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A Comparison of the Mathematics Curriculums in Turkey and Germany in the Context of Algebra Learning Domain[1]

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ABSTRACT

In this study, Turkey and German state of North Rhine-Westphalia mathematics curriculum have been examined in the context of the algebra learning area. The learning outcomes have been investigated similarities and differences of the in terms of quantity and quality; classified in terms of knowledge and cognitive process dimensions of the revised Bloom's taxonomy. This study, in which the qualitative research approach was adopted, was carried out with the descriptive screening. Document analysis technique has been used in the data collection process. As a result of the analysis, it is determined that the learning outcomes are similar in terms of knowledge dimension; difference in terms of cognitive process dimension. While it is seen that the outcomes in the Turkey curriculum are in the analyze step as the highest cognitive level; Germany curriculum also includes outcomes fof higher level skills such as evaluate and create. Moreover, it is seen that the outcomes are mainly included in the procedural knowledge and apply step for both curriculums. Although it is seen that the outcomes in the Turkey curriculum are quantitatively higher than the outcomes in the Germany curriculum, it has been determined that the outcomes in terms of content are of a nature to cover each other. When the implementation suggestions were examined, it was seen that they were included in both curriculums in a detailed and rich way. On the other hand, when the distribution of the themes used in comparative education studies in the literature according to the countries was investigated, it was determined that the studies mostly focused on the elements of the curriculum (aim/goal, content, educational situations and testing situations). In the process of making sense of basic mathematical concepts and acquisition of high-level cognitive skills, teaching environments and programs can be designed by considering real life problems and process-based teaching models. On the other hand, since technology is a part of life, the education process can be integrated with digital games and/or stories, and programs and learning outcomes can be prepared in a way that includes teaching materials with digital content. In addition, different types of representation can be used in the mathematics teaching process and appropriate activities can be designed.

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INTRODUCTION

Each country has its own education system, and the education systems of other countries are examined in order to renew this system in line with the needs of the age and to eliminate its deficiencies. In this process, where the similarities and differences between countries are determined and appropriate results are drawn for their own education systems, we come across comparative education.

Although comparative education is perceived as an interdisciplinary field that uses some tools and perspectives of other disciplines and approaches educational issues in terms of comparison, it has started to be accepted as a separate discipline over time (Manzon, 2011). Thanks to comparative education, facts, trends and problems related to education can be analyzed and resolved from a broader perspective (Iliman-Püsküllüoğlu & Hoşgörür, 2017). Comparative education, whose main purpose is to solve educational problems, deals with education in the cultural, economic, political and social context, and allows the development of ideas that will form the basis of the education policies of countries (Türkoğlu, 1984). As a matter of fact, each country has its own education policy, and this policy considers it appropriate to make some innovations in the education system in order to meet the needs of the society, increase the quality of the services offered to the individual, and strengthen the relations between students, teachers and parents (Kuzu, Kuzu, & Gelbal, 2019).

When comparative education studies are examined, it is seen that there is more focus on mathematics and science education (Sadak, İncikabı & Pektaş, 2021). It is thought that it is important for individuals to renew themselves in the mathematics and science education so that they can think more innovative, more creative, more flexible, and more productive and act more planned, more skeptical and more competitive. These two fields comes to the fore in curriculums due to its features such as objectivity, the universality of its own truths and the perceived relationship of each nation to the desire for economic development (Atweh et al., 2007). Mathematics education is used in raising individuals to think critically, creatively, multi-dimensionally, to solve problems and to make sound decisions when necessary (Sezgin-Memnun, 2013); Science education (Council of Higher Education[CoHE]-World Bank, 1997) is of great importance in understanding and interpreting the environment they live in, and in producing new knowledge using existing knowledge. In this context, the developments and advances within the scope of the International Mathematics and Science Trends Study (TIMSS), which was conducted to help the development of mathematics and science education teaching, can be shown as an important reason for conducting comparative education studies. In addition, the developments and advances within the scope of the Program for International Student Assessment (PISA), which aims to evaluate the level of students in transferring the knowledge and skills acquired at schools to daily life, may be another important reason. As a matter of fact, with the performances obtained from such international large-scale exams, countries can see their own place in international platforms and make evaluations by comparing their education systems with other countries (Doğan & Barış, 2010).

Curriculum and the learning outcomes in this curriculum have an important place in the developing of individuals' abstract thinking, performing high performance in the cognitive domains and having high-level thinking skills. Focusing on desired knowledge and skills in individuals, the preparation of a curriculum that is compatible and complementary to each other and the learning outcomes suitable for this curriculum is closely related to the effective passing of the teaching and learning process (Kuzu, Çil, & Şimşek, 2019). Thus, taking into account the needs of the age, structuring the learning outcomes for the skills required by the changing world, will enable more permanent and effective learning (Çil, Kuzu, & Şimşek, 2019). It is very important for the learning outcomes to be clear, understandable and clear, to contain a single educational action, and to write hierarchically from concrete to abstract and from simple to complex as the upper class levels rise (Kuzu, Çil, & Şimşek, 2019).

