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Tooth Tip Interference and Stress Analysis of High Contact Ratio Spur Gear Pairs Using an Optimized Design Tool

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ABSTRACT

Gears are structural elements that function to transmit power in mechanical systems. The power transmission is carried out by the rolling motion of the teeth pair. The average number of teeth that come into contact while the gears are running is called the contact ratio and this value is generally between 1 and 2 for gears with standard profile. As the number of teeth of the gear increases, the contact ratio may exceed 2, and the gears with a contact ratio greater than 2 are called high contact ratio (HCR) gears. For HCR gears, there is a higher risk of tooth tip interference when compared to the standard gears and they require contact point calculation to avoid interference. In this study, a mathematical tool is developed using MATLAB to analyze and avoid tool tip interference in HCR spur gears. The addendum diameter, dedendum diameter, pressure angle and modulus values are optimized using the developed mathematical tool in order to obtain an HCR geometry with minimal volume and no tip interference. In addition, the spur gear stresses are calculated using the AGMA standards. According to the results, it has been analytically proven that the load carrying capacity of the HCR spur gear is higher than that of the standard gear with the same diameter and volume. The developed optimization tool provided accurate and optimized geometries for the analyzed HCR spur gears.

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1. Introduction

The contact ratio in spur gears can be defined as the average number of teeth in contact between two meshing gears. In standard spur gears, this ratio is approximately between 1.2 and 1.6, and it is generally desired to be at least 1.3-1.4 for a smooth power transmission [1]. For the high contact ratio (HCR) spur gears, this ratio is between 2 and 3, which is meaning that the number of teeth in contact is at least two. In standard gears, tooth root stress analysis is performed according to a single tooth, but in HCR gears, this analysis is performed by sharing the total load on two teeth.

The contact ratio varies depending on the tooth diameter and pressure angle of the gears. When these values are brought to the desired level, it is possible to increase the contact ratio above 2. However, the increased tooth diameter in HCR gears increases the risk of tip interference. For this reason, a geometric optimization is needed for the HCR gears. The parameters that can be optimized are the module, tooth diameter, tooth root diameter, pressure angle, base diameter, backlash, and profile shift.

In the literature, there are several studies on the geometric optimization and force analysis in HCR gears. In a study conducted by Özgüven [2], a software is developed for calculating the dynamic forces on teeth and dynamic transmission errors, as well as the dynamic factors for gears and pinions in spur gears. Rackov et al. [3] demonstrated the Generalized Particle Swarm (GPS) optimization process to create an HCR spur gear pair without tooth tip interference or undercut teeth. By increasing tooth number and decreasing pressure angle to fix the maximum gear diameter, Rameshkumar et al. [4] replaced normal contact ratio (NCR) gears with HCR gears in 35-Ton military vehicles. They discovered that this increased load carrying capacity, decreased the noise and vibration, and slightly improved power-to-weight ratio.

Mohanty [5] employed an analytical approach to determine the load distribution in HCR gears. In the stated study an equivalent stiffness is determined using the bending, axial compression and contact stiffness.

Marimuthu and Muthuveerappan [6] performed finite element analysis in order to determine the exact load carrying capacity of an asymmetrical HCR spur gear set. In their study, the effect of backlash and friction between the gears are neglected. It is stated that asymmetrical HCR gears provided an improved load carrying capacity.

In the literature, there are also studies conducted on the design [7], efficiency [8], static and dynamic loading conditions [9], determination the critical loading points [10], nonlinear dynamic analysis [11], and the modification of the gear profiles [12] of HCR gears.
Within the scope of this study, tip interference analysis is performed for HCR gears by rotating the gear in a virtual environment developed in MATLAB. Addendum, dedendum, pressure angle, profile shift and backlash values are optimized by assigning a number of constraints, such as tip interference and sufficient gap between the teeth. In addition to the geometric optimization, the optimized HCR gear pair is compared to a normal contact ratio (NCR) gear pair in terms of the load carrying capacity.

2. Method

Spur gears work by rolling the pinion and gear with mutually pairing involute flanks. These involute flanks are formed by the projection of the base diameter of the gears depending on the angle scanned by the end points, as shown in Figure 1.
While evaluating the gear geometries, the formulations specified in the AGMA 913-A98 standard [13] are followed, and firstly, the diameters determining the gear surface are divided into certain intervals from the root diameter of the gear to the tip diameter. Pressure angle and tooth thickness are calculated, and then the involute values of the obtained pressure angles are determined. With the help of the values obtained, the geometric profiles of the gears are optimized by employing a MATLAB code. During the optimization, several boundary values are used such as the contact ratio, the tip interference and the limit value of the top land thickness to prevent case crashing during carburizing of hardened gears. In Figure 2 and Figure 3, the geometric dimensions used for the optimization calculations are summarized for the gear tooth profile and gear root profile, respectively.

