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Rough Groups Over Rough Sets

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Abstract: In this paper, the author focuses on the algebraic structure of the rough groups, especially those based on the rough sets. The author proposes a new definition of the rough group, based on Pawlak's rough sets. This new definition addresses some shortcomings of previous definitions. The author then provides a detailed description of the structure of the rough groups.

Rough Kümeler Üzerinde Rough Gruplar

Anahtar Kelimeler
Rough Set, Approximation Space, Rough Group, Rough Subgroup

Öz: Bu makalede yazar, kaba grupların, özellikle kaba kümelere dayalı olanların cebirsel yapısına odaklanmaktadır. Yazar, Pawlak'ın kaba kümelerine dayanan yeni bir kaba grup tanımı öneriyor. Bu yeni tanım, önceki tanımların bazı eksikliklerini gidermektedir. Yazar daha sonra kaba grupların yapısını ayrıntılı ele almaktadır.

1. INTRODUCTION

The theory of rough sets, an extension of the classical set theory, was introduced first by Pawlak [6]. The theory deals with the approximations of any subsets of the given universal set by traceable subsets which are called upper and lower approximation subsets of the addicted rough sets. Useful details of the rough set theory used in the further sections are given in this section with a new extended definition. Successful applications of the theory in a wide range of problems including basic algebraic notions, quasi-Boolean algebras, Boolean algebras of the topological spaces, rough algebraic structures, topological rough algebras, quasi-Boolean algebras of the topological spaces, etc., have properly demonstrated its versatility and practicality [1, 3].

In order to have the deep mathematical meanings for further developments, rough sets theory needed to contain universal algebraic structures. For this purpose, some investigations have been made by Biswas and Nanda in 1994 [2]. They give the first definitions of rough group, rough subgroup and some properties of them. Subsequently Miao and colleagues in their paper [5] renew the definition of rough group given by Biswas and Nanda. The main work of Miao and et al. was to improve some deficiencies in the definition of rough group given by Biswas and Nanda such as the lack of associativity property, the trivial rough subgroups of a rough group, and the intersection of any two rough subgroups of a rough group should satisfy the rough subgroup property too.

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2. APPROXIMATION SPACES

In the theory of rough sets, approximation spaces are the main concepts. In this section, the structure of Pawlak's approximation space is restated (see in [7]) which is used in the next sections.

Let U be a non-empty finite set and R be an equivalence relation over the set U ($R \subseteq U \times U$). Then the pair (U, R) is called as an *approximation space*. In an approximation space (U, R), U is called the *universe* and R is called the *underlying relation* of the approximation space (U, R). Since R is an equivalence relation on U, it separates U into disjoint subsets of its own. It is a quotient set of the universe U and denoted by U/R. Elements of U/R is called the equivalence classes. For every $x \in U$ the equivalence class containing x is denoted by $[x]_R$ and equals to $\{y \in U \mid xRy\}$. The set of all equivalence classes is also called a *classification* of U induced by underlying relation R. Further, if a classification $\{E_1, E_2, \dots, E_n\}$ of U is given then it is not needed to construct corresponding underlying equivalence relation R any more.

Every union of equivalence classes of the underlying equivalence relation for an approximation space is called *definable* (or *exact*) set [6, 2]. In general, the empty set is considered to be a definable set. For an arbitrary subset A of U, if it is not a definable set then A could not be described precisely by using the underlying relation of the approximation space (U, R). In this case, there are some characterization of A to give idea of its structure. They are lower and upper approximations:

$$\begin{aligned}\underline{apr}_R(A) &= \{x \in U \mid [x]_R \subseteq A\}\\ &= \bigcup\{[x]_R \mid [x]_R \subseteq A\}\end{aligned}$$

and

$$\begin{aligned}\overline{apr}_R(A) &= \{x \in U \mid [x]_R \cap A \neq \emptyset\}\\ &= \bigcup\{[x]_R \mid [x]_R \cap A \neq \emptyset\}.\end{aligned}$$

Here, the lower approximation $\underline{apr}_R(A)$ is the greatest definable set which is contained in A, and the upper approximation $\overline{apr}_R(A)$ is the least definable set which contains A in the approximation space (U, R).