While above points to be considered while preparing the learning outcomes are valid for each course, the appropriate classification approach should also be used, taking into account the content of the relevant curriculum and the structure of the learning outcomes. Bloom's taxonomy, which is one of the classification approaches, is a one-dimensional systematic classification type arranged according to a certain hierarchical structure and complexity by Bloom in 1956. Considering that it is not sufficient to examine the achievements in the mathematics curriculum in one dimension, as in other curricula, the achievements are classified in a two-dimensional systematic way consisting of knowledge and cognitive process dimensions. For this reason, the revised Bloom's Taxonomy acquisitions are more preferred in classification (Çelik, Kul, & Çalık-Uzun, 2018). It is thought that it would be more appropriate to use cognitive taxonomy since the mathematics curriculum includes more cognitive acquisitions. It is emphasized that the revised Bloom taxonomy is preferred more by scientists than cognitive taxonomies such as SOLO, Fink and Dettmer (Arı, 2013) and it is an effective taxonomy in interpreting the standards in mathematics (Näsström, 2009). Since Bloom's taxonomy (Bloom et al. (1956) is thought to be insufficient in acquiring in-depth information, its one-dimensional structure has been revised to a two-dimensional structure, knowledge and cognitive process, as a result of various criticisms over time (Anderson et al., 2001). Cognitive process dimension steps in the horizontal column of the revised Bloom taxonomy have passed from the noun form to the action form. In addition, considering that the synthesis step includes more complex mental processes than the evaluation step, its places were changed with the evaluation step and renamed and ordered as remember, understand, apply, analyze, evaluate and create (Anderson et al., 2001). Retrieval of relevant information from long-term memory in the cognitive process dimensions, in the remembering step, creating meaning from the instructional message as a verbal, written or graphical communication in the comprehension step, applying or using the process in a given situation in the application step, separating the material into its components in the analysis or analysis step, and There are actions to determine how the parts are related to each other/whole, to make judgments based on criteria and standards in the evaluation step, to bring the elements together in a consistent or functional structure in the creation or creation step, and to rearrange the elements in a new pattern or structure (Anderson et al., 2001). The knowledge dimension, which was added in order to express the cognitive terminology more clearly, took place in the vertical column of the taxonomy and consisted of factual, conceptual, procedural and metacognitive knowledge steps (Krathwohl, 2002). From the knowledge dimension, in the factual knowledge step, the basic elements that they must know in a discipline or in which they will solve the problem, the interrelationships between the basic elements that will work together among the broad structures in the conceptual knowledge step, how to do something in the procedural information step, methods and techniques, criteria for using skills, algorithms In the metacognitive knowledge step, there is information about cognition in general, such as the individual's awareness and knowledge of his own cognition process (Anderson et al., 2001). In the knowledge dimension what students know, and how they think in the cognitive process dimension are investigated and allowed to see the process from a student perspective (Kuzu, Çil, & Şimşek 2019). In both the knowledge and cognitive process dimensions, each step includes the other steps below, and as one goes to the higher levels, the abstraction, complexity and scope increase (for detailed information, see Anderson et al., 2001; Köğce et al., 2009; Krathwohl, 2002; Kuzu, Çil, & Şimşek, 2019).

Classifying the learning outcomes in the curriculum with this taxonomy allows what to teach and how to evaluate (Anderson et al., 2001). In the literature, it is possible to come across studies examining mathematics curriculum outcomes according to the revised Bloom taxonomy (e.g., Çelik et al., 2018; Çil et al., 2019; Hasić & Romano, 2018; Kablan, Baran, & Hazer, 2013; Kácovský et al., 2022; Kuzu, Çil, & Şimşek, 2019; Pizà-Mir, 2022). It is seen that the studies examining mathematics curriculum outcomes in the context of comparative education mostly focus on similarities and differences in terms of content and scope (e.g., Böke, 2002; Çiçek, Kuzu, & Çalışkan, 2021; Duygu, 2013; Galo, 2008; Özkan, 2006). On the other hand, it has been determined that there are limited number of comparative studies examining from taxonomic aspects (e.g., Bozkurt, Çırak-Kurt, & Tezcan, 2020). On the study conducted by Weissbach (2018), the PISA results of Turkey and Germany until 2015 were compared and it was stated that Germany was close to the average of the Organization for Economic Cooperation and Development (OECD) in mathematics in 2000 and that its scores in all fields increased in recent years. Although it was observed that Turkey's PISA scores increased in all areas until 2012, it was determined that it was below the OECD average. On the other hand, studies comparing the mathematics curriculum of Germany, which is a successful country in many fields, especially in engineering and technology, and which performs better than Turkey in international large-scale examinations (e.g. PISA, TIMSS), are still limited (e.g., Çiçek et al., 2021). Germany is a country located in the middle of Europe, neighboring many developed countries and attracting attention with its strong economy. As mentioned above, Germany has achieved more success in international exams (like PISA and TIMSS) than Turkey. It is thought that a comparison of the Turkish education system and the German education system, which has different characteristics compared to the Turkish education, will contribute to the development of the education system of Turkey, which is trying to enter the European Union. In addition, the reason for choosing the state of North Rhine-Westphalia is that it is the most populated state in Germany.

When TIMSS exams are examined in the context of "numbers", "algebra", "geometry", "data and probability" learning areas, it is seen that Turkish students have the lowest performance in algebra and geometry according to the results of 2019. According to the results of 2015, Turkish students have the two lowest performances. On the other hand, when the 5-8 mathematics curriculum in the Turkish education system is examined, there are "Numbers and Operations", "Algebra", "Geometry and Measurement", "Data Processing" and "Probability". It is seen that it consists of five learning areas and these areas are parallel to the TIMSS learning areas. Of these learning areas, algebra is included in all secondary school classes except grade 5. (MoNE, 2018). It is essential to find the equations established by describing symbols and letters, or to find the relationship between unknowns. It is based on" (Yenilmez & Avcu, 2009). Algebra acts as a common bridge and language between the sub-fields of mathematics and the elements of other disciplines in terms of conceptual and theoretical aspects, by providing individuals with an abstract thinking structure (Erbaş, Çetinkaya, & Ersoy, 2009). Algebra has an important position not only in mathematics, but also in every field and every stage of life, and algebra and algebraic thinking are used everywhere, from solving problems in daily events to solving problems in other sciences (Hawker & Cowley, 1997). Moreover, it has been observed that there is no comparison in terms of both the algebra learning area and the revised Bloom taxonomy. It is thought that it is important to work on the algebra learning area, which has an important place in helping individuals get abstract thinking, and also the revised Bloom's taxonomy, which is an effective taxonomy in interpreting the standards in mathematics.

In this study, firstly, the distribution of the themes used in studies comparing the mathematics curriculum of Turkey and other countries between the years 2002-2021 by countries was examined. Then, 5-8 mathematics curriculum in Turkey and German North Rhine-Westphalia (Nordrhein-Westfalen [NRW]) state have been examined in the context of algebra learning area and they were comparednd in line with following research questions.

1) What is the distribution of the outcomes in the 5-8 mathematics curriculum of Turkey and Germany in terms of knowledge and cognitive process dimensions of the revised Bloom's taxonomy in the context of algebra learning domain?

2) What are the similarities and differences between the sub-learning areas of the algebra learning field in the 5-8 mathematics curriculum of Turkey and Germany?

3) The learning outcomes of the algebra learning area in the 5-8 mathematics curriculum of Turkey and Germany;

• What are the similarities and differences in terms of number and content?

• What are the similarities and differences between the implementation proposals?

• What is the distribution of the revised Bloom's taxonomy in terms of knowledge and cognitive process dimensions?

METHOD

In this section, information about the research design, research instruments and processes, data analysis are presented in detail.

Research Design

Since this study aims to compare the mathematics curriculum in Turkey and Germany in the context of algebra learning area, it is a cross-national comparative education study in terms of subject. This study, in which the qualitative research approach was adopted, was carried out with the descriptive model. This model is a research approach that aims to describe a past or present situation as it is (Karasar, 2012). In this study, in the descriptive process, the literature on the subject (Ministry of National Education [MoNE], 2018; Ministerium für Schule und Bildung des Landes Nordrhein-Westfalen [MSB NRW], 2019) and the official websites of the countries were scanned.

