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On Symmetric Bi-Derivations of Lattice Implication Algebras

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Abstract

In this paper, we introduced the notion of symmetric bi- derivations on lattice implication algebra and investigated some related properties. Also, we characterized the Fix_D (L), and Ker_D (L) by symmetric bi-derivations. Additionally, we proved that if D is a symmetric bi-derivation of a lattice implication algebra, every filter F is D-invariant.

1. INTRODUCTION

The concept of lattice implication algebra was proposed by Y. Xu [13], in order to establish an alternative logic knowledge representation. In his paper, Xu combined lattice and implication algebras and created a new algebraic structure. Then lattice implication algebra, being an important non-classical algebra, has been studied by many researchers. Y. Xu and K. Y. Qin [10] discussed the properties of lattice implication H algebras and gave some equivalent conditions about H lattice implication algebras. Y. Xu and K. Y. Qin [11] defined the notion of filters in a lattice implication algebra and obtained their properties.

Lee and Kim introduced in [5] the notion of derivation in lattice implication algebra and considered its properties. Then Yon and Kim introduced in [4] the notion of f-derivation in lattice implication algebra similarly.

Motivated by the notion of symmetric bi-derivation, symmetric left bi- derivation and derivations on various logic algebras (see [1], [3], [6], [7] , [10]) ; in this paper we introduced the notion of symmetric bi-derivation in lattice implication algebra. We gave the properties of a symmetric bi-derivation D in lattice implication algebra, and also the properties of its trace. We also defined the fixed set FixD(L) and KerD, and showed that every filter in the lattice implication L is D-invariant for D being a symmetric bi-derivation.

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2. PRELIMINARIES

A lattice implication algebra is an algebra (L; ∧, ∨, r →, 0, 1) of type (2, 2, 1, 2, 0, 0) where (L, ∧, ∨, 0, 1) is a bounded lattice, " r " is an order- reversing involution and " → " is a binary operation, satisfying the following axioms for all x, y, z ∈ L:

(I1) $x \to (y \to z) = y \to (x \to z)$.

(I2) $x \to x = 1$.

(I3) $x \to y = y' \to x'$.

(I4) $x \to y = y \to x = 1 \Rightarrow x = y$

(I5) $(x \to y) \to y = (y \to x) \to x$.

(L1) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$.

(L2) $(x \wedge y) \to z = (x \to z) \vee (y \to z)$.

If L satisfies conditions (I1)-(I5), we say that L is a quasi-lattice implication algebra. A lattice implication algebra L is called lattice H implication algebra if it satisfies $x \vee y \vee ((x \wedge y) \to z) = 1$ for all x, y, z ∈ L.

In the sequel the binary operation"→" will be denoted by juxtaposition. We can define a partial ordering"≤" on a lattice implication algebra L by $x \leq y$ if and only if $x \to y = 1$.

In a lattice implication algebra L, the following hold [11]:

(U1) $0 \to x = 1$, $1 \to x = x$ and $x \to 1 = 1$.

(U2) $x \to y \leq (y \to z) \to (x \to z)$.

(U3) $x \leq y$ implies $y \to z \leq x \to z$ and $z \to x \leq z \to y$.

(U4) $x' = x \to 0$.

(U5) $x \vee y = (x \to y) \to y$.

(U6) $((y \to x) \to y')' = x \wedge y = ((x \to y) \to x')'$.

(U7) $x \leq (x \to y) \to y$.

In a lattice H implication algebra L, the following holds:

(U8) $x \to (x \to y) = x \to y$.

(U9) $x \to (y \to z) = (x \to y) \to (x \to z)$.

A non-empty subset F of a lattice implication algebra L is called a filter of L if it satisfies:

(F1) $1 \in F$,

(F2) $x \in F$ and $x \to y \in F$ imply $y \in F$, for all x, y ∈ L.

Deftnition 2.1. Let L be a lattice implication algebra. A mapping D(., .) :

$L \times L \to L$ is called symmetric if $D(x, y) = D(y, x)$ for all x, y ∈ L.

Deftnition 2.2. Let L be a lattice implication algebra. A mapping $d : L \to L$ defined by $d(x) = D(x, x)$ for all x ∈ L is called the trace of D(., .), where $D(., .) : L \times L \to L$ is a symmetric mapping.

