A Thematic Review of Quadratic Equation Studies in The Field of Mathematics Education

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Quadratic equations are one of the fundamental topics of the secondary school curriculum. Current research in algebra education indicates an upswing in interest in quadratic equations research; however, the general trend of quadratic equation studies is not known. Research, which systematically analyzes quadratic equation studies in the field of mathematics education, will be a guide for the researchers who plan to research this subject matter. Furthermore, displaying the content of quadratic equation studies will be a guide for mathematics teachers in finding the resources to reach alternative teaching methods. This study aimed to review both descriptively and thematically the quadratic equation studies in the field of mathematics education conducted in the period of 2000 to 2021. For this aim, both descriptive and thematic content analysis were used. 42 articles and 9 theses conducted between 2000 and 2021 were examined and analyzed. The findings of this study displayed that although current research in algebra education showed interest in quadratic equation studies, the number of the studies was quite scarce, and the studies still lack sufficient scope. Particularly, more studies should be conducted by considering mathematics teachers' and pre-service mathematics teachers' knowledge for teaching quadratic equations from different perspectives. This study will provide a more collective understanding of quadratic equation studies in the field of mathematics education.

Introduction

As one of the fundamental topics of the secondary school curriculum, quadratic equations are one of the procedurally and conceptually challenging concept for students. The studies indicate that many students face problems in learning quadratic equations and their performance is mainly poor (Harripersaud, 2021). The conceptual meaning of quadratic equations is generally ignored in the teaching of quadratic equations. Students usually tend to follow rote-learned procedural rules without thinking their meaning to solve quadratics (Tall, de Lima, & Healy, 2014; Vinagrodava & Wiest, 2007). The research revealed that students made different types of errors such as calculation errors, algebraic errors, comprehension errors, transformation errors, simplification errors, factorization errors or missing roots errors (e.g.,

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Didiş & Erbaş, 2015; Tendere & Mutambara, 2020; Zakaria & Maat, 2010). Several research indicates that students struggle particularly with solving some kinds of quadratic equations (e.g., Clinch, 2018; Vaiyavutjamai, Ellerton & Clements, 2005). Clinch (2018) expressed those factoring quadratic equations in the form of $ax^2 + bx + c$ with $a \neq 1$, that is, a leading coefficient other than 1, was much more difficult for students. Vaiyavutjamai, Ellerton and Clements (2005) pointed out students' difficulties with the quadratic equations in the form $x^2 = K$ ($K > 0$) and $(x - a)(x - b) = 0$ (where a and b can be any real numbers). On the other hand, instead of displaying students' errors and challenges in solving quadratic equations, several studies also focused on students' conceptions of quadratic equations (e.g., Lòpez, Robles, & Martínez-Planell, 2016; Tall et al., 2014) which aimed to gain insight into students' understanding in solving quadratic equations.

There are different methods used for solving quadratic equations such as factoring, completing the square, quadratic formula, and graphing (Bossé & Nandakumar, 2005; Harripersaud, 2021). Harripersaud (2021) expresses that although the quadratic formula seems that it has an advantage compared to other methods because it applies to all quadratics, each method has its advantages and disadvantages. Bossé and Nandakumar (2005) point out the ineffectiveness of the factoring method and pay attention to some significant advantages regarding completing the square and the quadratic formula methods. They stated that while factoring quadratics in the form of $ax^2 + bx + c = 0$ (if possible) is easy when "a or c" are units or prime, factoring quadratics can quite complex when the leading coefficient and the constant include many pairs of factors such as $a = 36$ and $c = 24$. The traditional teaching of quadratic equations usually focuses on factoring, quadratic formula and completing the square methods. Furthermore, teachers tend to overemphasize the use of the standard form of quadratic equations. These methods offer effective algorithms help to reach desired results (Harripersaud, 2021); however, students struggle to gain conceptual understanding. Therefore, teachers should be aware that solving quadratic equations is not based on procedures (Tendere & Mutambara, 2020). They should be aware of alternative methods and alternative ways of teaching. Use of a historical perspective of quadratic equation, manipulatives, or technological tools provide students with alternative perspectives and help them learn meaningfully and conceptually (e.g., Allaire & Bradley, 2001; Clifford & Son, 2018; Edwards & Chelst, 2019). Allaire and Bradley (2001) explain that the subject becomes more attractive to the typical students when using historical approaches. Clifford and Son (2018) suggest that teachers can use al-Khwarizmi's geometric proof, Babylonian and Euclidean methods to teach solving quadratic equations as historical methods. Furthermore, Vinagrodava and Wiest (2007) introduce an activity which describes a way to offer ideas regarding the use of algebra tiles in solving a quadratic equation.

