http://communications.science.ankara.edu.tr

Commun.Fac.Sci.Univ.Ank.Ser. A1 Math. Stat.
Volume 72, Number 4, Pages 976–991 (2023)
DOI:10.31801/cfsuasmas.1255830
ISSN 1303-5991 E-ISSN 2618-6470

Research Article; Received: February 24, 2023; Accepted: July 9, 2023

SUSTAINABLE METHOD FOR TENDER SELECTION USING LINEAR DIOPHANTINE MULTI-FUZZY SOFT SET

Jeevitha KANNAN[1] and Vimala JAYAKUMAR[2]

[1,2]Department of Mathematics, Alagappa University, Karaikudi, INDIA

Abstract. Tender selection is a fundamental issue for the success of construction projects since it contributes to the overall outline's performance. In real-life problems, the decision-makers cannot express certain crisp data, so there is uncertainty and vagueness in the values. In this paper, a sustainable technique is proposed to find desirable tenderers coherently and fairly under the needed circumstances. This paper presents three methods of an algorithmic approach to evaluate the tendering process and rank the tenderers. The attributes are expressed as Linear Diophantine Multi-Fuzzy Soft numbers (LDMFSN) since the existence of reference parameters makes the DM freely choose their grade values. Some of the rudimentary properties of LDMFSN are presented. An illustrative example is demonstrated to validate our proposed method. The uniqueness of the result in all three algorithms shows the effectiveness of our proposed approach.

1. Introduction

The tender selection process is one of the most vital processes in the construction industry. Many of the selection processes are associated with the lowest bid price method. But, the lower price bid does not assure the best outcomes. Hence, many researchers have recognised the prominent criteria for the selection process. The selection of an appropriate tenderer is directly correlated with the success of the project. So, it is essential to explore a suitable tender for a successful outcome. Many researchers studied the tender selection process using fuzzy theory.

The focus of research has been on bid evaluation models and indication systems. Researchers concentrate on various indicators in their studies of bid evaluation systems [9, 16, 17]. In decision-making problems, there are a large number of

2020 *Mathematics Subject Classification.* 03E72, 08A72, 94D05, 90B50.

Keywords. Linear Diophantine fuzzy, tender selection, threshold fuzzy, comparision matrix, score value.

[1] kjeevitha991@gmail.com; 0000-0001-7846-9173

[2] vimaljey@alagappauniveristy.ac.in-Corresponding author; 0000-0003-3138-9365.

uncertainties in the information. To solve such ambiguity, Zadeh [33] introduced the idea of a fuzzy set. For the purpose of scoring contractors in the study of bid assessment models, AHP [4], Fuzzy-AHP [8] and Fuzzy AHP-SMART [20] were employed. Jamili [13] proposed a novel fuzzy approach for the tender selection problem. Later, another fuzzy support desicion model [1] was constructed for selection of contractors. Membership grades are not sufficient to handle some problems resulting in the origination of an intuitionistic fuzzy set [2].Using Intuitionistic fuzzy information, many decision model [18, 29] were established for bid selection. Later, some generalizations like Pythagorean fuzzy set [32], q-rung orthopair fuzzy set [31] are introduced. These fuzzy sets have immense applications in the tender selection process. But, it has some limitations on their membership grades.

In order to relax these deficiencies, Riaz and Hashmi [24] introduced a Linear Diophantine Fuzzy Set (LDFS) with the inclusion of reference parameters. The existence of reference parameters extends the space of grade values. The practical advantages of LDFS attracted the attention of several researchers in various scientific fields, and a number of icon works were written as a result. A. Iampam [12] addressed using LDFS with a variety of Einstein aggregation approaches for MCDM issues. Later, S. Ayub [3] used decision-making to create LDF relations and related algebraic characteristics. Kamac [14] created the intricate LDFS and provided a description of the cosine similarity metric and its intended uses. By incorporating the concept of soft rough sets for use in material handling equipment, Riaz et al. [25] extended the LDFS. The use of spherical linear Diophantine fuzzy sets with modelling uncertainty in MCDM was addressed by Hashmi et al. Riaz et al. [23] built prioritised AOs for linear Diophantine fuzzy numbers (LDFNs) and used them to choose third-party logistic service providers.

Maji [19] investigated the fuzzy soft set theory, which got the boom in recent times. Later, it was extended to the formulation of the hybrid models named Fuzzy soft group [22], Intuitionistic Fuzzy soft group [26] Intuitionistic Fuzzy Soft Set (IFSS) [7], Pythagorean Fuzzy Soft Set (PFSS) [21]. Hussain [10] investigated q-Rung Orthopair Fuzzy Soft Set(q-ROFSS) with aggregation operators. Further, the multi-fuzzy soft set [30] is a fusion of multi-fuzzy set [27] and fuzzy soft set, which has a extensive application in many fields. The mixture of the multi-valued approach and the parametric approach helps to express the problems that are not able to be expressed in other existing fuzzy models. Begam [5, 6] introduced a novel approach to lattice ordered multi-fuzzy soft set based decision-making. In this paper, the implementation of a multi-fuzzy set in the LDFS is presented. The hybrid concept of LDMFSS was initiated along with the LDMFSN and some of its properties are discussed.

1.1. **Motivation and Inspiration.** The following is an explanation of the intended objectives of this research:

(1) A superior mathematical model beyond membership and non-membership grades is LDFS, which does deal with these limitations. It helps the DM select the grade values at their discretion. A general method for coping with uncertainty is a soft set. Based on these benefits, this study introduces the novel hybrid concept of linear Diophantine multi-fuzzy soft set(LDMFSS) in order to fill the research gap.
(2) This theory was found to be more useful for dealing with uncertain values in decision analysis. Our analysis of the literature revealed that there is no research investigating tender selection employing LDF data. Hence, the main intent of this paper is to apply the LDMFSS decision model to the selection of tenders.
(3) This paper presents three different algorithms to solve the LDMFSS model using a comparison matrix, score function, and threshold fuzzy. The illustrative example is presented, and the result is analysed using three different algorithms to show the validity of our proposed method.

The outline that follows is the organisational structure of the manuscript: Section 2 focuses on several fundamental concepts, such as FS, IFS, PFS, and LDFS. In Section 3, the new hybrid concepts of LDMFS and LDMFSS were established. LDMFSN and some of its properties are discussed. Section 4 comprises a case study for tender selection and three different algorithms. A comparative study of the results of three proposed algorithms was initiated. In the end, Section 5 summarised the conclusion of this study.

2. Preliminaries

Throughout this paper $\daleth$ is used as a universal set and Π is a parameter set.

