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Research Article

Multi-Objective De Novo Programming with Type-2 Fuzzy Objective for Optimal System Design

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ABSTRACT

De Novo Programming, which is also known as Optimal System Design, regulates the resource amount of constraints depending upon the budget. Mostly, this process is managed using traditional methods, fuzzy methods and hybrid methods. When considered from this point of view, there is no certain method for the solution of De Novo Programming problems. An approach for solving the Multi-Objective De Novo Programming has been recommended using Type-2 Fuzzy Sets in this research. Without exceeding the budget in the recommended approach, Type-2 membership function for each objective function has been defined applying positive and negative ideal solutions. The solution phase of this approach, called Multi-Objective De Novo Programming with Type-2 Fuzzy Objective, has been shown step by step on the illustrative problem. Then, this illustrative problem has been solved with regards to five different approaches in the literature and the results have been compared.

Keywords: De Novo Programming, Fuzzy Set Theory, Optimal System Design, Type-2 Fuzzy Sets

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1. Introduction

Mathematical models that allow the analysis and assessment of decision-making processes aim to achieve a specific goal under system constraints. Getting the maximum benefit from the goals is directly related to the constraints. In other words, the objectives are realized to the extent that the constraints allow, which is related to the correct planning of the resource levels of the constraints. As a result of solving Traditional Linear Programming problems, it is an expected result that the slack and/or surplus variables of the constraints are greater than zero. This result shows that constraint resources are not used at full capacity, which means that constraints are not active. In this case, resource usage amounts are not used at optimal level although the problem is optimized. Therefore, it is obvious that an optimal result does not occur. According to Zeleny (1984), due to the definition of optimality, slack and surplus variables must be equal to zero, A system that does not use all the resources at full capacity can not be considered an optimal system. If high-efficiency systems are to be mentioned, all the resources must be used at full capacity.

The traditional methodology to mathematical programming is predicated on the premise that resources and constraints are predetermined in the production model. However, the De Novo approach, also known as optimal system design, does not enforce resource constraints because it assumes that the majority of the required resources can be obtained at a reasonable cost. The sole constraint is the amount of money available, i.e., the budget required to purchase the resources needed (Babic and Pavic, 1996). Resources are limited since the budget which is a key component of De Novo, determines their maximum quantity (Babic et al., 2018). De novo programming is generally helpful for handling difficulties involving optimal system design. Because right-hand resource availability is unclear, it plans to build a portfolio of resource availability levels by allocating a budget based on resource price. Furthermore, because the available system budget can be modified depending on resource price variations, de novo programming can enable more flexible resource planning schemes (Gao,2018). In addition to offering solutions to multi-objective problems, de novo programming also gives a more optimal system design suggestion based on the available budget in order to boost production yields and maximize the usage of raw materials (Afli et al., 2019). Traditional concepts of optimality put an emphasis on the evaluation of a pre-existing system. The goal of new optimality concepts is to build optimal systems. The goal is to reduce or even eliminate tradeoffs, not to measure and evaluate them. An optimal system should have no tradeoffs. De Novo programming is a methodology for reshaping feasible sets in linear systems (Fiala,2011).

The first study on De Novo Programming, realizing the optimal system design, was conducted by Zeleny (1976). Afterward, De Novo assumption was built on solid foundations through scientific studies done by Zeleny (1981), Zeleny (1984) , Zeleny (1986), and Zeleny (1990).

The mathematical model of the De novo assumption, which is easily applied to Multiobjective Linear Programming problems as well as to single objective Linear Programming, is summarized below.

$$Max\ Z_k(x): \sum_{j=1}^{n} C_{kj}\, x_j$$

$$Min\ W_s(x): \sum_{j=1}^{n} C_{sj}\, x_j$$

Subject to (1)

$$\sum_{j=1}^{m} a_{ij} x_j \le b_i,$$

$x_j \ge 0$, i=1,2,...,m, j=1,2,...,n, k=1,2,...,l and, s=1,2,...,r.

where;

C_{kj}:*k*-th maximization-oriented objective function coefficients,

C_{sj}: *s*-th minimization-oriented objective function coefficients,

x_j:j-th decision variable,

a_{ij}: technological coefficients,

b_i:i-th constraint's resource amount.

In the Multiobjective Linear Programming problem (1), the "b_i" of the constraint functions $\sum_{j=1}^{m} a_{ij} x_j \le b_i$ can be rearranged according to the De Novo assumption. Where p_i is the unit price of the source b_i, Multiobjective De Novo Programming model occurs by adding the budget constraint $\sum_{i=1}^{m} p_i b_i \le B$ to (1). Furthermore, the budget constrained Multi-Objective De Novo Programming Model is reformed by refulation of $\sum_{j=1}^{m} p_i(a_{ij}) x_j \le (p_i) b_i$ as below.

$$Max\ Z_k(x): \sum_{j=1}^{n} C_{kj}\, x_j$$

$$Min\ W_s(x): \sum_{j=1}^{n} C_{sj}\, x_j$$

Subject to (2)

$$\sum_{j=1}^{m} v_j x_j \otimes B,$$

where

$x_j \ge 0$, j=1,2,...,n, $k = 1,2,..,l$ and $s = 1,2,...,r$.

is obtained. Where,

C_{kj}:k-th maximization-oriented objective function coefficients,

C_{sj}: s-th minimization-oriented objective function coefficients,

x_j:j-th decision variable,

a_{ij}: technological coefficients,

b_i:i-th constraint's resource amount,

p_i:cost of the ith resource,

$v_j = \sum_{i=1}^{m} p_i a_{ij}$: unit cost of producing the product

B:Budget,

$\otimes$:"= " or "$\leq$ " .

The difference of Multiobjective De Novo programming from Multiobjective Linear programming as a formulation arises only from the budget constraint. However, according to the De Novo assumption, positive ideal solutions of the objective functions are better thanks to the amount of resources restructured with the budget constraint.

When we build our systems optimally, that is, when we calculate the right levels of resources, we eliminate conflicts between objectives, tradeoffs become needless, all objectives perform at their maximum possible levels, and the rationale for traditional MCDM is eliminated (Zeleny,1987). De Novo formulation not only determines the best mix of outputs but also achieves the best combination of inputs (Tabucannon, 1988). Bypassing trade-offs, the De Novo approach seeks an "optimal system" by rearranging the feasible solution set to achieve optimality for all related criteria at the same time (Zhuang and Hocine,2018). De Novo programming aims to break limitations by releasing various constraints in order to find the optimal solution (Chen, 2014). According to Zeleny (1984), the reformation of system limits and constraints based on goals should be included in the design, redesign, and optimization of a system. System design is a process of creating options rather than selecting them.

In this study, there is a proposal made for the solution of Multi-Purpose De Novo programming problems based on the Zimmermann (1978) approach by using type-2 membership functions defined by Maali and Mahdavi-Amiri (2014). The contents of this paper are organised in 5 sections: In section 2, in terms of theoretical development, a literature summary including basic methods from Type-1 Fuzzy Multi-Objective Linear programming to Type-2 Fuzzy Multi-Objective Linear programming is presented. Afterward, Multi-Objective Linear Programming with Type-2 Fuzzy Sets Approach is given. In section 3, a new approach is suggested from Multi-Objective De Novo Programming Type-2 perspective. The results of the approach proposed are obtained on an illustrative example in section 4 and these are interpreted in section.

2. De Novo Programming Literature Survey in the Perspective of Fuzzy Set Theory

Since Zeleny (1976), researches continue for Multi-Objective De Novo Programming problems, which do not have a definite solution method. For the solution of Multi-Objective De Novo Programming, “optimum-path ratio” has been proposed (suggested) by Zeleny (1986). Later, Shi (1995) developed six “optimal-path” ratios for optimal system design (proposed). In addition, Traditional/ Fuzzy MODM methods are frequently used in researching the solution of Multi-Objective De Novo Programming problems. A literature summary from the perspective of MODM and Fuzzy Sets for the methods used in solving Multi-Objective De Novo Programming problems in this study is below.