The studies indicate that there is a lack of attention to quadratic equation studies in mathematics education (e.g., Sönnerhed, 2021). On the other hand, there are no research that systematically analyzed quadratic equation studies in the field of mathematics education. It is important to be aware of the general trend of the quadratic equation studies to recognize what kinds of studies are necessary for this area. Identifying the general trend of studies on quadratic equations will provide guidance for the researchers who plan to research on this subject matter. Displaying the content of quadratic equation studies will guide for mathematics teachers in finding the resources to reach alternative teaching methods. Considering the importance of quadratic equations in the secondary mathematics curriculum, this research aimed to review both descriptively and thematically the quadratic equation studies in the field of mathematics education conducted at different levels in the period of 2000 to 2021. This study seeks answers to the following research questions:

(1) How were quadratic equation studies in the field of mathematics education distributed according to publication year?
(2) What were the general methodological features of quadratic equation studies in the field of mathematics education?
(a) What were the research types and designs of the quadratic equation studies?
(b) How were the participants profiled in the quadratic equation studies?
(c) What data collection tools were used in the quadratic equation studies?
(3) How was the general thematic focus of quadratic equation studies in the field of mathematics education?

This study will provide a more collective understanding of quadratic equation studies in the field of mathematics education. Moreover, by showing the general trend of the quadratic equation studies, this study will give information on what kinds of studies are conducted and what kinds of studies are still missing in the mathematics education literature.

Method

Research Model

This study aims to examine the studies published on quadratic equations, to identify the general trends of quadratic equation studies and to depict the common tendencies in a comprehensive and interpretive way; therefore, both the descriptive and thematic content analysis (meta-synthesis) were used. Descriptive and thematic content analysis are types of content analysis (Çalık & Sözbilir, 2014). Through the descriptive content analysis, the studies are systematically reviewed to determine and describe the general trends (Bellibaş, 2018; Çalık & Sözbilir, 2014). On the other hand, thematic content analysis is more critical than descriptive content analysis. Through the thematic content analysis, the studies conducted in a specific content area are synthesized to produce generalizable results (e.g., Çalık & Sözbilir, 2014).

Selection of Studies

In this study, the quadratic equation studies conducted between 2000 and 2021 were examined. Since the studies for 2022 were not completed at the time of the study, it was decided to examine the studies until 2022. To reach all the studies that have been carried out both nationally and internationally, the well-known databases "ERIC, JSTOR, Springer LINK, Taylor & Francis, Science Direct, Ulakbim, Google Scholar, Wiley Online Library, Scopus, ProQuest Dissertations and Theses Global and Council of Higher Education Thesis Center (YÖK Ulusal Tez Merkezi)" were looked for. While searching the studies, the main keywords "quadratic equations, second-degree equations, complete the square and factoring" were used both in Turkish and in English. However, since there were studies on quadratic equations in both mathematics and mathematics education field while using these keywords, the quadratic equation studies in the field of mathematics education were distinguished. For this reason, based on these keywords, all the accessed studies were initially carefully scanned. Afterwards, only the quadratic equation studies published regarding the field of mathematics education were considered. A few quadratic equation studies conducted in the field of mathematics education could not be accessed because there was no open access for them. Moreover, if there is an article produced from the thesis of the same authors, to avoid duplication, their theses were preferred to examine. Only the studies published in Turkish and in English were also determined. In total 42 articles and 9 theses conducted between 2000 and 2021 were found and

51 studies were considered, in total, for the analysis. The appendix (Examined Studies) includes all studies that were considered.

Data Analysis

All studies were initially organized, namely, the articles numbered from A1 to A42 and the theses numbered from T1 to T9 according to their publication year. The studies were read before analysis. At this stage, three types of research were recognized: (i) Empirical studies that report data from research and include full Introduction, Methods, Results, and Discussion sections (ii) Theoretical studies that report only theoretical information rather than reporting data and (iii) Textbook analysis study. Firstly, the quadratic equation studies were categorized according to these research types. If a study could not evaluate under these research types, it was coded as other research. To identify the general trends of quadratic equation studies, the descriptive content analysis was conducted. All studies were coded according to publication years. To identify the general methodological features of quadratic equation studies, the empirical studies were coded according to participants, research method, research design and data collection tools. Then all studies were interpreted with a critical view through thematic content analysis (meta-synthesis). For the thematic content analysis, the studies were examined according to their research focus. Before analyzing the focus of the empirical studies, they initially were grouped according to their participants. After the focus of the studies were analyzed, the studies with similar aims were associated with each other. Similarly, the focus of the theoretical studies and textbook analysis studies were coded, and then the studies with a similar focus were associated. The studies were categorized by two researchers in mathematics education with an agreement of 0.85. The research categorized differently was discussed by reaching an agreement.

Findings

The initial analysis of studies on quadratic equations showed that there were three types of research. As presented in Table 1, 26 of the articles and six of the theses were Empirical Research;13 of the articles and one of the theses were purely theoretical research.

