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Bearing Capacity Equation for Shallow Foundations on Unsaturated Soils

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ABSTRACT

For shallow foundations resting on unsaturated soils, matric suction increases effective stresses, resulting in enhanced bearing capacity. However, this boost in bearing capacity is ignored in practice for the sake of simplicity, thus compromising economy. That is why this study aims to consider the beneficial effect of unsaturated conditions on bearing capacity. This is achieved by applying limit analysis using the finite element method on unsaturated soils and investigating the problem parametrically for shallow foundations. The suction effect is taken into account in the formulation of the limit analysis based on effective stress principle. The numerical outcomes are verified by comparing them with available experimental data from the literature. The findings from the parametric study emphasize that the influence of suction on bearing capacity is determined by the friction angle. Moreover, the effects of a varying suction profile are contingent on the foundation width. Based on the results derived from the numerical analyses, a modified bearing capacity equation is introduced. This equation showcases a very good coefficient of determination, effectively encompassing the effects of the soil suction profile. Consequently, the proposed procedure can be considered as a convenient yet precise tool for estimating bearing capacity in engineering practice.

Keywords: Bearing capacity, effective stress, unsaturated soil, matric suction, finite element limit analysis.

1. INTRODUCTION

Design of shallow foundations involve the consideration of ultimate and serviceability limit states. The main problem corresponding to an ultimate limit is the bearing capacity. That is why there are a number of seminal studies for calculating bearing capacity [1-4]. The general

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practice in bearing capacity calculations is to assume either saturated or dry conditions for the underlying soil layers, even though unsaturated soils account for almost 40% of Earth's land surface. As bearing capacity of unsaturated soils are greater, this practice yields uneconomic results. Therefore, accounting for unsaturation in bearing capacity computations would be economically beneficial.

The most common analytical methods used in the determination of bearing capacity are limit equilibrium, limit analysis and method of characteristic lines. Among these, limit analysis involves the upper bound and the lower bound theorems based on the classical theory of plasticity. Use of limit analysis method for analyzing stability is difficult when geometry and loading conditions are complex [5]. Therefore, combining the advantages of finite element method (FEM) with the theory of limit analysis presents a powerful option for evaluating stability and failure problems. The theoretical basis of finite element limit analysis method was originated from the studies of Lysmer [6] and Bottero et al. [7]. Subsequently, Sloan [5, 8] presented a new numerical scheme for solving lower bound and upper bound limit analysis problems using FEM and linear programming. Kim et al. [9] introduced pore water pressure into FELA formulation and applied it to the analysis of slope stability problems. This formulation can be straightforwardly extended to unsaturated soils with the aid of effective stress principle.

Mechanical behavior of unsaturated soils started to attract the attention of researchers around 1950s and the primary focus was on the applicability of effective stress principle. Researchers questioned the applicability of effective stress principle in some features of soil behavior like collapse [10-12]. These arguments were predominately based on an equivalent linear elastic theoretical background [13]. Therefore, up until early 2000s most models were based on two independent stress variables, until it was reasoned that collapse involves non-recoverable deformations and therefore arguments questioning the validity of effective stress principle are incorrect [14].

Over the recent two decades, experimental measurements of the bearing capacity of shallow foundations resting on unsaturated soils have been reported by several researchers [15-19]. However, due to the complexity of the problem and the relatively small number of experiments reported so far, as well as to the fact that existing experiments are generally performed on small scale footings, comprehensive understanding of the behavior of foundations on unsaturated soils is still lacking.

Fredlund et al. [20] proposed a relationship for calculating shear strength of unsaturated soils as a function of independent stress variables of net stress ($\sigma - u_a$) and suction ($u_a - u_w$) as:

$$\tau = c' + (\sigma - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \qquad (1)$$

where c' is cohesion, ϕ' is friction angle, and ϕ^b is the friction angle of the unsaturated soil because of the contribution of suction. Moreover, σ is total stress, u_a and u_w are air and water pressures, respectively.

Yet there has been limited research on unsaturated bearing capacity. Oloo et al. [21] modified Terzaghi's general bearing capacity equation and adjusted the cohesion term to consider the effect of matric suction on the ultimate bearing capacity (q_{ult}), as given in Eq. (2).

$$q_{ult} = [c' + (u_a - u_w)\tan\phi^b]N_c + qN_q + 0.5B\gamma N_\gamma \tag{2}$$

where q is the overburden pressure, B is the width of the footing, γ is soil unit weight, and N_c, N_q, N_γ are bearing capacity factors.

This method is referred to as total cohesion concept. Similarly, Oh and Vanapalli [22] adopted a similar approach and modified the cohesion term to account for the contribution of matric suction. More recent effective-stress based works use cohesion correction strategy to consider suction effects [23, 24]. Their general form of bearing capacity equation is as follows:

$$q_{ult} = [c' + \chi(u_a - u_w)\tan\phi']C_f N_c + qN_q + 0.5B\gamma N_\gamma \tag{3}$$

However, the majority of available methods often need a number of tuning or fitting parameters such as C_f which require material-specific calibration (Akbari Garkani et al., [24]) and this drawback limits their efficiency and practicality. On the other hand, Vo and Russell [25] solved the problem of bearing capacity of shallow foundations on unsaturated soils using Slip Line Method and proposed dimensionless bearing capacity charts. Inspired by apparent cohesion concept Vo and Russell [25] used dimensionless parameters which are originally defined by Martin [26] for soils with depth dependent cohesion values. Likewise, the effective-stress based relation of Tang et al. [27] modifies cohesion by considering the frictional contribution of suction.

Recently, Ghasemzadeh and Akbari [28] developed a limit equilibrium solution based on independent stress variables approach. Simultaneous with the modification of soil cohesion to take the influence of soil suction into account, a new term is introduced into Terzaghi's bearing capacity equation to consider the contribution of suction.