The solution of the Multi-Objective De Novo Programming problem was first made by Tabocanon (1988) using the MODM methods, compromise constraint method, global criterion method, and step method. Bare and Mendoza (1990) used the STEP method for the compromise solution of the designing Forest Plan problem in a de novo environment. Using interval coefficients and a generalized upper bounding structure, Kim et al. (1993) constructed the 0-1 bicriteria knapsack problem. Zhang et al. (2009) developed the Inexact de Novo programming approach for planning water resource systems. To select suppliers for the automotive industry, Huang and Hu (2013) created a two-stage solution approach employing De Novo Programming and Fuzzy Analytic Network Process-Goal Programming.Umarusman (2013) proposed Minmax Goal Programming to solve the Multi-Objective De Novo programming problem. Banik and Bhattacharya (2018) suggested a weighted goal programming approach for solving multi-objective de novo programming problems. Zhuang and Hocine (2018) carried out the solution of multi-criteria de novo programming problems by Meta-Goal Programming. A Monte-Carlo-based interval de novo programming method for an optimal system design under uncertainty was put forth by Gao et al. (2018). Modified Goal Programming is the approach used by Afli et al., (2019) for the solution of multi-objective de novo programming. Umarusman (2019) offered a satisfactory solution for solving Multi-Objective De Novo Programming problems using Lexicographic Goal Programming. Solomon and Mokhtar (2021) developed a technique for determining the optimal solutions to a Multiobjective De Novo Programming problem using rough interval coefficients. Banik and Bhattacharya (2022) put forward a general method of solving multi-objective de novo programming problems using the echelon system.

Li and Lee (1990a) searched Multi-Objective De Novo programming problems in a fuzzy setting for the first time. They proposed a solution depending on the set of positive and negative ideal solutions. Li and Lee (1990b) proposed Multi-Criteria De Novo Programming model with fuzzy parameters in terms of the possibility concept of a fuzzy set. Lai and Hwang (1992) solved the Single-Objective De Novo Programming problem using Chanas (1983)’s non-symmetric approach. Lee and Li (1993) developed an approach for Multi-Criteria De Novo Programming by utilizing fuzzy goals and fuzzy coefficients. Sasaki et al. (1995) suggested using the GA to solve a De Novo programming problem with a fuzzy goal. Chen and Hsieh (2006) suggested a new fuzzy Multi-Stage De-Novo programming problem and its Genetic Algorithm approach. Employing De Novo Programming, Chakraborty and Bhattacharya (2012) proposed a new approach a of solving a multi-stage and multi-objective decision-making problem. Miao et al. (2014) presented an interval-fuzzy De Novo programming technique for for determining how to plan water-resource management systems in the face of uncertainty. Khalifa (2018) proposed Fuzzy Goal Programming to solve of Fully Fuzzy Multi-Criteria De Novo Programming Problems. Saeid et al. (2018) developed a Fuzzy Multiple objective De Novo Linear Programming (FMODNLP) model in order to reformulate the MOPP with fuzzy parameters. Using Luhandjula's compensating µ–operator, Bhattacharya and Chakraborty (2018) provided an alternative approach for solving the General Multi-Objective De-Novo Programming Problem in a fuzzy environment in one step. Sarah and Khalili-Damghani (2019) proposed Fuzzy Type 2 De-Novo programming is used to allocate resources and create targets. Banik and Bhattacharya (2019) offered a one-step method to solve a General De Novo Programming Problem utilizing a Min-max Goal Programming technique with all fuzzy numbers as parameters. Khalifa (2019) proposed an approach for solving the Multi-Objective De Novo Programming problem which has possibilistic objective function coefficients. Umarusman (2020) developed a new Fuzzy De Novo Programming approach based on the possibility concept of Fuzzy Set Theory. A Monte-Carlo-based interval fuzzy De Novo programming (MC-IFDP) method for land-use planning under uncertainty was developed by Gao et al. in 2021. Furthermore, a comprehensive literature review was conducted by Haasan (2021), including the solution methods and application areas used in Multi-Objective De Novo Programming studies. As a result of the literature search conducted by Haasan (2021), the ratio of explanatory/numerical examples is 41%. This rate shows that the search for a solution method for De Novo Programming is still ongoing.

3. Type-2 Fuzzy Sets

With the Fuzzy Set Thoey by Zadeh (1965), as it can explain the gradual transition from membership to non-membership from a different perspective to the concept of uncertainty, it has offered substantial benefits in many areas. Type-2 fuzzy sets, an extension of classical fuzzy sets named Type-1 fuzzy sets, were proposed by Zadeh (1975). According to Mizumoto and Tanaka (1976), the fuzzy set of type 2 can be identified by a fuzzy membership function whose grade (or fuzzy grade) is a fuzzy set in the unit interval [0, 1] rather than a point in [0, 1]. Thus, these type-2 fuzzy sets enable the idea that a fuzzy set's members need not necessarily have membership grades in the range [0, 1], but rather that the degree of membership for each member is a fuzzy set in and of itself (John, 1998). Type-2 fuzzy sets are more effective in modeling uncertainty and imprecision. Because their membership functions are completely crisp, Type-1 fuzzy sets can not directly model uncertainty. Type-2 fuzzy sets, on the other hand, can model uncertainty because their membership functions are also fuzzy. Type-1 fuzzy sets have two-dimensional membership functions, whereas type-2 fuzzy sets have three-dimensional membership functions. It is the new third-dimension of type-2 fuzzy sets that adds more degrees of freedom, allowing for direct modeling of uncertainty (Mendel, and. John, 2002). The basic definitions of An type-2 fuzzy sets $\tilde{A}$ are below (Mendel, and. John, 2002; Mendel et al., 2006;Wu and Mendel,2007):

Definition 1: A type-2 fuzzy set, denoted $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$, where $x \in X$ and $u \in J_x \subseteq [0,1]$, i.e.

$\tilde{A} = \{((x,\mu), \mu_{\tilde{A}}(x,u)) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\}$

where $0 \le \mu_{\tilde{A}}(x,u) \le 1$ is the type-2 membership function, J_x is primary membership of $x \in X$ which is the domain of the secondary membership function $\mu_{\tilde{A}}(x)$. $\tilde{A}$ can be expressed as

$\tilde{A} = \int_{x \in X} \int_{x \in X} \mu_{\tilde{A}}(x,u)/(x,u) = \int_{x \in X} \left(\int_{x \in} \mu_{\tilde{A}}(x,u)/x \right)$

Where $J_x \subseteq [0,1]$ is the primary membership at x, and $\int_{x \in X} \left(\int_{x \in X} \mu_{\tilde{A}}(x,u)/x \right)$ is indicates the second membership at x. ∫∫ denotes the union over all admissible x and u. For discreate space ∫ is replaced by Σ.

Definition 2: Let $\tilde{A}$ be a type-2 fuzzy set in the universe of discourse X reperesent by type-2 membership function $\mu_{\tilde{A}}(x,u)$. If all $\mu_{\tilde{A}}(x,u) = 1$, then $\tilde{A}$ is called an interval type-2 fuzzy set. An interval type-2 fuzzy set can be regarded as special case of the type-2 fuzzy set, which is defined as folows;

$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x,u) = \int_{x \in X} \left(\int_{u \in J_x} 1/(x,u) \right)/x$

Definition 3: Uncertainty in the primary memberships of type-2 fuzzy set $\tilde{A}$ consist of a bounded region called the footprint of uncertainty footprint (FOU), which is the union of all primary membership may be expressed as follows;

$FOU\ (\tilde{A}) = \cup_{x \in X} J_x$

The FOU of $\tilde{A}$ is bounded a lower membership function (LMF) $\mu_{\tilde{A}}^{L}(x)$ and an upper membership function (UMF) $\mu_{\tilde{A}}^{U}(x)$. Thu, an interval type-2 fuzzy set is bounded by two classical fuzzy sets. The membership grade of each element of an interval type-2 fuzzy set is an interval $[\mu_{\tilde{A}}^{L}(x), \mu_{\tilde{A}}^{U}(x)]$.

