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# Investigation of the Relationship Between Chaos Data and €/$ Exchange Rate Index Data with RQA Method

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**ABSTRACT** A time series data contains a large amount of information in itself. Chaos data and volatility data which calculated by any time series are also derivative information included in the same time series. According to these assumptions, it is very important to question the ability of chaos and volatility information to affect each other, and which information affects and which information is affected. It is very important to determine the causes of volatility, which is an important result indicator for the finance literature, and especially with this study, it was tried to determine whether the chaos data is in a causal relationship with volatility. If some of the chaos data can be identified as the cause of volatility, the detected chaos data can be used in other research as a leading indicator of volatility. The data set used in the study is the daily €/$ exchange rate index between 01.01.2005 and 10.11.2022. In the study, time series of chaos data were created with Windowed RQA method and Hatemi-J asymmetric causality analysis research was carried out between these time series and €/$ exchange rate index volatility. The findings of the study conclude that the chaos data LnRR, LnEntr and LnLAM could be used as leading indicators of the €/$ exchange rate index volatility.

**KEYWORDS**
Recurrence quantification analysis
Chaos theory
€/$ exchange rate index
Volatility

## INTRODUCTION

Traditional methods in determining the fundamental variables in time series containing economic data are generally insufficient because they require time series to be stationary. Making time series stationary can cause data loss and make it difficult to examine long-term behavior (Engle and Granger 1987). RQA is a method that can be applied to stationary and nonlinear time series with insufficient number of data (Kamphorst et al. 1987).

The literature indicates that RQA has found a wide range of applications in other fields of science. In the field of finance, it has been used recently, and the number of related studies is quite limited. The aim of this study is to investigate the causality relationship between the chaos data obtained from the €/$ exchange rate index using the RQA method and volatility. Through this aim, acording to the literature review, RQA Method and Hatemi-J Asymmetric Causality Analysis are included in the study. In the 5th part of the study, how the chaos data are prepared is explained and in the 6th part, the findings obtained by applying the Hatemi-J asymmetric causality analysis to the chaos data are presented. Within the framework of the findings, it can be concluded that the chaos data LnRR, LnEntr and LnLAM can be used as leading indicators of the €/$ exchange rate index volatility. As a result of the obtained results; It has been proven that the RQA method can be used in financial decision processes.

Although the number of studies applying the RQA methodology to the data obtained from the financial time series is increasing day by day in the literature, there are not many studies. In this study, it is tried to examine the relationship between volatility and chaos data, which is an important result indicator in the finance literature with using RQA. Some of the studies on financial time series are summarized below:

Belaire-Franch (2004) examined the time series behavior of simulated data from a financial market model with Lux and Marchesi (1999)'s interacting intermediaries. All RQA descriptors outperformed all nonlinear tests in terms of the number of rejections of the null hypothesis of linearity for the realization of the artificial financial market. Thus, it has been shown in the study that this new framework provides a useful complementary tool for testing complexity in financial data.

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Karagianni and Kyrtsou (2011) investigated the dynamics between US inflation and the Dow Jones Index using a set of nonlinear methods, including RQA, and found evidence in favor of negative nonlinear links between the natural dynamics of inflation and stock returns. Sasikumar and Kamaiah (2014) examined India's two major stock market indices, BSE Sensex and CNX Nifty. The analysis by applying RQA to two time series covering 2002 and 2013 provides conclusive evidence that the Indian stock market is inherently chaotic.

Celik and Afsar (2010)'s study considering the daily return series of the ISE 100 index between 1986 and 2008, concluded that the index movements are 25% based on internal dynamics and are predictable. However, when the ISE 100 index is analyzed by periods, it is seen that the periods of 1991-1995 and 2006-2008 are the periods in which deterministic tendencies are most intense, and the period of 1996-2000 is the period with the weakest predictability.

Niu and Zhang (2017) used MWPE (Multiscale Weighted Permutation Entropy) and RQA methods in their study in which they examined the price fluctuations in exchange rates between 2006 and 2016 in 8 different economies. According to their empirical results; They found that while some economies, such as South Korea, Hong Kong, and China, showed lower and weaker activity in their foreign exchange markets, JPY/USD indicates a higher complexity and the Japanese foreign exchange market has a relatively higher activity. Niu and Zhang (2017) also suggested that the financial crisis increased market efficiency in foreign exchange markets.