On the other hand, Jahanandish et al. [29] calculated the bearing capacity of foundations resting on unsaturated soils by the method of Zero Extension Lines based on the principle of effective stress. To account for the effect of unsaturation, empirical relations were proposed to adjust $N\gamma$ as a function of suction. Similarly, Ajdari and Esmail-Pour [19] performed experiments on small-scale physical model footings and then adjusted $N\gamma$ for different magnitudes of suction to fit the experimental data. Much like numerous other existing methods, this approach requires the utilization of fitting parameters specific to the soil being studied.

Clearly, the influence and contribution of suction to bearing capacity need to be well understood and methods based on numerical analyses provide a powerful medium for this purpose.

Accordingly, present work adopts a numerical approach and utilizes Finite Elements Limit Analysis (FELA) which is a powerful yet effective tool. The most important advantage of FELA over nonlinear stress-strain analyses is that it can directly determine the capacity with low computational effort. For this purpose, Optum G2 [30] software is used in this study. The optimal limit solution is determined by incorporating the Mohr-Coulomb yield criterion. After determining the trend of bearing capacity boost induced by matric suction, the bearing capacity equations are modified to take the effects of suction into account.

2. METHOD OF ANALYSIS

Kim et al. [9] introduced steady-state pore-water pressures to the formulation of FELA based on the effective stress principle. Both in lower-bound and upper-bound formulations, pore pressures were introduced as auxiliary nodal variables. In lower-bound analysis, effective stresses were used for applying yielding constraints, while total stresses were applied for imposing stress boundary conditions. As an example, two-dimensional differential equations of equilibrium are as follows:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = \frac{\partial {\sigma_x}'}{\partial x} + \frac{\partial u_w}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \tag{4a}$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = \frac{\partial {\sigma_y}'}{\partial y} + \frac{\partial u_w}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = \gamma \tag{4b}$$

In which, u_w is pore-water pressure and stress variables with prime superscript denote effective stress variables while those without prime are total stresses.

Correspondingly, Kim et al. [9] employed effective stresses to enforce yield condition and flow rule in upper bound solutions. They also considered the work done by pore-pressure in the formulation. The procedure suggested by Kim et al. [9] can be easily extended to unsaturated soils using Bishop's [31] equation of effective stress.

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w) = (\sigma - u_a) + \chi s \tag{5}$$

Here, χ denotes the effective stress parameter under partially saturated conditions and s is suction. Analogous to Terzaghi's [1] bearing capacity equation in unsaturated state, u_w in Eq. 3 is replaced with $u_a - \chi s$ as given below:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = \frac{\partial {\sigma_x}'}{\partial x} + \frac{\partial (u_a - \chi s)}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \tag{6a}$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = \frac{\partial {\sigma_y}'}{\partial y} + \frac{\partial (u_a - \chi s)}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = \gamma \tag{6b}$$

The bearing capacity of shallow foundations were evaluated using the upper and lower bound theorems of plasticity in finite element simulations (FELA). The analyses were performed using OptumG2 [30] software which uses second order cone programming (SOCP) to solve the optimization problems of limit analysis. Originally Optum G2 [30] that the effective stress parameter (χ) is equal to the degree of saturation (S_r). However, in the present research, Khalili and Khabbaz [32] relation is preferred which empirically determines the magnitude of χ as function of suction ratio:

$$\chi = \begin{cases} \left(\frac{(u_a - u_w)}{(u_a - u_w)_b}\right)^{-0.55} & \text{if } (u_a - u_w) > (u_a - u_w)_b \\ 1 & \text{if } (u_a - u_w) < (u_a - u_w)_b \end{cases} \tag{7}$$

where, $(u_a - u_w)_b$ is the air-entry suction for drying paths or the air-expulsion suction for wetting paths.

3. VALIDATION

For the purpose of validation, bearing capacities obtained by the FELA analyses conducted in this study are compared with the results of model footing tests available in literature. Rojas et al. [15] conducted in-situ plate load tests in seven test pits on lean clay using a steel plate with a diameter of 0.31m. In those tests, matric suction was measured by tensiometers that are installed at depths of 0.1m, 0.3m, 0.6m and 0.9m below the model footing. These experiments are modeled as axis-symmetric problems in OptumG2. The footing was modeled as a weightless rigid element, whereas the soil obeys the Mohr-Columb yielding criterion. The initial number of elements was 1000 which was eventually increased to 5000 using adaptive meshing.

The boundary conditions utilized in this study align with what are commonly referred to as "standard fixities". Specifically, along the vertical boundaries of the domain, tangential forces are released while normal forces exist. On the contrary, the bottom boundary permits the presence of both tangential and normal forces. A multiplier distributed force was applied at the surface of the foundation. Its magnitude was determined through an optimization procedure. The resulting optimized distributed load represents the bearing capacity of the foundation. Table 1 summarizes the geometrical dimensions and material properties used in these analyses.

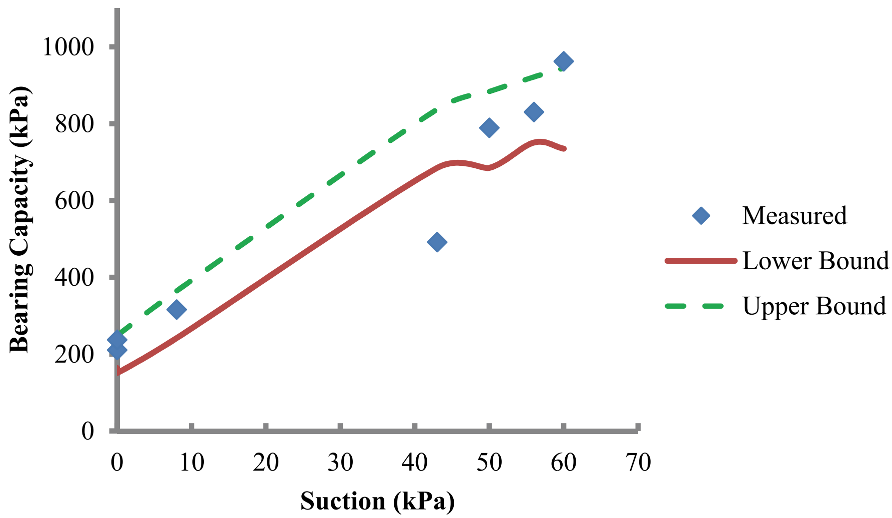


Figure 1 - The measured bearing capacities together with upper-bound and lower bound analysis results

Table 1 - Details of bearing capacity test and material properties

Footing Diameter (m)	0.31
Footing Depth (m)	0
Air entry suction (kPa)	18
Angle of internal friction of soils (degrees)	26
Cohesion (kPa)	3
Dry unit weight of soil (kN/m^3)	15.6

Bearing capacities obtained from lower-bound and upper bound limit analyses are compared with the results of the model tests in Figure 1. Apparently, there is good agreement between experimental results and numerical calculations since the measured test results are located between lower and upper bounds as expected. Figure 2 shows a zoomed view of shear dissipation as a typical example of the resulting failure mechanism in axisymmetric lower bound finite element analyses.