Table 1. The Findings regarding the research types

Studies		Empirical Research	Study Number	Theoretical Research	Study Number	Textbook Analysis Research	Study Number	Other	Study Number	Total
Articles		26	A1, A2, A3, A6, A7, A8, A10, A12, A13, A14, A16, A17, A18, A20, A21, A23, A24, A27,A28, A30, A31, A33,A34, A35 A36, A38	13	A5, A9, A11, A15, A19, A22, A25, A26, A29, A37, A40, A41, A42	2	A4, A32	1	A39	42
Thesis	PhD	3	T1, T3, T4	-	-	-	-	-	-	3
	MS	3	T6, T8, T9	1	T7	2	T2, T5	-	-	6
Total		32		14		4		1		51

Furthermore, two of the articles and two of the theses were textbook analysis research, specifically, the review of quadratic equation sections in books or the comparison of quadratic equation units in different countries' textbooks. One research was evaluated as other research since it does not exactly fit any of these classifications.

The findings have presented in two sections. Firstly, to display the general trend in quadratic equation studies, descriptive content analysis will be reported. Then, to gain a better understanding of the focus of the quadratic equation studies, the thematic content analysis will be reported in depth.

Findings of descriptive content analysis

The distribution of the studies by years displayed that the number of the studies increased in the last few years (see Figure 1). The number of analyzed studies was most in 2019, followed by 2021.
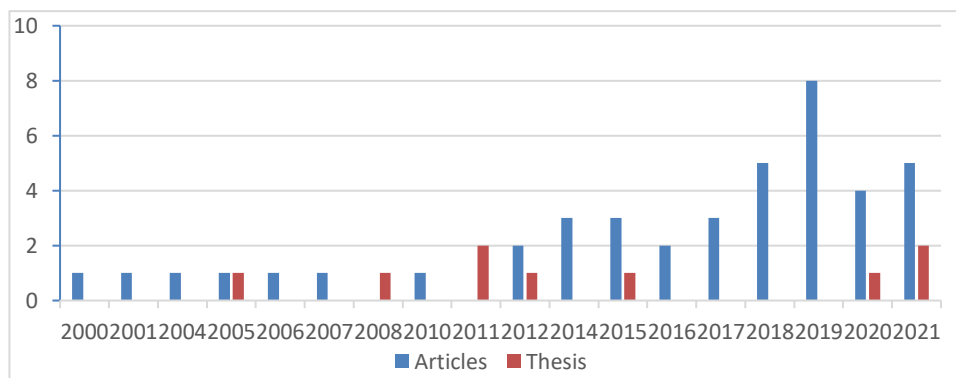

Figure 1. The distribution of the studies according to publication years

When the research method and research design of studies were examined, it was seen that most of the studies used qualitative research methods, particularly case study (see Table 2).

Table 2. The research method and research design of empirical studies and textbook analysis studies

Research Method of Empirical Research	Research Design	Articles f	Thesis f	Total f	Study Number
Qualitative Research	Case Study	11	-		A2, A3, A7, A8, A10, A14, A16, A17, A18, A20, A21, A23, A24, A30, A35, A36, T6, T4
	Action Research	-	1	18	
	Exploratory	2	-		
	Descriptive	1	-		
	Unspecified	2	1		
Quantitative Research	Experimental Research (Pre-post test design)	-	1	2	A13, T9
	Unspecified	1	-		
Qualitative& Quantitative	-	2	3	5	A6, A28, T3, T5, T8
Design Research	-	1	1	2	A31, T1
Not directly specified	-	8	1	9	A1, A4, A12, A27, A32, A33, A34, A38, T2

Among 18 studies that used qualitative approaches, 11 studies were reported using case studies as a research design. Moreover, only two studies reported using the quantitative research method were found. Furthermore, while 5 of the studies reported that they utilized both qualitative and qualitative research methods, 9 of them did not specifically report their research methods and design in their studies. So, these were remarked as "not directly specified". However, when their research methods were examined, it was observed that many of them had qualitative method and some of them had mixed method. In addition, the research method of the two studies (one article and one thesis) was design research.

When the participants of the empirical studies were analyzed, it was observed that quadratic equation studies were generally conducted with secondary school/high school students (see Table 3). As presented in Table 3, there were 17 studies whose participants were only secondary school/high school students. However, there were also 5 other studies whose participants included teachers and secondary school students together. There was only 3 study conducted with pre-service mathematics teachers and two studies conducted with only teachers. Moreover, there were 2 studies conducted with tutors of the College of Education, and one study conducted with mathematics lecturers having mathematics education background and no mathematics education background as experts.