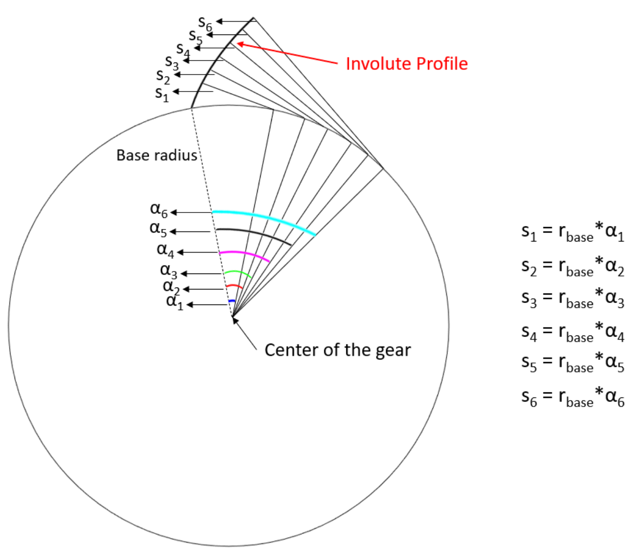

Figure 1. *The method of involute profile evaluation.*

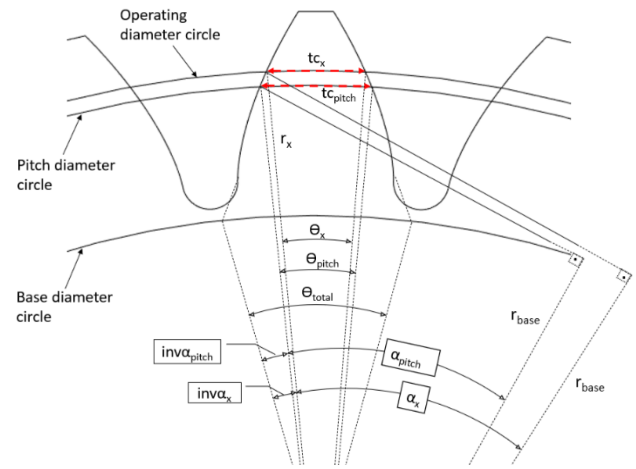

Figure 2. *Gear tooth profile evaluation.*

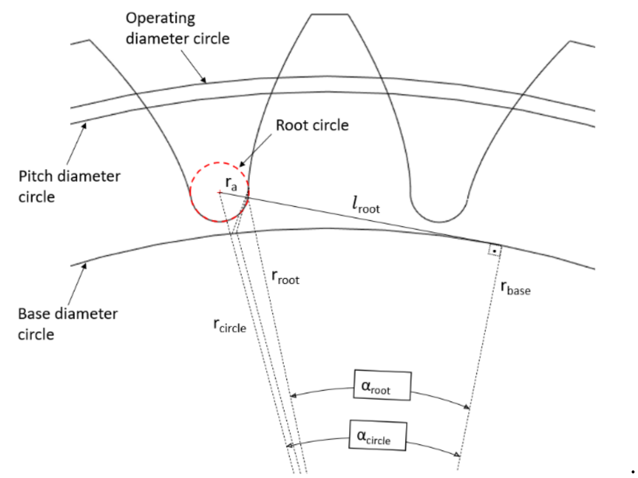

Figure 3. *Gear root profile evaluation.*

2.1 Constraints

2.1.1 Contact ratio

The contact ratio is usually between 2 and 3 for HCR gears, and this value can be expressed as the ratio of the minimum length between the point of contact where the contact starts and the point where the contact is interrupted to the diametral pitch of the gear in the transverse section. Figure 4 shows the contact points in the transverse section for the opposing pinion and gear.
By the nature of high-contact, the HCR gears transfer the load through the contact of two or three tooth pairs at the same time, and the contact points of the contacting tooth pairs are shown in Figure 4.
Active profile is the region formed by the contact points of the mutually working tooth pairs, and if the contact points are