Facchini et al. (2019) studied the changes in price volatility after the modification in 2002 using a combination of RP and RQA in their study which is about the UK electricity supply industry. According to the findings of the study, after the modification, short-term price volatility decreased significantly between 2001 and 2008, long-term price volatility was not affected by the change, a dynamic regime change occurred in the price, and shorter GC (Gate Closure) intervals made easier short-term predictions of electricity demand and on the supply side, it facilitates reliability. In the study, the relationship between the GC, which is closer to real time, and the decreasing price fluctuations in the wholesale market is revealed.

Wu et al. (2020) investigated the volatility spread between the crude oil, natural gas and coal futures market and the carbon emissions market using RP and RQA. According to the findings from the study, it was seen that the volatility spread between the coal market and the carbon emission market was stronger than the others. Based on this, industries need to switch from coal to natural gas or oil in order to avoid the risk from the carbon emission market, and it is concluded that this behavior will lead to a reduction in carbon emissions.

Baki (2022b) examined how the dynamic properties of Bitcoin changed over time using RQA. In the study, it was concluded that Bitcoin became more unpredictable, more random, more unstable, more irregular and less complex in 2021.

Baki (2022a) analyzed the USD/TRY and EUR/TRY exchange rates using nonlinear and chaotic time series analysis methods. In the study, RQA and CRQA were used to determine how the chaotic characteristics of the exchange rates changed over time, and it was concluded that the exchange rate market became more unpredictable, more irregular and more unstable after 2014.

In the study, it was tried to determine the chaotic structures occurring in the foreign exchange markets and the interaction of this chaotic structure with volatility. Since the 2008 Mortgage crisis and the Covid-19 outbreak are the most important events that have deeply affected the global financial markets, the data set is formed from the end-of-day values of the cross currency index of the units $ (United States Dollar) and € (European Union Euro) money between 01.01.2005 - 01.11.2022 in order to keep it within the scope.

The main problem of the study is to determine whether the chaos or volatility started earlier. If it can be determined that the chaos structure started before the volatility in the foreign exchange market, measures can be taken against volatility with the chaos data which will be explained in the following sections. For this reason, RQA studies will be carried out on the obtained data set and a volatility time series will be created with the historical volatility calculation method over the same time series. In addition, time series will be created on the chaos data to be calculated and the Hatemi-J asymmetric causality test queries will be performed on the derived time series. If a causality can be determined from chaos data to volatility, it will be assumed that investors in the foreign exchange market begin to exhibit different behaviors before volatility begins.

## METHODOLOGY

### RQA (Recurrence Quantification Analysis)

According to Schumpeter, capitalism is inherently a form of change, and the economy is not static and can never be static (Schumpeter 1976; Orlando and Zimatore 2018). However, time series analyzes used in the literature accept the precondition of being stationary for the time series, if the time series is not stationary, it is requested to make the time series stationary. The stagnation process of the time series, on the other hand, causes data loss, especially in financial time series, and makes the interpretation of the results difficult (Engle and Granger 1987). Kamphorst et al. (1987) developed a method of visualization by transforming one-dimensional time series into two-dimensional with a delay of $j$ in order to facilitate the research due to the stationarity problem in time series. In the method developed by Eckmann, the $x_i$ time series is formed by a matrix with the $x_j$ time series of the same time series with $j$ delay. The repetitions detected by running the cascading $\mathcal{H}$ (Heaviside) function on the obtained matrix are converted to 1, and 0 if there is no repetition. Visual graphics can also be obtained by coloring the matrix consisting of 1 and 0. The resulting square matrix image is called the refresh matrix ($R_{ij}$). The mathematical derivation method of the renewal matrix is as shown in Equation (1).

$$R_{ij} = \Theta\left(\varepsilon_i \| \overrightarrow{x_i} - \overrightarrow{x_j} \|\right), \quad \overrightarrow{x_i} \in R^m, \quad i,j = 1,2,\ldots,N \qquad (1)$$