The initial membership function, or FOU, is the complete union of initial membership functions and designates a limited area of uncertainty. According to Namvar and Bamdad (2021), FOU is defined by the upper and lower membership functions, UMF and LMF, respectively, each of which is T1 FS.

3.1. Type-2 Fuzzy Multiobjective Linear Programming

Fuzzy Sets Theory proposed by Zadeh (1965) has revealed important changes and developments in mathematical programming as in many areas. Especially, with the fuzzy decision-making process developed by Bellman and Zadeh (1970), a large number of "fuzzy mathematical programming" techniques and approaches to these techniques have emerged. Studies on examining Single/Multiple Objective Linear Programming problems in fuzzy environment were started by Zimmermann (1975), Zimmermann (1976) and Zimmermann (1978). In this study, the transition from Type-1 Fuzzy Multi-Objective Linear programming to Type-2 Fuzzy Multi-Objective Linear programming depending on positive and negative ideal solutions is through using Zimmermann (1978) methodology.

The positive and negative ideal solution sets obtained from the solution of Multiobjective Linear Programming (1) are $I^{*} = [Z_1^{*}, \dots, Z_l^{*};\ W_1^{*}, \dots, W_r^{*}]$ and $I^{-} =$

$[Z_1^-, \dots, Z_l^-;\ W_1^-, \dots, W_r^-]$ respectively, using these solutions as fuzzy membership function for each objective function.

For maximization-oriented objectives;

$$\mu(Z_k(x)) = \begin{cases} 1 & , \quad Z_k(x) \geq Z_k^* \\ \frac{Z_k(x)-Z_k^-}{Z_k^*-Z_k^-} & , \quad Z_k^- \leq Z_k(x) \leq Z_k^* \\ 0 & , \quad Z_k(x) \leq Z_k^- \end{cases} \tag{3}$$

For minimization-oriented objectives;

$$\mu(W_s(x)) = \begin{cases} 1 & , \quad W_s(x) \leq W_s^* \\ \frac{W_s^- - W_s(x)}{W_s^- - W_s^*} & , \quad W_s^* \leq W_s(x) \leq W_s^- \\ 0 & , \quad W_s(x) \geq W_s^- \end{cases} \tag{4}$$

are defined as above. Fuzzy Multi-Objective Linear Programming based on Zimmermann (1978)'s approach are established as below using (3) and (4).

$Max\ \lambda$

Subject to (5)

$$\lambda \leq \frac{Z_k(x) - Z_k^-}{Z_k^* - Z_k^-},$$

$$\lambda \leq \frac{W_s^- - W_s(x)}{W_s^- - W_s^*}$$

$$x \in X = \{x | Ax \leq b, x \geq 0\},$$

$x \geq 0, \lambda \in [0;1], k = 1,2,..,l$ and $s = 1,2,\dots,r$.

where λ is definned as $\lambda = \min_i \mu_i(x) = \min_{k,s}\big(Z_k(x), W_s(x)\big)$. In (5), the "normalized degree" of the distances of each objective function to its positive ideal solutions is minimized, while at the same time maximizing the satisfaction of the objective function in the range [0,1].

Since membership functions Type-1 fuzzy sets are completely crisp, they cannot directly model uncertainties. Moreover, type-2 fuzzy sets can model such uncertainties because their membership functions are also fuzzy. Type-2 fuzzy sets have three-dimensional membership functions as opposed to type-1 fuzzy sets' two-dimensional membership functions. In type-2 fuzzy sets, the newly introduced third dimension offers additional degrees of freedom that enable direct modeling of uncertainties (Mendel and John, 2002). Uncertain optimization using IT2FS is a new approach to dealing with the linguistic uncertainty of a Type-1 fuzzy set. Because it must handle FLP problems with more complicated parameters, this approach implies higher computational complexity than deterministic, stochastic, or classical FLP models, necessitating either computations or new algorithm (Garcia,2011). Solving single- or multi-objective type-1 and interval type-2 FLP problems is one of the applications of all existing defuzzification methods. All coefficients and numbers used in general interval type-2 FLP problems are interval type-2 fuzzy numbers. To solve FLP problems, numerous methods have been presented, such as ranking or ordering methods and approximation techniques, and each method has its own pros and limitations; hence, deciding which method is the best would be tough (Javanmard and Nehi, 2019).

Within the framework of this study, it was determined that the first study on Single/Multiple Objective Linear Programming approaches from the Type-2 perspective was made by Garcia (2009) as a result of the research conducted in the databases of Emerald insaght, IEEE Xplore, JSTOR, Google Scholar, ScienceDirect, Springer, Taylor&Francis, TRDizin. A brief literature summary for Single/Multiple Objective Linear Programming approaches developed from the Type-2 perspective is below.

Garcia (2009) used Interval Type-2 Fuzzy Sets to offer two general methods for dealing with uncertainty in the Right Hand Side parameters of a Linear Programming model. Garcia (2011) proposed solving an LP problem with uncertain right-hand side parameters as Interval Type-2 Fuzzy sets. Dinagar and Anbalagan (2011) used a two-phase method to solve Type-2 fuzzy linear programming problems. In their 2014 paper, Jin et al. proposed a Robust Inexact Joint-optimal α cut interval Type-2 Fuzzy Boundary Linear Programming. Srinivasan and Geetharamani (2016) solved a Linear Programming model using Perfectly normal Interval Type-2 Fuzzy Sets Right Hand Side (PnIT2FS RHS) parameters using ranking values and PnIT2FS arithmetic operations. Javanmard and Nehi (2017) suggested a new ranking function method for solving interval type-2 fuzzy linear programming problem. Using interval type-2 fuzzy variables, Kundu et al. (2019) suggested a method for solving linear programming network problems with constraints.

The Fuzzy Multi-Objective Linear Programming problem was first studied from the perspective of type-fuzzy sets by Maali and Mahdavi-Amiri (2014). Maali and Mahdavi-Amiri (2014) put forward two solution strategies based on Lai and Hwang (1992)'s approach using upper membership function and lower membership function. The first one uses a maxmin approach, while the second one views the objective as an aggregation of membership functions. Afterwards, Bigdeli and Hassanpour (2015) developed an interactive approach for solving type-2 fuzzy multi-objective linear programming problems using the type-2 membership function of Maali and Mahdavi-Amiri (2014).

According to Maali and Mahdavi-Amiri (2014), if the appropriate membership function for the ith objective in (5) is close to $\mu_i(x)$ primary membership, restricted by Upper Membership Function and Lower Membership Function then the FOU corresponding to the fuzzy membership function is can defined. The FOU of the type-2 membership function for the maximization and minimization directional objective functions is defined as follows.