3. THE SYMMETRIC BI-DERIVATION ON LATTICE IMPLICATION ALGEBRAS

The following definition introduces the notion of symmetric bi-derivation for lattice implication algebras.

Deftnition 3.1. Let L be a lattice implication algebra and let D(., .) :

L × L ⟶ L be a symmetric mapping. We call D a symmetric bi-derivation on L if it satisfies;

D(x ⟶ y, z) = (x ⟶ D(y, z)) ∨ (D(x, z) ⟶ y) for all x, y, z ∈ L.

It is clear that a symmetric bi-derivation on L also satisfies;

D(x, y ⟶ z) = (D(x, y) ⟶ z) ∨ (y ⟶ D(x, z)) for all x, y, z ∈ L.

Example 3.1. Let L:= {0, a, b, 1} be a set with the Cayley table;

x	x'
0	1
a	b
b	a
1	0

$\rightarrow$	0	a	b	1
0	1	1	1	1
a	b	1	1	1
b	a	b	1	1
1	0	a	b	1

For any x ∈ L we have x' = x ⟶ 0 and the operations ∨ and ∧ on L are defined as : x ∨ y = (x ⟶ y) ⟶ y and x ∧ y = ((x' ⟶ y') ⟶ y')'. Then (L; ∧, ∨, r ⟶, 0, 1) is a lattice implication algebra. The mapping D(., .) : L × L ⟶ L will be defined by D(x, y) = x' ⟶ (y' ⟶ a) for every x, y ∈ L,

i.e.,

$$D(x,y) = \begin{cases} a, & if\ x = 0\ \ and\ y = 0, \\ b, & if\ (x = a\ \ and\ y = 0)\ \ or\ (x = 0\ \ and\ y = a), \\ 1, & otherwise \end{cases}$$

Then we can see that D is a symmetric bi-derivation on L.

Example 3.2. Let L := {0, a, b, 1} be a set with the Cayley table;

x	x'
0	1
a	b
b	a
1	0

$\rightarrow$	0	a	b	1
0	1	1	1	1
a	b	1	1	1
b	a	b	1	1
1	0	a	b	1

For any x ∈ L we have x' = x ⟶ 0 and the operations ∨ and ∧ on L are defined as : x ∨ y = (x ⟶ y) ⟶ y and x ∧ y = ((x' ⟶ y') ⟶ y')'. Then (L; ∧, ∨, r ⟶, 0, 1) is a lattice implication algebra. The mapping D(., .) :

L × L ⟶ L will be defined as :

$$D(x,y)=\begin{cases} x, & if\ y=0,\\ y, & if\ x=0,\\ b, & if\ x=a\ and\ y=a,\\ 1, & otherwise\end{cases}$$

It is easy to check that D is a symmetric bi-derivation on L.

Example 3.3. Let $(B; \wedge, \vee, ', 0, 1)$ be a Boolean algebra. If we define

$$x \longrightarrow y = x' \vee y$$

for every $x, y \in B$, then $(B; \wedge, \vee, r, \longrightarrow, 0, 1)$ is a lattice implication algebra. For a fixed element a in B, if we define a map $D : B \times B \longrightarrow B$ by

$$D(x, y) = x \vee y \vee a$$

for every $x, y \in B$, then D is a symmetric bi-derivation of B.

Proposition 3.2. Let L be a lattice implication algebra and d be the trace of symmetric bi-derivation D on L. Then the followings hold:

i) $D(1, x) = 1$ for all $x \in L$.

ii) $d(1) = 1$

Proof. i) Let $x, y \in L$. Then we have

$$\begin{aligned} D(1, x) &= D(x \longrightarrow 1, x)\\ &= (x \longrightarrow D(1, x)) \vee (D(x, x) \longrightarrow 1)\\ &= (x \longrightarrow D(1, x)) \vee 1 = 1 \end{aligned}$$

Therefore we get $D(1, x) = 1$.

ii) It is clear from i).

Proposition 3.3. Let L be a lattice implication algebra and d be the trace of symmetric bi-derivation D on L. Then the followings hold:

i) $D(x, y) = D(x, y) \vee x$ for all $x, y \in L$

ii) $d(x) = d(x) \vee x$ for all $x \in L$.