combined on a line, the resulting line is called line of action (LA).
The point where the first gear contact starts is called SAP (Start of active profile), and from this moment, the two-tooth pair contact starts. This point where the gears contact is called SFTC (Start of first tooth contact).
Meanwhile, the contact of the tooth pair next to the first tooth pair is the second tooth pair that is in contact. This second contact point is called SSTC (Start of second tooth contact). The load transfer is provided by these two tooth pairs for a certain period of time. Then, the third pair of teeth ends the contact and then the first pair of teeth comes out of the contact. This point is shown by ESTC (End of second tooth contact) in Figure 4.
The third pair of teeth, whose contact ends, are separated at the EAP (End of active profile), where the active profile and the contact area end. The parameter of p_b which is illustrated in Figure 4 represents the base pitch and corresponds to the linear length between two adjacent teeth. The ratio of line of action to base pitch is called contact ratio.

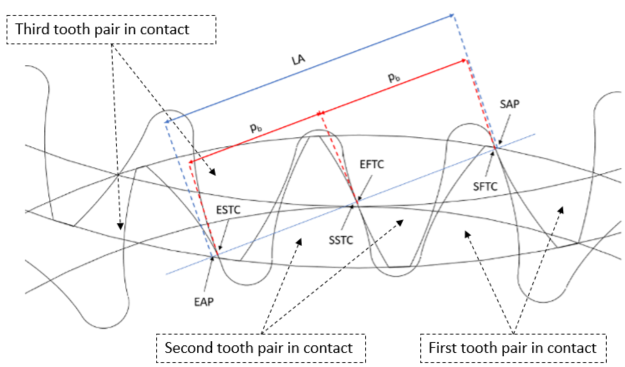

Figure 4. *Line of action of the HCR gear pair.*

Detailed equations on how to formulate the contact ratio can be found in the AGMA 913 A-98 standard [13]. As can be seen from the formulations, the factors that increase or decrease the contact ratio of the gear pair are the diameter of the addendum and the pressure angle of the gear. Dedendum diameter can be used as a factor if the gear is undercut or tip interference does not occur, and it does not directly affect the calculation of the coupling ratio.

2.1.2 Interference

In order to obtain an HCR gear, tooth diameter should be increased and the pressure angle should be reduced. This means that HCR gears in the same module have thinner and longer teeth than NCR gears, which is increasing the possibility of end interference. There must be a continuous clearance between the tip diameter of the pinion gear and the root diameter of the gear during the operation of the gear pair. This situation is shown in Figure 5.

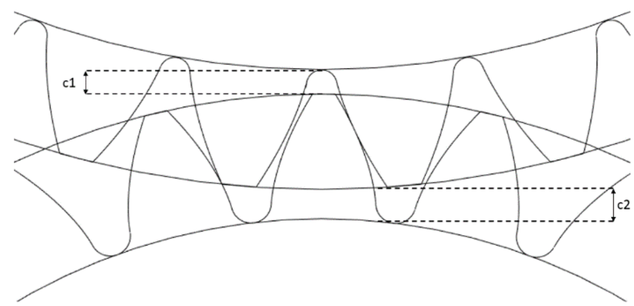

Figure 5. *Tip-root clearance between pinion and gear.*

Theoretically, c1 and c2 clearance values specified in Figure 5 are sufficient for tip-root interference, but in cases where real situations are considered, this value should be determined by considering manufacturing tolerances. In addition to the tip-root interference, the tip-flank interference is also one of the issues to be considered, and in this study, the efforts are mainly performed to prevent this situation.
A number of numerical implementations are carried out in the MATLAB code by gradually increasing the angle scanned by the gear until it enters and exits the contact region. The points with high risk of interference are determined. After performing iterations, an optimal gear geometry with no interference is presented to the user.
Figure 6 shows the condition with a high probability of tooth tip-involute face interference. Several improvements have been performed to prevent tip-face interference, considering backlash enhancement, tooth thinning, profile shifting, root modification and root crown. In this study, the interference is specifically prevented by using backlash boost and profile shift.

