.00)	(0.45,0.73,0.73,0.96)	(0.38,0.65,0.65,0.88)
C3	(0.11,0.29,0.29,0.53)	(0.22,0.45,0.45,0.70)	(0.18,0.40,0.40,0.65)
C4	(0.23,0.47,0.47,0.70)	(0.15,0.35,0.35,0.61)	(0.19,0.42,0.42,0.68)
C5	(0.51,0.81,0.81,1.00)	(0.41,0.71,0.71,0.93)	(0.31,0.59,0.59,0.83)
C6	(0.17,0.40,0.40,0.70)	(0.17,0.40,0.40,0.70)	(0.12,0.27,0.27,0.48)
C7	(0.20,0.43,0.43,0.70)	(0.22,0.45,0.45,0.70)	(0.22,0.45,0.45,0.70)

Step 6-7-8: Determine positive ideal solution and negative ideal solution using Eqs. (12-14). Then calculate the distance of each alternative from positive ideal solution and negative ideal solution through Eqs.(15 and 16).

Finally, the closeness coefficient of each alternative is calculated using Eq. (17). Results can be seen in Table 12.

Table 12: The distances of suppliers from fuzzy positive and negative ideal solutions and the fuzzy closeness coefficient CCi for all suppliers.

	d+	d-	CC	Ranking
S1	0.235	0.244	0.5096	1
S2	0.234	0.23	0.4961	2
S3	0.238	0.189	0.4419	3

It can be seen clearly in Table 12, according to type-1 fuzzy TOPSIS solution, the best supplier is Supplier 1.

3.2 Type-2 fuzzy TOPSIS solutions

In the first step, the importance criteria are determined by DMs using linguistic terms as Table 5 and interval T2FSs in Table 4. Decision matrix in Table 5 is composed of three alternatives S_i (i=1,2,3) and seven criteria (C1, C2,..., C7) mentioned previously. In the second step, using Table 2 and Table 5, T2F weights ($\tilde{\tilde{w}}_1$) for the evaluation criteria are obtained given in Table 13.

Table 13: Type-2 fuzzy weights ($\tilde{\tilde{w}}_1$) for the evaluation criteria.

$\tilde{\tilde{w}}_1$	((0.90,1.00,1.00,1.00,1.00,1.00),(1.00,1.00,1.00,1.00,0.90,0.90))
$\tilde{\tilde{w}}_2$	((0.70,0.80,0.80,0.90,1.00,1.00),(0.80,0.80,0.80,0.90,0.90,0.90))
$\tilde{\tilde{w}}_3$	((0.20,0.40,0.40,0.60,1.00,1.00),(0.30,0.40,0.40,0.50,0.90,0.90))
$\tilde{\tilde{w}}_4$	((0.80,0.90,0.90,1.00,1.00,1.00),(0.90,0.90,0.90,1.00,0.90,0.90))
$\tilde{\tilde{w}}_5$	((0.70,0.90,0.90,1.00,1.00,1.00),(0.80,0.90,0.90,1.00,0.90,0.90))
$\tilde{\tilde{w}}_6$	((0.70,0.90,0.90,1.00,1.00,1.00),(0.80,0.90,0.90,1.00,0.90,0.90))
$\tilde{\tilde{w}}_7$	((0.50,0.70,0.70,0.80,1.00,1.00),(0.60,0.70,0.70,0.80,0.90,0.90))

The next step is to determine the most appropriate supplier for the textile firm with T2FSs procedures. To do this, three DMs DM_k (k=1,2,3) evaluated three alternative supplier S_i (i=1,2,3) with respect to evaluation criteria (C1,..., C7), respectively. Evaluation scores of the alternatives are presented in Table 14.

Table 14: Evaluation scores of the alternatives.

		C1	C2	C3	C4	C5	C6	C7
DM1	S1	H	M	M	VH	MH	H	H
	S2	H	M	H	MH	H	M	H
	S3	H	M	H	H	MH	L	H
DM2	S1	M	H	H	H	VH	M	H
	S2	H	MH	VH	H	M	MH	VH
	S3	VH	MH	M	H	ML	VH	VH
DM3	S1	MH	MH	ML	H	H	MH	MH
	S2	MH	MH	MH	M	H	H	MH
	S3	M	M	H	MH	H	ML	MH

Based on Eqs. (20-22), T2F weighted evaluation matrix is obtained. Using Eqs.(23-24), the ranks, Rank($\tilde{\tilde{v}}_{ij}$), for alternatives are obtained shown in Table 15.

Table 15: The ranks for the alternatives.

	S1	S2	S3
C1	7.62	8.39	8.21
C2	6.95	6.64	6.31
C3	4.79	5.40	5.20
C4	8.59	7.34	8.05
C5	8.06	7.53	6.85
C6	7.19	7.19	6.01
C7	6.92	7.06	7.06

Then, using Table 15 and Eqs. (25 and 26), the ranks for the positive ideal and negative ideal solutions are determined given in Table 16.

Table 16: The ranks for the positive ideal and negative ideal solutions.

	C1	C2	C3	C4	C5	C6	C7
(+) ideal	7.88	7.62	6.08	6.26	7.09	5.61	6.08
(-) ideal	7.19	6.85	5.22	5.61	6.15	4.95	5.98

Using Eqs. (27 and 28), the distances from the positive ideal and negative ideal solutions are obtained in Table 17. Finally, using Eqs. (29), the closeness index and the rankings results are calculated and given in Table 17. According to Table 17, Supplier 1 is the most appropriate supplier for textile firm.