Proof. i) Let $x, y \in L$. Then we have

$$\begin{aligned} D(x, y) &= D(1 \longrightarrow x, y)\\ &= (1 \longrightarrow D(x, y)) \vee (D(1, y) \longrightarrow x)\\ &= D(x, y) \vee (1 \longrightarrow x)\\ &= D(x, y) \vee x \end{aligned}$$

So we have $D(x, y) = D(x, y) \vee x$.

Also, we can get $x \leq D(x, y)$ for all $x, y \in L$.

ii)Let $x \in L$. Then we have

$$\begin{aligned} d(x) &= D(x, x) \\ &= D(1 \rightarrow x, x) \\ &= D(x, x) \vee (1 \rightarrow x) \\ &= d(x) \vee x \end{aligned}$$

We proved $d(x) = d(x) \vee x$ and as above we can obtain $x \leq d(x)$ for all $x \in L$.

Proposition 3.4. Let L be a lattice implication algebra and d be the trace of symmetric bi-derivation D on L. Then

$$d(x) \rightarrow y \leq x \rightarrow y \leq x \rightarrow d(y) \text{ for all } x, y \in L.$$

Proof.Let L be a lattice implication algebra and d be the trace of symmetric bi-derivation D on L.

From Proposition 3.3(ii) we have $x \leq d(x)$ and $y \leq d(y)$.

Using (U3); $x \leq y$ implies $y \rightarrow z \leq x \rightarrow z$ and $z \rightarrow x \leq z \rightarrow y$.

We obtain $d(x) \rightarrow y \leq x \rightarrow y$ and $x \rightarrow y \leq x \rightarrow d(y)$.

We can conclude $d(x) \rightarrow y \leq x \rightarrow y \leq x \rightarrow d(y)$ for all $x,y \in L$.

Proposition 3.5. Let L be a lattice implication algebra and d be the trace of symmetric bi-derivation D on L. Then

i)$D(d(x) \rightarrow x, x) = 1$ for all $x \in L$.

ii) $d(d(x) \rightarrow x) = 1 = d(x \rightarrow d(x))$ for all $x \in L$.

Proof. i) By using the definition of symmetric bi-derivation D of a lattice implication algebra and properties of a lattice implication algebras (I2), (U5) and (U1);

$$\begin{aligned} D(d(x) \rightarrow x, x) &= (d(x) \rightarrow D(x, x)) \vee (D(d(x), x) \rightarrow x) \\ &= (d(x) \rightarrow d(x)) \vee (D(d(x), x) \rightarrow x) \\ &= (1 \rightarrow (D(d(x), x) \rightarrow x)) \rightarrow (D(d(x), x) \rightarrow x) \\ &= (D(d(x), x) \rightarrow x) \rightarrow (D(d(x), x) \rightarrow x) = 1 \end{aligned}$$

ii) By using the definition of trace of a symmetric bi-derivation D of a lattice implication algebra and properties of a lattice implication algebras

$$\begin{aligned} d(d(x) \rightarrow x) &= D(d(x) \rightarrow x, d(x) \rightarrow x) \\ &= (d(x) \rightarrow D(x, d(x) \rightarrow x)) \vee (D(d(x), d(x) \rightarrow x) \rightarrow x) \\ &= [d(x) \rightarrow ((d(x) \rightarrow d(x)) \vee (D(x, d(x)) \rightarrow x))] \vee (D(d(x), d(x) \rightarrow x) \rightarrow x) \\ &= [d(x) \rightarrow (1 \vee (D(x, d(x)) \rightarrow x))] \vee (D(d(x), d(x) \rightarrow x) \rightarrow x) \\ &= [d(x) \rightarrow 1)] \vee (D(d(x), d(x) \rightarrow x) \rightarrow x) \\ &= 1 \vee (D(d(x), d(x) \rightarrow x) \rightarrow x) \quad =1 \end{aligned}$$

This proves the first part; $d(d(x) \rightarrow x) = 1$.

The second equality is obvious since $d(x \rightarrow d(x)) = d(1) = 1$ which was

the result that we obtained at the end of the proof of Proposition 3.3 ii).