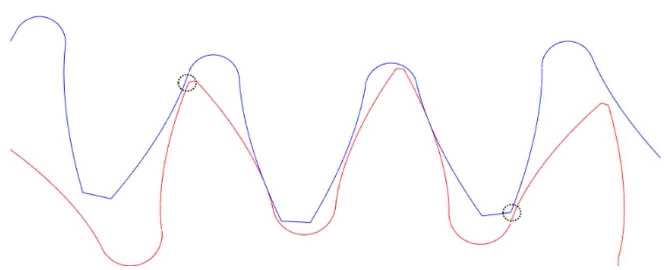

Figure 6. *Tooth tip-Involute face interference point.*

2.1.3 Top land thickness

Top land thickness is a critical factor for smooth load transfer, especially for spur gears with hardened surfaces used in aviation industry. Increasing the case hardness values of the surfaces is a method of obtaining high allowable stress values and is carried out by absorbing carbon from the surface in special furnaces. If the top land thickness is lower than the required case hardness depth, a case crash occurs and a highly brittle region is formed at the tip of the gear, which is increasing the risk of crack formation and the failure of the gear. Figure 7 shows a case with cracked gear in top land.

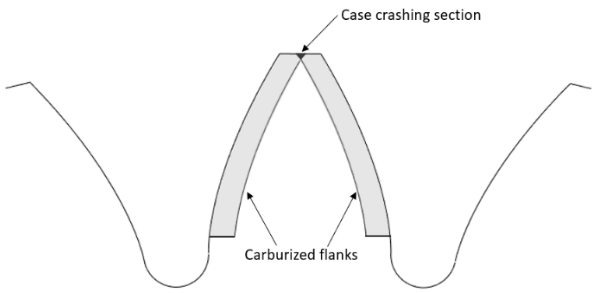

Figure 7. *Case hardness crash at the top land.*

There are a few methods that can be applied to prevent case crash in the top land region. For example, applying a copper plating process to top land can reduce the case hardness depth or increase the top land thickness. Compared to other processes, increasing the top land thickness is a method that does not require heat treatment and prevents case crash at the design stage of the gear. Figure 8 shows dimensions for the calculation of the top land thickness.

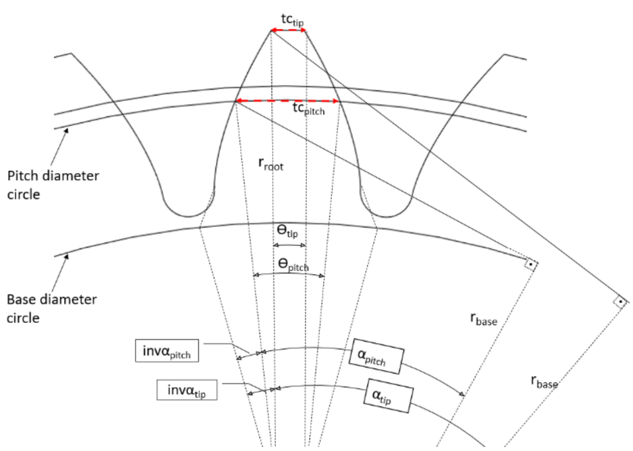

Figure 8. *Top land thickness evaluation.*

2.2 Load sharing and stress analysis

With the previously explained design method and boundary conditions, optimization process is carried out with the developed MATLAB code to obtain an HCR gear that would not allow any tip interference. Load distribution analysis is performed on gears meshing simultaneously under the obtained geometry and a sample loading condition. Since the contact ratio in standard gears is between 1 and 2, the load is transferred with a single tooth. As a general approach, the load acts from the HPSTC (Highest point of single tooth contact) point on the tooth as specified in the AGMA standard. By drawing a normal curve to the base diameter of the tooth from this point where the load is acting, a Lewis parabola is formed with the center point of the peak. Then, the load angle and radius are obtained, and the critical point where the maximum stress will occur is determined. Detailed geometric formulations in the determination process can be obtained from the AGMA 908 B-98 standard [14].

3. Results and discussion

The stress on spur gears can be divided into two parts, where one of them is the contact and the other is the bending stress. Contact stress is achieved by using systems such as normal load, speed and lubrication, which are formed on the surface based on the Hertzian contact theory. Bending stress, on the other hand, is the type of stress calculated by using the tangential load and geometrical factors, which are the highest at the critical point at the base of the tooth. The formulations specified in the AGMA 2101-C95 standard [15] is used within the scope of this study while calculating the stress analytically. The distribution of the loads used in the analysis is assumed to be equal for the meshing teeth. The stiffness values of the teeth should be calculated in order to determine the load distribution on the teeth in detail, and it requires other assumptions such as keeping the total normal load constant and the friction coefficient the same in each tooth [5]. Within the scope of this study, it is assumed that the total load will be equally distributed on the 2 meshing gears of the HCR gears.