Definition 1. *[33] The fuzzy set $\mathcal{F}$ on $\daleth$ is a mapping μ: $\daleth \rightarrow [0,1]$ where $\mu(x)$ represents the degrees of elements in $\daleth$ and it is represented in the form*

$$\mathcal{F} = \{(x, \mu(x))/x \in \daleth\}$$

Definition 2. *[27] Let $K = \{1,2,...k\}$ be the set of indices. The multi-fuzzy set $\mathcal{M}$ with dimension k on $\daleth$ is symbolized as follow:*

$$\mathcal{M} = \{(x, \mu^K(x))/x \in \daleth\}$$

where $\mu^K = (\mu_1, \mu_2, \mu_3, ..., \mu_k)$, $\mu_l : \daleth \rightarrow [0,1]$ for every $l \in K = \{1,2,...k\}$ and the collection of all multi-fuzzy set of dimension k over $\daleth$ is represented by $M_kFS(\daleth)$.

Definition 3. *[19] A pair $(\mathcal{F}, \Pi)$ is called a multi-fuzzy soft set of dimension k over $\daleth$, where $\mathcal{F}$ is a mapping given by $\mathcal{F} : \Pi \rightarrow M_kFS(\daleth)$, where Π is a parameter set.*

Definition 4. *[2] The intuitionistic fuzzy set (IFS) on $\daleth$ is represented as*

$$\mathcal{I} = \{x, \langle \mu_{\mathcal{I}}(x), \nu_{\mathcal{I}}(x)\rangle : x \in \daleth\}$$

where $\mu_{\mathcal{I}}(x), \nu_{\mathcal{I}}(x)$ are degrees of membership and non-membership which belongs to [0,1] subject to the condition $0 \leq \mu_{\mathcal{I}}(x) + \nu_{\mathcal{I}}(x) \leq 1$.

Definition 5. *[32] The Pythagorean fuzzy set (PFS) on* $\beth$ *is in the mathematical form*

$$\mathcal{P} = \{x, \langle \mu_{\mathcal{P}}(x), \nu_{\mathcal{P}}(x)\rangle : x \in \beth\}$$

where $\mu_{\mathcal{P}}(x), \nu_{\mathcal{P}}(x)$ *are degrees of membership and non-membership which belongs to [0,1] subject to the condition* $0 \leq \mu^2_{\mathcal{P}}(x) + \nu^2_{\mathcal{P}}(x) \leq 1$.

Definition 6. *[31] The q-rung orthopair fuzzy set (q-ROFS) on* $\beth$ *is in the mathematical form*

$$\mathcal{R} = \{x, \langle \mu_{\mathcal{R}}(x), \nu_{\mathcal{R}}(x)\rangle : x \in \beth\}$$

where $\mu_{\mathcal{R}}(x), \nu_{\mathcal{R}}(x)$ *are degrees of membership and non-membership which belongs to [0,1] subject to the condition* $0 \leq \mu^q_{\mathcal{R}}(x) + \nu^q_{\mathcal{R}}(x) \leq 1$.

Definition 7. *[24] A linear Diophantine fuzzy set* $\mathcal{L}$ *on* $\beth$ *is a structure symbolized as:*

$$\mathcal{L} = \{(x, \langle \mu_{\mathcal{L}}(x), \nu_{\mathcal{L}}(x)\rangle, \langle \tau_{\mathcal{L}}(x), \eta_{\mathcal{L}}(x)\rangle) : x \in \beth\}$$

where, $\mu_{\mathcal{L}}(x), \nu_{\mathcal{L}}(x), \tau_{\mathcal{L}}(x), \eta_{\mathcal{L}}(x) \in$ *[0, 1] are degrees of membership, non-membership and their reference parameters respectively. These grades satisfy the condition* $0 \leq \tau_{\mathcal{L}}(x)\mu_{\mathcal{L}}(x) + \eta_{\mathcal{L}}(x)\nu_{\mathcal{L}}(x) \leq 1$ *for all* $x \in \beth$ *with* $0 \leq \tau_{\mathcal{L}}(x) + \eta_{\mathcal{L}}(x) \leq 1$.

3. Linear Diophantine Multi-Fuzzy Soft Set

Definition 8. *[15,28] Let K be the set of indices. A linear Diophantine multi-fuzzy set* $\mathcal{J}$ *on* $\beth$ *with dimension k is the set of ordered sequences in the form*

$$\mathcal{J} = \left\{(x, \langle \mu^K_{\mathcal{J}}(x), \nu^K_{\mathcal{J}}(x)\rangle, \langle \tau^K_{\mathcal{J}}(x), \eta^K_{\mathcal{J}}(x)\rangle) : x \in \beth\right\}$$

where,

$$\begin{aligned}
\mu^K_{\mathcal{J}}(x) =& (\mu^1_{\mathcal{J}}(x), \mu^2_{\mathcal{J}}(x), \mu^3_{\mathcal{J}}(x), ..., \mu^k_{\mathcal{J}}(x)) \\
\nu^K_{\mathcal{J}}(x) =& (\nu^1_{\mathcal{J}}(x), \nu^2_{\mathcal{J}}(x), \nu^3_{\mathcal{J}}(x), ..., \nu^k_{\mathcal{J}}(x)) \\
\tau^K_{\mathcal{J}}(x) =& (\tau^1_{\mathcal{J}}(x), \tau^2_{\mathcal{J}}(x), \tau^3_{\mathcal{J}}(x), ..., \tau^k_{\mathcal{J}}(x)) \\
\eta^K_{\mathcal{J}}(x) =& (\eta^1_{\mathcal{J}}(x), \eta^2_{\mathcal{J}}(x), \eta^3_{\mathcal{J}}(x), ..., \eta^k_{\mathcal{J}}(x))
\end{aligned}$$

and $\mu^K_{\mathcal{J}}(x), \nu^K_{\mathcal{J}}(x), \tau^K_{\mathcal{J}}(x), \eta^K_{\mathcal{J}}(x)$ *are collection of multi membership, multi non-membership and multi reference parameters values respectively. Along that, it satisfies the condition*

$$0 \leq \mu^l_{\mathcal{J}}(x)\tau^l_{\mathcal{J}}(x) + \nu^l_{\mathcal{J}}(x)\eta^l_{\mathcal{J}}(x) \leq 1$$
$$0 \leq \tau^l_{\mathcal{J}}(x) + \eta^l_{\mathcal{J}}(x) \leq 1$$

for every $l \in K = \{1, 2, 3..., k\}$.
The collection of all linear Diophantine multi-fuzzy set of dimension k over $\beth$ *is denoted by* $LDM_kF(\beth)$.

Remark 1. *The term $\mu^K(x)$ represents collection of memberships of k dimension whereas $\mu^l(x)$ is the element of $\mu^K(x)$ and it represents the membership value of x for the l^{th} dimensional parameter.*

Definition 9. *Let Π be the set of parameters. Define a map $\mathfrak{J} : \Pi \to LDM_kFS(\beth)$. Then the ordered pair $(\mathfrak{J}, \omega)$ is claimed to be linear Diophantine multi-fuzzy soft set of dimension k and it is of the structure*

$$\{(\omega_i, \mathfrak{J}(\omega_i)) : \omega_i \in \Pi\}$$

where $\mathfrak{J}(\omega_i)$ is a $LDM_kFS(\beth)$.