Table 3. The participants of the empirical studies

Participants	Types	Article f	Thesis f	Total f	Study Number
Teachers	• Secondary school/ High school mathematics teachers	2	-	2	A1, A14
Students	• Secondary school/High school students	13	4	17	A2, A3, A8, A10, A12, A13, A16, A21, A23, A28, A30, A33, A36, T3, T4, T8, T9
Tutors	• College of Education tutors	2	-	2	A6, A18
Students & Teachers	• Students and teachers of secondary schools/high schools	4	1	5	A7, A31, A34, A38, T6
Undergraduate Students	• Preservice elementary mathematics teachers studying at mathematics education programs	3	-	3	A17, A20, A35
	• University students in other programs	1	1	2	A27, T1
Expert	• Mathematics lecturers having mathematics education background and mathematics lecturers having no mathematics education background.	1	-	1	A24

The analysis of data collection tools used by quadratic equation studies revealed that various data collection tools were preferred in the studies as presented in Table 4. Interviews were the most preferred data collection tools for the studies and in 19 of the empirical studies, interviews were used to collect data. Furthermore, tests were another frequently used data collection tool in these studies. 15 of the empirical studies utilized the different test types. The tests generally included open-ended questions about quadratic equations. In eight studies tests and interviews were used together and in two studies questionnaires and interviews were used together. Different types of interviews such as structured, semi-structured or task-based were utilized in the studies, and these interviews were mainly conducted to get depth information about the answers obtained by test or questionnaire. In five studies, a questionnaire was preferred to collect the data as a primary data source. Although the research aims of the two studies were different, they (A6 and A18) used the same questionnaire because the authors of these studies were the same. However, the intended use and the structure of the questionnaire used in the studies were generally different. While some questionnaires used in the studies included Likert-type questions, others involved open-ended quadratic equation questions. In four studies, problems, problem-solving activities or tasks were used as primary tools to collect data. Teaching scenario tasks and video recording of lessons were used in the studies conducted with mathematics teachers. Furthermore, in two studies quadratic equation tasks were used to collect data.

Table 4. Data collection tools used in empirical studies

Data Collection Tools	Articles f	Thesis f	Total f	Study Number
Interview (e.g., semi structured, structured, task/activity based)	15	4	19	A2, A3, A7, A8, A12, A14, A16, A17, A20, A21, A27, A28, A33, A36, A38, T1, T4, T6, T8
Test (e.g., achievement test, performance test, open-ended questions)	12	3	15	A3, A7, A8, A10, A12, A16, A23, A28, A30, A34, A36, A38, T3, T8, T9
Questionnaire	5	-	5	A6, A13, A14, A18, A33
Teaching scenario task	1	-	1	A1
Observations	2	1	3	A3, A14, T6
Video recording of lessons	1	-	1	A34
Problem solving activities or tasks	4	-	4	A2, A24, A33
Scale (attitude towards mathematics, attitude toward technology scale)	-	2	2	T8, T9
Students' written work	2	-	2	A17, A27
Students' reflective journal	1	-	1	A35
Instruction activities	1	-	1	A31
Concept map	1	-	1	A33
Mathematics exams grade points average	-	1	1	T9

The Findings of Thematic Content Analysis

The focus of the theoretical quadratic equation studies

As presented in Table 5, the theoretical quadratic equation studies showed that the focus of five studies was to introduce a classroom activity on quadratic equations. While two of these studies specifically focused on completing the square method (A19, A37), one of them focused on finding the meaning of quadratic formula as $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ through a GeoGebra activity (A11). Furthermore, one study (A9) presented a classroom activity that helps students solve quadratic equations efficiently in the stations, and another study (A25) presented an activity that requires students to find linear and quadratic equations in visual patterns. On the other hand, the analysis revealed that three studies addressed historical approaches concerning quadratic equations. While one of the studies (A26) presented the historical development of quadratic equations through periodic examples, another (A41) offered an alternative perspective to solve quadratic equations by focusing on geometric techniques from ancient Babylonia, classical Greece, medieval Arabia, and early modern Europe. By displaying the various geometric approaches, the purpose of the study (A41) was to offer alternative perspective to symbolic approaches. Differently, another study (A42) focused on reinventing the quadratic equation formula through a Babylonian approach. Moreover, one study (A19), which presented an activity for completing the square method, allowed students to explore completing the square method from a historical perspective and introduced al-Khwarizmi's geometrically based proof of completing the square. One study (A22) focused on the factorization method and presented an alternative strategy to factor quadratic expressions in the form of $ax^2 + bx + c$. The focus of the two studies was to introduce different ways of solving quadratic equations. However, while one's (A5) displayed the ways a quadratic equation can be solved by paying more attention to the quadratic formula, the other's (A15) main goal was to offer an alternative method based on the area model. On the other hand, the purpose of the one study (A29) was to respond to the questions asked by middle school mathematics teachers

regarding irrational numbers, square roots and solving quadratic equations and to provide a meaningful explanation of each concept. The first question discussed in the study was "How do we introduce irrational numbers without simply telling students that $\sqrt{2}, \sqrt{3}$ and $\sqrt{18}$?" and the second question discussed was "Does $\sqrt{(25)} = 5$ or $\sqrt{(25)} = \pm5$? and If the first one is correct, why does the equation $x^2 = 25$ have two solutions, $x = \pm5$?". Contrary to other theoretical studies, the focus of the study coded A40 was different. This study did not directly focus on quadratic equations, and it intended to clear up the principles and philosophies of secondary mathematics teaching in Japan. For this aim, the study focused on quadratics and described two lessons observed in a third-grade class at a junior high school in Tokyo. The thesis study (T7) displayed two well-organized lesson plans for teaching completing the square, one of which used an algebraic approach and the other used a geometric approach.