Considering lower and upper approximations as two operators from the power set of universes U to itself then they are duals of each other which means one of them can be found from the other by using the following formulas:

$$\begin{aligned}\sim \underline{apr}_R(A) &= \overline{apr}_R(\sim A)\\ \sim \overline{apr}_R(A) &= \underline{apr}_R(\sim A)\end{aligned}$$

where $\sim A$ denotes the set of complement of A in the universe U.

Now for any subsets A, B of the universe U, it can be given the list of some properties satisfied dually by $\underline{apr}_R$ and $\overline{apr}_R$ together:

$(LU1)$ $$\begin{aligned}\underline{apr}_R(U) &= U,\\ \overline{apr}_R(\emptyset) &= \emptyset,\end{aligned}$$

$(LU2)$ $$\begin{aligned}\underline{apr}_R(A \cap B) &= \underline{apr}_R(A) \cap \underline{apr}_R(B),\\ \overline{apr}_R(A \cup B) &= \overline{apr}_R(A) \cup \overline{apr}_R(B),\end{aligned}$$

$(LU3)$ $$\begin{aligned}\underline{apr}_R(A \cup B) &\supseteq \underline{apr}_R(A) \cup \underline{apr}_R(B),\\ \overline{apr}_R(A \cap B) &\subseteq \overline{apr}_R(A) \cap \overline{apr}_R(B),\end{aligned}$$

$(LU4)$ $A \subseteq B$ implies $\underline{apr}_R(A) \subseteq \underline{apr}_R(B)$ and $\overline{apr}_R(A) \subseteq \overline{apr}_R(B)$

$(LU5)$ $$\begin{aligned}\underline{apr}_R(\emptyset) &= \emptyset,\\ \overline{apr}_R(U) &= U,\end{aligned}$$

$(LU6)$ $\underline{apr}_R(A) \subseteq A \subseteq \overline{apr}_R(A)$

$(LU7)$ $\overline{apr}_R\left(\underline{apr}_R(A)\right) \subseteq A \subseteq \underline{apr}_R\left(\overline{apr}_R(A)\right),$

$(LU8)$

$$\begin{aligned}\underline{apr}_R(A) &\subseteq \underline{apr}_R(\underline{apr}_R(A)),\\ \overline{apr}_R(\overline{apr}_R(A)) &= \overline{apr}_R(A),\end{aligned}$$

$(LU9)$ $$\begin{aligned}&\overline{apr}_R(A) \subseteq \underline{apr}_R\left(\overline{apr}_R(A)\right),\\ &\overline{apr}_R\left(\underline{apr}_R(A)\right) \subseteq \underline{apr}_R(A),\end{aligned}$$

$(LU10)$ $\underline{apr}_R(\sim A \cup B) \subseteq \sim \underline{apr}_R(A) \cup \underline{apr}_R(B),$

$(LU11)$ $\underline{apr}_R(A) \subseteq \overline{apr}_R(A).$

In an approximation space (U, R), for any two definable subsets A and B of U with $A \subseteq B$, the pair (A, B) is called a *rough set* [4]. On the set of all rough sets of the approximation space (U, R), the set-theoretic operators can be defined component wise by using the standard set operators. For any rough sets (A_1, B_1) and (A_2, B_2), the following definitions can be stated

union of rough sets:

$$(A_1, B_1) \cup (A_2, B_2) = (A_1 \cup A_2, B_1 \cup B_2)$$

intersection of rough sets:

$$(A_1, B_1) \cap (A_2, B_2) = (A_1 \cap A_2, B_1 \cap B_2)$$

set inclusion of rough sets:

$(A_1, B_1) \subseteq (A_2, B_2)$ iff $A_1 \subseteq A_2$ and $B_1 \subseteq B_2$

difference of rough sets:

$$(A_1, B_1) \backslash (A_2, B_2) = (A_1 \backslash A_2, B_1 \backslash B_2)$$

set complement of a rough set:

$$\sim (A_1, B_1) = (U, U) \backslash (A_1, B_1) = (\sim B_1, \sim A_1)$$

The set of all rough sets with the above set-theoretic operators is called a *rough set algebra.*

Now for any rough set (A', A''), there is a subset A of the universe U such that $\underline{apr_R}(A) = A'$ and $\overline{apr}_R(A) = A''$.

2.1. Definition

Let (U, R) be an approximation space and A be a subset of the universe U. Then the triple $(A, \underline{apr_R}(A), \overline{apr}_R(A))$ is called the *rough set of* A and denoted by $\mathcal{R}(A)$.

It is obvious that if A is a definable subset of U then so A' and A''. Furthermore $A = A' = A''$. In this case, it can be considered that any definable subset A of U as a rough set of A, $\mathcal{R}(A) = (A, A, A)$. The operations $\cup$, $\cap$, and $\backslash$ given above can be defined similarly over the set $\mathcal{U} = \{\mathcal{R}(A) | A \in \mathcal{P}(U)\}$ of all rough sets of subsets in the approximation space (U, R).

For two rough sets $\mathcal{R}(A)$ and $\mathcal{R}(B)$, the rough inclusion could be defined as follows $\mathcal{R}(A) \subseteq \mathcal{R}(B)$ iff $A \subseteq B$. If $\mathcal{R}(A) \subseteq \mathcal{R}(B)$ then it can be said A is a rough subset of B. For A and B, A is rough equal to B if and only if A is a rough subset of B and vice versa.

It is clear that with rough inclusion and the operations $\cup$,$\cap$,$\backslash$, and $\sim$ defined above, the set $\mathcal{U}$ can be considered as a rough set algebra.

3. ROUGH GROUPS

According to the given structural knowledges in former section, basic algebraic rough group structure can be introduced with the following definition over rough sets of subsets in an approximation space.

3.1. Definition

Let (U, R) be an approximation space and $*$ be a binary operation defined on U. Let G be a subset of the universe U. If the following properties are satisfied then $\mathcal{R}(G)$ is called a *rough group:*

(i) for every $x, y, z \in \overline{apr}_R(G)$,

$$x * (y * z) = (x * y) * z,$$

(ii) $a_1 * a_2 * \cdots * a_n \in \overline{apr}_R(G)$,

where $a_1, a_2, \ldots, a_n \in G$,

(iii) there exists an element $e \in \overline{apr}_R(G)$, called *rough identity*, for all $a \in G$, $a * e = e * a = a$,

(iv) for all $a \in G$ there exists an element $b \in G$, called *rough inverse*, such that $a * b = b * a = e$.

A rough group $\mathcal{R}(G)$ with the binary operation $*$ is sometimes denoted by $(\mathcal{R}(G), *)$. $(\mathcal{R}(G), *)$ is called *Abelian* if $a * b = b * a$ for all $a, b \in G$. Further, the element $a_1 * a_2 * \cdots * a_n$ for $a_1, a_2, \ldots, a_n \in G$ is denoted by $\prod_{j=1}^{n} a_j$.

3.2. Proposition

Let (U, R) be an approximation space, $\mathcal{R}(G)$ be a rough group and e be the rough identity of $\mathcal{R}(G)$. Then $G \cap \overline{apr}_R(\{e\}) \neq \emptyset$.

Proof. Since $e \in \overline{apr}_R(G)$, $[e]_R \cap G \neq \emptyset$ and $[e]_R = \overline{apr}_R(\{e\})$ then the result is obvious.