Definition 10. *The linear Diophantine multi-fuzzy soft set*

$$\mathfrak{J} = \{\big(\omega_i, (x, \langle\mu^K_{\mathfrak{J}(\omega_i)}(x), \nu^K_{\mathfrak{J}(\omega_i)}(x)\rangle, \langle\tau^K_{\mathfrak{J}(\omega_i)}(x), \eta^K_{\mathfrak{J}(\omega_i)}(x)\rangle) : \forall x \in \beth, \omega_i \in \Pi\big)\}$$

is claimed to be absolute linear Diophantine multi-fuzzy soft set if $\mu^K_{\mathfrak{J}(\omega_i)}(x) = 1, \nu^K_{\mathfrak{J}(\omega_i)}(x) = 0, \tau^K_{\mathfrak{J}(\omega_i)}(x) = 1, \eta^K_{\mathfrak{J}(\omega_i)}(x) = 0$.
i.e, $\mu^l_{\mathfrak{J}(\omega_i)}(x) = 1, \nu^l_{\mathfrak{J}(\omega_i)}(x) = 0, \tau^l_{\mathfrak{J}(\omega_i)}(x) = 1, \eta^l_{\mathfrak{J}(\omega_i)}(x) = 0$, for every $l \in K$, $\omega_i \in \Pi$.

Definition 11. *The linear Diophantine multi-fuzzy soft set*

$$\mathfrak{J} = \{(\omega_i, (x, \langle\mu^K_{\mathfrak{J}(\omega_i)}(x), \nu^K_{\mathfrak{J}(\omega_i)}(x)\rangle, \langle\tau^K_{\mathfrak{J}(\omega_i)}(x), \eta^K_{\mathfrak{J}(\omega_i)}(x)\rangle)) : \forall x \in \beth, \omega_i \in \Pi\}$$

is said to be null linear Diophantine multi-fuzzy soft set if $\mu^K_{\mathfrak{J}(\omega_i)}(x) = 0, \nu^K_{\mathfrak{J}(\omega_i)}(x) = 1, \tau^K_{\mathfrak{J}(\omega_i)}(x) = 0, \eta^K_{\mathfrak{J}(\omega_i)}(x) = 1$.
i.e., $\mu^l_{\mathfrak{J}(\omega_i)}(x) = 0, \nu^l_{\mathfrak{J}(\omega_i)}(x) = 1, \tau^l_{\mathfrak{J}(\omega_i)}(x) = 0, \eta^l_{\mathfrak{J}(\omega_i)}(x) = 1$, for every $l \in K$, $\omega_i \in \Pi$.

Definition 12. *The linear Diophantine multi-fuzzy soft number(LDM_kFSN) is expressed as $\mathfrak{J}(\omega_i) = \langle\mu^K_{\mathfrak{J}(\omega_i)}(x), \nu^K_{\mathfrak{J}(\omega_i)}(x)\rangle, \langle\tau^K_{\mathfrak{J}(\omega_i)}(x), \eta^K_{\mathfrak{J}(\omega_i)}(x)\rangle$ and it satisfies the following conditions:*
(i) $0\leq \mu^l_{\mathfrak{J}(\omega_i)}(x)\tau^l_{\mathfrak{J}(\omega_i)}(x) + \nu^l_{\mathfrak{J}(\omega_i)}(x)\eta^l_{\mathfrak{J}(\omega_i)}(x) \leq 1$ and
(ii) $0 \leq \tau^l_{\mathfrak{J}(\omega_i)}(x) + \eta^l_{\mathfrak{J}(\omega_i)}(x) \leq 1$, where $l = 1, 2, 3, ..., k$

Definition 13. *Considering two LDMFSN*

$$\mathfrak{J}(\omega_1) = \langle\mu^K_{\mathfrak{J}(\omega_1)}(x), \nu^K_{\mathfrak{J}(\omega_1)}(x)\langle, \langle\tau^K_{\mathfrak{J}(\omega_1)}(x), \eta^K_{\mathfrak{J}(\omega_1)}(x)\rangle$$
$$\mathfrak{J}(\omega_2) = \langle\mu^K_{\mathfrak{J}(\omega_2)}(x), \nu^K_{\mathfrak{J}(\omega_2)}(x)\langle, \langle\tau^K_{\mathfrak{J}(\omega_2)}(x), \eta^K_{\mathfrak{J}(\omega_2)}(x)\rangle$$

and defined some operations as follows:

$$(i)\ \mathfrak{J}(\omega_1) \cup \mathfrak{J}(\omega_2) = \langle max\{\mu^l_{\mathfrak{J}(\omega_1)}, \mu^l_{\mathfrak{J}(\omega_2)}\}, min\{\nu^l_{\mathfrak{J}(\omega_1)}, \nu^l_{\mathfrak{J}(\omega_2)}\}\rangle, \langle max\{\tau^l_{\mathfrak{J}(\omega_1)}, \tau^l_{\mathfrak{J}(\omega_2)}\}, min\{\eta^l_{\mathfrak{J}(\omega_1)}, \eta^l_{\mathfrak{J}(\omega_2)}\}\rangle,\ for\ every\ l \in K$$
$$(ii)\ \mathfrak{J}(\omega_1) \cap \mathfrak{J}(\omega_2) = \langle min\{\mu^l_{\mathfrak{J}(\omega_1)}, \mu^l_{\mathfrak{J}(\omega_2)}\}, max\{\nu^l_{\mathfrak{J}(\omega_1)}, \nu^l_{\mathfrak{J}(\omega_2)}\}\rangle, \langle min\{\tau^l_{\mathfrak{J}(\omega_1)}, \tau^l_{\mathfrak{J}(\omega_2)}\}, max\{\eta^l_{\mathfrak{J}(\omega_1)}, \eta^l_{\mathfrak{J}(\omega_2)}\}\rangle\ for\ every\ l \in K$$