Table 5. The focus of the theoretical studies

		Study Number	f
Presenting/Introducing/Describing an activity on quadratic equations	▪ A classroom activity that helps students solve quadratic equations efficiently in the stations.	A9	5
	▪ A Geogebra activity to find the meaning of the quadratic formula	A11	
	▪ An activity to introduce al-Khwarizmi's geometrically based proof of completing the square	A19	
	▪ An activity presents finding linear and quadratic equations in visual patterns	A25	
	▪ An activity describes a way solving quadratic equations by completing squares using algebra tiles	A37	
Introducing historical approaches	▪ Presenting the historical development of quadratic equations through periodic examples	A26	3
	▪ Offering an alternative perspective to solve quadratic equations by displaying geometric techniques from ancient Babylonian, classical Greece, medieval Arabia, and early modern Europe	A41	
	▪ Reinventing the quadratic equation formula which solves the general quadratic equation through a Babylonian approach	A42	
Presenting a technique to factor quadratic expressions in the form of $ax^2 + bx + c$		A22	1
Introducing different ways of solving quadratic equations		A5, A15	2
Offering answers to teachers' questions about irrational numbers, square roots, and simple quadratic equations		A29	1
Quadratics in Japanese		A40	1
Developing well organized lesson plans for teaching completing square		T7	1

The focus of the empirical quadratic equation studies

The focus of the empirical studies was classified according to their participants as presented in Table 6.

Table 6. The focus of the empirical quadratic equation studies

Participants		The focus	Study number	f
Students		Investigating students' cognitive process, understanding, algebraic reasoning or conceptions	A10, A12, A16, A21, A31, A33, A38*, T3, T4	9
		Determining/Investigating students' performance, difficulties, errors, failures, inabilities, and misconceptions	A3, A7*, A23, A28, A30, A34*, A36, T6*,	8
		Describing categories of students' critical thinking skills	A8	1
		Examining students' argumentation-based problem-solving process	A2	1
		Investigating the effect of technology-supported methods/technology-based treatments	T8, T9	2
		Development of the android based instructional media of algebraic tiles for quadratic equation and validation of the media with high school students.	A13	1
Teachers		Investigating teachers' analyses and responses students' errors in solving quadratic equations	A1	1
		Investigating teachers' mathematical knowledge for teaching	A14	1
		Investigating teachers' teaching approach to solve second degree equations	A34*, A38*, T6*	3
		Investigating teachers' opinions regarding students' errors and misconceptions	A7*	1
Tutors		Understanding tutors' conceptions in teaching completing square	A6	1
		Identifying tutors' knowledge, skills and practice in the teaching quadratic equations using completing the square	A18	1
Experts		Developing structure sense tasks on quadratic equations for high school algebra and validation of these tasks by experts who are mathematics lecturers having mathematics education background and no mathematics education background.	A24	1
Undergraduate students	Pre-service mathematics teachers	Exploring pre-service mathematics teachers' conceptions concerning the use of the history of mathematics, particularly, Al-Khwarizmi's method of completing the square while teaching	A20	1
		Exploring the impacts of using the history of solving quadratic equations (the method of completing the square) in teaching pre-service mathematics teachers' knowledge about this topic	A35	1

	Determining mathematics teacher candidate's mathematical reasoning ability in solving quadratic equation problems.	A17	1
University students	Investigating students' understanding of quadratic equations by applying Action–Process–Object–Schema theory (APOS)	A27	1
	Investigating the connections between the fundamental concepts for solving factorable quadratic equations that students may develop.	T1	1

Note. A7, A31, A34, A38 and T6 are the studies whose participants are both teachers and students. However, since research design of the study A31 was design research, there was not a specific aim of the study for the teacher participants, and teachers were only the implementer of the instruction activities in classrooms. Therefore, four studies (A7, A34, A38 and T6) appear twice in table and the table includes 36 studies rather than 32.

The studies conducted with students showed that they mostly focused on investigating either students' cognitive process, understanding, algebraic reasoning, conceptions or students' performance, difficulties, errors, failure, inabilities, and misconceptions. For example, while one of the studies (A10) described students' algebraic reasoning in solving quadratic equation questions, another study (A21) explored students' conceptions concerning quadratic equations. Another study (A3) determined students' error types in solving quadratic equation problems and the factors that caused students to make the errors. Moreover, the study (A30) examined eleventh-grade students' failures and inabilities of quadratic equations and functions.