Data Collection

Document analysis technique has been used in the data collection process. Document review is expressed as a meticulous and systematic examination of the content of all documents, including printed and electronic materials (Wach, 2013). Document review carried out in five main stages: Accessing documents, checking authenticity, understanding documents, analyzing data, and using data (Forster, 1994). In the accessing documents and checking authenticity processes, the official electronic pages of the ministries of education of the countries were investigated and the curriculums were obtained from here. In the understanding documents process, these documents and education system of both countries detailed investigations have been made. In this direction, some information has been obtained. Although the general framework of Germany curriculum is formed by the ministry, schools have the right to define a unique pedagogical concept. In this context, school learning program can be formed by the decisions taken by the school administration and teachers based on the core curriculum. Although the core curriculum is narrow in scope, as the name suggests, it leaves a wide range of action and decision to teachers and schools with the determination of the general framework. In the using data process, for the mathematics curriculum in Turkey, the 2018 secondary school mathematics curriculum was used as a document. For the mathematics curriculum in Germany, the sample school program, which was created with reference to the core curriculum prepared for the Gymnasium secondary education, which is located in the state of North Rhine-Westphalia, where the most populated, was used as a document. In this study, 23 outcomes under the title of "Algebra" learning area in Turkey's 5-8 mathematics curriculum (6th grade: M.6.2.1.1-M.6.2.1.3; 7th grade: M.7.2.1.1-M.7.2.1.3, M.7.2.2.1-M.7.2.2.4, 8th grade: M.8.2.1.1-M.8.2.1.4, M.8.2.2.1-M.8.2.2.6, M.8.2.3.1-M.8.2.3.3) were examined. 5-8 in the Germany curriculum, although there are 25 outcomes in total under the "Arithmetic/Algebra" learning area title, only nine of these 25 outcomes are for algebra, and so these nine outcomes (5th-6th grades: IK4, IK6, IK7; 7th-8th grades: IK4-IK7, IK9, IK10) were taken into account. Analyzing data process took place in four stages and was presented in detail in the "Data Analysis" section.

Data Analysis

The data analysis process in this study was carried out in four step. First of all, the sub-learning areas and outcomes of the algebra learning area of the German mathematics curriculum were translated into Turkish independently by two foreign language experts whose mother is Turkish and whose foreign language is German. It is stated by Hambleton (2005) that it is necessary for the translators to be competent in both languages and to have a good command of the cultural structure of both languages. Sample translation texts regarding the translation process are presented in detail in Table 1.

Table 1. *Sample translation texts on the translation process*

Learning Area	Grade level	Original Learning Outcome Text	FLE	Translation 1/ Translation 2
Arithmetic\Algebra	5-6	IK4. Die Schülerinnen und Schüler verbalisieren Rechenterme unter Verwendung von Fachbegriffen und übersetzen Rechenanweisungen und Sachsituationen in Rechenterme.	1	IK4. Öğrenciler, teknik kavramları/terimleri kullanarak cebirsel ifadeleri sözlü olarak ifade eder, aritmetik talimatları ve sözel olarak verilen durumları cebirsel ifadelere çevirir.
			2	IK4. Öğrenciler, teknik terimleri kullanarak aritmetik terimleri sözlü hale getirir ve aritmetik talimatları ve olgusal durumları aritmetik terimlere çevirir.
	7-8	IK10. Die Schülerinnen und Schüler wählen algebraische Lösungsverfahren für lineare Gleichungssysteme zielgerichtet aus und vergleichen die Effizienz unterschiedlicher Lösungswege.	1	IK10. Öğrenciler doğrusal denklem sistemleri için uygun cebirsel çözüm yöntemleri seçer ve farklı çözüm yöntemlerinin verimliliğini karşılaştırır.
			2	IK10. Öğrenciler, lineer denklem sistemleri için cebirsel çözüm yöntemlerini hedefli bir şekilde seçer ve farklı çözüm yöntemlerinin verimliliğini karşılaştırır.

In line with the main purpose of this study, only the sub-learning areas and learning outcomes belonging to the "Algebra" learning area of the "Arithmetic/Algebra" learning area in the German mathematics curriculum were taken into account. IK: Inhaltsbezogene Kompetenzen (Skills/learning outcomes related to the content); FLE: Foreign language expert

Borsa et al. (2012) suggest that both versions obtained at the end of the translation process should be compared and examined whether there are words and complex sentences that would make it difficult to understand. In this context, the intelligibility of both translation versions obtained and the extent to which they matched the original translation were evaluated by experts and made into a single version. Experts in this process is in the form: A professor who is an expert in mathematics education whose native language is Turkish; a PhD student who is expert in the areas of Curriculum and Instruction whose native language is both German and Turkish; a foreign language expert whose native language is Turkish, whose foreign language is German. To what extent the Turkish translation obtained is understandable linguistically; the extent to which the translation corresponds to the original text in terms of meaning and content has been evaluated with expert opinions. As the results of the expert reviews, it is observed that the average score is distributed three and above. The average scores for the outcomes during the evaluation process of the translations are presented in detail in Table 2.

Table 2. *Average scores regarding the evaluation process of translations*

		5-6			7-8					
Grade level/Learning outcome		IK4	IK6	IK7	IK4	IK5	IK6	IK7	IK9	IK10
5-8	L	4,8	4,2	4,6	3,6	4,4	4,8	4,6	4,4	4
	MC	4	3,6	4,2	3,4	4,6	4,8	4,4	4	3,8

Linguistic (L) 1: Never understandable, 2: Not understandable, 3: Undecided, 4: Understandable, 5: Completely understandable; Meaning annd Content (MC) 1: Never correspond, 2: Not correspond, 3: Undecided, 4: Correspond, 5: Completely correspond

In the second stage of the analysis process, studies comparing the mathematics curriculum of Turkey and other countries between the years 2002-2021 were analyzed with descriptive analysis. The data obtained in the descriptive analysis process are presented by organizing, interpreting and classifying, summarizing and interpreting in terms of predetermined themes. In addition, there are four stages in this analysis process: “creating the framework, processing the data in terms of thematic framework, defining the findings and interpreting the findings” (Yıldırım & Şimşek, 2018). In this process, the themes determined by Çiçek et al. (2021) (see Table 3) were used, and the inter-rater reliability was found to be .94 with the reliability calculation prepared by Miles and Huberman (1994). It is recommended that the inter-rater reliability be at least 80% (Miles & Huberman, 1994) and since it is higher than .80, it is seen to be reliable. The resulting disagreements were discussed again by the researchers and the reliability was calculated as .100 by providing a consensus.