$$
\begin{aligned}
&(iii)\ \mathfrak{J}(\omega_1)\oplus\mathfrak{J}(\omega_2) = \langle \mu^l_{\mathfrak{J}(\omega_1)}+\mu^l_{\mathfrak{J}(\omega_2)}-\mu^l_{\mathfrak{J}(\omega_1)}\mu^l_{\mathfrak{J}(\omega_2)}, \nu^l_{\mathfrak{J}(\omega_1)}\nu^l_{\mathfrak{J}(\omega_2)}\rangle,\\
&\qquad \langle \tau^l_{\mathfrak{J}(\omega_1)}+\tau^l_{\mathfrak{J}(\omega_2)}-\tau^l_{\mathfrak{J}(\omega_1)}\tau^l_{\mathfrak{J}(\omega_2)}, \eta^l_{\mathfrak{J}(\omega_1)}\eta^l_{\mathfrak{J}(\omega_2)}\rangle \ for\ every\ l\in K\\
&(iv)\ \mathfrak{J}(\omega_1)\otimes\mathfrak{J}(\omega_2) = \langle \mu^l_{\mathfrak{J}(\omega_1)}\mu^l_{\mathfrak{J}(\omega_2)}, \nu^l_{\mathfrak{J}(\omega_1)}+\nu^l_{\mathfrak{J}(\omega_2)}-\nu^l_{\mathfrak{J}(\omega_1)}\nu^l_{\mathfrak{J}(\omega_2)}\rangle,\\
&\qquad \langle \tau^l_{\mathfrak{J}(\omega_1)}\tau^l_{\mathfrak{J}(\omega_2)}, \eta^l_{\mathfrak{J}(\omega_1)}+\eta^l_{\mathfrak{J}(\omega_2)}-\eta^l_{\mathfrak{J}(\omega_1)}\eta^l_{\mathfrak{J}(\omega_2)}\rangle \ for\ every\ l\in K\\
&(v)\ \mathfrak{J}(\omega_1)\le\mathfrak{J}(\omega_2)\ iff\ \mu^l_{\mathfrak{J}(\omega_1)}\le\mu^l_{\mathfrak{J}(\omega_2)}, \nu^l_{\mathfrak{J}(\omega_1)}\ge\nu^l_{\mathfrak{J}(\omega_2)},\\
&\qquad \tau^l_{\mathfrak{J}(\omega_1)}\le\tau^l_{\mathfrak{J}(\omega_2)}, \eta^l_{\mathfrak{J}(\omega_1)}\ge\eta^l_{\mathfrak{J}(\omega_2)}\ for\ every\ l\in K\\
&(vi)\ \mathfrak{J}(\omega_1)^c = \langle \nu^l_{\mathfrak{J}(\omega_i)}(x), \mu^l_{\mathfrak{J}(\omega_i)}(x)\rangle, \langle \eta^l_{\mathfrak{J}(\omega_i)}(x), \tau^l_{\mathfrak{J}(\omega_i)}(x)\rangle\ for\ every\ l\in K\\
&(vii)\ \alpha\mathfrak{J}(\omega_1) = \langle 1-(1-\mu^l_{\mathfrak{J}(\omega_1)})^\alpha, (\nu^l_{\mathfrak{J}(\omega_i)})^\alpha\rangle,\\
&\qquad \langle 1-(1-\tau^l_{\mathfrak{J}(\omega_1)})^\alpha, (\eta^l_{\mathfrak{J}(\omega_i)})^\alpha\rangle\ for\ every\ l\in K\\
&(viii)\ \mathfrak{J}(\omega_1)^\alpha = \langle (\mu^l_{\mathfrak{J}(\omega_i)})^\alpha, 1-(1-\nu^l_{\mathfrak{J}(\omega_i)})^\alpha\rangle,\\
&\qquad \langle (\tau^l_{\mathfrak{J}(\omega_i)})^\alpha, 1-(1-\eta^l_{\mathfrak{J}(\omega_i)})^\alpha\rangle\ for\ every\ l\in K
\end{aligned}
$$

Example 1. *Let* $\mathfrak{J}(\omega_1) = \langle(0.7,0.6),(0.3,0.5)\rangle, \langle(0.8,0.7),(0.1,0.2)\rangle$ *and* $\mathfrak{J}(\omega_2) = \langle(0.8,0.5),(0.1,0.5)\rangle, \langle(0.8,0.9),(0.1,0.1)\rangle$ *Then*

(1) $\mathfrak{J}(\omega_1)\cup\mathfrak{J}(\omega_2) = \langle(0.8,0.6),(0.1,0.5)\rangle, \langle(0.8,0.9),(0.1,0.1)\rangle$

(2) $\mathfrak{J}(\omega_1)\cap\mathfrak{J}(\omega_2) = \langle(0.7,0.5),(0.3,0.5)\rangle, \langle(0.8,0.7),(0.1,0.2)\rangle$

(3) $\mathfrak{J}(\omega_1)\oplus\mathfrak{J}(\omega_2) = \langle(0.94,0.8),(0.03,0.25)\rangle, \langle(0.96,0.97),(0.01,0.08)\rangle$

(4) $\mathfrak{J}(\omega_1)\otimes\mathfrak{J}(\omega_2) = \langle(0.56,0.30),(0.37,0.75)\rangle, \langle(0.64,0.63),(0.19,0.28)\rangle$

(5) $(0.2)\mathfrak{J}(\omega_1) = \langle(0.21,0.17),(0.79,0.87)\rangle, \langle(0.30,0.21),(0.63,0.72)\rangle$

(6) $\mathfrak{J}(\omega_1)^{0.1} = \langle(0.96,0.95),(0.09,0.07)\rangle, \langle(0.98,0.96),(0.01,0.02)\rangle$

Proposition 1. *Let* $\mathfrak{J}(\omega_1)$, $\mathfrak{J}(\omega_2)$ *be two LDMFSNs. Then the following operations holds.*

(1) $\mathfrak{J}(\omega_1)\oplus\mathfrak{J}(\omega_2) = \mathfrak{J}(\omega_2)\oplus\mathfrak{J}(\omega_1)$
(2) $\mathfrak{J}(\omega_1)\otimes\mathfrak{J}(\omega_2) = \mathfrak{J}(\omega_2)\oplus\mathfrak{J}(\omega_1)$
(3) $\mathfrak{J}(\omega_1)\oplus(\mathfrak{J}(\omega_2)\oplus\mathfrak{J}(\omega_3)) = (\mathfrak{J}(\omega_1)\oplus\mathfrak{J}(\omega_2))\oplus\mathfrak{J}(\omega_3)$
(4) $\mathfrak{J}(\omega_1)\otimes(\mathfrak{J}(\omega_2)\otimes\mathfrak{J}(\omega_3)) = (\mathfrak{J}(\omega_1)\otimes\mathfrak{J}(\omega_1))\otimes\mathfrak{J}(\omega_3)$
(5) $\alpha(\mathfrak{J}(\omega_1)\oplus\mathfrak{J}(\omega_2)) = (\alpha\mathfrak{J}(\omega_1)\oplus\alpha\mathfrak{J}(\omega_2))$
(6) $(\mathfrak{J}(\omega_1)\otimes\mathfrak{J}(\omega_2))^\alpha = \mathfrak{J}(\omega_1)^\alpha\oplus\mathfrak{J}(\omega_2)^\alpha$
(7) $\alpha\mathfrak{J}(\omega_1)\oplus\beta\mathfrak{J}(\omega_1) = (\alpha+\beta)\mathfrak{J}(\omega_1)$

Proof. The proof follows from above definition. □

Definition 14. Comparison Matrix: *The Comparison Matrix is a matrix having the columns as a elements in universe and rows as a set of parameters. The elements* *in the matrix(α_{ij}) corresponding the element x_i and the parameter ω_j is defined by the number*

$$\alpha_{ij} = \frac{1}{2}[(\Gamma_{ij} - \sigma_{ij}) + (\gamma_{ij} - \phi_{ij})]$$

where,

$\Gamma_{ij} = \sum_{l=1}^{k}\{how\ many\ times\ \mu^{l}_{\mathcal{K}(\omega_j)}(x_i)\ exceeds\ \mu^{l}_{\mathcal{K}(\omega_j)}(x_j)\}$

$\sigma_{ij} = \sum_{l=1}^{k}\{how\ many\ times\ \nu^{l}_{\mathcal{K}(\omega_j)}(x_i)\ exceeds\ \nu^{l}_{\mathcal{K}(\omega_j)}(x_j)\}$

$\gamma_{ij} = \sum_{l=1}^{k}\{how\ many\ times\ \tau^{l}_{\mathcal{K}(\omega_j)}(x_i)\ exceeds\ \tau^{l}_{\mathcal{K}(\omega_j)}(x_j)\}$

$\phi_{ij} = \sum_{l=1}^{k}\{how\ many\ times\ \eta^{l}_{\mathcal{K}(\omega_j)}(x_i)\ exceeds\ \eta^{l}_{\mathcal{K}(\omega_j)}(x_j)\}$

Definition 15. Total Score of an Object:

The Score of an attribute x_i is given by

$$S_i = \sum_j \alpha_{ij}$$

where α_{ij} is computed from the comparision matrix.