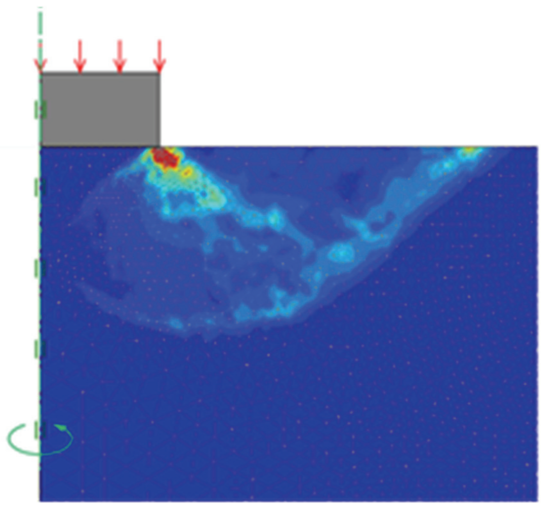


Figure 2 - Shear dissipation contour in axisymmetric lower-bound analysis of validation experiments

4. PARAMETRIC ANALYSIS

Once the method of analysis is validated, it is used for a parametric study of unsaturated bearing capacity of shallow foundations. For this purpose, lower bound analyses are preferred as the results obtained by lower-bound FELA are strictly lower bound and therefore safe. To consider the influence of the differing suction profiles on the results, analyses are performed for two different suction profiles. These are uniform and linear suction profiles and the obtained results are presented in the following two sections.

4.1. Uniform Suction Profile

In order to determine the effect of matric suction on bearing capacity, initially a uniform suction profile is assumed for the zone of influence below the foundation (Figure 3a). A parametric study was conducted with the assumed suction profile in which effective friction angle (ϕ'), cohesion (c') and χs are varied. The magnitudes of parameters used in the parametric study are given in Table 2. View of a typical failure mechanism of strip foundation overlain by refined mesh are shown in Figure 4.

Table 2 - The values of the parameters that are varied in the analyses

Parameter	Values used in the analyses
ϕ' (deg)	5°, 10°, 15°, 20°, 25°, 30°, 35°, 40°, 45°
c' (kPa)	0, 10, 20, 30, 40, 50, 75, 100
χs_o (kPa)	0, 50,100, 150, 200, 250, 300, 350, 400, 450, 500, 550, 600
ρ (kN/m^2)	-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
B (m)	1, 2, 3, 4, 5

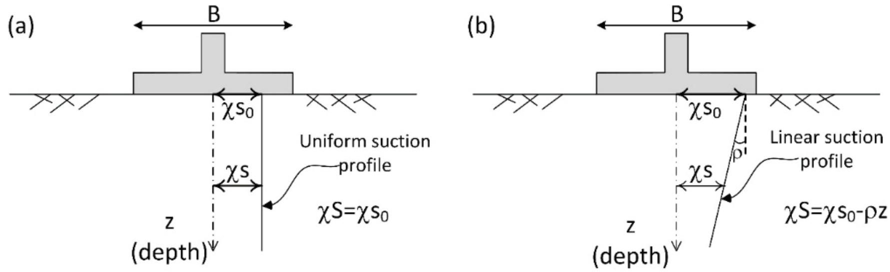


Figure 3 - Suction (χs) profiles under a footing (a) uniform suction profile (b) linear suction profile

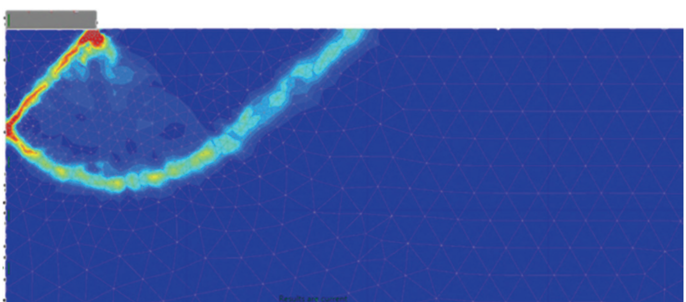


Figure 4 - Failure pattern (total dissipation) in the case of ϕ'=10, c'=100 kPa, χs=50 kPa

The concentration of present study on strip foundations stems from their widespread practical use, particularly in scenarios where the problem can be simplified as a plain strain situation. Additionally, it is important to note that many established bearing capacity equations for saturated soils were originally developed specifically for strip foundations. These equations were subsequently extended to encompass other foundation shapes like rectangular or circular by incorporating shape correction factors.

For uniform suction profiles, Figure 5 presents the results of the parametric study in the form of χs versus q_{ult} relationships for specific magnitudes of ϕ' and c′. It is observed that for constant values of ϕ' and c′, relationships between χs and q_{ult} are linear. Moreover, it is important to note that the slopes of these linear χs-q_{ult} relationships are the same when ϕ' is kept constant and is not affected by the magnitude of cohesion.