Table 3. *Themes used in the compared studies (Çiçek et al., 2021)*

A	Comparison of the program elements (aim/goal, content, educational situations and testing situations)
B	Comparison of the program in terms of educational philosophies/vision
C	omparison of the program in terms of subjects/learning areas, sub-learning areas and outcomes
D	Comparison of the program in terms of general features

E	Comparison in the context of pedagogical content knowledge components
F	Comparison in terms of paradigm reflections

In the third stage of the analysis process, the similarities and differences between the sub-learning areas of the algebra learning area of the mathematics curriculum applied in Turkey and Germany; the number and content of the outcomes and the implementation suggestions for the outcomes were examined with descriptive analysis. In the fourth stage of the analysis process, the learning outcomes of the algebra area of the mathematics curriculum implemented in Turkey and Germany were examined in terms of knowledge and cognitive process dimension of the revised Bloom taxonomy. In this process, which has been carried out with descriptive analysis, a two-dimensional structure consisting of knowledge and cognitive process dimensions created by Krathwohl (2002) has been used as the coding key. In the classification of 23 outcomes belonging to the algebra learning area of the mathematics curriculum in Turkey, it has been taken into account (MoNE, 2018) the classification of Kuzu, Çil, & Şimşek (2019). In the classification of nine outcomes (MSB NRW, 2019) belonging to the algebra learning area implemented in the German state of North Rhine-Westphalia, a total of five experts, including two mathematics education experts, two assessment and evaluation experts, one education expert, have contributed by independent of each other. In the classification of the learning outcomes, not only the educational action in the sentence, that is, the verb stem, but the entire outcome sentence is taken into account. In the classification of the learning outcomes, not only the educational action in the sentence, that is, the verb stem, but the entire outcome sentence is taken into account. In this process Miles and Huberman (1994) method (Agreement/Total Agreement) was used and if the inter-rater reliability was .80 and above the classification of the learning outcome is completed. On the other hand, for the outcomes where consensus was not reached or a low level of consensus was reached, the experts came together and at the end of the discussion process, a common denominator was reached and the final step of the outcomes was decided. If five experts choose the relevant step corresponding to the outcome:1; if four experts choose: .80; if three experts choose: .60; if two experts choose: .40; if an expert chose: .20; a score of .00 if no expert chose it (Table 4).

Table 4. *Distribution of expert opinions on the classification of the outcomes of the algebra learning area of the German mathematics curriculum in terms of knowledge and cognitive process dimensions*

	Knowledge Dimension								Cognitive Process Dimension											
	IC				D				IC						D					
	F	Co	P	M	F	Co	P	M	R	U	Ap	An	E	Cr	R	U	Ap	An	E	Cr
IK4	,40		,60				1,00			,40	,60						1,00			
IK6		,40	,60			,20	,80			,20	,80						1,00			
IK7			1,00				1,00				1,00						1,00			
IK4	,20	,60	,20			1,00				,20	,40	,40					,20	,80		
IK5	,20	,40	,40		,20		,80				,20			,80			,20			,80
IK6		,60	,40			1,00						,40	,20	,40				,80		,20
IK7			1,00				1,00				,20		,80						1,00	
IK9			1,00				1,00			,20	,20	,60					,20	,80		
IK10			1,00				1,00			,40			,60						1,00	

IC: Independent coding; D: Discussion; F: Factual, Co: Conceptual, P: Procedural, M: Metacognitive; R: Remember, U: Understand, Ap: Apply, An: Analyze, E: Evaluate, Cr: Create

Research Ethics

This study is an international comparative education study in terms of its subject, and there is a human or animal subject within the scope of the research; Data collection methods such as experiments, observations, questionnaires or interviews were not used. For this reason, it was not necessary to obtain an ethics committee report.

FINDINGS AND INTERPRETATION

In this section, first of all, the studies comparing the curricula applied in Turkey and other countries between the years 2002-2021 were examined, and the distribution of the themes covered in these studies on the basis of countries and years was presented. Then, the comparison of the 5th-8th mathematics curriculum applied in Turkey and the German state of North Rhine-Westphalia in the context of the algebra learning area is explained in line with the sub-problems of the research.

The number of comparative education studies for mathematics curriculum conducted in Turkey between 2002 and 2021 and their distribution by country, subject and year

In this section, the studies conducted in Turkey between 2002-2021 comparing the mathematics curriculum of Turkey and other countries are examined; A total of 29 academic studies, including 11 articles (Altıntaş & Görgen, 2014; Bacakoğlu & Işık-Tertemiz, 2021; Batur et al., 2021; Bozkurt et al., 2020; Çiçek et al., 2021; Erdoğan et al., 2016; Güzel et al., 2010; Kul & Aksu, 2016; Sugandi & Delice, 2014; Tan-Şişman & Karataşlı, 2020; Yağan, 2020) and 18 master's theses (Abid, 2017; Böke, 2002; Çetinbağ, 2019; Çoban, 2011; Duygu, 2013; Erbilge, 2019; Galo, 2008; Güzel, 2010; Ismail-Amet, 2021; Karakaya, 2021; Karataşlı, 2019; Kaytan, 2007; Özkan, 2006; Öztürk, 2020; Serçe, 2020; Sugandi, 2015; Tezcan, 2016; Uğur-Arslan, 2015), were found. In these studies, it was found that 4 master's theses (Güzel, 2010; Sugandi, 2015; Tezcan, 2016; Karataşlı, 2019) were presented as articles (Bozkurt et al., 2020; Güzel et al., 2010; Tan-Şişman & Karataşlı, 2020; Sugandi & Delice, 2014) at the same time. Among these four studies conducted as both thesis and article, the thesis was considered. Because the studies were only considered once, the total number of studies was set at 25. The distribution of comparative education studies conducted in Turkey for the mathematics curriculum between 2002-2021 according to countries, themes and years is presented in detail in Table 5.