Definition 16. Score function for LDMFSN:

Let $\mathfrak{I}(\omega_j) = (\langle\mu^{K}_{\mathfrak{I}_{\omega_j}}, \nu^{K}_{\mathfrak{I}_{\omega_j}}\rangle, \langle\tau^{K}_{\mathfrak{I}_{\omega_j}}, \eta^{K}_{\mathfrak{I}_{\omega_j}}\rangle)$ be a LDMFSN over $\beth$. Then the score function of LDMFSN $\mathfrak{I}_{\omega_j}$ is characterized as

$$\Omega(\mathfrak{I}_{\omega_j}) = \sum_{l=1}^{k} \frac{1}{2}[(\mu^{l}_{\mathfrak{I}_{\omega_j}} - \nu^{l}_{\mathfrak{I}_{\omega_j}}) + (\tau^{l}_{\mathfrak{I}_{\omega_j}} - \eta^{l}_{\mathfrak{I}_{\omega_j}})]$$

where $l \in K = \{1, 2, ..., k\}$.

4. Overview of Proposed Approach

The proposed approach is a fuzzy-based bid evaluation of tenders in order to evaluate tenderers and obtain the optimal tenderer promptly under some instances. The attributes of the tenderers are expressed as LDMFSNs. After analyzing the literature, it was found that there are many criteria affecting the pre-evaluation for selecting an appropriate tenderer. Those factors may vary in their importance and strength of the effect in selecting process. Some may have a high effect and some may have a low effect depending on the project and place. With the help of some expertise, five important factors are identified which have high effects on our selection process. The factors are listed below in the Figure 1.

4.1. **Case study.**

A government tendering sector wants to select the most appropriate tender for the building project. After pre-evaluation, four companies $\{x_1, x_2, x_3, x_4\}$ are remained as alternatives for further evaluation. Based on above discussion, five criteria $\{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ were identified as the main factors affecting tender selection.

$$\omega_1 = Professional\ Activity$$
$$\omega_2 = Resource\ Availability$$
$$\omega_3 = Organizational\ Availability$$
$$\omega_4 = Quality$$
$$\omega_5 = Management\ capacity$$

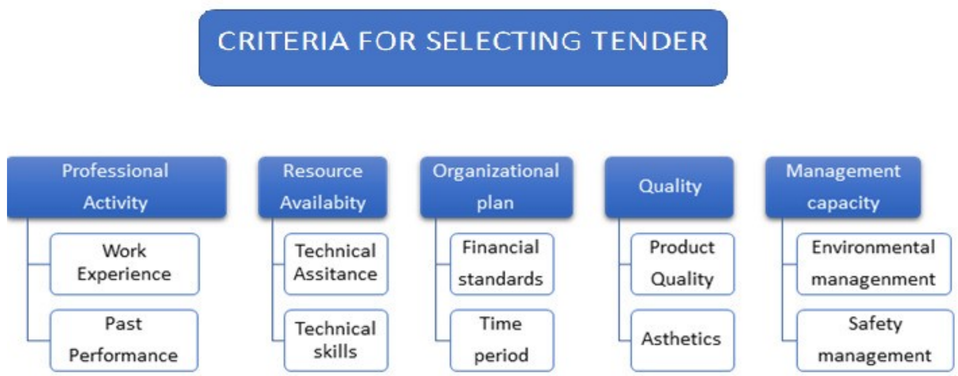

FIGURE 1. Selection criteria

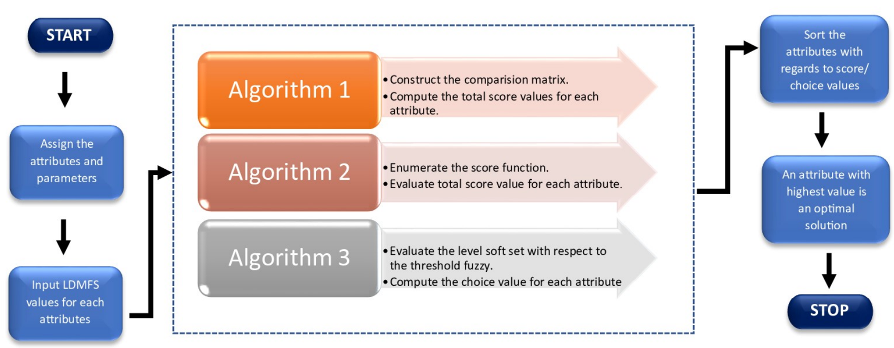

FIGURE 2. Flow chart diagram of algorithms 1, 2 and 3

We proposed three new algorithms for LDMFSS for selecting best tender. The graphical view of the algorithms are given in the Figure 2.

Algorithm 1: Algorithm based on comparison table

This novel and hybrid approach contains following steps:
Input:
(i) Define linear Diophantine multi-fuzzy soft values for each attributes $\{x_1, x_2, ..., x_n\}$ with respect to the parameters $\{\omega_1, \omega_2, ..., \omega_k\}$.
(ii) Input the above defined values $(\mathfrak{J}, \omega)$ in the table.
Calculations:
(iii) Construct the comparison matrix for $(\mathfrak{J}, \omega)$ with the knowledge of the given definition.
(iv)Evaluate the total score value of an object as defined in the previous section.
(v) Find the maximum score and rank the attributes.
Final decision:
(vi) An attribute with a high score is the required optimal solution.

Algorithm:2 Algorithm based on Score function:

Input:
(i) Define linear Diophantine multi-fuzzy soft values for each attributes $\{x_1, x_2, ..., x_n\}$ with respect to the parameters $\{\omega_1, \omega_2, ..., \omega_k\}$.
(ii)Input LDMFSN for each attributes $\{x_1, x_2, ..., x_n\}$.
Calculations:
(iii)Evaluate the score function value Ω_{ij} for each x_i by using the definition.
(iv)Enumerate the total score value of each attribute.
(v)Rank the attributes based on score values and find the maximum.
Final decision:
(vi) An attribute with a high score is the required optimal solution.