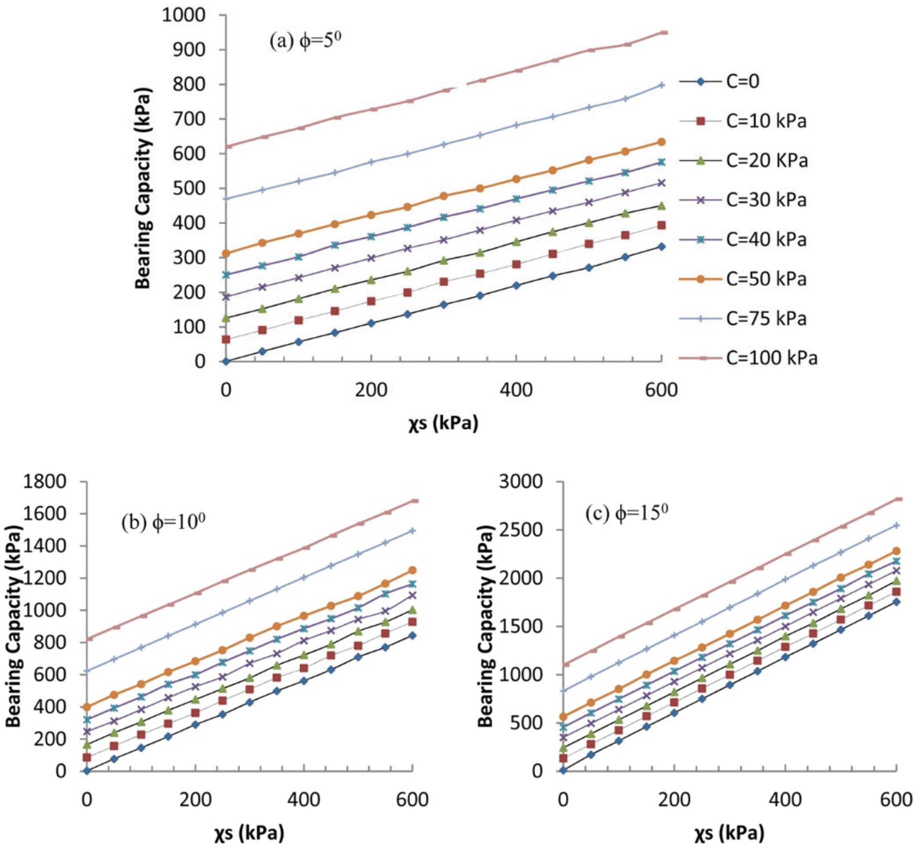


Figure 5 - The variations of bearing capacity versus suction for a constant suction profile and different combinations of c′ and ϕ′.

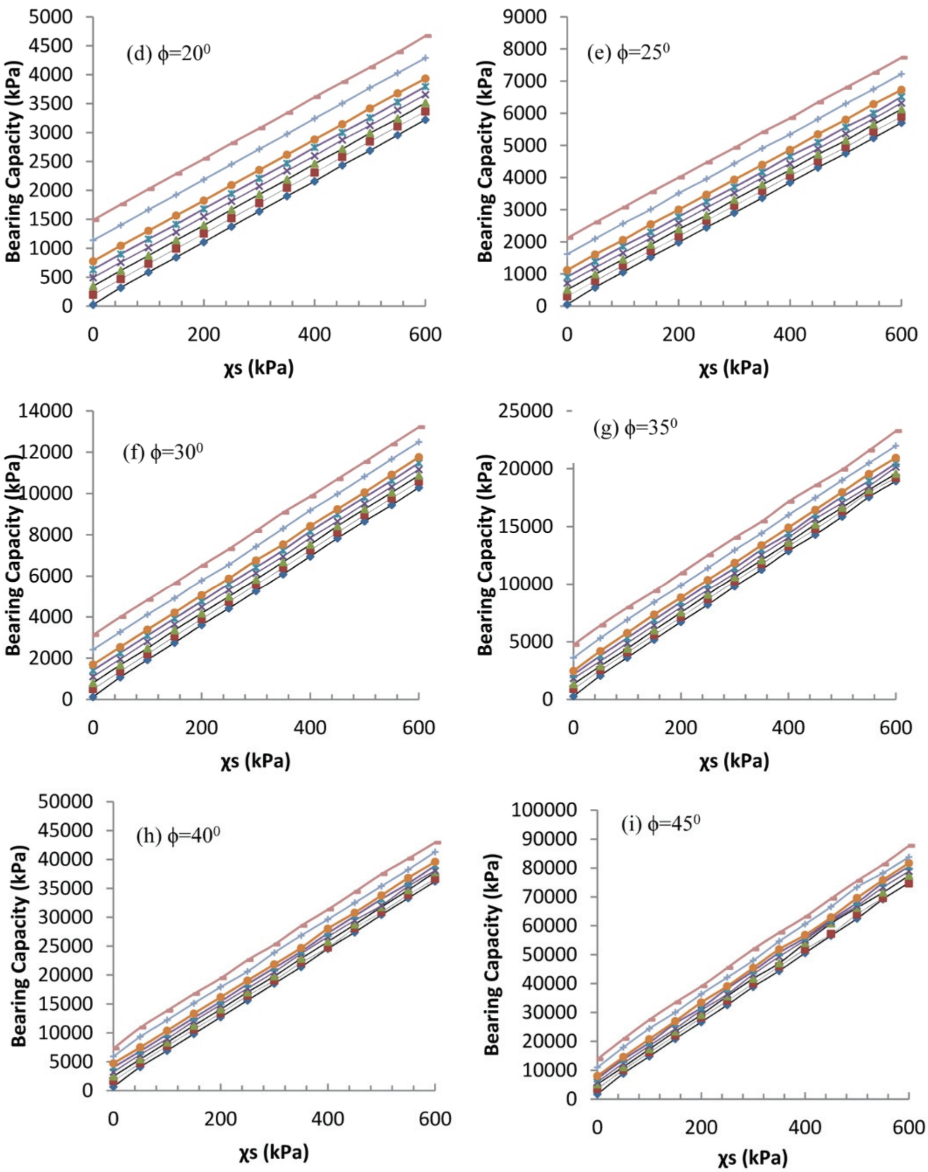


Figure 5 - The variations of bearing capacity versus suction for a constant suction profile and different combinations of c' and ϕ'. (continue)

It is possible to separate q_{ult} into its components based on the source of contribution, as shown in Eq. 8:

$$q_{ult} = q_0 + q_s \tag{8}$$

where q_o is the bearing capacity when χs is zero and q_s is the contribution of matric suction to bearing capacity. Then using the results of the parametric study, the magnitudes of q_s ($= q_{ult} - q_o$) are determined and plotted against their respective χs values in Figure 6. It is observed in Figure 6 that q_s-χs relationships for all magnitudes of ϕ′ are linear and they all pass through the origin. In other words, the amount of cohesion has no effect on the magnitude of q_s.