$$\Theta(x) = \left\{1, \quad x_j \geq x_i 0, \quad x_j < x_i\right\}$$

In the formula, '$\varepsilon_i$' defines the threshold distance, '$\Theta$' defines Heaviside stepping function, '$\overrightarrow{x_i}$' defines time series vector and '$\overrightarrow{x_j}$' defines delay time series vector. The resulting refresh graph can be applied to all stationary or non-stationary time series. In the renewal graph, the dark areas are considered to indicate that the two vectors converge, in other words, a repetition occurs on the time series, and the open areas are considered to indicate that there is no convergence or repetition between the two vectors (Celik and Afsar 2010). Although time series were visualized by Eckmann, mathematical analysis and visual description methods were developed by Zbilut and Webber Jr (2006) and the method was named RQA. While RQA was a method that was used to draw conclusions with topological (changes on the visual) analyzes in the early days, it was translated into mathematical models by the work of Zbilut, Webber, Marwan and Kurths. After this stage, it has become a more understandable model with the interpretation of various variables obtained by RQA. Some of the numerical data obtained by RQA are explained below, respectively (Marwan and Kurths 2002; Zbilut and Webber Jr 2006).

***RR (Recurrence Rate)*** It measures the repetition density as a percentage on the refresh graph obtained based on the RQA. The higher this ratio, the greater the number of repeated information on the time series. The RR ratio is explained on Equation (2).

$$RR = \frac{1}{N^2}\sum_{i,j=1}^{N} R(i,j) \tag{2}$$

The 'N' shown in equation (2) describes the recurrence points on the refresh graph.

***DET (Determinism Ratio)*** It is the value that measures the predictability of the time series as a percentage ratio. It is understood that the larger the DET measurement value, the more predictable the system on the time series is. The calculation of the DET value is explained in Equation (3).

$$DET = \frac{\sum_{l=l_{\min}}^{N} lP(l)}{\sum_{i,j=1}^{N} R(i,j)} \tag{3}$$

The length of the diagonal lines $P(l)$ formed in the refresh graph shown in the formula $l$ shows the diagonal line length frequency.

***Entr (Entropy)*** The entropy value calculated with the RQA structure is the disorder value defined as Shannon Entropy. It shows that as the Entr value increases, the disorder in the system increases, that is, the time series turns into a chaotic structure, and as it decreases, it shows that the disorder decreases. The calculation made using the equation (4) is shown below:

$$Entr = -\sum_{l=l_{\min}}^{N} p(l)\; lnp(l) \tag{4}$$

The $p(l)$ shown in equation (4) represents the probability of diagonal lines.

***LAM (Laminarity)*** It represents laminar flow in time series. The higher the LAM value in the time series, the more stationary the system is. The frequencies of the vertical lines are used to calculate the slide value and the calculation is shown in Equation (5),

$$LAM = \frac{\sum_{v=v_{\min}}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)} \tag{5}$$

The $v$ shown in equation (5) represents the vertical line length on the refresh graph, and $P(v)$ the vertical line length frequency.

Although it is possible to calculate many more variables in RQA, analyzes will be performed with the four variables described in this study. The calculation formulas of the other RQA variables that were not included in the study were not included in this study.

As it can be understood from the calculations of RQA variables, static results are obtained with time series data. When the previous RQA studies were examined, it was examined whether the system had a chaotic structure mostly through the obtained static variables. In order to transform this static structure of RQA into a dynamic structure, Zbilut et al. (2002) created the windowed RQA method with their study. With this proposition, it is shown how to switch from a static structure to a dynamic data set in RQA calculations. Unlike the normal RQA structure, the windowed RQA structure is divided into smaller time series using the window step number (s) and window size (m) parameters, and the RQA data are calculated over these newly created small time series. Derivative time series of the current time series based on RQA data can be created with the obtained RQA data. Windowed RQA has found use in the analysis of time series in many different fields of science, and in finance, Bastos and Caiado (2011); Piskun and Piskun (2011); Sasikumar and Kamaiah (2014); Soloviev and Belinskiy (2019); Soloviev et al. (2020); Baki (2022a) have been pioneering researchers using windowed RQA analysis techniques. After the windowed RQA technique was put into practice, apart from making inferences from RQA static data, dynamic chaos data were derived and different econometric analyzes were made with time-dependent chaos data indices. With this new situation, the effect of the chaos data obtained from the time series data on the same time series can be examined. Considering the studies examined in the literature review; structural breaks on time series and time series derived from chaos data were examined and the values of RQA data during structural break periods were tried to be interpreted. However, no causality research was conducted between time series and RQA data during structural break periods. In our study, it was aimed to find the traces of chaotic structure on the €/$ exchange rate, and to reveal the causal relationship between the exchange rate and the chaos data time series that obtained with windowed RQA. Thus, this study will present a different perspective to chaos research on time series.