3.3. Proposition

Let $\mathcal{R}(G)$ be a rough group, then the following properties hold;

(i) G has a unique rough identity element,

(ii) every element of G has a unique rough inverse,

(iii) for any $a, b, x, y \in G$,
$x * a = x * b$ implies $a = b$
$a * y = b * y$ implies $a = b$.

3.4. Definition

Let G be a rough group and H be a nonempty subset of G. If $\mathcal{R}(H)$ is a rough group with the same binary operation on U, then $\mathcal{R}(H)$ is called a *rough subgroup* of $\mathcal{R}(G)$.

3.5. Definition

Let G be a subset of the universe U, then the set
$$\{a_1 * a_2 * \cdots * a_n | a_1, a_2, \ldots, a_n \in G\}$$
is called *exact set* of G and denoted by $\tilde{G}$.

3.6. Example

Let S_3 be the symmetric group on $X = \{1,2,3\}$ and α_i be functions on X with restrictions $\alpha_i(1) \neq \alpha_i(2)$ and $\alpha_i(1) \neq \alpha_i(3)$ where $i = 1, \ldots, 6$. These functions are all given below,

$$\alpha_1(x) = \begin{cases} 1 & if \quad x = 1 \\ 2 & if \quad x = 2,3 \end{cases},$$

$$\alpha_2(x) = \begin{cases} 1 & if \quad x = 1 \\ 3 & if \quad x = 2,3 \end{cases},$$

$$\alpha_3(x) = \begin{cases} 2 & if \quad x = 1 \\ 1 & if \quad x = 2,3 \end{cases},$$

$$\alpha_4(x) = \begin{cases} 2 & if \quad x = 1 \\ 3 & if \quad x = 2,3 \end{cases},$$

$$\alpha_5(x) = \begin{cases} 3 & if \quad x = 1 \\ 2 & if \quad x = 2,3 \end{cases},$$

$$\alpha_6(x) = \begin{cases} 3 & if \quad x = 1 \\ 1 & if \quad x = 2,3 \end{cases},$$

Let U be the set of functions $\{\alpha: X \to X \,|\alpha(1) \neq \alpha(2), \alpha(1) \neq \alpha(3)\}$, using above notation $U = S_3 \cup \{\alpha_i | i = 1, \ldots, 6\}$, and $\{E_1, E_2, E_3, E_4\}$ be a classification of the universe U where $E_1 = \{\alpha_1, \alpha_2, \alpha_3\}$, $E_2 = \{(1), (13), \alpha_4\}$, $E_3 = \{(23), (12), \alpha_5\}$, and $E_4 = \{(123), (132), \alpha_6\}$. Function composition can be considered as an operation on U.

Therefore, for the subset $G = \{(13), (12), (123), (132)\}$ of U, $(\mathcal{R}(G), o)$ is a rough group with $\overline{apr}(G) = E_2 \cup E_3 \cup E_4$, $\underline{apr}(G) = \emptyset$ and $\tilde{G} = S_3$. For the subset $H = \{(13)\}$ of G, $\mathcal{R}(H)$ is a rough subgroup of $\mathcal{R}(G)$ with $\overline{apr}(H) = E_2$, $\underline{apr}(G) = \emptyset$ and $\tilde{H} = \{(1), (13)\}$.

3.7. Definition

Let $\mathcal{R}(G)$ be a rough group and $\mathcal{R}(H)$ be a rough subgroup of $\mathcal{R}(G)$. Then $\mathcal{R}(H)$ is called *rough cyclic subgroup* of $\mathcal{R}(G)$ if there exists an element $x \in G$ such that $H \subseteq \langle x \rangle$. In particular, if $\mathcal{R}(G)$ is a rough cyclic subgroup of itself then it is called *rough cyclic group* of U.