Proposition 3.6. Let a symmetric map $D : L \times L \to L$ defined for all $x, y, z \in L$ by $D(x \to y, z) = x \to D(y, z)$ on $L \times L$. Then D is a bi- derivation on $L \times L$.

Proof. For all $y \in L$, $D(1, y) = D(D(1, y) \to 1, y) = D(1, y) \to D(1, y) = 1$. It follows

$$x \to D(x, y) = D(x \to x, y) = D(1, y) = 1.$$

for every $x, y \in L$. Since $x \le D(x, z)$ and $y \le D(y, z)$, we have

$$D(x, z) \to y \le x \to y \le x \to D(y, z).$$

Hence $D(x \to y, z) = x \to D(y, z) = (x \to D(y, z)) \vee (D(x, z) \to y)$, and

D is a bi-derivation.

Proposition 3.7. If $D : L \times L \to L$ is a symmetric bi-derivation, then D satisfies $D(x \to y, z) = x \to D(y, z)$ for all $x, y, z \in L$.

Proof. Let D be a symmetric bi-derivation and $x, y, z \in L$. Since $x \le D(x, z)$ and $y \le D(y, z)$, we have

$$D(x, z) \to y \le x \to y \le x \to D(y, z).$$

Hence $D(x \to y, z) = (x \to D(y, z)) \vee (D(x, z) \to y) = x \to D(y, z)$.

As a consequence of Propositions 3.6 and 3.7 we can state the following theorem.

Theorem 3.8. A map $D : L \times L \to L$ is a symmetric bi-derivation if and only if D is a symmetric map and it satisfies $D(x \to y, z) = x \to D(y, z)$ for every $x, y, z \in L$.

Proposition 3.9. A map D being a symmetric bi derivation defined on the lattice implication algebra L satisfies the following:

$$D(x, y \to z) = y \to D(x, z) \text{ for all } x, y, z \in L.$$

Proof. We will make use of the previous theorem 3.8 and the fact that D is symmetric.

$$\begin{aligned} D(x, y \to z) &= D(y \to z, x) \\ &= y \to D(z, x) \\ &= y \to D(x, z) \end{aligned}$$

Proposition 3.10. A map D being a symmetric bi derivation defined on the lattice implication algebra L satisfies the following: $D(x, y) = x' \to (y' \to D(0, 0))$ for every $x, y \in L$. That is, the value of D is determined by $D(0, 0)$.

Proof. For any $x, y \in L$, $D(x, y) = D(x'', y'') = D(x' \to 0, y' \to 0) = x' \to (y' \to D(0, 0))$.

Proposition 3.11. Let L be a lattice implication algebra where d is the trace of symmetric bi-derivation D on L. Then

$$d(x \to y) = x \to [x \to d(y)] \text{ for all } x, y, \in L.$$

Proof.

$$d(x \to y) = D(x \to y, x \to y)$$
$$= x \to D(y, x \to y)$$
$$= x \to D(x \to y, y)$$
$$= x \to [x \to D(y, y)]$$
$$= x \to [x \to d(y)]$$

Furthermore, in a lattice H implication algebra with the additional equality $x \to (x \to y) = x \to y$ we get;

$$d(x \to y) = x \to d(y)$$

Definition 3.12. Let D be a symmetric bi-derivation of the lattice implication algebra L and d be the trace of D. We can define a set FixD(L) by

$$Fix_D(L) := \{x \in L | d(x) = x\}.$$

Proposition 3.13. Let d be the trace of the symmetric bi-derivation D of the lattice implication algebra L, then we have;

$$(dodod....od)(x) = x \text{ for all } x \in L.$$

Proof. The proof is obvious by the definition of the trace of D.

Proposition 3.14. Let D be a symmetric bi-derivation of the lattice H implication algebra L.

i) if $x \in L$, $y \in FixD(L)$ then $x \to y \in FixD(L)$.

ii) if $y \in FixD(L)$ then $x \vee y \in FixD(L)$ $\forall x \in L$.