In the first stage of this study, derivative time series were created with RR, DET, Entr and LAM values obtained as a result of windowed RQA applied on the €/$ exchange rate time series. In the second stage of the research, causality analyzes will be made between the €/$ variable and the RR, DET, Entr and LAM variables, and a causality research will be carried out between the variables. In the causality research, Hatemi-J asymmetric causality analysis was preferred, which allows to understand the effects of negative and positive shocks of the variables. The main reason for choosing the Hatemi-J asymmetric causality analysis among the causality analyzes is; It is the desire to investigate how the negative or positive shocks experienced in the chaos data affect the volatility separately. In order to convey the subject better, the Hatemi-J asymmetric causality analysis is briefly explained in the next section.

## Hatemi-J Asymmetric Causality Analysis

The concept of causality is a set of models that try to explain the correlation between two variables that depend on the stationarity problem on time series. According to the idea first put forward by Granger in 1969; If $x$ and $y$ are two different time series, $y$ time series lagged values by $t$ can explain $x$ time series, then the hypothesis that $y$ time series is the cause of $x$ time series is accepted. However, it is not known whether the shocks in the $y$ time series are positive or negative. Therefore, it will not be possible to determine whether a positive situation in the $y$ time series or whether a negative situation explains the x time series. In order to eliminate this problem, Hatemi-j (2012) extracted the negative and positive shocks on the time series and derived two different time series, negative and positive, from one time series. By performing a Vector Autoregressive analysis on these derived time series, he was able to reach causality results due to positive and negative shocks (Hatemi-j 2012; Mert and Çağlar 2019). In order to better understand the subject, the mathematical propositions of the Hatemi-J asymmetric causality analysis will be briefly explained. Let $x_t$ and $y_t$ be time series which we think there is a causal relationship between them. Accordingly, the time series can be written as shown in equations (6) and (7),

$$x_t = x_{t-1} + \varepsilon_t = x_0 + \sum_{i=1}^{t} \varepsilon_{xi} \tag{6}$$

$$y_t = y_{t-1} + \varepsilon_t = y_0 + \sum_{i=1}^{t} \varepsilon_{yi} \tag{7}$$

In here, while $x_0$ and $y_0$ are the initial values of both time series, $\varepsilon_{xi}$ and $\varepsilon_{yi}$ are the error terms of time series. The resulting error terms are converted to negative and positive shock data as shown in Equation (8).

$$\varepsilon^{+}_{xi} = (\ \varepsilon_{xi}\ ,\ 0),\ \ \varepsilon^{-}_{xi} = (\varepsilon_{xi}\ ,\ 0) \tag{8}$$

$$\varepsilon^{+}_{yi} = (\varepsilon_{yi}\ ,\ 0),\ \ \varepsilon^{-}_{xi} = (\varepsilon_{yi}\ ,\ 0)$$

and from here we obtain Equation (9),

$$\varepsilon_{xi} = \varepsilon^{+}_{xi} + \varepsilon^{-}_{xi} and \varepsilon_{yi} = \varepsilon^{+}_{yi} + \varepsilon^{-}_{yi} \tag{9}$$

After the error terms of the time series are divided into positive and negative series, equations (6) and (7) can be modified and written as equations (10) and (11).

$$x_t = x_{t-1} + \varepsilon_t = x_0 + \sum_{i=1}^{t} \varepsilon^{+}_{xi} + \sum_{i=1}^{t} \varepsilon^{-}_{xi} \tag{10}$$

$$y_t = y_{t-1} + \varepsilon_t = x_0 + \sum_{i=1}^{t} \varepsilon^{+}_{yi} + \sum_{i=1}^{t} \varepsilon^{-}_{yi} \tag{11}$$

Positive and negative models are obtained from the structure modified as equations (10) and (11).

$$x^{+}_t = \sum_{i=1}^{t} \varepsilon^{+}_{xi}\,, \quad x^{-}_t = \sum_{i=1}^{t} \varepsilon^{-}_{xi}\ , \quad y^{+}_t = \sum_{i=1}^{t} \varepsilon^{+}_{yi}\,,\ y^{-}_t = \sum_{i=1}^{t} \varepsilon^{-}_{yi} \tag{12}$$

Let's build a model as seen in equation (13), which is completely different from the models created in equation (12), and let's assume that this model is valid.