The FOU of the type-2 membership function for $Z_k(x)$;

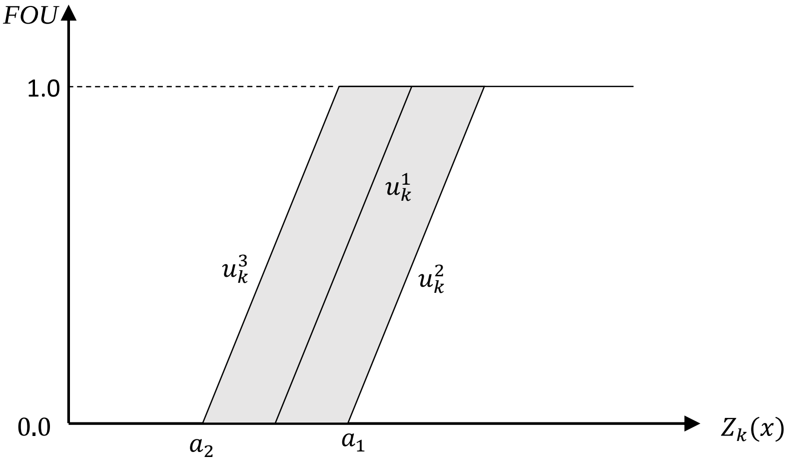

Figure 1. The FOU of the type-2 membership function for $Z_k(x)$

The Upper Membership Function (UMF) and the Lower Membership Function (LMF), represented by $u_k^2(Z_k(x))$ and $u_k^3(Z_k(x))$, respectively, are two type-1 membership functions that determine the FOU bounds for the type-2 membership function $u_k^1(Z_k(x))$. The following is the type-2 fuzzy membership function of $u_k(Z_k(x))$:

$$u_k(Z_k(x)) = \begin{cases} (0,0,0) & , \quad Z_k(x) \le a_2 \\ (u_k^1(Z_k(x)), (u_k^2(Z_k(x)), (u_k^3(Z_i(x)) & , \quad a_2 \ge Z_k(x) \ge a_1 \\ (1,1,1) & , \quad Z_k(x) \ge a_1 \end{cases} \tag{6}$$

The FOU of the type-2 membership function for $W_s(x)$;

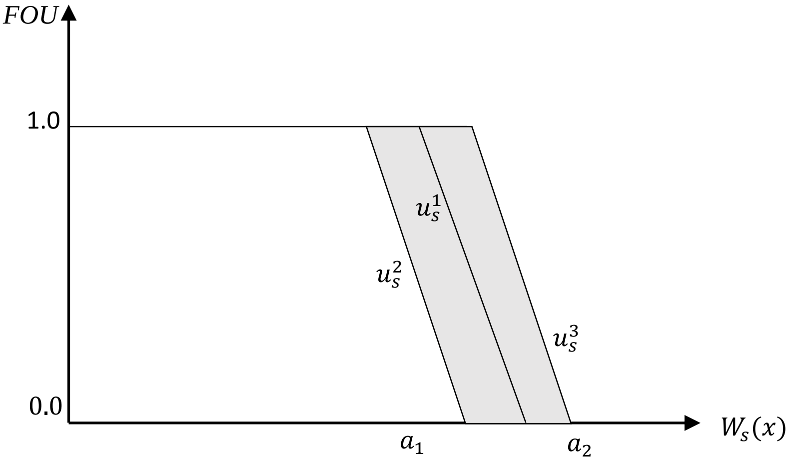

Figure 2. The FOU of the type-2 membership function for $W_s(x)$

The Upper Membership Function (UMF) and the Lower Membership Function (LMF), represented by $u_s^2(W_s(x))$ and $u_s^3(W_s(x))$, respectively, are two type-1 membership functions that determine the FOU bounds for the type-2 membership function $u_s^1(W_s(x))$. The following is the type-2 fuzzy membership function of $u_s(W_s(x))$:

$$u_s(W_s(x)) = \begin{cases} (0,0,0) & , \quad W_s(x) \ge a_2 \\ (u_s^1(W_s(x)), (u_s^2(W_s(x)), (u_s^3(W_s(x)) & , \quad a_1 \ge W_s(x) \ge a_2 \\ (1,1,1) & , \quad W_s(x) \le a_1 \end{cases} \tag{7}$$

According to The Upper Membership Function (UMF) and the Lower Membership Function (LMF) given above, the Zimmermann (1978) approach is organized as follows.

$$Max\ \left[u_1(Z_1(x), u_2\left(Z_2(x)\right), \dots, u_l(Z_l(x), u_1(W_1(x), u_2\left(W_2(x)\right), \dots, u_r(W_r(x)\right]$$

Subject to (8)

$$\sum_{j=1}^{m} a_{ij}x_j \leq b_i,$$

$x_j \geq 0$, i=1,2,...,m.

In (6) and (7), Maali and Mahdavi-Amiri (2014) considered triangular type-2 functions, and turned each type-2 function into three type-1 functions. Additionaly, in (8), the objective function is to maximize the largest membership value, $u_1(Z_1(x)$ and $(u_s^1\left(W_s(x)\right)$, as well as maximizing mean value of upper bound $\left[\frac{u_k^1(x)+u_k^2(x)}{2}\right]$ and $\left[\frac{u_s^1(x)+u_s^2(x)}{2}\right]$,and mean value of lower bound $\left[\frac{u_k^1(x)+u_k^3(x)}{2}\right]$ and $\left[\frac{u_s^1(x)+u_s^3(x)}{2}\right]$.

According to these limits, the objective function in (8) is

$$Max\ \left[u_k^1(x), \frac{u_k^1(x)+u_k^2(x)}{2}, \frac{u_k^1(x)+u_k^3(x)}{2}, u_s^1(x), \frac{u_s^1(x)+u_s^2(x)}{2}, \frac{u_s^1(x)+u_s^3(x)}{2}\ \right].$$

According to these regulations, (8) is written as follows.

$Max\ \lambda$

Subject to (9)

$$\lambda \leq u_k^1(x), \lambda \leq \frac{u_k^1(x)+u_k^2(x)}{2}, \lambda \leq \frac{u_k^1(x)+u_k^3(x)}{2},$$
$$\lambda \leq u_s^1(x), \lambda \leq \frac{u_s^1(x)+u_s^2(x)}{2}, \lambda \leq \frac{u_s^1(x)+u_s^3(x)}{2},$$
$$\sum_{j=1}^{m} a_{ij}x_j \leq b_i,$$
$x \geq 0, \lambda \in [0;1], k = 1,2,..,l$ and $s = 1,2,\dots,r$.

Here, $\lambda = \min_{k,s}\left(u_k^1(x), \frac{u_k^1(x)+u_k^2(x)}{2}, \frac{u_k^1(x)+u_k^3(x)}{2}, u_s^1(x), \frac{u_s^1(x)+u_s^2(x)}{2}, \frac{u_s^1(x)+u_s^3(x)}{2}\right)$. The maxmin approach (9) prioritizes maximizing the least possible membership value, allowing the strength of the model to be determined by the least membership value. Plus, a Pareto optimal solution can be revealed by the result obtained from (9). Maali and Mahdavi-Amiri (2014) followed the following path to test whether the solution obtained from (9) is a pareto-optimal solution: The following problem is resolved by performing a Pareto test if (9), which has multiple optimal solution;

$$Max \sum_i \varepsilon_i$$

Subject to; (10)

$$Z_i(x) + \varepsilon_i = Z_i(x^*),$$

$Ax \leq b,$

$x \geq 0, \varepsilon_i \geq 0, i = 1, \dots, k.$

Following the resolution of problem (9), x^* is the Pareto optimal solution for (5) if $\varepsilon_i = 0$ for all i; otherwise $\bar{x}$, a solution for (9), is the Pareto optimal solution.

3.2. Formulation of Multiobjecitve De Novo Programming with type-2 fuzzy objective

The first approach to solving Multi-Objective De Novo Programming problems in fuzzy environment was developed by Li and Lee (1990a). This approach consists of two phases: Phase 1 is based on the Zimmermann (1978) methodology, using positive and negative ideal solutions. Then, whether the solution obtained from Phase-1 is the only (unique) solution is tested with Phase-2. In this study, the phase-1 of the Li and Lee (1990a) approach was taken to form the basis for the proposed new approach for optimal system design. In Phase-1, $I^* = [Z_1^*, \dots, Z_l^*;\ W_1^*, \dots, W_r^*\ ]$ and $I^- = [Z_1^-, \dots, Z_l^-;\ W_1^-, \dots, W_r^-\ ]$ are the positive and negative ideal solution sets obtained from the solution of Multiobjective De Novo Programming (2), respectively. Later, fuzzy membership functions are created using (3) and (4) for each objective function. The phase-1 of Li and Lee (1990a]) is below.