Algorithm:3 Algorithm-based threshold fuzzy:

Input:
(i) Define linear Diophantine multi-fuzzy soft values for each attributes $\{x_1, x_2, ..., x_n\}$ with respect to the parameters $\{\omega_1, \omega_2, ..., \omega_k\}$.
(ii) LDMFSS$(\mathfrak{J}, \omega)$ and parameter weights $\phi(\omega_i)$ are taken as input.
Calculations:
(iii)Induced fuzzy soft set $\Delta_{\mathfrak{J}}$ [7, 19] is computed.
(iv) A threshold fuzzy set $\lambda : A \to [0, 1]$ which is the mid-level decision rule is chosen for decision making.
(v) Evaluate the level soft set L$(\Delta_{\mathfrak{J}}, \lambda)$ with respect to threshold fuzzy.

(vi) Enumerate the choice value c_i for x_i
(vii) Sort the attributes in regards to the choice value.
Final decision:
(viii) An attribute with a high score is the required optimal solution.

4.2. **Solution for algorithm 1:** According to the company and selection criteria given in Figure **??**, our expert team provides their preference for the attributes. It can be converted in the form of LDMFSN. Defined values for each attributes $\{x_1, x_2, x_3, x_4\}$ are given in the Table 1.

TABLE 1. Linear Diophantine multi-Fuzzy soft set

$(\mathfrak{J}, \omega)$	ω_1	ω_2	ω_3	ω_4	ω_5
x_1	⟨(0.71,0.68)	⟨(0.41,0.32)	⟨(0.52,0.73)	⟨(0.39,0.59)	⟨(0.56,0.67)
	(0.30,0.39)⟩	(0.69,0.54)⟩	(0.41,0.29)⟩	(0.62,0.45)⟩	(0.45,0.35)⟩
	⟨(0.80,0.92)	⟨(0.76,0.83)	⟨(0.96,0.92)	⟨(0.84,0.94)	⟨(0.79,0.80)
	(0.17,0.04)⟩	(0.18,0.12)⟩	(0.01,0.02)⟩	(0.15,0.03)⟩	
x_2	⟨(0.82,0.73)	⟨(0.59,0.72)	⟨(0.73,0.82)	⟨(0.49,0.52)	⟨(0.72,0.64)
	(0.24,0.31)⟩	(0.45,0.33)⟩	(0.30,0.24)⟩	(0.53,0.51)⟩	(0.29,0.37)⟩
	⟨(0.82,0.90)	⟨(0.76,0.80)	⟨(0.85,0.88)	⟨(0.91,0.94)	⟨(0.68,0.84)
	(0.14,0.08)⟩	(0.22,0.13)⟩	(0.12,0.09)⟩	(0.07,0.04)⟩	(0.30,0.12)⟩
x_3	⟨(0.32,0.42)	⟨(0.92,0.84)	⟨(0.76,0.54)	⟨(0.41,0.22)	⟨(0.57,0.68)
	(0.69,0.59)⟩	(0.11,0.20)⟩	(0.30,0.46)⟩	(0.52,0.73)⟩	(0.50,0.40)⟩
	⟨(0.89,0.72)	⟨(0.96,0.82)	⟨(0.82,0.87)	⟨(0.79,0.85)	⟨(0.90,0.85)
	(0.10,0.26)⟩	(0.03,0.14)⟩	(0.17,0.11)⟩	(0.20,0.17)⟩	(0.07,0.11)⟩
x_4	⟨(0.35,0.75)	⟨(0.60,0.75)	⟨(0.80,0.75)	⟨(0.32,0.67)	⟨(0.72,0.83)
	(0.60,0.27)⟩	(0.40,0.30)⟩	(0.35,0.30)⟩	(0.70,0.39)⟩	(0.29,0.19)⟩
	⟨(0.85,0.82)	⟨(0.80,0.72)	⟨(0.90,0.91)	⟨(0.91,0.87)	⟨(0.87,0.82)
	(0.14,0.15)⟩	(0.19,0.26)⟩	(0.09,0.08)⟩	(0.07,0.11)⟩	(0.11,0.15)⟩

The comparison matrix is computed in the Table 2. Then, the score values for each attributes are calculated. Ranking of the attributes are as follows: $x_2 > x_3 >$

TABLE 2. Comparision matrix

$(\mathfrak{J}, \omega)$	ω_1	ω_2	ω_3	ω_4	ω_5	Score values
x_1	$\frac{5}{2}$	$\frac{-3}{2}$	$\frac{3}{2}$	1	-3	$\frac{1}{2}$
x_2	4	1	3	$\frac{5}{2}$	-1	$\frac{19}{2}$
x_3	-1	5	0	$\frac{-7}{2}$	$\frac{3}{2}$	2
x_4	$\frac{-9}{2}$	$\frac{-9}{2}$	-5	$\frac{1}{2}$	$\frac{3}{2}$	-12

$x_1 > x_4$. Clearly, it shows that the company x_2 is most suitable for the selection.

The graphical representation of the outcomes shown in the Figure 3

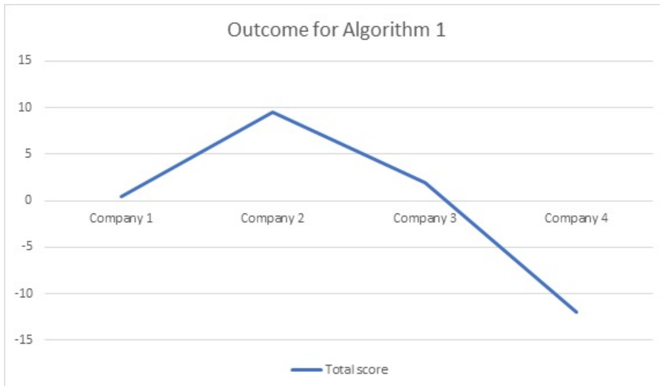

FIGURE 3. Graph of total score values using algorithm 1

4.3. **Solution for algorithm 2:** LDMFSN are taken as a input from the Table 1. The score function values of all attributes are enumerated in the Table 3.

The total score values of each attributes are S(x_1)= 4.252, S(x_2)= 5.12, S(x_3) = 4.165, S(x_4)= 2.93. Ranking of the attributes are as follows: $x_2 > x_1 > x_3 > x_4$. It is shown that the company x_2 is the best company for our selection. The final outcome of algorithm 2 is shown in the Figure 4.

TABLE 3. Score function value

Ω_{ij}	ω_1	ω_2	ω_3	ω_4	ω_5
x_1	1.105	0.36	1.2	0.762	0.825
x_2	1.25	0.85	1.265	0.855	0.9
x_3	0.355	1.53	0.975	0.345	0.96
x_4	0.175	0.21	0.545	0.75	1.25

4.4. **Solution of algorithm 3:** Taking the values from the Table 1 as a input. The induced soft set is given in the Table 4. The mid values are obtained as follows: $mid_\Delta = \{(\omega_1, 1.9905), (\omega_2, 1.9825), (\omega_3, 1.987), (\omega_4, 1.987), (\omega_5, 2.002)\}$. The mid-level soft set with choice values is given in the Table 5.