Table 5. *The Distribution of comparative education studies conducted in Turkey for the mathematics curriculum between 2002-2021 by countries, themes and years*

	A	B	C	D	E	F
Germany	Güzel (2010)	Güzel (2010)	Çiçek et al. (2021)			
USA	Çoban (2011) Duygu (2013)	Duygu (2013)	Batur et al. (2021) Tezcan (2016)			
Australia	Karataşlı (2019) Yağan (2020)		Karataşlı (2019)	Yağan (2020)		
Belgium	Özkan (2006)					
Indonesia						Sugandi (2015)
Estonia	Serçe (2020)					
South Korea	Altıntaş & Görgen (2014) Duygu (2013)	Duygu (2013)	Batur et al. (2021) Uğur-Arslan (2015)	Altıntaş & Görgen (2014)	Kul & Aksu (2016)	
Hong Kong-China	Duygu (2013) Erbilge (2019)	Duygu (2013)				
England	Böke (2002) Çoban (2011) Kaytan (2007)			Kaytan (2007)		
Libya	Abid (2017)	Abid (2017)				
Canada	Çetinbağ (2019) Erbilge (2019) Güzel (2010) Öztürk (2020) Serçe (2020)	Güzel (2010)	Karakaya (2021)			
Kosova			Galo (2008)			
Singapore	Duygu (2013) Kaytan (2007) Özkan (2006) Serçe (2020)	Duygu (2013)	Bacakoğlu & Işık-Tertemiz (2021) Batur et al. (2021) Erdoğan et al. (2016) Karakaya (2021) Tezcan (2016) Uğur-Arslan (2015)	Kaytan (2007)	Kul & Aksu (2016)	
New Zealand	Duygu (2013)	Duygu (2013)	Batur et al. (2021)			
Greece	İsmail-Amet (2021)		İsmail-Amet (2021)			

A: Comparison of the program elements (aim/goal, content, learning experiences and evaluation); B: Comparison of the program in terms of educational philosophies/vision; C: Comparison of the program in terms of subjects/learning areas, sub-learning areas and achievements; D: Comparison of the program in terms of general features; E: Comparison in the context of pedagogical content knowledge components; F: Comparison in terms of paradigm reflections.

When Table 5 is examined, it is noticeable that some studies compare more than one theme (e.g.

Karataşlı, 2019; Yağan, 2020), some themes are included in more than one study for the same country (e.g. Böke, 2002; Çoban, 2011; Kaytan, 2007 for thema A in the study on England), and in some studies more than one country is compared at the same time (e.g. Güzel, 2010; Duygu, 2013). Moreover, Table 5 shows that comparative education studies conducted in Turkey between 2002 and 2021 for the mathematics curriculum were most frequently conducted on the countries Singapore (f=11), Canada (f=6) and South Korea (f=5). Considering that more than one comparison was made in some studies, it is seen that 13 comparisons have been made on Singapore, while this number is seven for South Korea and Canada. In the studies on Singapore and South Korea, it is seen that comparisons are made for five themes. It has been determined that there is no study on the F theme for these two countries with the most studies on. In studies on Germany, USA, Austria, Canada and New Zealand, comparisons were made for three themes. The study on these four countries, excluding Austria, focused on the A, B and C themes; In Austria, it was determined that the A, C and D themes were focused on. In the studies on Hong Kong-China, England, Libya and Greece, it is seen that comparisons are made for two themes. For Indonesia, Estonia and Kosovo, comparison was made on only one theme. On the theme A of the study on Estonia; it is seen that the study on Kosovo was carried out on the C theme. The point that draws attention here is that the study on Indonesia is on the F theme. As a matter of fact, no comparative education study has focused on the F theme, except for the study on Indonesia. The number of themes used in studies comparing mathematics curriculum of Turkey and other countries and their distribution by country are presented in detail in Table 6.

Table 6. *The number of themes in the mathematics curriculum studies conducted in Turkey between 2002-2021 and their distribution by country*

	Germany	USA	Australia	Belgium	Indonesia	Estonia	South Korea	Hong Kong-China	England	Libya	Canada	Kosovo	Singapore	New Zeland	Greece	TNTU
A	✓	✓✓	✓✓	✓		✓	✓✓	✓✓	✓✓✓	✓	✓✓✓✓✓		✓✓✓✓	✓	✓	26
B	✓	✓					✓	✓		✓	✓		✓	✓		8
C	✓	✓✓	✓				✓✓				✓	✓	✓✓✓✓✓✓	✓	✓	16
D			✓				✓		✓				✓			4
E							✓						✓			2
F					✓											1
TNS	3	5	4	1	1	1	7	3	4	2	7	1	13	3	2	

TNS: It gives the total number of studies. Some studies included more than one theme at the same time.
TNTU: It gives the total number of themes used. Some themes were included in more than one study for the same country. In addition, in some studies, more than one country was discussed for the same theme.

Looking at Table 6, it was found that the most (f=26) comparisons are made from the elements of the program (aims/goals, content, learning experiences and evaluation). At the same time, it was noted that there were quite a lot of comparison studies (f=16) made in terms of the subjects/learning areas, sub-learning areas and achievements of the program. On the other hand, it has been observed that there are also comparative studies in terms of the educational philosophies/vision of the program (f=8) and the general features of the program (f=4). In addition, it was found that there are also studies in the literature that make comparisons in the context of pedagogical content knowledge components (f=2) and paradigm reflections of the programs (f=1)

Findings on the similarities and differences between the sub-learning areas of the algebra learning area in the 5 th-8th mathematics curriculum of Turkey and Germany

In this section, the similarities and differences between the sub-learning areas of the algebra learning field in the 5th-8th mathematics curriculum of Turkey and Germany are examined, and the findings are presented in Table 7.

Table 7. *Sublearning areas of the algebra learning areas of the mathematics curriculum in Turkey and Germany*

Country	Learning Area (LA)	Grade	Sub-Learning Area (SLA)
Turkey	Algebra	5th grade	-
		6th grade	SLA1. Algebraic Expressions
		7th grade	SLA1. Algebraic Expressions SLA2. Equality and Equation
		8th grade	SLA1. Linear Equations SLA2. Algebraic Expressions and Identities SLA3. Inequalities
Germany	Arithmetic/Algebra	5th and 6th grade	SLA1. Conceptualization: Arithmetic Term
		7th and 8th grade	SLA1. Term and Variable: Variables as Variable, as Placeholder and as Unknown; Term Transformations SLA2. Solutions Methods: Algebraic and Graphical Solution Methods (Linear Equations and Systems of Linear Equations with Two Variables, Elementary Fractional Equations)

In accordance with the main aim of this study, only the sub-learning areas and learning outcomes belonging to the "Algebra" learning area of the "Arithmetic/Algebra" learning area in the German mathematics curriculum were taken into account.