$Max\ \lambda$

Subject to (11)

$$\lambda \leq \frac{Z_k(x) - Z_k^-}{Z_k^* - Z_k^-},$$

$$\lambda \leq \frac{W_s^- - W_s(x)}{W_s^- - W_s^*}$$

$$\sum_{j=1}^{m} v_j x_j \otimes B,$$

$x_j \geq 0, \lambda \in [0,1], j = 1,2,..,n, k = 1,2,..,l$ and $s = 1,2,\dots,r.$

The difference of (11) from the Zimmermann (1978) approach is only due to the budget constraint. In this study, the Multi-Objective De Novo Programming with type-2 fuzzy set formulation recommended for the solution of Multi-Objective De Novo Programming (2) with respect to (6) and (7) used in the approach of Maali and Mahdavi-Amiri (2014) is below.

$Max\ \lambda$

Subject to (12)

$$\lambda \leq u_k^1(x), \lambda \leq \frac{u_k^1(x) + u_k^2(x)}{2}, \lambda \leq \frac{u_k^1(x) + u_k^3(x)}{2},$$

$$\lambda \leq u_s^1(x), \lambda \leq \frac{u_s^1(x) + u_s^2(x)}{2}, \lambda \leq \frac{u_s^1(x) + u_s^3(x)}{2},$$

$$\sum_{j=1}^{m} v_j x_j \otimes B,$$

$x \geq 0, \lambda \in [0;1]$, $k = 1,2,..,l$ and $s = 1,2,\dots,r$.

Here, $\lambda = \min_{k,s}\left(u_k^1(x), \frac{u_k^1(x)+u_k^2(x)}{2}, \frac{u_k^1(x)+u_k^3(x)}{2}, u_s^1(x), \frac{u_s^1(x)+u_s^2(x)}{2}, \frac{u_s^1(x)+u_s^3(x)}{2}\right)$. The proposed approach (12) adjust maximizing the least possible membership value, allowing the strength of the model to be determined by the least membership value. Further, whether the solution obtained here is a Pareto solution can be investigated using (10). In (12), depending on the budget constraint, the "normalized degree" of the distances of each objective function from its positive ideal solution is minimized, while at the same time, the satisfaction of the objective function is maximized in the range [0,1]. In addition, using (10), it can be investigated whether the solution obtained from (12) is a pareto solution.

The recommended Multi-Objecitve De Novo Programming with type-2 fuzzy set steps for Optimal System Design are below.

Step 1: Determine the Positive and Negative Ideal Solutions for $Z_k(x) ve\ W_s(x)$ (k=,...,l and s=1,...r). In this step, positive and negative ideal solutions are determined for each objective function. Positive ideal solution set is $I^* = [Z_1^*, Z_2^*, \dots, Z_l^*;\ W_1^*, W_2^*, \dots, W_r^*]$ and negative ideal solution set is $I^* = [Z_1^-, Z_2^-, \dots, Z_l^-;\ W_1^-, W_2^-, \dots, W_r^-]$.

Step 2: Search for optimality with respect to positive ideal solutions. In accordane with the definition of optimality given at this stage, the decision variables that determine the positive ideal solutions of each objective function are taken into consideration.

$\underline{x^*}$ *is an optimal solution to the VMP if* $x^* \in X$ *and* $Z_k(\underline{x^*}) \geq Z_k(x)$ (*and/or* $W_s(\underline{x^*}) \leq W_s(x)$) *for all* $\underline{x^*} \in X$.

If $\underline{x^*}$ is optimal, go to Step 7. Otherwise, go to Step 3.

Step 3: Based on the positive and negative ideal solutions obtained in step 1, define the FOU of type-2 membership function for Z_k *and* W_s using (6) and (7). At this stage, the determination of the membership function is under the control of the decision maker.

Step 4: Determine $\underline{x^*}$using (12) according to the defined the FOU of type-2 membership function for Z_k *and* W_s.

Step 5: Use (10) to test whether $\underline{x^*}$ obtained in Step 4 is a pareto solution.

Step 6: Decision stage. At this stage, the decision variables obtained from Step 4 and Step 5 are compared.

if $\varepsilon_i = 0$ for all i, then the Pareto optimal solution is x^*. Therefore, $x^*$is a unique optimal solution. Otherwise, the optimal solution is not unique.

Step 7: Stop.

Diagram of the proposed Multiobjective De Novo Programming process with a type-2 fuzzy objective is in Figure 3.

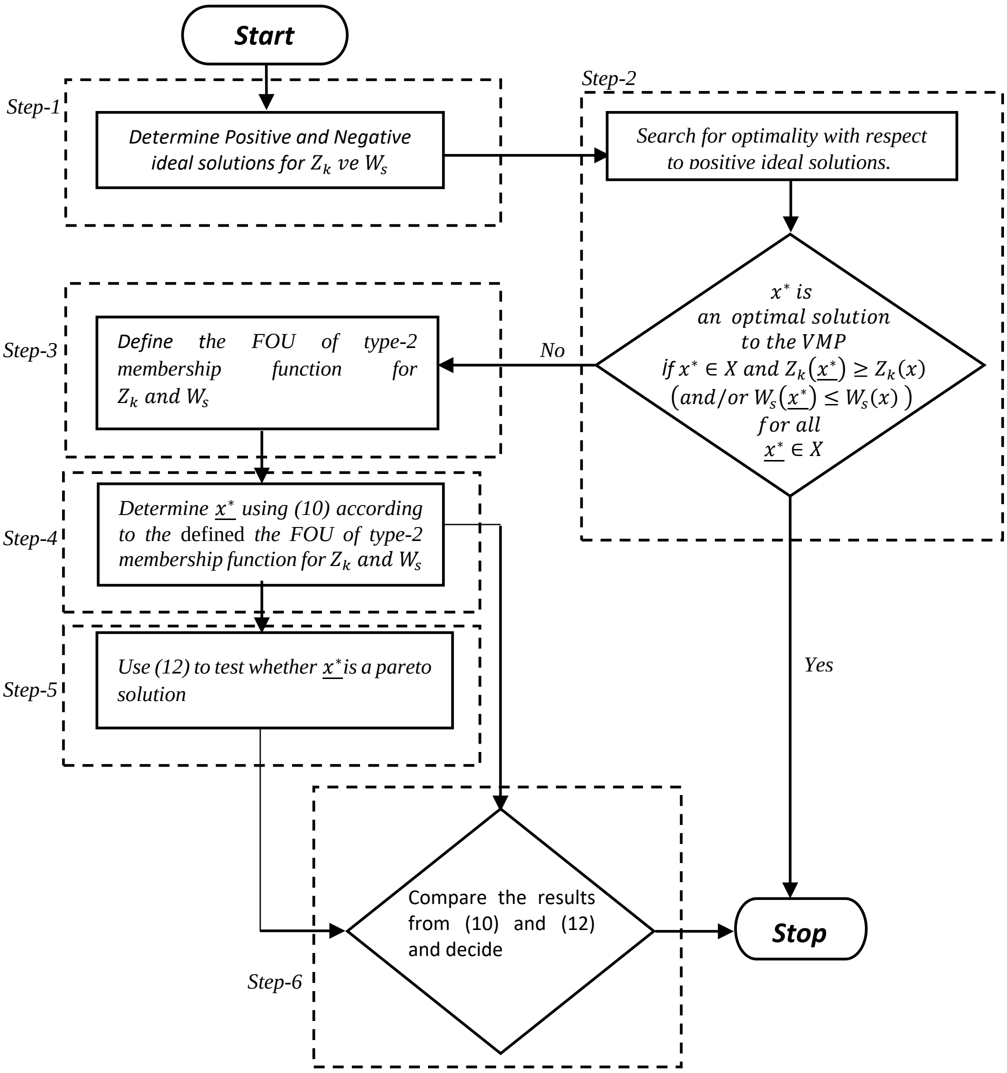

Figure 3. A flowchart of Multiobjective De Novo Programming with type-2 fuzzy objective

4. An Illustrative Example

Zeleny (1986)'s "problem of optimal design" has been used in some studies (paper) that include methods developed/used for the solution of Multi-Objective De Novo Programming problems. In this study, Zeleny (1986)'s problem was used to compare the results of the proposed approach with some of the previous approaches. The problem was given as the Multi-Objective Linear Programming problem in (P1), then it was rearranged in (P2) according to the De Novo assumption, taking into account the unit prices of the resources.