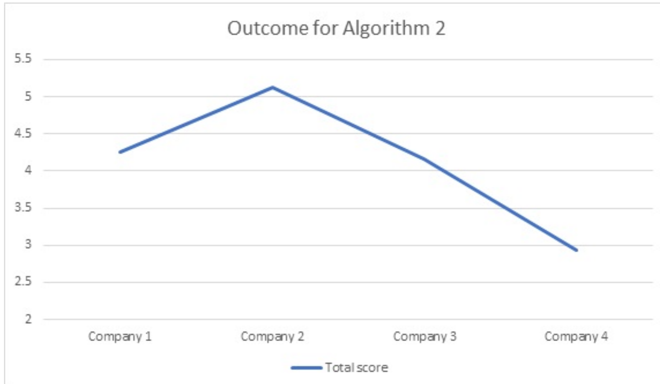

FIGURE 4. Graph of total score values using algorithm 2

TABLE 4. Induced fuzzy soft set

Δ	ω_1 $\phi(\omega_1)$=(0.6,0.4)	ω_2 $\phi(\omega_2)$=(0.5,0.5)	ω_3 $\phi(\omega_3)$=(0.3,0.7)	ω_4 $\phi(\omega_4)$=(0.6,0.4)	ω_5 $\phi(\omega_5)$=(0.6,0.4)
x_1	2	1.96	1.942	2.004	1.996
x_2	2.02	2.02	2.021	2.004	1.982
x_3	1.996	2.01	2.001	1.924	2.04
x_4	1.946	1.94	1.983	2.016	1.99

TABLE 5. mid-level soft set with choice values

Δ	ω_1	ω_2	ω_3	ω_4	ω_5	choice value
x_1	1	0	0	1	0	2
x_2	1	1	1	1	0	4
x_3	1	1	1	0	1	3
x_4	0	0	0	1	0	1

Ranking of the attributes are as follows: $x_2 > x_3 > x_1 > x_4$. Likewise, x_2 is the required optimal solution. The Figure 5 shows the final outcomes for algorithm 3.

4.5. **Discussion.** The previously mentioned three findings demonstrate that the bid x_2 is the one that is economically most advantageous. The innovation and robustness of our suggested methods are demonstrated by the distinctiveness of the results of the three algorithms. The picture 6 displays a comparison of our three suggested algorithm outcomes.

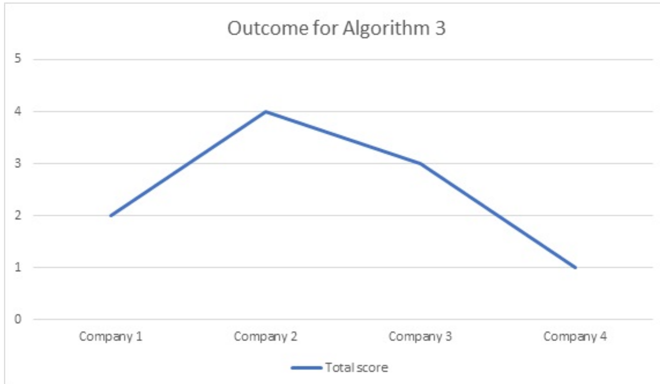

FIGURE 5. Graph of total score values using algorithm 3

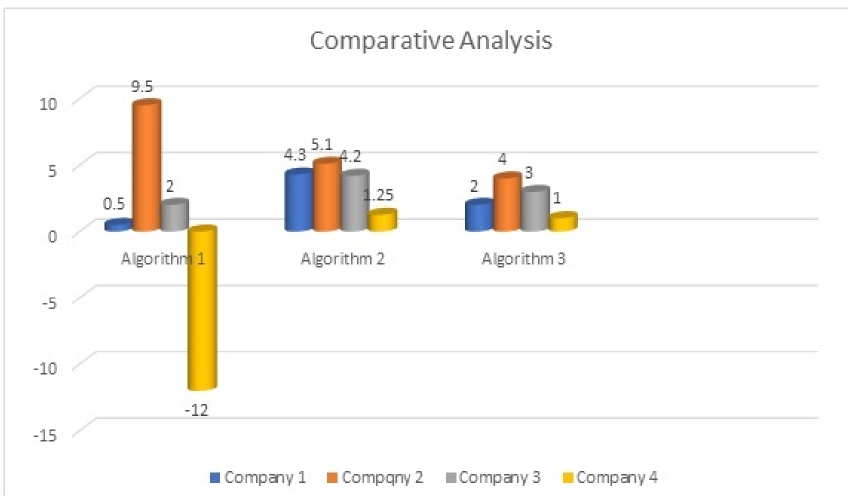

FIGURE 6. Comparision between Algorithm 1, 2 and 3

5. CONCLUSION

In this paper, the concept of LDMFSN is introduced. Some of the properties are discussed. The main motive of this paper is to find the most suitable tender for the government based on the same selection criteria. There are three different algorithms, like those based on comparison matrix, score function, and threshold fuzzy, that are initiated. Our proposed methods are used to sort the tenderers and select the optimal one among them. The illustrative example and the obtained results show the effectiveness of our proposed algorithms. In the future, the work will be extended to similarity measures and entropy measures between two LDMFSS.

Author Contribution Statements Jeevitha Kannan: Conceptualization and design of the study, writing, methodology. Vimala Jayakumar: Reviewing, editing, and validation.

Declaration of Competing Interests All authors declare no conflicts of interest in this paper.

Acknowledgements This paper is presented in the Conference ICAMM-2023. The article has been written with the joint financial support of RUSA-Phase 2.0 grant sanctioned vide letter No.F 24-51/2014-U, Policy (TN Multi-Gen), Dept. of Edn. Govt. of India, Dt. 09.10.2018, DST-PURSE 2nd Phase programme vide letter No. SR/PURSE Phase 2/38 (G) Dt. 21.02.2017 and DST (FIST - level I) 657876570 vide letter No.SR/FIST/MS-I/2018/17 Dt. 20.12.2018.