Proof. i) Let D be a symmetric bi-derivation of the lattice H implication algebra. By using Proposition 3.9 and the fact that $y \in FixD(L)$ we have $d(x \to y) = x \to d(y) = x \to y$. Therefore, we get $x \to y \in FixD(L)$.

ii) Let D be a symmetric bi-derivation of the lattice H implication algebra. By using Proposition 3.9 and the fact that we have $y \in FixD(L)$ we have

$$d(x \vee y) = d((x \to y) \to y$$
$$= (x \to y) \to d(y)$$
$$= (x \to y) \to y$$

Hence $d(x \vee y) = x \vee y$. Therefore, we get $x \vee y \in FixD(L)$.

Proposition 3.15. Let D be a symmetric bi-derivation of the lattice H implication algebra L; for $x, y \in L$

If $x \leq y$ and $x \in FixD(L)$ then $y \in FixD(L)$.

Proof. We have $x, y \in L$ and $x \leq y$ so that $x \to y = 1$ and $x \in FixD(L)$.

$$d(y) = d(1 \to y)$$
$$= d((x \to y) \to y)$$
$$= d((y \to x) \to x)$$
$$= d(y \vee x)$$
$$= y \vee x$$

by Proposition 3.11 $x \in FixD(L)$ implies $y \vee x \in FixD(L)$

$$d(y) = (y \to x) \to x$$
$$= (x \to y) \to y = 1 \to y = y$$

So we get $d(y) = y$ and we have proved $y \in FixD(L)$.

Deftnition 3.16. Let L be a lattice implication algebra. A nonempty subset A of L is said to be D-invariant if $D(A, A) \subseteq A$ where $D(A, A) =\{D(x, x)|x \in A\}$.

Proposition 3.17. Let D be a symmetric bi-derivation of the lattice im- plication algebra L. Then every filter A is D-invariant.

Proof. Let $y \in D(A, A)$ then $y = D(x, z)$ for some $x, z \in A$. We have $x \leq D(x, z)$ and $z \leq D(x, z)$ from Proposition 3.3. So $x \to D(x, z) = 1$ and $z \to D(x, z) = 1$. Since $x, z \in A$ and A is a filter we have $D(A, A) \subseteq A$.

Deftnition 3.18. Let D be a symmetric bi-derivation of the lattice impli- cation algebra L, and let d be the trace of D. We can define KerD ;

$$KerD := \{x \in L|D(x, x) = d(x) = 1\}$$

Proposition 3.19. Let D be a symmetric bi-derivation of the lattice im- plication algebra L, and let d be the trace of D.

$$\text{If } y \in KerD \text{ then } x \vee y \in KerD\ \forall x \in L.$$

Proof. $y \in KerD$ and using the definition of symmetric bi-derivation D of lattice implication algebra we have

Since

$$\begin{aligned} D(y, x \vee y) &= D(x \vee y, y) \\ &= D((x \to y) \to y, y) \\ &= [(x \to y) \to D(y, y)] \vee [D(x \to y, y) \to y] \\ &= [(x \to y) \to 1)] \vee [D(x \to y, y) \to y] \\ &= 1 \vee [D(x \to y, y) \to y] \\ &= 1 \end{aligned}$$

we have $D(y, x \vee y) = 1$.

Therefore,

$$\begin{aligned} d(x \vee y) &= D(x \vee y, x \vee y) \\ &= D((x \to y) \to y, x \vee y) \\ &= [(x \to y) \to D(y, x \vee y)] \vee [D(x \to y, x \vee y) \to y] \\ &= [(x \to y) \to 1] \vee [D(x \to y, x \vee y) \to y] \\ &= 1 \vee [D(x \to y, x \vee y) \to y] \\ &=1 \end{aligned}$$

Hence, we get the result that is $x \vee y \in KerD$, $\forall x \in L$

Deftnition 3.20. Let L be a lattice implication algebra. Then for a fixed element $a \in L$ let us define a map $da : L \to L$ such that $da(x) = D(x, a)$ for every $x \in L$.

Theorem 3.21. For each $a \in L$ the map da defined above is a derivation of L.

Proof. For a fixed element $a \in L$ let us define a map $da : L \to L$ such that

$da(x) = D(x, a)$ for every $x \in L$. Now for every $x, y \in L$, we have

$$
\begin{aligned}
da(x \to y) &= D(x \to y, a) \\
&= (x \to D(y, a)) \wedge (D(x, a) \to y) \\
&= (x \to da(y)) \wedge (da(x) \to y)
\end{aligned}
$$

So da is a derivation of L. So we can say that for each element $a \in L$ the map da defined above is a derivation of L.