$$z^{+}_t = x^{+}_t y^{+}_t \tag{13}$$

In here, the causality relationship between $x^{+}_t$ and $y^{+}_t$ variables will be determined by the $p$ delayed Var model.

$$z^{+}_t = v + A_1 z^{+}_{t-1} + \ldots + A_p z^{+}_{t-1} + \mu^{+}_t \tag{14}$$

In equation (14), $z^{+}_t$ denotes 2x1 variable vector, 2x1 denotes constant vector, $\mu^{+}_t$ denotes 2x1 vector of error terms and $A_p$ denotes 2x2 parameters matrix created for delay $p$. The results of the Var model are interpreted with the results of the Wald test statistics and the hypotheses are accepted or rejected.

## THE DATA SET PREPARATION

In accordance with the purpose of the research, the daily €/$ exchange rate index was obtained from the Yahoo/finance website between 02/01/2005 - 10/11/2022. The structure of the mentioned data set is shown in Figure 1.

In order to generate RR, DET, Entr and LAM data, which are chaos indicators, Coco et al. (2020) prepared by (CRQA) software was used. Although the CRQA software package was originally prepared for Cross RQA structures, instead of choosing different

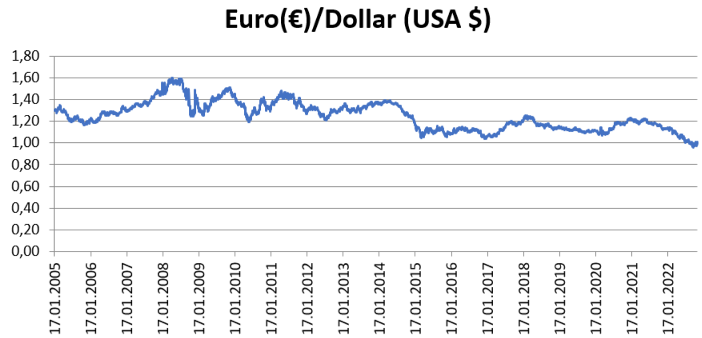


**Figure 1** €/$ Daily Exchange Rate, Source: Yahoo

variables, it turns into an RQA structure if both variables are the same. In addition, the ease of use and the reliability of the tested analysis results were effective in our preference for this software package Coco and Dale (2014). The mentioned CRQA software package runs on the R package program. In order to obtain windowed RQA results in the CRQA software package, the delay number (d), the embedding degree of the phase space (n) and the critical threshold value diameter (r), which are a requirement of the RQA structure, must be determined. In order to obtain these data, it will be necessary to run the "optimizeParam" module, which is also included in the same software package.

When the module that mentioned was run, it was determined that d=1, n=1 and r=0.01. After the necessary parameters were prepared, the window size (m) and window step number (s) values were determined for the windowed RQA. Since daily exchange rate data were used in the research, it was thought that it would be appropriate to produce chaos data as daily data, and s=1, that is, the number of window steps was determined as 1 day. For the appropriate window size, the 'windowdrp' module was run and it was determined that the smallest suitable window size would be m=10. Chaos data of the €/$ exchange rate time series were obtained with all the parameters obtained, and the graphics of the chaos data are presented below.

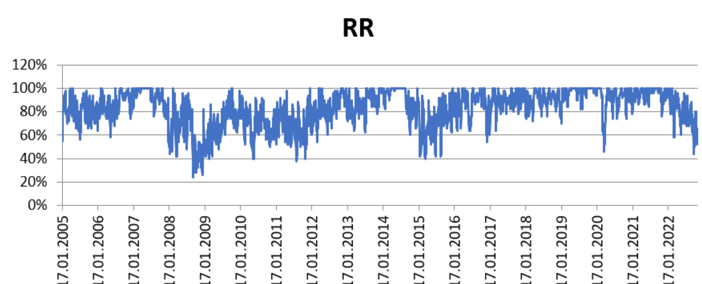


**Figure 2** Daily RR Time Series

The RR value shows the number of repetitions of an information on the time series, and as this value decreases, it shows that the repetition of the information decreases. When Figure 2 is examined, the rate of recurrence before the 2008 crisis approached 100% and decreased to 20% with the onset of the crisis.

DET data determines the deterministic structure of the time series, that is, its predictability. While DET data, like RR data, had a high value before 2008, it was at low levels until 2014. Afterwards, it entered an upward trend again until 2020, the predictability of the time series decreased with the pandemic crisis.