Gear geometry is created using a standard tooth profile, which can be defined as the macro geometry obtained in the HCR gear design and analysis processes specified in the above sections. These processes determine the maximum diameter of the gear by keeping the number of teeth and the module constant. The resulting geometry is analyzed with the same level of load as the load condition on the HCR gear.

For the NCR case, the optimal gear geometry indicated in Figure 9 has been obtained. Afterwards, the optimal HCR geometry is determined for the same diameter of NCR gear pair, as shown in Figure 10. As can be seen in Figure 10, the tooth profile of the optimized HCR gear is significantly different from the optimized NCR gear. The main differences can be summarized as the general curved profile of the tooth, the thickness of the tooth, and the length of the top land region on the top of the tooth. The macro-geometric parameters of the optimized NCR and HCR gears are compared in Table 1.

After completing the geometric optimization, the optimized NCR and HCR gear pairs are exposed to the same loading conditions given in Table 2, and critical bending stresses are calculated. According to the results obtained, optimized NCR and HCR pinions are exposed to approximately 675 MPa and 467 MPa, respectively.

With this analytical study, an HCR gear pair is obtained by converting a spur gear pair with a normal contact ratio. During the processes, the number of modules and teeth are preserved, and the load carrying capacity is increased. The main difference of this study from the other HCR studies in the literature is to obtain HCR profile by preserving the number of teeth and diameter, and keeping the transmission ratio constant during increasing the transmitted power capacity in the gearbox.

4. Conclusion

In this study, the tip interference analysis of the HCR gear pair, which has the same values of diameter, module and tooth number as the NCR gear pair, is performed and the necessary pressure angle, addendum and dedendum diameter, backlash and profile shift values are optimized using MATLAB software. Afterwards, the HCR and NCR gears with the same maximum diameter are compared in terms of load carrying capacities, when the maximum bending stress values formed at the tooth base in Table 2 are examined. It has been observed that the bending stress in the HCR gear pair is 30.8% less than the NCR gear pair. This means that the load carrying capacity of the HCR gear with the data given in Table 1 is quite better than the NCR gear.

This theoretical study demonstrates the superiority of the HCR gears in terms of the load carrying capacity. In order to implement this study into the real life, gear manufacturing capabilities need to be further developed compared to today's circumstances, because the gear cutting tools used to create the gear profile need special manufacturing for each case.

There are a few limitations in this study. The allowable stress values used to control the stress levels obtained as a result of contact and stress analysis are taken from the AGMA standard. As a future work, it is necessary to determine experimental allowable stress values with devices such as single tooth bending machines in order to determine more accurate values for the HCR gear pairs. In addition, an equal load distribution is assumed for the contacting teeth pairs. For a more realistic approach, the exact load distribution between the teeth can be determined using finite element analysis and can be verified with experimental measurements on the teeth deformations. Nevertheless, this study provides important results to demonstrate the high capability of the HCR gears and shows directions for an appropriate HCR gear design.

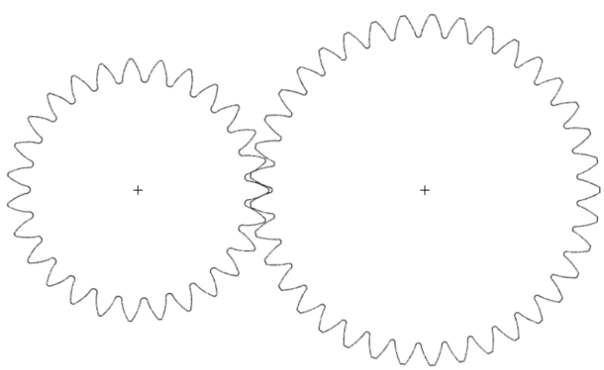

Figure 9. *Standard gear pair with 27 and 37 teeth, 2.5 mm module, and 25 degrees pressure angle.*

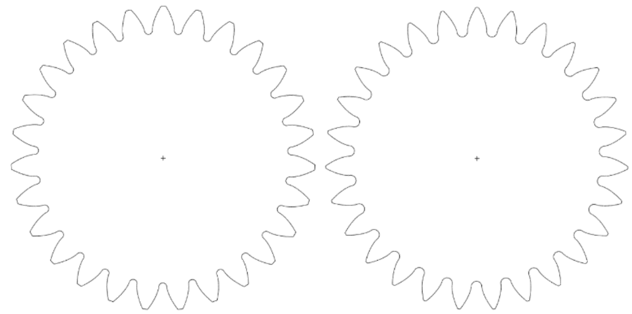

Figure 10. *NCR (at the left) and HCR (at the right) gear geometry with the data given in Table 1.*