References

[1] Akcay, C., Manisali, E., Fuzzy decision support model for the selection of contractor in construction works, *Revista de la Construction*, 17 (2018), 258–266. DOI: 10.7764/RDLC.17.2.258

[2] Atanassov K.T, Intuitionistic fuzzy sets, *Fuzzy Set System*, 20 (1986), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3

[3] Ayub, S., Shabir, M., Riaz, M., Aslam, M., Chinram, R., Linear Diophantine fuzzy relations and their algebraic properties with decision making, *Symmetry*, 13(6) (2021), 945. https://doi.org/10.3390/sym13060945

[4] Al-Harbi, K.M.A.S., Application of the AHP in project management, *International Journal of Project Management*, 19(1) (2001), 19–27. https://doi.org/10.1016/S0263-7863(99)00038-1

[5] Begam, S.S., Vimala, J., Application of lattice ordered multi-fuzzy soft set in forecasting process, *Journal of Intelligent & Fuzzy Systems*, 36(3) (2019), 2323–2331. 10.3233/JIFS-169943

[6] Begam, S.S., Vimala, J., Preethi, D., A novel study on the algebraic applications of special class of lattice ordered multi-fuzzy soft sets, *Journal of Discrete Mathematical Sciences and Cryptography*, 22(5) (2019), 883–899. DOI: 10.1080/09720529.2019.1685239

[7] Das, S., Kar, S., Intuitionistic Multi Fuzzy Soft Set and Its Application in Decision Making, International Conference on Pattern Recognition and Machine Intelligence, Lecture Notes in Computer Science, Springer, Berlin, Heidelberg, 2013, 587–592. https://doi.org/10.1007/978-3-642-45062-4-82

[8] Deng, H., Multi-criteria analysis with fuzzy pairwise comparison, *International Journal of Approximate Reasoning*, 21(3) (1999), 215–231. https://doi.org/10.1016/S0888-613X(99)00025-0

[9] Hatush, Z., Skitmore, M., Contractor selection using multi-criteria utility theory: An additive model, *Building and Environment*, 33(23) (1998), 105–115. https://doi.org/10.1016/S0360-1323(97)00016-4

[10] Hussain, A., Ali, M.I., Mahmood, T., Munir, M., q-Rung orthopair fuzzy soft average aggregation operators and their application in multi-criteria decision-making, *International Journal of Intelligent Systems*, 35(4) (2020), 571–599. https://doi.org/10.1002/int.22217

[11] Hashmi, M.R., Tehrim, S.T., Riaz, M., Pamucar, D, Cirovic G, Spherical linear Diophantine fuzzy soft rough sets with multi-criteria decision making, *Axioms*, 10(185) (2021). https://doi.org/10.3390/axioms10030185

[12] Iampan, A., Garcia, G.S., Riaz, M., Athar Farid, H.M., Chinram, R., Linear Diophantine fuzzy Einstein aggregation operators for multi-criteria decision-making problems, *Journal of Mathematics*, 2021 (2021), 1–31. https://doi.org/10.1155/2021/5548033

[13] Jamili, A., Tender participation selection problem with fuzzy approach, *Advances in Industrial Engineering*, 54(4) (2020), 355–364. 10.22059/JIENG.2021.325197.1772

[14] Kamaci, H., Linear Diophantine fuzzy algebraic structures, *J Ambient Intell Human Comput*, 12 (2021), 10353–10373. https://doi.org/10.1007/s12652-020-02826-x

[15] Kannan, J., Garg, H., Jayakumar, V., Hananf, A., Aty, A., Haleem, A., Linear Diophantine multi-fuzzy aggregation operators and its application in digital transformation, *Journal of Intelligent & Fuzzy Systems*, 45(2) (2023), 3097–3107. DOI: 10.3233/JIFS-223844

[16] Lambropoulos, S., The use of time and cost utility for construction contract award under European Union Legislation, *Building and Environment*, 42(1) (2007), 452–463. https://doi.org/10.1016/j.buildenv.2005.08.002

[17] Lai, K.K., Liu, S.L., Wang, S.Y., A method used for evaluating bids in the Chinese construction industry, *International Journal of Project Management*, 22 (2004), 193-201. https://doi.org/10.1016/S0263-7863(03)00009-7

[18] Liu, B., Huo, T., Wang, X., Shen, G.Q., Chen, Y., The decision model of the intuitionistic fuzzy group bid evaluation for urban infrastructure projects considering social costs, *Canadian Journal of Civil Engineering*, 40(3) (2013), 263-273. 10.1139/cjce-2012-0283

[19] Maji, P.K., Biswas, R., Roy, A.R., Fuzzy soft sets, *J. Fuzzy Math.*, 9(3) (2001), 589–602.

[20] Padhi, S.S., Mohapatra, P.K.J., Contractor selection in government procurement auctions: A case study, *European Journal of Industrial Engineering*, 3(2) (2009), 170–186. DOI: 10.1504/EJIE.2009.023604

[21] Peng X.A., Yang, Y., Song, J., Pythagoren fuzzy soft set and its application, *Computer Engineering*, 41 (2015), 224-229. https://doi.org/10.1016/j.jksuci.2021.08.010

[22] Reeta, A.J., Vimala, J., A study on distributive and modular lattice ordered fuzzy soft group and its duality, *Appl. Math. J. Chin. Univ.*, 31 (2016), 491-502.

[23] Riaz, M., Farid, H.M.A., Aslam, M., Pamucar, D., Bozanic, D., Novel approach for third-party reverse logistic provider selection process under linear Diophantine fuzzy prioritized aggregation operators, *Symmetry*, 13 (2021), 1152. https://doi.org/10.3390/sym13071152

[24] Riaz, M., Hashmi, M.R., Masooma, R., Linear Diophantine fuzzy set and its applications towards multi-attribute decision-making problems, *Journal of Intelligent and Fuzzy Systems*, 37 (2019), 5417–5439. https://doi.org/10.3233/JIFS-190550

[25] Riaz, M., Hashmi, M.R., Kalsoom, H., Pamucar, D., Chu, M., Linear Diophantine fuzzy soft rough sets for the selection of sustainable material handling equipment, *Symmetry*, 12 (2020), 1215. https://doi.org/10.3390/sym12081215

[26] Rajareega, S., Vimala, J., Operations on complex intuitionistic fuzzy soft lattice ordered group and CIFS-COPRAS method for equipment selection process, *Journal of Intelligent & Fuzzy Systems*, 41(5) (2021), 5709–5718. DOI: 10.3233/JIFS-189890

[27] Sebastian, S., Multi-fuzzy sets, *International Mathematical Forum*, 5 (2010), 2471–2476.

[28] Vimala, J., Garg, H., Jeevitha, K., Prognostication of myocardial infarction using lattice ordered linear Diophantine multi-fuzzy soft set, *Intelligent Journal of Fuzzy System*, (2023). https://doi.org/10.1007/s40815-023-01574-2

[29] Wang, Y., Xi, C., Zhang, S., Zhang, W., Yu, D., Combined approach for government e-tendering using GA and TOPSIS with intuitionistic fuzzy information, *PloS One*, 10 (2015), e0130767. 10.1371/journal.pone.0130767

[30] Yang, Y., Tan, X., Meng, C., The multi-fuzzy soft set and its application in decision making, *Applied Mathematical Modelling*, 37 (2013), 4915–4923. https://doi.org/10.1016/j.apm.2012.10.015

[31] Yager, R.R., Generalized orthopair fuzzy sets, *IEEE Transactions on Fuzzy Systems*, 25(5) (2017), 1222–1230. https://doi.org/10.1109/TFUZZ.2016.2604005

[32] Yager, R.R, Pythagorean membership grades in multicriteria decision making, *IEEE Transactions on Fuzzy Systems*, 22(4) (2014), 958–965. https://doi.org/10.1109/TFUZZ.2013.2278989

[33] Zadeh, L.A., Fuzzy sets, *Information Controls*, 8 (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X