According to Table 7, it is seen that there is a difference between the sub-learning areas of the algebra learning area of the 5th-8th mathematics curriculum of the countries. It can be observed that the number of sub-learning areas in the Germany curriculum is less than the Turkey curriculum. While there is no algebra sub-learning area in the 5th grade in the Turkey curriculum; there is one algebra sub-learning area in the 6th grade; two in the 7th grade; three in the 8th grade, so six sub-learning areas are determined for four grade levels in total. In the Germany curriculum, it is seen that there is no separation for the 5th and 6th grades and a sub-learning area for algebra is determined for both grade levels. It is observed that there is no separation in the same way for the 7th and 8th grades, and there are two sub-learning areas for algebra for both grade levels. Although the sub-learning areas of the Turkey curriculum differ quantitatively, it can be said that the sub-learning areas mostly overlap. For istance, the "SLA1. Algebraic Expressions" sub-learning area of the 6th grade level of the curriculum in Turkey and the "SLA1. Conceptualization: Arithmetic Term" sub learning area of the 5th-6th grade level of the Germany curriculum overlap each other. M.6.2.1.1. learning outcome of the SLA1 in Turkey curriculum correlates with the IK4 learning outcome SLA1 in Germany curriculum. On the other hand, it can be said that the number of sub-learning areas covered in the Germany program is small, but the content is broad. For istance, the "SLA2. Solutions Method: Algebraic and Graphical Solution Methods" sub learning area of the 7th-8th grade level of the Germany curriculum while it includes the expressions in the Turkey curriculum, it also includes the relationship with concepts such as area-volume.

Findings on the similarities and differences in the number and content of algebra learning outcomes in the 5th-8th mathematics curriculum of Turkey and Germany

In this section, the similarities and differences in the number and content of algebra learning outcomes in the 5th-8th mathematics curriculum of Turkey and Germany are examined, and the findings are presented in Table 8. As mentioned before, only the sub-learning areas and learning outcomes belonging to the "Algebra" learning area of the "Arithmetic/Algebra" learning area in the German mathematics curriculum were taken into account.

According to Table 8, it is seen that Turkish and German mathematics curriculum have a similar approach in terms of algebra learning outcomes. Learning outcomes that should be taught to students in both curricula are expressed in short, clear and simple present tense. While the outcomes in the Turkey curriculum are separated on a class basis, there is no such separation in the Germany curriculum. The learning outcomes in the Germany curriculum are included by combining two grade levels, like 5th-6th and 7th-8th grade. In addition, before the learning outcomes are given in the Germany curriculum, the 5th-6th and 7th-8th topics and concepts of the grade are included. In the Turkey curriculum, the subject headings are given separately for each grade level and the learning outcomes are listed under each subject. There are quantitatively more learning outcomes in the Turkey curriculum compared to the Germany curriculum. There are 23 learning outcomes in the learning area of algebra in the Turkish mathematics curriculum for the 5th-8th class levels. Although there are 25 learning outcomes in total under the "Arithmetic/Algebra" learning area in the Germany curriculum, only 9 of them are for algebra. The Turkey curriculum assigns more learning outcomes but the Germany curriculum includes the content of the Turkey curriculum.

Table 8. *Learning outcomes of the algebra learning area of the mathematics curriculum in Turkey and Germany*

Turkey	Germany
5th Grade Learning Outcomes - **6th Grade Learning Outcomes** **SLA1. Algebraic Expressions** M.6.2.1.1. Writes an algebraic expression suitable for a verbally given situation and a verbal situation suitable for a given algebraic expression. M.6.2.1.2. Calculates the value of the algebraic expression for different natural number values that the variable will take. M.6.2.1.3. Explain the meaning of simple algebraic expressions. **7th Grade Learning Outcomes** **SLA2. Algebraic Expressions** M.7.2.1.1. Makes addition and subtraction operations with algebraic expressions. M.7.2.1.2. Multiplies an algebraic expression by a natural number. M.7.2.1.3. Expresses the rule of the number patterns with a letter, finds the desired term of the pattern whose rule is expressed with a letter. **SLA3. Equality and Equation** M.7.2.2.1. Understands the principle of conservation of equality. M.7.2.2.2. Recognizes an equation with a first degree unknown and sets up an equation with a first degree unknown in accordance with given real-life situations. M.7.2.2.3. Solves first degree equations with one unknown. M.7.2.2.4. Solves problems that require establishing an equation with a first degree unknown. **8th Grade Learning Outcomes** **SLA4. Algebraic Expressions and Identities** M.8.2.1.1. Understands simple algebraic expressions and writes them in different formats. M.8.2.1.2. Multiplies algebraic expressions. M.8.2.1.3. Explain identities with models. M.8.2.1.4. Factors algebraic expressions. **SLA5. Linear Equations** M.8.2.2.1. Solves first degree equations with one unknown. M.8.2.2.2. Recognizes the coordinate system with its properties and shows ordered pairs. M.8.2.2.3. Expresses how one of the two variables, which have a linear relationship between them, changes depending on the other, with a table and an equation. M.8.2.2.4. Draws the graph of linear equations. M.8.2.2.5. Creates and interprets equations, tables and graphs of real life situations with linear relationships. M.8.2.2.6. Explain the slope of the line with models, relate linear equations and graphs with slope. **SLA6. Inequalities** M.8.2.3.1. Writes mathematical sentences suitable for daily life situations involving inequality with a first degree unknown. M.8.2.3.2. Represents inequalities with a first degree unknown on the number line. M.8.2.3.3. Solves inequalities with a first degree unknown.	**5th and 6th Grade Learning Outcomes** **SLA1. Conceptualization:** Arithmetic Term The students, IK4. Verbalize arithmetic terms by using technical terms and translate arithmetic instructions and factual situations into arithmetic terms. IK6. Use variables to describe simple factual relationships and to phrase calculation rules. IK7. Put numbers into expressions with variables and calculate their value. **7th and 8th Grade Learning Outcomes** **SLA2. Term and Variable:** Variables as Variable, as Placeholder and as Unknown; Term Transformations **SLA3. Solutions Methods:** Algebraic and Graphical Solution Methods (Linear Equations and Systems of Linear Equations with Two Variables, Elementary Fractional Equations) The students, IK4. Interpret variables as variables to describe relations, as placeholders in terms and arithmetic laws, and as unknowns in equations and systems of equations. IK5. Set up terms as a calculation rule for relations and for calculating areas and volumes. IK6. Set up equations and inequalities to phrase conditions in factual situations. IK7. Transform terms, including fractional terms, purposeful, and correct incorrect term transformations. IK9. Determine solution sets of linear equations and systems of linear equations as well as fractional equations using suitable methods and interpret them in the factual context. IK10. Select algebraic solution methods for systems of linear equations purposefully and compare the efficiency of different solution methods.