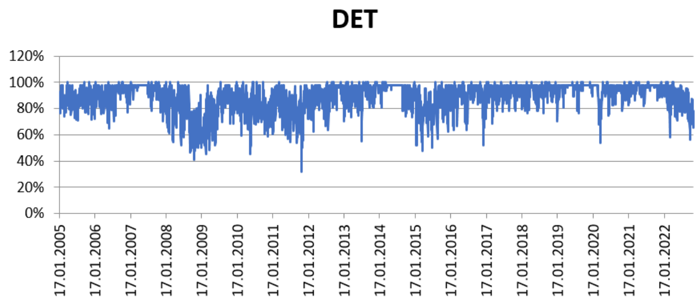


**Figure 3** DET Time Series

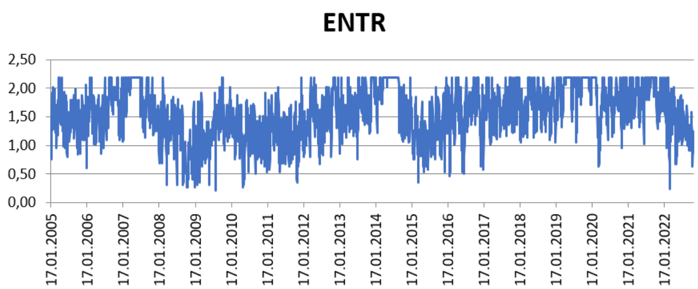


**Figure 4** Entr Time Series

Entr is a measure of the disorder on the time series. If the information in the time series is in the same direction, the Entr variable will approach zero, and if the information differs, the value of the Entr variable will increase. According to the random walk hypothesis in financial markets, if Entr data is interpreted, the efficient market should be in a high entropy state. Otherwise, since all investors in the financial market will have the same opinion, the predictability of prices will increase.

When Figure 4 is examined, the entropy of the €/$ exchange rate reached the highest levels before the 2008 crisis, and after the crisis, the complexity in the market started to decrease.

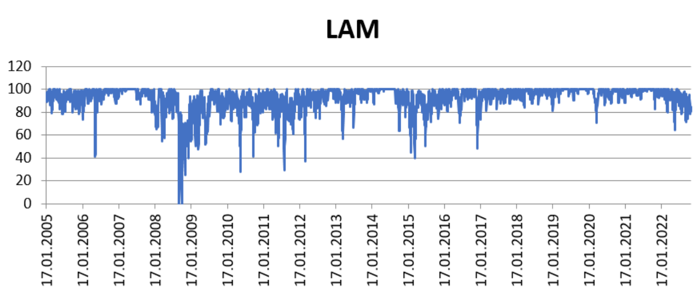


**Figure 5** LAM Time Series

LAM data is an indicator of the stationarity of the time series. In the RQA literature, it is stated that LAM data is a suitable data for detecting the exit from chaos (Orlando and Zimatore 2018). When Figure 5 is examined, the LAM value decreased to its lowest level at the end of 2009 and showed a rapid increase after that. For this reason, it is necessary to carefully examine the proposition that sudden and large increases in the LAM value can be the points of exit from the chaos of the time series.

After creating the chaos data to be used in the research, historical volatility time series were created from the €/$ exchange rate index data. The reason for using historical volatility instead of logarithmic return is to investigate whether chaos data has an effect on volatility, which is a measure of risk in the market. For this purpose, it was decided to use the historical volatility time series with the thought that more accurate results would be obtained, and Equation (15) was used in the calculation of this series.

$$\sigma = |P_n - P_{n-1}| \quad (15)$$

In here, by calculating the absolute value of the difference between the price of P and the price of the previous period, the historical volatility time series is created and shown in Figure 6.