Table 17: The distances of suppliers from fuzzy positive and negative ideal solutions and the fuzzy closeness coefficient CCi for all suppliers.

	S1	S2	S3
d+	0.995	1.400	1.912
d-	2.205	1.720	1.021
C*	0.689	0.551	0.348
Ranking	1	2	3

Table 18 shows type 1 fuzzy TOPSIS and type 2 fuzzy TOPSIS solutions in term of the closeness index.

Table 18: The comparison of T1FT and T2FT solutions in term of the closeness index.

	S1	S2	S3
T1FT	0.5096	0.4961	0.4419
T2FT	0.6890	0.5510	0.3480

As can be seen Table 18, both methods, type 1 fuzzy TOPSIS and type 2 fuzzy TOPSIS, indicate S1 is the best supplier whereas S1 has bigger closeness index according to type 2 fuzzy TOPSIS.

Considering Table 3, the closeness index of S1 obtained with type 1 fuzzy TOPSIS indicates that S1 can be *recommended with low risk*, on the other hand, type 2 fuzzy TOPSIS score is classified as *approved*.

3.3 Sensitivity analysis

In this section, sensitivity analysis is conducted for T2F TOPSIS method to observe the effect of weight of criteria on the closeness index. To do this, firstly, the weight configurations for different cases shown in Table 19 are utilized.

Then, the closeness indices C* are estimated for each case using Eqs. (20-29). Table 20 illustrates the computed C* for each case.

According to the sensitivity analysis, as seen in Figure 4, ranking among the alternative suppliers can change due to different importance level of criteria.

Thus, the sensitivity analysis indicates that determining correct importance level of criteria is very vital.

Table 19: Importance level of criteria for different cases.

	C1	C2	C3	C4	C5	C6	C7
Case 1	M	M	M	M	M	M	M
Case 2	VH	VH	M	M	M	M	M
Case 3	VH	VH	VH	M	M	M	M
Case 4	VH	VH	VH	VH	M	M	M
Case 5	VH	VH	VH	VH	VH	M	M
Case 6	VH	VH	VH	VH	VH	VH	M
Case 7	VH	VH	VH	VH	VH	VH	VH

Table 20: The closeness index (C*) for each case.

	S1	S2	S3
Current Solution	0.689	0.551	0.348
Case 1	0.561	0.623	0.405
Case 2	0.544	0.640	0.413
Case 3	0.416	0.723	0.494
Case 4	0.485	0.594	0.505
Case 5	0.531	0.590	0.442
Case 6	0.561	0.619	0.405
Case 7	0.560	0.620	0.407

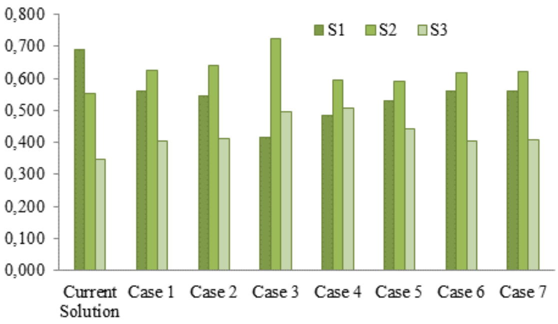

Figure 4: Sensitivity analyses.

4 Concluding remarks and future works

Supply chain management ensures many benefits to the organization such as reducing production costs, maximizing revenue, improving customer service, minimizing inventory levels, and increasing in competitiveness, customer satisfaction and profitability. SS is one of the most essential decisions due to the fact selection of appropriate suppliers significantly reduces purchasing costs. In the literature, SS has been considered as a MCDM problem and a wide range of mathematical methods have been presented to provide sufficient and accurate solutions. Multi-criteria decision-making methods provide a solution that decision-makers can select the best one in limited alternatives [40].

There always exists uncertainty and imprecision in real-life [39]. T2FSs are used in literature because of the fact that T1FSs are unable to deal with high complexity and uncertainty. Zadeh [11] presented T2FSs theory in 1975 as an extension of the concept of an ordinary fuzzy set called as a T1FS in order to overcome the limitations of T1FSs theory. Although T2FSs are more difficult to utilize than T1FSs, it is preferred by researchers to take into consideration uncertainty.

In this paper, TOPSIS method for multi-criteria group decision making within the environment of interval T2FSs have presented to handle the vagueness of the information. The proposed method is applied to SS process of a textile firm in Turkey. After giving the solutions of the type-1 fuzzy TOPSIS, same problem is solved through using T2F TOPSIS method. We compare type-1 fuzzy TOPSIS and T2F TOPSIS solutions. Considering quality, purchasing cost, delivery performance, customer relationships, payment options, technical capability, and references, three potential suppliers have been evaluated by three DMs. Solution indicated that supplier 1 is the most appropriate supplier in term of TOPSIS method under type-1 fuzzy set environment. According to TOPSIS method under T2FS environment, supplier 1 is also the best solution. Comparing T1FT with T2FT, supplier 1 had bigger closeness index according to T2FT. The results of the sensitivity analysis indicated weights of evaluation criteria are vital parameters affecting best alternative indicated by T2FT-TOPSIS. As a result, if uncertainty level in decision making environment is high, type-2 fuzzy multi criteria decision making methods give better and more proper solutions than type-1 fuzzy multi criteria decision making methods.

Type 2 Fuzzy multi criteria decision making methods can be used in any decision making problems involving high degree of uncertainty in term of selection criteria such as personnel selection in human resources department, product selection in procurement department, location selection in strategic planning department etc. Future research efforts can be devoted to the application of other MCDM methods such as ELECTRE, AHP, VIKOR, MOORA etc. under T2FSs.

5 References

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