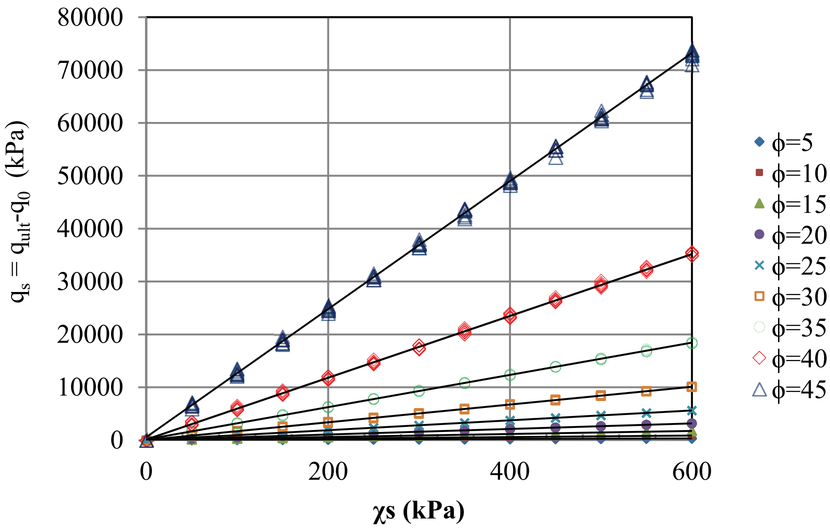


Figure 6 - The variation of q_s with χs for different values of the friction angle for a soil layer with constant χs profile

Table 3 - Slopes of the q_s-χs lines($N_{\chi s}$)at various friction angle values

ϕ′	N_c*	$N_{\chi s}$	$N_c \times \tan\phi'$
5	6.20	0.55	0.54
10	8.20	1.43	1.44
15	10.81	2.88	2.90
20	14.50	5.28	5.28
25	20.40	9.33	9.51
30	29.64	16.80	17.11
35	43.02	30.51	30.12
40	64.79	59.03	54.36
45	116.17	121.47	116.17
		R^2=0.999	

* Computed through Lower Bound Finite Element Analysis in the current study

The slopes of the q_s-χs lines at different friction angle values ($N_{\chi s}$), as ascertained through the performed parametric analysis, are detailed in Table 3. This table also includes the N_c values, which are also determined from the findings of the current research, albeit under conditions of zero suction. The values of N_c multiplied by tanϕ are also presented in the table. It is evident from the table that the $N_{\chi s}$ values closely align with the respective Nc × tanϕ values, exhibiting a high coefficient of determination ($R^2 = 0.999$).

Hence, $N_{\chi s}$, which determines the contribution of χs to bearing capacity, can be expressed as follows:

$$N_{\chi s} = N_c \times \tan\phi' \tag{9}$$

Clearly, q_s can be computed using the formula provided in Eq. 10:

$$q_s = \chi s \, N_c \tan\phi' \tag{10}$$

Consequently, in the case of a constant suction profile, the bearing capacity can be determined as:

$$q_{ult} = [c' + \chi s \tan\phi'] N_c + qN_q + 0.5B\gamma N_\gamma \tag{11}$$

Clearly, the present research outcomes arising from a comprehensive array of parametric analyses have yielded valuable insights. As expounded in the introduction, a notable gap existed in the comprehensive understanding of adapting the bearing capacity equation to account for the effects of suction. Previous researchers harbored varying viewpoints on which specific bearing capacity term necessitated modification and how to implement adjustments for suction. The present study bridges this gap and establishes that, for a consistent suction profile, adjusting cohesion relative to suction proves sufficient.

Moreover, while certain researchers concentrated on the cohesion term and employed specific tuning parameters tailored to individual materials, this approach significantly curtailed their practical applicability and user-friendliness. However, the findings of the present research indicate that no distinct tuning parameter is required. Instead, incorporating χs tanϕ' into the effective cohesion (c'), as delineated in Equation 11, proves to be satisfactory. Thus, the most significant advantage of the proposed method lies in its simplicity: obtaining an accurate measurement of χs stands as the sole prerequisite, negating the need to incorporate adaptive strength parameters for unsaturated conditions.

4.2. Linear Suction Profile

A soil profile with linearly varying χs is assumed (Figure 3b) to consider the influence of a variable suction profile on bearing capacity. Vo and Russell [25] found that when there is infiltration or evaporation at ground surface, χs profile could be approximated by a linear function within the zone of influence, and the errors associated with this approximation are negligible. Accordingly, the value of χs at any depth can be calculated as follows:

$$\chi s = \chi s_0 - \rho z \tag{12}$$

where, χs_0 is the value of χs at ground surface and ρ is the rate of decrease of χs with depth. Positive ρ values signify a decline in soil suction as depth increases. This arises when the groundwater level is beneath the surface and infiltration is absent. Conversely, negative ρ values correspond to scenarios where the soil faces surface infiltration. Consequently, points nearer to the surface exhibit lower suction compared to those at greater depths.