On the other hand, while one study (A8) described high school students critical thinking skills in solving quadratic equation problems, another (A2) investigated 10th-grade students' argumentation-based problem-solving processes in the context of quadratic equations. Furthermore, three studies conducted with students (two theses and one article) included the effect of the use of technology. One thesis (T8) investigated the effect of technology-based treatments on students' attitudes towards mathematics and technology, and students' performance in the graph of quadratic equations. Another thesis (T9) examined the effect of technology-based and traditional methods on the teaching of quadratic equations and functions through realistic problem situations. The aim of the technology-related article (A13) was to develop the android-based instructional media of algebraic tiles for quadratic equations.

The focus of the studies conducted with teachers gathered in four different categories. Three of the studies (A34, A38, T6) investigated teachers' teaching approach to solving second-degree equations and one of them (A14) examined teachers' mathematical knowledge for teaching. Furthermore, one of the studies (A1) focused on teachers' analyses and responses to students' errors in solving quadratic equations, other (A7) focused on teachers' opinions regarding students' errors and misconceptions.

There were two studies conducted with tutors, written by the same authors, and focused on the completing the square method. While one of them (A6) aimed to understand tutors' conceptions in teaching completing square, the other (A18) aimed to identify tutors' knowledge, skills and practice in the teaching quadratics using completing the square. On the other hand, in the study conducted with the experts (A24), the researchers developed three quadratic equation tasks based on the characteristic of structure sense and then aimed to validate the appropriateness of these tasks by mathematics lecturers as experts when dealing with the tasks.

There were five studies at the undergraduate level. Three of them were conducted with students who were studying in the mathematics teacher education programs and two of them were implemented with students who were studying other programs. The content of the two studies

conducted with similar. While one of them (A20) focused on pre-service mathematics teachers' conceptions concerning Al-Khwarizmi's method of completing the square while teaching, the other (A35) explored the impacts of using the history of mathematics in teaching quadratic equations, particularly, the historical context of solving quadratic equations using completing the square, on pre-service mathematics teachers' knowledge. Differently, the other study (A17) implemented with pre-service mathematics teachers aimed to determine pre-service mathematics teachers' mathematical reasoning ability in solving quadratic equation problems. On the other hand, the aims of the studies conducted with university students were different from each other. One study (A27), which conducted with university students, examined beginning university students' and science and engineering students' understanding of quadratic equations in one variable by applying Action–Process–Object–Schema theory (APOS). Another study (T1), which was a thesis, focused on how university students could develop connections between essential concepts for solving factorable quadratic equations and investigated university students' understanding of factorable quadratic equations from a symbolic and graphical perspective.

When the focus of all empirical studies was reviewed, four studies (A6, A18, A20 and A35) focused on particularly completing the square method. Furthermore, while the content of the two studies (A20, A31) was based on the history of mathematics, the content of three studies (A13, T8, T9) was based on the use of technology.

The focus of the textbook analysis studies

As presented in Table 7, the aim of the three studies was similar, which aimed to the comparison of quadratic equation sections of different countries' mathematics textbooks.

Table7. The aim of the mathematics textbooks analysis studies

The target/aim of the studies	Study Number	f
A comparison of quadratic equation sections of different countries' textbooks	A32, T2, T5	3
Analyzing quadratic equation tasks and content in Swedish textbooks	A4	1

The aim of the article study (A32) was to compare quadratic equations sections of Korean and American textbooks in terms of topics, contents, and mathematics items. Furthermore, while one of the thesis studies (T2) compared the quadratic equation unit of secondary school mathematics textbooks from the U.S. and Singapore in terms of content coverage in grade level, another (T5) described and compared the units on quadratics in Singapore, Turkey and the IBDP mathematics textbooks in terms of the perspectives of content, organization, and presentation style, by using content analysis as a research method. Unlike these studies, one study (A4) focused only on Swedish mathematics textbooks and analyzed the content and tasks of eight Swedish mathematics textbooks' quadratic equation sections.

Discussion and Conclusions

This study aimed to review both descriptively and thematically the quadratic equation studies in the field of mathematics education conducted at different levels in the period of 2000 to 2021.

When the trend of quadratic equation studies was examined, it was observed that the number studies has started to increase in the last few years. This finding indicates that current research in algebra education shows interest in quadratic equation studies. However, the studies still lack

sufficient scope, and the number of the studies was still scarce.