The reason for this difference is that the outcomes in the Turkey curriculum are given in more detail and the Germany curriculum more generally. In the Germany curriculum, some learning outcomes are combined and expressed as a single learning outcome. In other words; a learning outcome in the Germany curriculum is expressed in two or three different learning outcomes sentences in the Turkey curriculum. It is noteworthy that the 6th grade algebra learning outcomes of the Turkey curriculum are both quantitatively and qualitatively similar to the 5th and 6th grade algebra learning outcomes of the Germany curriculum. When the 7th and 8th grade learning outcomes of both countries are examined, it is seen that there are quantitative differences. In addition, when the relationship between the uses of algebra in real life situations is examined, it is seen that both countries include activities and experiences related to real life situations at the level of learning outcomes.

Findings on the similarities and differences between the implementation suggestions for the learning outcomes of the algebra learning area in the 5th-8th mathematics curriculum of Turkey and Germany

In this section, the similarities and differences between the implementation suggestions for the learning outcomes of the algebra learning area in the 5th-8th mathematics curriculum of Turkey and Germany are examined, and the findings are presented in Table 9.

Table 9. *Implementation suggestions for the algebra learning area of the mathematics curriculum in Turkey and Germany*

Turkey	Germany
• Appropriate models are used in addition and subtraction with algebraic expressions. • Studies/activities to find the rule by transforming the relationships in daily life situations or shape patterns into patterns are also included. • Scales or similar equilibrium models are used to show that equality is maintained in addition and subtraction. • Studies on multiplication with algebraic expressions with models are included. • Studies on associating real-life situations with locating on the coordinate system are included. • Appropriate information and communication technologies are used when necessary.	• Area formulas and perimeter formulas in different variants enable a first, clear encounter with terms and term transformations. • First set up terms with one variable for illustrative situations (matches, packaging tape, pattern...) and calculate values. • Set up and solve equations through systematic experimentation, tables, graphs and equivalent transformation (scale model). • Problem solving exercises with equations (number puzzle, age puzzle, daily life situations). • Use spreadsheet to check substitution equality and clarify variable aspect. • Investigation of term transformations with computerized algebra systems (Computer Algebra Systems: CAS).

According to Table 9, it is seen that the implementation suggestions prepared for the algebra learning outcomes of both countries are given in detail and richly in both curricula. In addition, it is noteworthy that the implementation suggestions of both countries are similar. For example, using appropriate models in transactions, including studies for associating them with real-life situations, using appropriate information and communication technologies (computer algebra systems, spreadsheets, ...) are the common points of the implementation suggestions of both countries.

Distribution of the learning outcomes of the algebra learning area in the 5th-8th mathematics curriculum of Turkey and Germany in terms of knowledge and cognitive process dimension of the revised Bloom's Taxonomy

In this section, the distribution of the learning outcomes of the algebra learning area in the 5th-8th mathematics curriculum of Turkey and Germany in terms of knowledge and cognitive process dimension of the revised Bloom's Taxonomy are examined, and the findings are presented in Table 10.

Table 10. *Percentage distribution of the Turkish and german mathematics curriculum on the classification of the learning outcomes of algebra learning area in terms of knowledge and cognitive process dimensions*

	Turkey						Germany					
	R	U	Ap	An	E	Cr	R	U	Ap	An	E	Cr
F	0	0	0	0	0	0	0	0	0	0	0	0
CoO	0	26,09	0	0	0	0	0	0	0	22,22	0	0
P	0	0	52,17	17,39	0	4,35	0	0	33,33	11,11	22,22	11,11
M	0	0	0	0	0	0	0	0	0	0	0	0

F: Factual, Co: Conceptual, P: Procedural, M: Metacognitive; R: Remember, U: Understand, Ap: Apply, An: Analyze, E: Evaluate, Cr: Create

When Table 10 is examined, it is seen that the learning outcomes in the 5th-8th mathematics curriculum in Turkey and Germany are concentrated in the conceptual and procedural steps in terms of knowledge dimension. In terms of the cognitive process dimension, it was observed that the learning outcomes in the Turkey curriculum concentrated on the steps of understand, apply, analyze and create, while the learning outcomes in the Germany curriculum focused on the steps of apply, analyze, evaluate and create. Although it is seen that the learning outcomes in both Turkish (52.17%) and German (33.33%) curriculum are predominantly in the procedural apply step; It is noteworthy that the outcomes in the procedural evaluation (22.22%) and procedural creation (11.11%) steps in the Germany curriculum are higher than in the Turkey curriculum. In addition, it was determined that the outcomes in the conceptual knowledge level in the Turkey curriculum were more cognitively at the understanding level (26.09%), while the outcomes in the Germany curriculum in the conceptual knowledge level were more concentrated on the cognitive analyze step (22.22%). On the other hand, it has been observed that the number of learning outcomes in the procedural analyze step of the Turkey curriculum (17.39%) is higher than the number of learning outcomes in the procedural analyze step of the Germany curriculum (11.11%). In this context, while it is seen that the learning outcomes of the algebra learning area of the 5th-8th mathematics curriculum in Turkey and Germany are similar in terms of knowledge dimension; it has drawn attention that the Germany curriculum is prepared for higher level skills in terms of cognitive process dimension.

CONCLUSION, DISCUSSION and SUGGESTIONS

In this study, the studies comparing the mathematics curriculum of Turkey and other countries between 2002-2021 were examined within the framework of the determined themes and it was seen that these studies were mostly carried out with Singapore. At the same time, when examined in terms of the theme used, it was determined that the studies mostly focused on the elements of the program (aim/goal, content, educational situations and testing situations). As a matter of fact, the goal constitutes the most basic element of a program and gives direction to the content and clarifies the teaching process. In addition, determining the goal, choosing the methods and strategies suitable for the goal and the content, and evaluating the extent to which the target has been achieved are also very important in the education process. Learning outcomes, which are one of the elements of the program, have a special importance as they are the starting point for other elements. In addition, it is a necessity for the formation of a consistent education program to determine the outcomes correctly, to try to give them to the students as determined, to guide the measurements and to use them as criteria in the evaluation (Bümen, 2006).

This may be an indication of why comparative education studies in the literature mostly focus on the elements of the curriculum. On the other hand, although it had a very low performance in 1985, the fact that it has become a perfect system since 2000 with the reform movements (Bakioğlu & Göçmen, 2013) may be the reason why the Singapore education system is the most studied. In addition, its high performance in mathematics in international exams such as TIMSS and PISA (Mullis et al., 2016; Mullis et al., 2020; Mullis et al., 2015; OECD, 2010-2019) may be cited among the reasons why Singapore's prefered.