$Max. Z_1: 50x_1 + 100x_2 + 17{,}5x_3$

$Max. Z_2: 92x_1 + 75x_2 + 50x_3$

$Max. Z_3: 25x_1 + 100x_2 + 75x_3$

Subject to; (P1)

$12x_1 + 17x_2 \leq 1400$

$3x_1 + 9x_2 + 8x_3 \leq 1000$

$10x_1 + 13x_2 + 15x_3 \leq 1750$

$6x_1 + 16x_3 \leq 1325$

$12x_2 + 7x_3 \leq 900$

$9.5x_1 + 9.5x_2 + 4x_3 \leq 1075$

$x_1, x_2, x_3 \geq 0$

In (P1), the source unit prices of each constraint are respectively $p_1 = 0.75, p_2 = 0.6, p_3 = 0.35, p_4 = 0.5, p_5 = 1.15$ and $p_6 = 0.65$. Considering this information, the budget allocated for (P1) is \$4658.75. Accordingly, Using (2), the Multi-Objective De Novo Programming model for (P1) is set up as follows.

$Max. Z_1: 50x_1 + 100x_2 + 17{,}5x_3$

$Max. Z_2: 92x_1 + 75x_2 + 50x_3$

$Max. Z_3: 25x_1 + 100x_2 + 75x_3$

Subject to; (P2)

$23.475x_1 + 42.675x_2 + 28.7x_3 \leq 4658{,}75$

$x_1, x_2, x_3 \geq 0$

The solution of (P2) using the proposed approach is step by step below.

Step 1: The positive and negative ideal solutions obtained from the solution of (P2) are given below.

$Z_1^* = 10916{,}813;\ x_1 = 0;\ x_2 = 109{,}1681;\ x_3 = 0,$

$Z_1^- = 0;\ x_1 = 0;\ x_2 = 0;\ x_3 = 0,$

$Z_2^* = 18257{,}933;\ x_1 = 198{,}45795;\ x_2 = 0;\ x_3 = 0,$

$Z_2^- = 10;\ x_1 = 0;\ x_2 = 0;\ x_3 = 0,$

$Z_3^* = 12174{,}433;\ x_1 = 0;\ x_2 = 0;\ x_3 = 162{,}325790,$

$Z_3^- = 0;\ x_1 = 0;\ x_2 = 0;\ x_3 = 0.$

With these results, positive and negative ideal solution sets for (P2) are created below:

$I^* = [10916{,}813; 18257{,}933; 12174{,}433]$ and $I^- = [0; 0; 0]$.

Step 2: The positive ideal solution determined for each objective function in Step-1 was obtained according to the different values of the decision variables. Because of the definition of optimality, the result from (P2) is not optimal. Proceed to Step-3.

Step 3: Using the positive and negative ideal solution sets, Type-2 membership functions for three objective functions can be formed as follows:

The type-2 fuzzy membership for $u_1(Z_1(x))$:

$$u_1^1(Z_1(x)) \geq 8000) = 1, u_1^1(Z_1(x) \leq 3000) = 0,$$

$$u_1^2(Z_1(x) \geq 10916) = 1, u_1^2(Z_1(x) \leq 6000) = 0,$$

$$u_1^3(Z_1(x) \geq 4500) = 1, u_1^3(Z_1(x) \leq 0) = 0.$$

and

$$u_1\big(Z_1(x)\big) = \begin{cases} (0,0,0) & , \quad Z_1(x) \leq 0 \\ u_1^1(Z_1(x); u_1^2(Z_1(x)); u_1^3(Z_1(x)) & , \quad 0 \leq Z_1(x) \leq 10916. \\ (1,1,1) & , \quad Z_1(x) \geq 10916 \end{cases}$$

Here;

$$u_1^1(Z_1(x)) = \frac{Z_1(x) - 3000}{5000}; u_1^2(Z_1(x)) = \frac{Z_1(x) - 6000}{4916};\ u_1^3(Z_1(x)) = \frac{Z_1(x) - 0}{4500}$$

In Figure 4, type-2 fuzzy membership function for $Z_1(x)$ is shown.

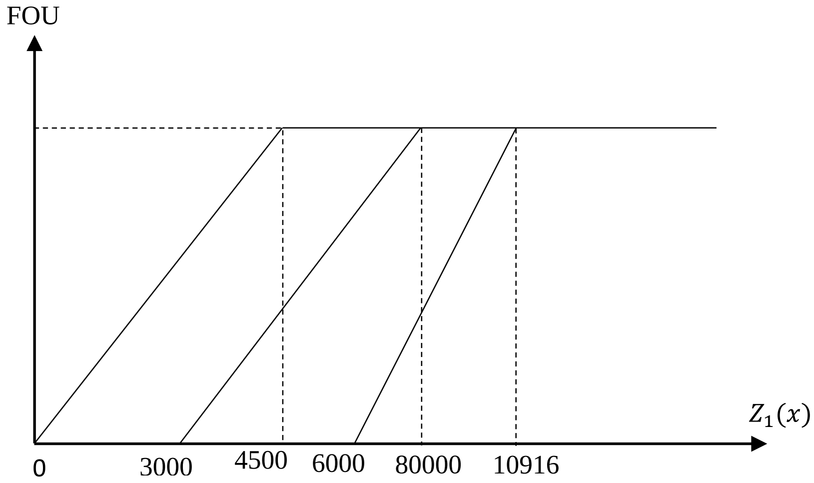

Figure 4. The FOU of type-2 membership function for $Z_1(x)$

The type-2 fuzzy membership for $u_2(Z_2(x))$:

$$u_2^1(Z_2(x) \geq 15000) = 1, u_2^1(Z_2(x) \leq 5000) = 0,$$

$$u_2^2(Z_2(x) \geq 13000) = 1, u_2^2(Z_2(x) \leq 18257) = 0,$$

$$u_2^3(Z_2(x) \geq 10000) = 1, u_2^3(Z_2(x) \leq 0) = 0,$$

and

$$u_2\big(Z_2(x)\big) = \begin{cases} (0,0,0) & , \quad Z_2(x) \leq 0 \\ u_2^1(Z_2(x); u_2^2(Z_2(x)); u_2^3(Z_2(x)) & , \quad 0 \leq Z_2(x) \leq 18257 \\ (1,1,1) & , \quad Z_2(x) \geq 18257 \end{cases}$$

Here;

$$u_2^1\big(Z_2(x)\big) = \frac{Z_2(x) - 5000}{10000}; u_2^2\big(Z_2(x)\big) = \frac{Z_2(x) - 13000}{5257};\ u_2^3\big(Z_2(x)\big) = \frac{Z_2(x) - 0}{10000}.$$

Type-2 fuzzy membership function for $Z_2(x)$ is in Figure 5.