Proposition 3.22. Let D be a symmetric bi-derivation of the lattice im- plication algebra L.

Then $D(x \vee y, z) = D(x, z) \vee D(y, z)$ and $D(x \wedge y, z) = D(x, z) \wedge D(y, z)$ for every $x, y, z \in L$.

Proof. Let $x, y, z \in L$. Then we have

$$
\begin{aligned}
D(x \vee y, z) &= D(x'' \vee y'', z) \\
&= D((x' \wedge y')', z) \\
&= D((x' \wedge y') \to 0, z) \\
&= (x' \wedge y') \to D(0, z) \\
&= (x' \to D(0, z)) \vee (y' \to D(0, z)) \\
&= D(x' \to 0, z) \vee D(y' \to 0, z) \\
&= D(x'r, z) \vee D(y'r, z) \\
&= D(x, z) \vee D(y, z)
\end{aligned}
$$

We can prove the case of meet operation in the similar way.

Proposition 3.23. Let D be a symmetric bi-derivation of the lattice im- plication algebra L. D is monotone, That is if $x1 \le x2$ and $y1 \le y2$, then $D(x1, y1) \le D(x2, y2)$, for every $x1, x2, y1, y2 \in L$.

Proof. For every $x1, x2, y1, y2 \in L$, we have

$D(x1 \vee x2, y1) = D(x1, y1) \vee D(x2, y1)$ from Prop 3.21

since $x1 \le x2$, we have $x1 \vee x2 = x2$

$D(x2, y1) = D(x1, y1) \vee D(x2, y1)$

$D(x1, y1) \le D(x2, y1)$ (*)

Again,

$D(y1 \vee y2, x2) = D(y1, x2) \vee D(y2, x2)$ from Prop 3.21 since $y1 \le y2$, we have $y1 \vee y2 = y2$

$D(x2, y1) = D(x2, y1) \vee D(x2, y2)$

$D(x2, y1) \le D(x2, y2)$ (* *)

Therefore by using (*) and (* *), we get $D(x1, y1) \le D(x2, y2)$.

Proposition 3.24. Let D be a symmetric bi-derivation of the lattice implication algebra L. Then

$$D(x', x) = D(x, x') = 1 \text{ for every } x \in L$$

Proof. For every $x \in L$, we have

$D(x', x) = D(x \rightarrow 0, x) = x \rightarrow D(0, x) = x \rightarrow D(x, 0)=D(x \rightarrow x, 0)=D(1, 0) = 1$

Proposition 3.25. Let D be a symmetric bi-derivation of the lattice implication algebra L.

$$D(y, x) = 1 \text{ for every } x, y \in L \text{ with } x' \leq y$$

Proof. For every $x, y \in L$, we know that $x' \leq y$ implies $x' \vee y = y$.

$$D(y, x) = D(x' \vee y, x) = D(x', x) \vee D(y, x) = 1 \vee D(y, x) = 1$$

CONCLUSION

The aim of this work was to study maps on lattice implication algebras and more specifically the derivations and f-derivations defined on implication algebras. Then this work aims to define a new type of derivation in lattice implication algebras, the notion of symmetric bi-derivations in this algebraic structure. First of all in the first part some basic definitions needed for the readability of the work are given about the lattice implication algebras. Then in the second part the notion of derivation and f-derivation in lattice implication algebras; introduced respectively by Lee and Kim are observed. Main properties of these maps are listed in this part. In the third part the notion of symmetric bi-derivation of lattice implication algebras is defined; examples satisfying its properties are listed. Then some theorems and propositions that these symmetric bi-derivations satisfied in other algebraic structures like B-algebras and in lattices are proved for lattice implication algebras. Moreover the properties of the symmetric bi-derivation D in lattice implication algebra and also the properties of its trace are given; also are defined the fixed set and the Kernel of the map. The next step of the work can be some more detailed studies about other types of derivations in lattice implication algebras, generalized derivations can be for example studied in this algebraic structure.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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