The analysis of examined studies showed that the types of studies on quadratic equations can be collected under three types which were empirical, theoretical, and textbook analysis. The participants' analysis of empirical studies displayed that the quadratic equation studies were mostly conducted with high school students. Because the quadratic equation is taught as a subject in a secondary school curriculum, it is an expected finding. However, there are very few studies conducted with mathematics teachers and pre-service mathematics teachers. So, the scarcity of studies conducted with mathematics teachers and pre-service mathematics teachers is quite surprising. Three research with pre-service mathematics teachers and two research conducted with only teachers were found. One of the reasons why quadratic equation studies with pre-service mathematics teachers are not given sufficient attention may be related to the position of the topic in mathematics teacher education programs. The quadratic equations are not taught as a specific unit in the calculus courses, and it is practiced within other subjects of higher-level mathematics such as functions, function graphs or derivative. For this reason, the researchers may not have shown sufficient attention to investigate pre-service mathematics teachers' content knowledge with quadratic equations. Alternatively, like the research Huang and Kulm (2012), there may be some studies which examining pre-service mathematics teachers' knowledge of quadratic equations within their knowledge for teaching the concept of function rather than examining quadratic equations as a specific topic. Almost 17 years ago, in their study, Vaiyavutjamai and Clements (2006) paid attention to the scarcity of studies on teaching and learning quadratic equations in the literature. Similarly, almost eleven years ago, Olteanu and Holmqvist (2012) stated the existence of only a few articles on the teaching and learning of quadratic equations research. Despite the extensive search for reaching quadratic equation studies in the field of mathematics education for this study, it was observed that there was still very little research, particularly, on the teaching of quadratic equations. This finding indicates researchers does not give sufficient importance to teaching and learning quadratic equations. As the studies conducted with students pointed out, many students were unable to solve quadratic equations and they had various kinds of cognitive and procedural difficulties (e.g., Lòpez et al., 2016; Tall et al., 2014; Zakaria & Maat, 2010). One of the reasons for students' difficulties may be related to the teaching of quadratic equations. Therefore, the relationship between teaching and learning quadratic equations, particularly, students' difficulties, should be examined. Because mathematics teachers and pre-service mathematics teachers are the key elements for the teaching of quadratics, more studies with both mathematics teachers and pre-service teachers' teaching on this topic are needed.

The analysis of quadratic equation studies' research methods displayed that most of the studies used qualitative research methods, particularly, case study. In addition to that, interviews and tests were the most preferred data collection tools for the studies. The tests generally included open-ended questions about quadratic equations. The analysis of the studies displayed that the participants of the research which used interviews and tests were especially secondary school students. Because the aim of many studies conducted with students was either to identify students' errors and difficulties or to understand students' cognitive processes in depth, the use of open-ended questions and interviews was likely to be preferred in line with the aim of the studies. On the other hand, while five studies were a mix of qualitative and quantitative research (mixed method), only two studies were quantitative. One of the quantitative studies had an experimental design aiming to investigate the effect of technology-based methods and traditional methods on the teaching of quadratic equations and functions through realistic problem situations. There were also several studies which did not specify their research method in their research. But the design and data analysis processes implied that they were either

qualitative research or mixed research. There were also two research used design research as the research method. The new studies on this topic may be designed as quantitative or design research that will provide diversity to quadratic equation research.

Through a series of content analyses of quadratic studies published, it was observed that there were various foci of the theoretical studies such as introducing an activity, presenting a historical approach to quadratic equations, or introducing the different ways of solving quadratic equations. Many of these studies showed that they offer an alternative approach to the teaching of quadratics. Among them, some of the studies emphasize the use of the history of the quadratic equations in the teaching of the quadratic equations (Allaire & Bradley, 2001; Clark, 2012; Clifford & Son, 2018) and present various historical approaches for teaching such as Babylonian, Greece, medieval Arabia, and early modern Europe' methods. Fried (2001) explains that the use of the history of mathematics in teaching mathematics makes mathematics more interesting and understandable, and it also provides insight into concepts and problems. Fried (2001) also indicates that historical problems help students motivate the classroom topics. He exemplifies even if teachers use the non-historical explanation and exercises in the teaching of completing the square method for finding the roots of the quadratics equations, they might introduce the Babylonian problems to foster the discussion. Other studies focus on introducing several activities for teaching including Khwarizmi's geometric proof, GeoGebra activity, algebra tiles and visual patterns of linear and quadratics. These studies present a different aspect in the teaching of quadratics and point out the crucial gains for the students. For example, Vinogradova and Wiest (2007) suggests using a hands-on visual model to make accessible quadratics for middle school students, drawing attention to the activity that helps students to build a connection between algebraic procedures and the geometric meaning of the completing the square method. Edwards and Chelst's (2019) GeoGebra activity, which allows students the opportunity to discover both the symmetry axis embedded in the quadratic formula and the role of the term $\frac{\pm\sqrt{b^2-4ac}}{2a}$, helps students understand the components of the quadratic formula as well as develop a conceptual understanding of the quadratic formula. Kurz (2017) suggests that teachers should be encouraged to use problem-solving and hands-on practice to help students understand mathematics meaning. Quadratic equations are quite abstract topic for students. Like many topics of algebra, quadratic equations include a bunch of algebraic symbols, so they were not meaningful for many of the students. Because these suggested alternative approaches can help students build connections between algebraic procedures and their geometric meaning and help students gain a deep understanding of the meaning behind the symbols, they are good sources for mathematics teachers to use in their classrooms. To investigate the potential benefits of these activities empirically in the case of use in the teaching of quadratics, mathematics education researchers, who are interested in algebra teaching and learning, should be aware of these alternative approaches. When the aims of empirical studies were analyzed, it was observed that only one study used to explore pre-service mathematics teachers' conceptions concerning the use of the history of mathematics, particularly, Al-Khwarizmi's method of completing the square while teaching (Genç & Karataş, 2018). Therefore, these findings suggest that more studies both qualitatively and quantitatively should be conducted to examine whether mathematics teachers use alternative teaching approaches while teaching quadratic equations or not.