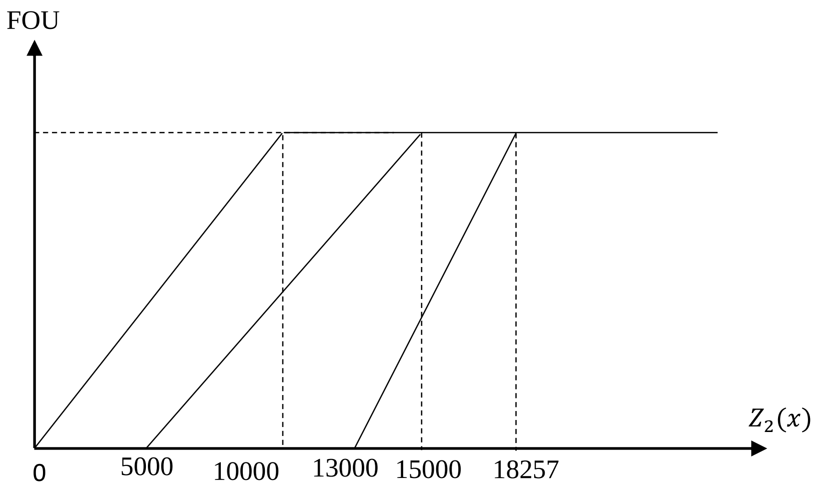

Figure 5. The FOU of type-2 membership function for $Z_2(x)$

The type-2 fuzzy membership for $u_3(Z_2(x))$:

$$u_2^1(Z_3(x) \geq 10000) = 1, u_3^1(Z_3(x) \leq 3000) = 0,$$

$$u_2^2(Z_3(x) \geq 12174) = 1, u_3^2(Z_3(x) \leq 7000) = 0,$$

$$u_3^3(Z_3(x) \geq 5000) = 1, u_3^3(Z_3(x) \leq 0) = 0,$$

and

$$u_3\big(Z_3(x)\big) = \begin{cases} (0,0,0) & , \quad Z_3(x) \leq 0 \\ u_3^1(Z_3(x); u_3^2(Z_3(x)); u_3^3(Z_3(x)) & , \quad 0 \leq Z_3(x) \leq 12174 \\ (1,1,1) & , \quad Z_3(x) \geq 12174 \end{cases}$$

Here;

$$u_3^1\big(Z_3(x)\big) = \frac{Z_3(x) - 3000}{7000}; u_3^2\big(Z_3(x)\big) = \frac{Z_3(x) - 7000}{5174}; \; u_3^3\big(Z_3(x)\big) = \frac{Z_3(x) - 0}{5000}.$$

Type-2 fuzzy membership function for $Z_3(x)$ is in Figure 6.

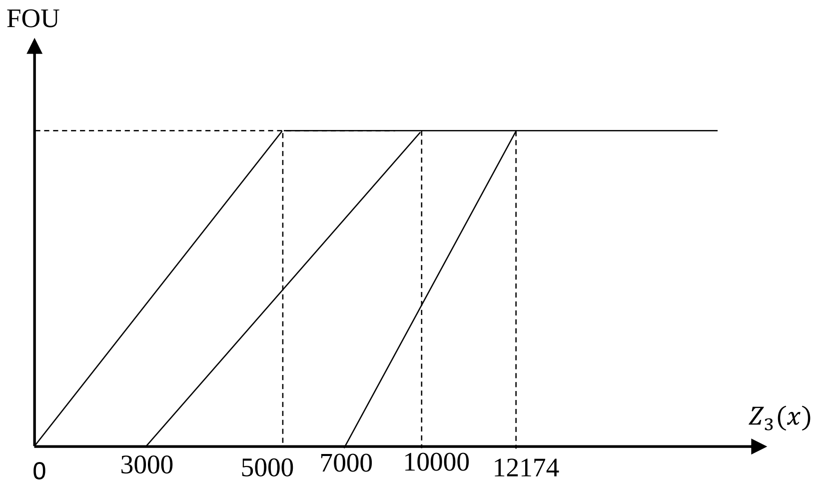

Figure 6. The FOU of type-2 membership function for $Z_3(x)$

Step 4: Type-2 fuzzy membership functions of the 3 objective functions above are arranged as follows, according to the proposed (12).

Max λ

Subject to (P3)

$$\frac{Z_1(x)-3000}{5000}\geq\lambda;\frac{\frac{Z_1(x)-3000}{5000}+\frac{Z_1(x)-6000}{4916}}{2}\geq\lambda;\frac{\frac{Z_1(x)-3000}{5000}+\frac{Z_1(x)-0}{4500}}{2}\geq\lambda$$

$$\frac{Z_2(x)-5000}{10000}\geq\lambda;\frac{\frac{Z_2(x)-5000}{10000}+\frac{Z_2(x)-13000}{5257}}{2}\geq\lambda;\frac{\frac{Z_2(x)-5000}{10000}+\frac{Z_2(x)-0}{10000}}{2}\geq\lambda$$

$$\frac{Z_3(x)-3000}{7000}\geq\lambda;\frac{\frac{Z_3(x)-3000}{7000}+\frac{Z_3(x)-7000}{5174}}{2}\geq\lambda;\frac{\frac{Z_3(x)-3000}{7000}+\frac{Z_3(x)-0}{5000}}{2}\geq\lambda$$

$23{,}475x_1+42{,}675x_2+28.7x_3<=4658{,}75$

$x_1,x_2,x_3\geq 0$ and $\lambda\in[0{,}1]$.

The decision variable value, objective function value and membership function values obtained from the solution of (P3) are given in Table 1.

Decision Variables	$Z_1(x^*)$	$Z_2(x^*)$	$Z_3(x^*)$
x_1^*	107,727829		
x_2^*	3,506307		
x_3^*	68,996780		
Objective Function Value	6944,4658	13623,7723	8218,5849
Budget	4658,75		
The type-2 Memberships	$u_1^1=0{,}7890$ $u_1^2=0.1921$ $u_1^3=1.0$	$u_2^1=0{,}8624$ $u_2^2=0{,}1187$ $u_2^3=1.0$	$u_3^1=0{,}7455$ $u_3^2=0{,}2355$ $u_3^3=1.0$
The mean values of lower bounds	$\frac{u_1^1(Z_1(\boldsymbol{x}^*))+u_1^2(Z_1(\boldsymbol{x}^*))}{2}=0{,}4905$	$\frac{u_2^1(Z_2(\boldsymbol{x}^*))+u_2^2(Z_2(\boldsymbol{x}^*))}{2}=0{,}4905$	$\frac{u_3^1(Z_3(\boldsymbol{x}^*))+u_3^2(Z_3(\boldsymbol{x}^*))}{2}=0{,}4905$
The mean values of upper bounds	$\frac{u_1^1(Z_1(\boldsymbol{x}^*))+u_1^3(Z_1(\boldsymbol{x}^*))}{2}=0{,}8945$	$\frac{u_2^1(Z_2(\boldsymbol{x}^*))+u_2^3(Z_2(\boldsymbol{x}^*))}{2}=0{,}9312$	$\frac{u_3^1(Z_3(\boldsymbol{x}^*))+u_3^3(Z_3(\boldsymbol{x}^*))}{2}=0{,}8728$
Max λ	0.4905		

Table 1. Solution for (P3)

Step 5: According to the result in Table 1, (P3) can have multiple optimal solutions. (10) is used to test this.

$Max\ \varepsilon_1+\varepsilon_2+\varepsilon_3$

Subject to; (P4)

$50x_1+100x_2+17{,}5x_3+\varepsilon_1=6944{,}4658$

$92x_1+75x_2+50x_3+\varepsilon_2=13623{,}7723$

$25x_1+100x_2+75x_3+\varepsilon_3=8218{,}5849$

$23{,}475x_1+42{,}675x_2+28{,}7x_3\leq 4658{,}75$

$x_1,x_2,x_3\geq 0.$

The decision variable values obtained from the solution of (P4) are respectively $x_1 = 107{,}037025, x_2 = 3{,}506939, x_3 = 68{,}99678, e_1 = 0{,}000266, e_2 = 0$ and $e_3 = 0$. Additionally, using these decision variables, objective function values are obtained as $Z_1 = 6944{,}4635, Z_2 = 13623{,}7727$ ve $Z_3 = 8218{,}5851$.