In order to assess the effect of suction variation on bearing capacity, numerous analyzes are performed and bearing capacities for different magnitudes of ρ are determined (q_ρ). These are later compared with the bearing capacities measured in models with uniform suction profiles (ρ=0) that have the same c'-ϕ'-χs_0 combinations ($q_{\rho=0}$). Then, the reduction in the magnitude of bearing capacity due to the variability of the suction profile (δq) can be expressed as follows:

$$\delta q = q_\rho - q_{\rho=0} \tag{13}$$

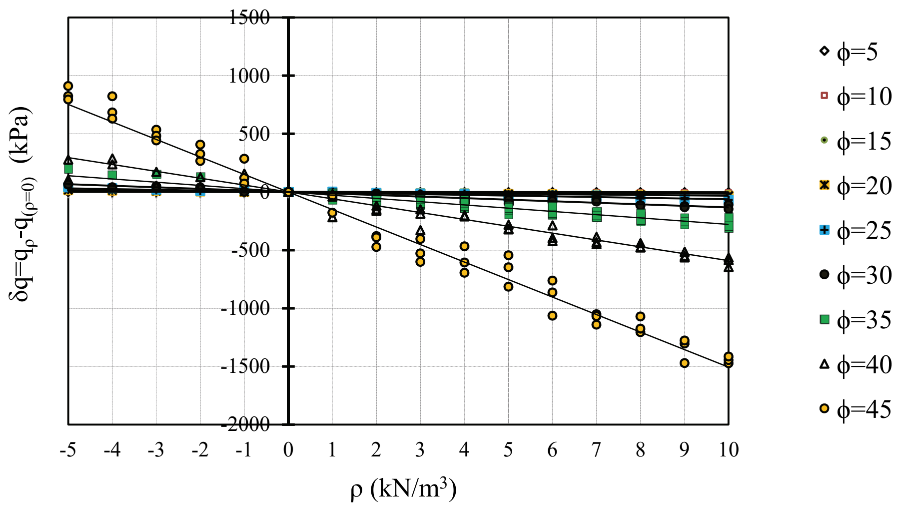


Figure 7 - Bearing capacity variations relative to χs gradient (ρ), (B=1 m)

Figure 7 illustrates the values of δq (for the foundation width B=1 m) plotted against ρ for various magnitudes of ϕ'. In addition to the data presented in Figure 7, further analyses were conducted, and while these analyses yield consistent trends, they are omitted here to prevent overcrowding. Evidently, the relationship between δq and ρ can be accurately described by a linear function with a zero intercept where the slope depends on the value of ϕ'. To investigate the influences of cohesion and χs_0 on the gradient of the parametric relationship between δq and ρ, a series of analyses were conducted. As illustrative instances, Figures 8 and 9 are presented. Upon observing Figure 8, it becomes evident that the cohesion value does not exert any discernible impact on the gradient of δq – ρ. Similarly, Figure 9 underscores the consistent nature of the gradient across various values of χs_0. Consequently, it can be deduced that the slope of δq – ρ remains unaffected by both cohesion and χs_0 values.

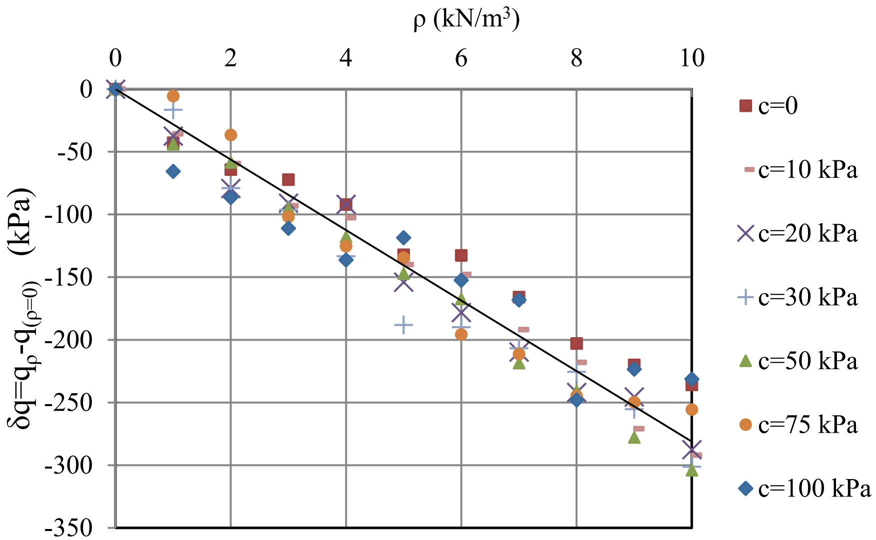


Figure 8 - Sensitivity analysis of the effect of cohesion value on δq – ρ relationship (ϕ′=35º, χs₀=50 kPa, B=1 m)