The content analysis of empirical studies displayed that the focus of studies differs according to their participants. Analyses revealed that most of the studies conducted with high school students investigated students' cognitive process, understanding and algebraic reasoning or determined students' performance, difficulties, errors, failure, inabilities, and misconceptions.

Because students at different levels display many conceptual and procedural difficulties in many algebra topics, the researchers particularly might have focused on secondary school students' understanding, difficulties, errors, failures, or misconceptions in quadratic equation topics. In addition, in two theses (Buran, 2005; İnce, 2008) and one article (Irianti & Qohar, 2019), the effect of technology-supported methods (technology-based treatments) was addressed. All these studies have valuable sources for mathematics teachers and pre-service mathematics teachers to understand students' possible errors and procedural and conceptual difficulties and to help students to overcome their low performance. The focus of the studies with teachers had some diversity such as examining teachers' mathematical knowledge for teaching, teachers' analyses, and responses to students' errors in solving quadratic equations, or teachers' teaching approach to solving second-degree equations. However, studies investigating teachers' use of students' obstacles in their teaching of quadratics do not exist and more studies are needed to address students' challenges and teachers' teaching together. Unlike studies conducted with teachers, no study was found examining pre-service mathematics teachers' knowledge for teaching quadratic equations. Although quadratic equations are not taught as a specific topic in the mathematics teacher education program, pre-service mathematics teachers are responsible to teach quadratic equations in secondary schools as future mathematics teachers. So, they must have both strong content knowledge and knowledge for teaching to address students' learning effectively in the classrooms. Several studies (e.g., Huang & Kulm, 2012) displayed that pre-service mathematics teachers had limited knowledge of algebra for teaching. For example, in Huang and Kulm's (2012) study, pre-service middle-grade mathematics teachers' knowledge of algebra, particularly, knowledge of function for teaching, was examined. Their findings revealed pre-service mathematics teachers' limited knowledge of algebra for teaching, their difficulty in the use of geometrical representations to present algebraic and arithmetic relations and their numerous mistakes in solving quadratic equations. Therefore, the studies especially examining pre-service mathematics teachers' content knowledge and pedagogical content knowledge for teaching quadratic equations should be carried out because they are responsible to teach this topic in the future. It would be useful for mathematics teacher educators to reveal what the pre-service mathematics teachers know and do not know about this topic. Among the three studies conducted with pre-service mathematics teachers, one study examined the mathematics pre-service mathematics teachers' mathematical reasoning ability in solving quadratic equation problems by emphasizing the importance of mathematical reasoning in learning mathematics materials. The other two studies (Clark, 2012; Genç & Karataş, 2018) focused on the importance of pre-service mathematics teachers to know completing the square method with its historical approach, particularly Al-Khwarizmi's completing the square method. Because of the contribution of the historical perspective of completing the square method to pre-service mathematics teachers' knowledge for teaching (Clark, 2012; Genç & Karataş, 2018), mathematics teacher educators should be aware of the historical approach of completing the square method and they should implement such alternative approaches in mathematics courses of mathematics teacher preparation programs.

There were two studies conducted with the College of Education tutors that examined tutors' conceptions and tutors' knowledge, skills and practice in the teaching of completing the square method. Both were written by the same authors based on the same data. Quadratic equations are a subject that links elementary algebra to calculus; therefore, implementing a study with tutors who are responsible for the teaching of quadratics at the university level has an important contribution to this area and the number of studies conducted with tutors can be increased. There was also one study conducted with experts which aimed to develop structure sense tasks on quadratic equations for high school algebra and validate these tasks by experts.

The content analysis regarding textbooks analysis displayed that the contents of the chapters on quadratics in different countries' mathematics textbooks displayed the approaches, the positions of the chapters and weights of the quadratic units, the time allocated and the grades the quadratics are taught were different (Sağlam, 2012). This finding indicated that the same topic can be taught differently in different countries. Therefore, increasing the number of studies that analyze the content of the quadratic equations in mathematics textbooks would be helpful for mathematics education researchers to understand how this topic is presented in detail and how students learn the content using textbooks in different countries.

Briefly, the findings of this study displayed a general trend of the quadratic equation studies as well as presented the focus of the studies in an interpretive way. The findings suggest that as one of the important topics of secondary school algebra, the topic of quadratic equations deserves more attention.

Ethical Declaration

All rules included in the "Directive for Scientific Research and Publication Ethics in Higher Education Institutions" have been adhered to, and none of the "Actions Contrary to Scientific Research and Publication Ethics" included in the second section of the Directive have been implemented.

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