3.8. Proposition

Let $\mathcal{R}(G)$ be a rough group and $\mathcal{R}(H)$, $\mathcal{R}(K)$ be rough subgroups of $\mathcal{R}(G)$, then the following properties hold;

(i) $\tilde{\tilde{G}} = \tilde{G}$,

(ii) $G \subseteq \tilde{G} \subseteq \overline{apr}(G)$,

(iii) $\overline{apr}(\tilde{G}) = \overline{apr}(G)$,

(iv) if $H \subseteq K$ then $\tilde{H} \subseteq \tilde{K}$. In particular, if $H = K$ then $\tilde{H} = \tilde{K}$.

Note that the converse of Proposition 3.8 (iv) is not true in general as in the following example.

3.9. Example

Let the universe U be as in Example 3.6 and $G = S_3$. If $H = \{(1), (123)\}$ and $K = \{(1), (132)\}$ are taken, then $\overline{apr}(H) = \overline{apr}(K) = E_2 \cup E_4$ and $\tilde{H} = \tilde{K} = \{(1), (123), (132)\}$. $\tilde{H} \subseteq \tilde{K}$ but $H \nsubseteq K$.

For a rough group $(\mathcal{R}(G), *)$ of the universe U, the exact set of G, $\tilde{G}$ is called *exact group* of the rough group $\mathcal{R}(G)$. For any two subsets H, K of U the set $\{a * b \mid a \in H, b \in K\}$ is denoted by $H * K$.

3.10. Proposition

Let $\mathcal{R}(G)$ be a rough group and $\mathcal{R}(H), \mathcal{R}(K)$ are rough subgroups of $\mathcal{R}(G)$. Then the following are satisfied:

(i) $\widetilde{H \cap K} \subseteq \tilde{H} \cap \tilde{K}$,

(ii) if G is abelian then $\widetilde{H * K} \subseteq \tilde{H} * \tilde{K}$.

Proof. (i) Let $x \in \widetilde{H \cap K}$, then there exist $a_1, a_2, \ldots, a_n \in H \cap K$ such that $x = a_1 * a_2 * \cdots * a_n$. Since $a_1, a_2, \ldots, a_n \in H$ and $a_1, a_2, \ldots, a_n \in K$ then $x \in \tilde{H}$ and $x \in \tilde{K}$. And so $x \in \tilde{H} \cap \tilde{K}$.

(ii) Let $x \in \widetilde{H * K}$, then there exist $x_j = a_j * b_j$ and $a_j \in H$, $b_j \in K$ for $j = 1,2, \ldots, n$ such that $x = x_1 * \cdots * x_n$. Since G is abelian, $x = (a_1 * a_2 * \cdots * a_n) * (b_1 * b_2 * \cdots * b_n) \in \tilde{H} * \tilde{K}$.

3.11. Theorem

Let H be a nonempty subset of G. $\mathcal{R}(H)$ is a rough subgroup of $\mathcal{R}(G)$ if and only if for all $a \in H$, $a^{-1} \in H$ and $\widetilde{H} \subseteq \overline{apr}(H)$.

Proof. If H is a rough subgroup then the condition holds. For the converse suppose that for all $a \in H$, $a^{-1} \in H$ and $\widetilde{H} \subseteq \overline{apr}(H)$. Since associativity holds in $\overline{apr}(G)$ for $*$, so in $\overline{apr}(H)$. It is clear that by our assumption, for all $a_1, a_2, \dots, a_n \in H$, $a_1 * a_2 * \cdots * a_n \in \widetilde{H} \subseteq \widetilde{G}$. And so for $a \in H$ and $a^{-1} \in H$, thus $a * a^{-1} = e \in \widetilde{H}$. Hence $\mathcal{R}(H)$ is a rough subgroup of $\mathcal{R}(G)$.

3.12. Proposition

Let $\mathcal{R}(G)$ be a rough group and $\mathcal{R}(H)$, $\mathcal{R}(K)$ are rough groups of $\mathcal{R}(G)$. If $\overline{apr}(H) \cap \overline{apr}(K) = \overline{apr}(H \cap K)$ then $\mathcal{R}(H \cap K)$ is a rough subgroup.