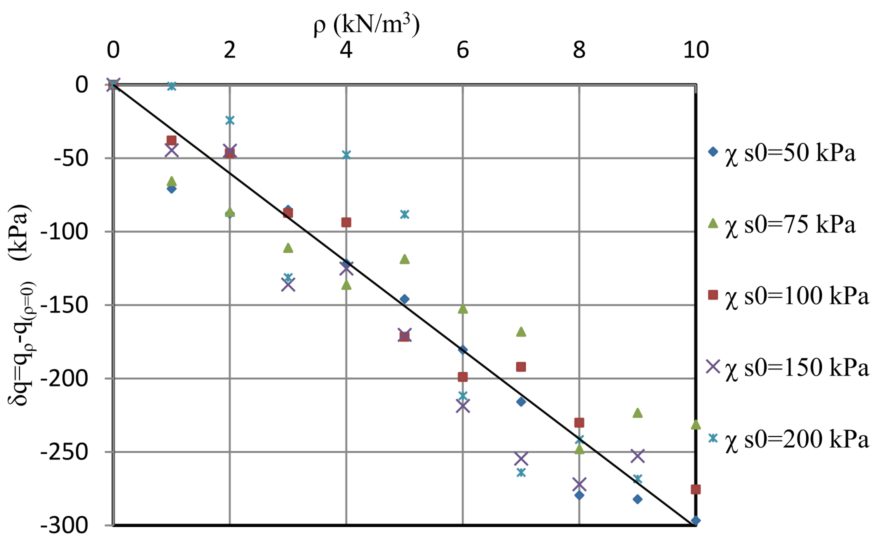
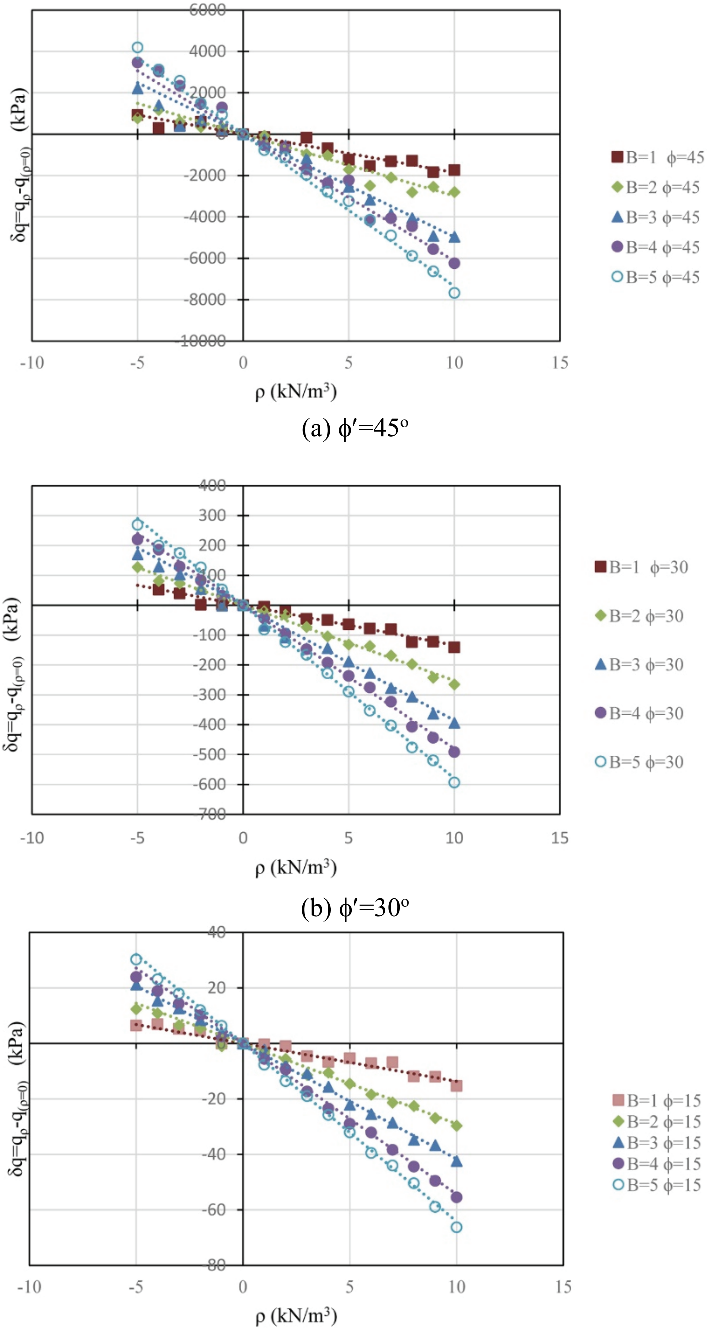


Figure 9 - Sensitivity analysis of the effect of χs₀ on δq – ρ relationship (ϕ′=35º, c=100 kPa, B=1 m)



(a) ϕ′=45°

(b) ϕ′=30°

(c) ϕ′=15°

Figure 10 - The changes in bearing capacity (δq) versus the gradient of χs (ρ) for different foundation widths (a) ϕ′=45°, (b) ϕ′=30°, (c) ϕ′=15°

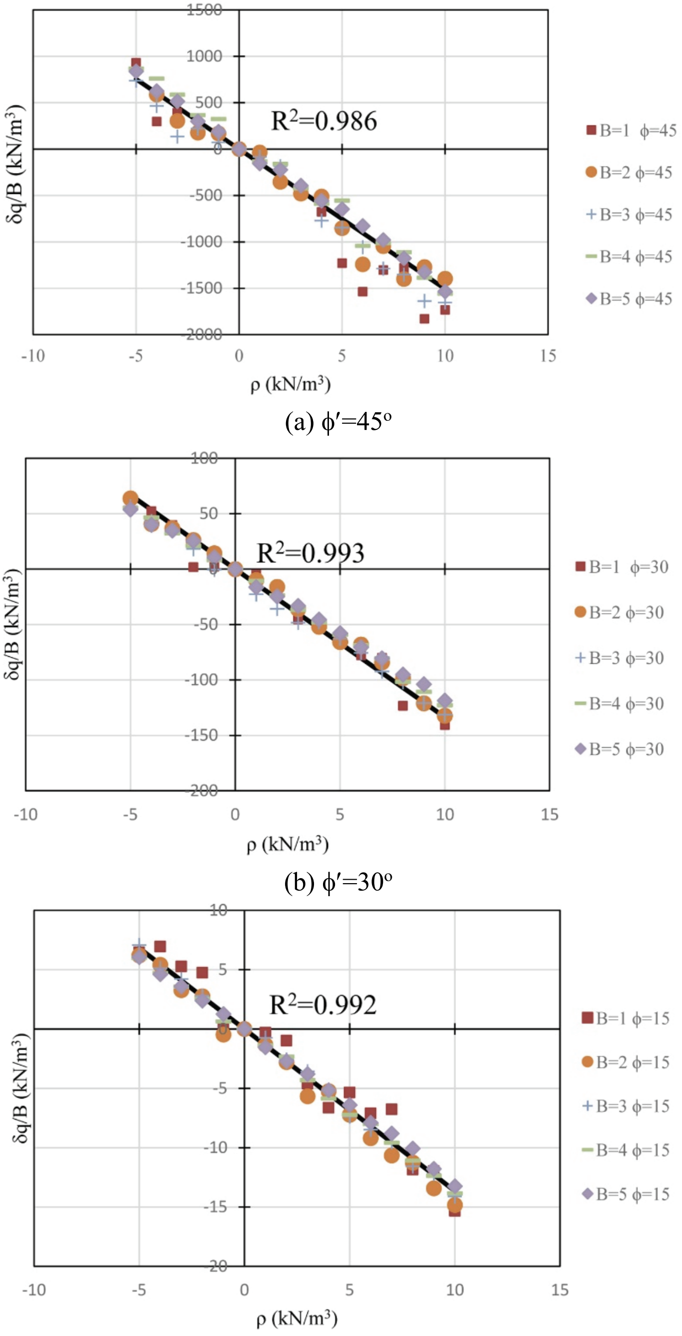


Figure 11 - The changes in normalized bearing capacity ($\frac{\delta q}{B}$) versus ρ for different foundation widths a) ϕ'=45°, (b) ϕ'=30°, (c) ϕ'=15°