In this study, the sub-learning areas of the algebra learning area in the 5-8 mathematics curriculum of Turkey and Germany were compared and it was seen that there were some differences between the programs. According to these differences, it can be said that although the Germany curriculum has fewer algebra sub-learning areas than the Turkey curriculum, the sub-learning areas mostly overlap in content. In addition, although there is no algebra learning area in the 5th grade in the Turkey curriculum, there is an algebra learning area in the 5th grade according to the Germany curriculum. At the same time, while the 6th, 7th and 8th grades in the Turkey curriculum have separate sub-learning areas of algebra learning, there is no separation for the 5th and 6th grades in the Germany curriculum, and there is no separation for the 7th and 8th grades in these grade levels. It has been noticed that there is a sub-learning area. It was determined that the number of algebra sub-learning areas increased as the grade levels increased in both Turkey and Germany curriculum.

The similarities and differences in terms of the number and content of the learning outcomes of the algebra learning area in the 5-8 mathematics curriculum of Turkey and Germany were examined, and it was seen that the outcomes in both Turkey and Germany curriculums were expressed in short, clear and broad time sentences and associated with daily life. In this way, it can be said that students will see algebra in real life problems and realize how important and necessary it is to learn algebra. On the other hand, if the differences between the two curriculums are taken into account, it can be said that the outcomes in the Turkish program are differentiated for each grade level, while the outcomes in the Germany curriculum are divided into 5-6 and 7-8 classes. Another difference between the programs is that the outcomes in the Turkey curriculum for algebra learning area are higher than the outcomes in the Germany curriculum. Although the number of outcomes is different, the outcomes mostly cover each other. The reason for this is that the outcomes in the Turkey program are given in more detail and some of the outcomes in the German program are combined and expressed as a single achievement. In this direction, Çetinbağ (2019) states that when similar outcomes are combined and expressed in the simplest form, it will prevent the formation of learning outcome density. However, while combining the outcomes, care should be taken not to allow another educational action to enter the area covered by an educational action, and overlap should not be allowed (Çil et al., 2019; Kennedy, 2006; Kuzu et al., 2019). When the grade level increases in the curriculums of both countries, the number of outcomes increases and the content of the outcomes becomes more intense. Considering that it is necessary to progress from simple subjects to complex subjects in order for individuals to experience a sense of outcome and to realize learning more efficiently, we can associate this situation with the principle of education from simple to complex. As a matter of fact, according to the study conducted by Kuzu et al. (2019), the organization of the outcomes according to the aim and goal of the curriculum, expressing them clearly and precisely, containing a single action, writing the outcomes at different grade levels hierarchically, from concrete to abstract and from simple to complex, teaches the learning outcomes and facilitates classification. In this context, it can be said that the outcomes in both Turkey and Germany were prepared by taking these features into account. On the other hand, when the application proposals for the outcomes of the algebra learning field of the Germany and Turkey curriculums are examined, it is seen that the application proposals for the outcomes in both programs are given in detail and richly. Considering that algebra takes place in all areas of life and is a necessity (Dede & Argün, 2003), it can be said that the implementation suggestions presented in the program are very important. In addition, considering the aims and goals of the curriculum, a teaching process can be planned and implemantation suggestions can be made with the help of concept cartoons, since associating abstract and incomprehensible concepts through a character will pave the way for more meaningful and permanent learning (Karaca et al., 2020).

In this study, the learning outcomes of the algebra learning area in the 5th -8th mathematics curriculum in Turkey and Germany were classified according to the revised Bloom's taxonomy in terms of knowledge and cognitive process. It was observed that the outcomes in both curriculums mainly on the conceptual and operational steps in terms of knowledge. In terms of cognitive process dimension, it was seen that there were differences as well as similarities between both programs. For example, it is seen that the outcomes in the Turkey and Germany curriculums are predominantly in the operational implementation phase. On the other hand, at the cognitive level of understanding the outcomes in the conceptual knowledge level in the Turkey curriculum; it is determined that the outcomes in the conceptual knowledge level in the Germany program are more at the analysis level. Although it is seen that the number of learning outcomes in the operational analysis step of the Turkey curriculum is higher than the number of outcomes in the operational analysis step of the Germany curriculum, it has attracted attention that the outcomes in the procedural evaluate and procedural create steps of the Germany curriculum are higher than those in the Turkey program. In this context, while it is seen that the outcomes of the algebra learning area of the 5-8 mathematics course curriculum in Turkey and Germany are similar in terms of knowledge; it has drawn attention that the Germany curriculum is prepared for higher level skills in terms of cognitive process dimension. The questions and/or outcomes prepared at lower cognitive levels lead students to memorization and increase their anxiety levels; It has been emphasized that the outcomes and/or questions prepared for high-level cognitive skills prepare the ground for using existing information and effective thinking and increase motivation (Doğanay & Ünal, 2006; Kuzu & Çalışkan, 2018). In this context, it can be emphasized that the curriculum outcomes and implementation suggestions are aimed at high-level cognitive skills. According to the study conducted by Kuzu (2020), it is stated that the use of multiple representations in concept teaching and problem solving process will contribute to the development of cognitive process skills, so different types of representation can be used in the mathematics teaching process and appropriate activities can be designed. In addition, in the process of making sense of concepts and acquiring high-level cognitive skills, teaching environments and programs can be designed by considering real life problems and process-based teaching models. As a matter of fact, it is emphasized that designing a learning environment and teaching process suitable for students' understanding in the education process is important in making sense of basic mathematical concepts (Kuzu et al., Sıvacı, 2018). On the other hand, it has been emphasized that a more permanent and effective learning environment will occur (Kuzu & Sıvacı, 2018), the narration will be easier and learning will take place with fun (Özüdogru, 2021) thanks to the integration of technology with digital games and/or stories and integrating it into the education process. In this context, the use of teaching materials with digital content can be included while preparing the programs and learning outcomes.

It is stated that comparative education is a field of study related to education, which includes the researches carried out to determine the similarities and differences by examining the education systems of the countries and to find solutions by comparing the countries that have faced similar problems (Tatlı & Adıgüzel, 2012). From this point of view, it is thought that it is important to focus on comparative education studies for the development of the education system. For this reason, it is recommended to researchers to conduct a comparative education study on the education systems of countries that have been more successful than our country in important exams such as TIMMS and PISA. It is also among the suggestions to compare the achievements of the same learning areas in the education programs of these countries.

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