Proof. Suppose that $\overline{apr}(H) \cap \overline{apr}(K) = \overline{apr}(H \cap K)$ and let $a_1, a_2, \dots, a_n \in H \cap K$, then $a_1 * a_2 * \cdots * a_n \in \widetilde{H \cap K} \subseteq \widetilde{H} \cap \widetilde{K} \subseteq \overline{apr}(H) \cap \overline{apr}(K) = \overline{apr}(H \cap K)$. The rest of proof follows easily.

3.13. Theorem

Let $\mathcal{R}(G)$ be an abelian rough group and $\mathcal{R}(H)$, $\mathcal{R}(K)$ are rough subgroups of $\mathcal{R}(G)$. If $\overline{apr}(H) * \overline{apr}(K) = \overline{apr}(H * K)$ then $\mathcal{R}(H * K)$ is a rough subgroup of $\mathcal{R}(G)$.

Proof. Suppose that $\overline{apr}(H) * \overline{apr}(K) = \overline{apr}(H * K)$ and let $a_1, a_2, \dots, a_n \in H * K$, then $a_1 * a_2 * \cdots * a_n \in \widetilde{H * K} \subseteq \widetilde{H} * \widetilde{K} \subseteq \overline{apr}(H) * \overline{apr}(K) = \overline{apr}(H * K)$. Now take $x \in H * K$. Then there exists $h \in H$, $k \in K$ such that $x = h * k$. Since $h^{-1} \in H$, $k^{-1} \in K$, $x^{-1} = h^{-1} * k^{-1} \in H * K$. Hence, $\mathcal{R}(H * K)$ is a rough subgroup of $\mathcal{R}(G)$ by Theorem 3.11.

Note that for a rough group $(\mathcal{R}(G), *)$ of the universe U, an element $x \in G$ and a natural number n, x^n is defined as $x * x * \cdots * x$ (n times). Similarly $(x^{-1})^n = x^{-n}$ is defined as $x^{-1} * x^{-1} * \cdots * x^{-1}$ (n times). Then the set $\langle x \rangle = \{x^n \mid n \in \mathbb{Z}\}$ can be defined. It is clear that $\mathcal{R}(\langle x \rangle)$ is a rough group contained in $\overline{apr}(G)$. Furthermore, $\langle x \rangle = \widetilde{\langle x \rangle} \subseteq \overline{apr}(\langle x \rangle)$.

3.14. Theorem

Let $\mathcal{R}(G)$ be a rough group and $\mathcal{R}(H)$ be a rough subgroup of $\mathcal{R}(G)$. Then $\mathcal{R}(H)$ is a rough cyclic subgroup of $\mathcal{R}(G)$ if and only if $\widetilde{H} = \langle x \rangle$ for an element x of G.

Proof. Suppose that $\widetilde{H} = \langle x \rangle$ for an element x of G, then it is clear that $H \subseteq \widetilde{H} = \langle x \rangle$. So $\mathcal{R}(H)$ is a rough cyclic subgroup of $\mathcal{R}(G)$. Now suppose that $\mathcal{R}(H)$ is a rough cyclic subgroup. If $H = \{e\}$ then $\widetilde{H} = \langle e \rangle$. If $H \neq \{e\}$ then there exists an element $y \in G$ such that $H \subseteq \langle y \rangle$. Now let $e \neq z \in H$, then there exists $n \in \mathbb{N}$ such that $z = y^n$. Let $k \in \mathbb{N}$ be the least positive integer such that $y^k \in H$. Then if taken $x = y^k$ then $\widetilde{H} = \langle x \rangle$.

3.15. Corollary

Every rough subgroup of a rough cyclic group is a rough cyclic subgroup.

In summary, there are many possibilities for further research involving rough groups and rough sets. One can explore the structure of cosets of rough groups, the homomorphisms between rough groups, and even the construction of rough rings or rough fields over rough sets.

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