Step 6 : The solution result of (P3) and the solution result obtained by using (10) are the same with negligible difference. Therefore, there is the unique optimal solution for (P3). When the mean values of the upper and lower limits of each objective function are examined, the mean value of lower limit of objective functions and " Max λ" value of (P3) are 0,4905. With this result, the power of the model was obtained at the lowest membership value. The result of the suggested approach can be summarized as follows: Decision variable values are $x^* = (107{,}037025;\ 3{,}506307;\ 68{,}99678)$ and objective function values are (6944,4658; 13623,7723; 8218,5849).

Furthermore, type-2 membership functions are $u_1(x^*) = (0{,}789;\ 0{,}1921;\ 1.0), u_2(x^*) = (0{,}8624;\ 0{,}1187;\ 1{,}0)$ and $u_3(x^*) = (0{,}7755;\ 0{,}2355;\ 1{,}0)$.

Step 7: Stop.

The de novo assumption allows resources to be rescheduled using the budget constraint. The optimal amounts determined for each resource with the proposed approach are given in Table 2. (↓) and (↑) show the decrease and increase in the optimal amount, respectively.

Resouce Amount	Initial amount	Proposed Optimal Amount
b_1	1400	1352,341167 (↓)
b_2	1000	906,71449 (↓)
b_3	1750	2157,811981(↑)
b_4	1325	1750,315454 (↑)
b_5	900	525,053144 (↓)
b_6	1075	1332,711412 (↑)
Budget	4658,75	4658,75

Table 2. Optimal resource amount for (P1)

With the proposed approach, the resource amounts of the constraints were determined without increasing or decreasing the budget. Besides, for the solution of (P2), a solution was made according to different methods, and a comparison was made with the proposed method. The theory of these methods is not given in the study. The results obtained are shown in Table 3.

	Li and Lee (1990a)'s two-phase approach		Li & Lee (1990b)'s possibistic two-phase approah	Banik & Bhattacharya (2019)	Umarusman (2020)	Suggested Solution
	Phase-1	Phase-2				
Z_1	7686,8729	7686,8108	9305,9750	12313,2250	12786,1650	6944,4658
Z_2	12855,9872	12855,9960	16564,4200	16443,3340	15751,2793	13623,7723
Z_3	8572,4022	8572,4063	8880,7500	9006,6125	9959,7175	8218,5849
x_1	92,4812	92,481361	139,76	132,2645	113,0579	107,727829
x_2	20,8964	20,895512	13,84	57	71,3327	3,506307
x_3	55,6097	55,610947	53,37	0	0	68,996780
b_1	1465,013599	1465	1912,4	2556,174	2569,3507	1352,341167
b_2	910,389229	910,391267	970,8	909,7935	981,168	906,71449
b_3	2030,611257	2030,619471	2378,07	2063,645	2057,9041	2157,811981
b_4	1444,642556	1444,663318	1692,48	793,587	678,3474	1750,315454
b_5	640,025294	640,022773	539,67	684	855,9924	525,053144
b_6	1299,526234	1299,524082	1672,68	1798,01275	1751,7107	1332,711412
Budget	4658,7501	4658,75	5403,207	5537,3841	5698,1572	4658,75

Table 3. Results obtained in terms of different approaches for (P2)

Although the budget of (P2) in Table 3 is 4658.75, as a result of solutions using Li and Lee's (1990b) two-stage probability approach, Banik and Bhattacharya (2019) and Umarusman (2020), the budget increased as follows, respectively: 13.41%, 15.51 and 17.9%. This increase had a direct impact on the amount of natural resources. This is because Li and Lee (1990b)'s possibistic two-phase approach is based on the possibilistic perspective of Banik and Bhattacharya (2019) approach and Umarusman (2020) approach type-1 fuzzy set theory. In other words, besides the objective functions, the budget is blurred in a possibilistic framework.

The solutions obtained from Li and Lee (1990a)'s two-phase approach are from the perspective of type-1 fuzzy set theory and provide solutions considering the positive and negative ideal solution sets of objective functions. In the proposed approach, membership functions are created and the solution is made by using positive and negative ideal solution sets within the framework of type-2 fuzzy sets. It seems more appropriate to compare the results of Li and Lee (1990a)'s two-phase approach and the approach proposed. This comparison is given in Table 4.

	Li and Lee (1990a)'s two-phase approach		**Suggested Solution**
	Phase-1	Phase-2	
Z_1	7686,8729	7686,8108	6944,4658
Z_2	12855,9872	12855,9960	13623,7723
Z_3	8572,4022	8572,4063	8218,5849
x_1	92,4812	92,481361	107,727829
x_2	20,8964	20,895512	3,506307
x_3	55,6097	55,610947	68,996780
b_1	1465,013599	1465	1352,341167
b_2	910,389229	910,391267	906,71449
b_3	2030,611257	2030,619471	2157,811981
b_4	1444,642556	1444,663318	1750,315454
b_5	640,025294	640,022773	525,053144
b_6	1299,526234	1299,524082	1332,711412
Budget	4658,7501	4658,75	4658,75
The fuzzy degree of satisfaction	0.7041315	0.7041315	0.4905165

Table 4. Optimal resource amount within the framework of Type-1 and Type-2 fuzzy sets

The solution obtained from phase-1 of Li and Lee (1990a) approach was tested using phase-2, and it was determined that the result obtained from phase-1 was a pareto-optimal solution. In addition, pareto test was performed using (10) for the proposed approach and it was determined that the obtained result was pareto-optimal. It is possible to compare Li and Lee (1990a) approach and the proposed approach in terms of objective function (Z_k), decision variable(x_j), resource amounts (b_i), budget and fuzzy degree of satisfaction.The aim here is to show that the proposed approach can be used in solving Multiobjective De Novo Programming problems, rather than discussing the superiority of the results over each other. According to the De Novo assumption, the decision variables determine the values of the objective functions and the amount of resources depending on the budget constraint. When the proposed solution is examined in terms of budget, it has not changed which is initially given. This shows that the proposed approach is in line with the de novo assumption from a budgetary perspective. The values of the decision variables and therefore the objective function are different due to the membership functions. This is to be expected.

5. Conclusion and Recommendations

In solutions based on the De Novo assumption, the amount of resources is replanned in a way that the constraints depend on the budget. With this planning, the level of performance of the objective functions increases. In Multi-Objective De Novo Programming problems, there is an increase in budget constraints in the solutions, made in the possibilistic framework, of fuzzy set theory. However, the budget remains unchanged in other methods in the literature. In the proposed approach, type-2 fuzzy membership functions were created for each objective function by using the positive and negative ideal solution set determined depending on the crisp budget for (P2). Then, using the proposed model (12), (P2) was solved and the value of type-2 membership functions was calculated in accordance with the solutions obtained. Accordingly, the power of (P2) emerged at the lowest membership value. Moreover, it was investigated whether the solution of (P2) was a pareto-optimal solution and it was determined that the solution obtained was the unique optimal solution. In addition to the proposed approach, the results of the other 5 methods are presented in Table 3. In Table 4, Li and Lee (1990a)'s two-phase approach and the results of the proposed approach and the degree of satisfaction of the approaches are given. The reason for the difference in the fuzzy degree of satisfaction in both approaches is due to the type-1 and type-2 membership functions. Although the proposed approach is made using the patero-optimality of its results (10), phase-2 of Li and Lee (1990a) can be taken into account in determining the pareto-optimal solution and a new two-phase approach within the framework of type-2 fuzzy sets, could be the subject of future research.

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