To evaluate the impact of the footing width (B) on the relationship between δq and ρ, supplementary analyses were conducted for varying B values. Figure 10 presents the correlations between δq and ρ across different B values. This figure highlights that an increase in the foundation width (B) leads to an augmentation in the slope of the δq – ρ relationship. Consequently, when dealing with foundations situated in areas with varying suction profiles in depth, the width of the foundation becomes a significant factor in influencing the contribution of matric suction to bearing capacity. As a result, it is imperative to incorporate foundation width into relevant formulations.

However, when the values of δq are divided by the foundation width (δq / B) and plotted against ρ, a notable observation can be made, as depicted in Figure 11. Across a constant ϕ value, all data points become normalized and coalesce along a singular line. This signifies that the slope of the δq/B – ρ relationship remains consistent for a specific ϕ value, which is denoted as Nρ. Consequently, it can be formulated as follows:

$$N_\rho(\phi') = \frac{\frac{\delta q}{B}}{\rho} \tag{14}$$

Consequently, if $N_\rho(\phi)$ values corresponding to the friction angle (ϕ) are accessible, it becomes possible to calculate the impact of a varying suction profile on the bearing capacity (δq) using the equation:

$$\delta q = \rho B \, N_\rho(\phi') \tag{15}$$

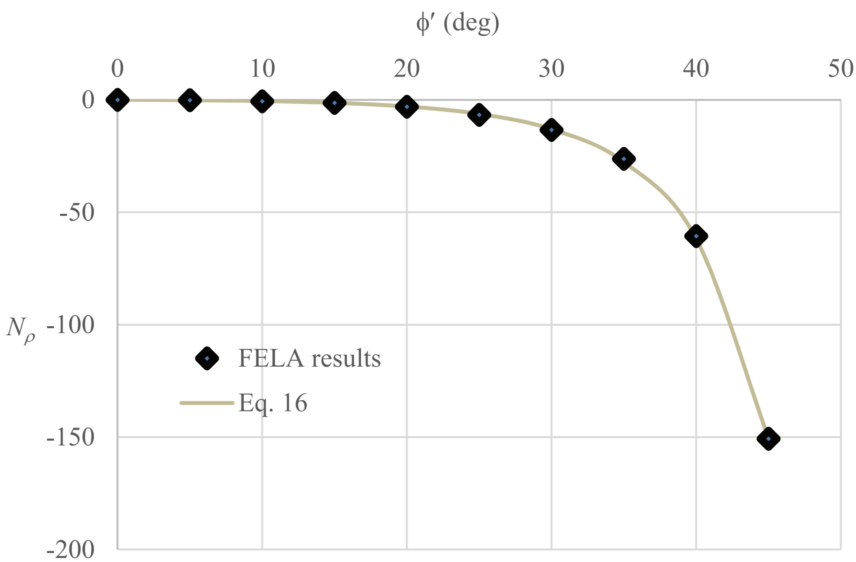


Figure 12 - The values of N_ρ versus ϕ'

The symbolic regression technique was used to develop an empirical relation for N_ρ as:

$$N_\rho = N_\rho(\phi') = tan\phi' \times \left(1 - 1.86e^{(4.4\tan\phi')}\right) \tag{16}$$

The comparison of values of N_ρ obtained by LBFE analyses and calculated using Eq. 16 are shown in Figure 12.

The bearing capacity equation for a shallow continuous foundation resting on an unsaturated soil profile becomes:

$$q_{ult} = [c' + \chi s \tan\phi']N_c + \rho B\, N_\rho + qN_q + 0.5B\gamma N_\gamma \tag{17}$$

5. CONCLUSIONS

The principle of effective stress together with FELA was used to parametrically investigate the influence of different suction profiles on bearing capacity shallow strip footings. The effectiveness of the employed numerical analysis was evaluated by comparing the obtained results with existing experimental data, revealing a notable concurrence.

A comprehensive parametric analysis was conducted by varying factors such as friction angle, cohesion, suction, rate of suction change with depth, and foundation width. The outcomes of these parameter variations indicated that cohesion does not impact the extent of increase in bearing capacity caused by suction. Instead, the primary influencing factor is the friction angle. Enhancing the friction angle leads to an amplified contribution of suction to the bearing capacity.

The research discovered that the augmentation in bearing capacity due to suction can be incorporated by modifying the conventional bearing capacity equations to consider the attributes of the suction distribution. The findings demonstrated that the influence of suction can be predicted by adjusting the cohesion term with respect to suction in the bearing capacity equation. This approach enables a highly accurate prediction of the bearing capacity. For linear suction profiles, the significant factors are the rate of suction variation with depth, friction angle, and foundation width. The equation proposed in this study effectively predicts the contribution of variable suction profiles on bearing capacity.

Although Khalili and Khabaz [32] relationship is preferred by the authors and used to determine the effective stress parameter (χ) to validate the numerical analyses, the method proposed in this study for calculating the bearing capacity of unsaturated soils is independent of how effective stress parameter is calculated.

Contrary to earlier equations presented so far, the proposed extended equation does not need additional calibration and soil specific material parameters. Therefore, the proposed method is practical and can be used for calculating bearing capacity of shallow footings resting on unsaturated soils provided that the distributions of suction and effective stress parameter are known.

A notable aspect of the proposed method is its reliance on effective stress parameters, allowing the continued applicability of saturated strength parameters. This characteristic underscores the practical simplicity of the method, as it obviates the need for measuring separate strength parameters in unsaturated conditions, making its application notably straightforward.

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