Table 1. *Macro-geometries for HCR and NCR spur gear pairs.*

	Macro-geometries			
	NCR Gear Pair		HCR Gear Pair	
	Pinion	Gear	Pinion	Gear
Number of teeth	27	37	27	37
Module, mm	2.5			
Addendum coefficient	1		1.35	
Dedendum coefficient	1.25		1	
Pressure angle, °	25		21.7	
Profile shift coefficient	0.1	-0.1	0.1	-0.1
Total Backlash, mm	0		0.2	
Contact Ratio	1.48		2.07	

Table 2. *Kinematic and stress analyses results of HCR and NCR spur gear pairs for the data given in Table 1.*

	Analysis results	
	HCR Pinion	NCR Pinion
Load point diameter, mm	74.1619	67.3694
Pressure angle at load point, °	32.2563	24.7607
Load angle, °	35.6291	23.0419
Critical height, mm	5.673	2.8685
Critical thickness, mm	4.8527	5.3839
Transmitted Load, N	100	
Geometry factor	0.0014	0.002
Bending stress, MPa	467.3758	675.278

References

[1] Turan S., "Yüksek Kavrama Oranına Sahip Düz Dişli Çiftlerinin Uç-Kök Girişim Analizi ve Yük Taşıma Kapasitesinin Belirlenmesi", Master's Thesis, (2023), TOBB University of Economics and Technology.

[2] Özgüven H.N., "A non-linear mathematical model for dynamic analysis of spur gears including shaft and

bearing dynamics", Journal of Sound and Vibration, 145, (1991), 239-260.

[3] Rackov M., Vers M., Kuzmanovic Z.K., "HCR gearing and optimization of its geometry", Advanced Materials Research, 633, (2013), 117-132.

[4] Rameshkumar M., Sivakumar P., Sundaresh S., Gopinath K., "Load Sharing Analysis of High-Contact-Ratio Spur Gears in Military Tracked Vehicle Applications", Gear Technology, 1, (2010), 115-126.

[5] Mohanty S.C., "Tooth load sharing and contact stress analysis of high contact ratio spur gear in mesh", Journal of the Institution of Engineers (India), Part MC, Mechanical Engineering Division, 84, (2003), 66-70.

[6] Marimuthu P., Muthuveerappan, G., "Investigation of load carrying capacity of asymmetric high contact ratio spur gear based on load sharing using direct gear design approach", Mechanism and Machine Theory, 96, (2016), 52-74.

[7] Podzharov E., Mozuras A., "Design of high contact ratio spur gears cut with standard tools", Gear Technology, 20, (2003), 34-37.

[8] Anderson N.E., Loewenthal S.H., "Efficiency of nonstandard and high contact ration involute spur gears", Journal of Mechanisms, Transmissions, and Automation in Design, 108, (1986), 119-126.

[9] Aydeniz A.İ., "Yüksek kavrama oranının düz alın dişli mekanizmalarında statik ve dinamik davranışa etkileri", Doctoral Dissertation, (2007), Istanbul Technical University.

[10] Thirumurugan, R., Muthuveerappan, G., "Critical loading points for maximum fillet and contact stresses in normal and high contact ratio spur gears based on load sharing ratio", Mechanics Based Design of Structures and Machines, 39, (2011), 118-141.

[11] Huang K., Yi Y., Xiong Y., Cheng Z., Chen H., "Nonlinear dynamics analysis of high contact ratio gears system with multiple clearances", Journal of the Brazilian Society of Mechanical Sciences and Engineering, 42, (2020), 98.

[12] Yildirim N., Gasparini G., Sartori S., "An improvement on helicopter transmission performance through use of high contact ratio spur gears with suitable profile modification design", Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, 222, (2008), 1193-1210.

[13] AGMA 913 A-98, "Method for specifying the geometry of spur and helical gears", American Gear Manufacturers Association, 1998.

[14] AGMA 908 B-98, "Geometry factors for determining the pitting resistance and bending strength of spur, helical and herringbone gear teeth", American Gear Manufacturers Association, 1989.

[15] AGMA 2101-C95, "Fundamental rating factors and calculation methods for involute spur and helical gear (Metric version)", American Gear Manufacturers Association, 1995.