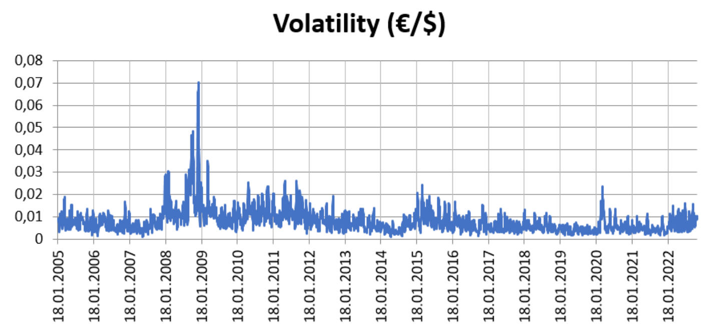


**Figure 6** €/$ Exchange Rate Volatility Time Series

## EMPIRICAL FINDINGS

The Hatemi-J asymmetric causality analysis, which was previously explained, will be performed in order to test whether the chaos data has an effect on the €/$ exchange rate volatility with the created data set. However, since the fundamental theorem of this analysis is the Var analysis, the stationarity problem of the time series should be questioned. Unit root tests are used to detect stationarity problems in time series. In this study, PP and ADF unit root tests were performed on time series and their results are shown in Table-1.

*Notes: (*)Significant at the 10%; (**)Significant at the 5%; (***) Significant at the 1%. and (no) Not Significant *MacKinnon (1996) one-sided p-values.*

As can be seen from the test results in Table-1, no unit root problem was detected in any of the variables to be used in the research, and accordingly, all of the variables were considered stationary at the level. In the next stage, the Var model was established and the most appropriate number of lags between the variables was determined and presented in Table-2.

The most appropriate lag number of the model created according to Table-2 was determined as 2 periods, and the results of the analysis were obtained with this lag number in the Hatemi-J asymmetric causality analysis and the results are presented in Table-3.

When the causality analysis results were examined, it was determined that both the negative shocks and positive shocks of the chaos data RR, DET, Entr and LAM data were not the cause of the negative and positive shocks of the €/$ exchange rate index volatility. However, on the contrary, the negative and positive shocks of the €/$ exchange rate index volatility are the cause of the negative and positive shocks of the RR, DET, Entr and LAM data. Depending on these determinations, while the volatility variable affects the chaos data, the chaos data does not affect the volatility.

**Table 1** Unit Root Test Results

| | | | | UNIT ROOT TEST TABLE (PP) | | |
|---|---|---|---|---|---|---|
| | **At Level** | | | | | |
| | | DET | ENTR | LAM | RR | Volatilite |
| With Constant | t-Statistic | -32.6300 | -16.7856 | -16.5460 | -10.3829 | -9.8304 |
| | **Prob.** | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | | *** | *** | *** | *** | *** |
| With Constant & Trend | t-Statistic | -33.6054 | -17.3222 | -17.2308 | -11.1659 | -10.2556 |
| | **Prob.** | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | | *** | *** | *** | *** | *** |
| | | | | UNIT ROOT TEST TABLE (ADF) | | |
| | **At Level** | | | | | |
| | | DET | ENTR | LAM | RR | STDAVDO |
| With Constant | t-Statistic | -7.5979 | -9.6085 | -7.3866 | -5.0824 | -14.5642 |
| | **Prob.** | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | | *** | *** | *** | *** | *** |
| With Constant & Trend | t-Statistic | -7.9746 | -10.0362 | -7.7480 | -5.2695 | -15.3171 |
| | **Prob.** | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0000 |
| | | *** | *** | *** | *** | *** |

**Table 2** The Most Appropriate Number of Lags

| Lag | LogL | LR | FPE | AIC | SC | HQ |
|---|---|---|---|---|---|---|
| 0 | 1885.041 | NA | 0.000189 | -2.896987 | -2.889032 | -2.894002 |
| 1 | 2106.520 | 441.9349 | 0.000135 | -3.231569 | -3.207707 | -3.222616 |
| 2 | 2170.273 | 127.0159* | 0.000124* | -3.323497* | -3.283727* | -3.308575* |

**Table 3** Hatemi-J Causality Analysis Results Between Chaos Data and Volatility

| | Wald İst. | %1 | %5 | %10 |
|---|---|---|---|---|
| entr +=>vol+ | 3.923 | 13.234 | 7.748 | 4.509 |
| entr +=>vol- | 2.529 | 11.237 | 8.170 | 4.642 |

**Table 3** Hatemi-J Causality Analysis Results Between Chaos Data and Volatility (continued)

| | | | | |
|---|---|---|---|---|
| entr -=>vol+ | 3.735 | 10.172 | 7.891 | 5.428 |
| entr -=>vol- | 2.324 | 11.765 | 7.072 | 5.143 |
| Vol+ => entr+ | 25.132(*) | 13.321 | 7.943 | 5.802 |
| Vol+ => entr- | 35.223(*) | 11.632 | 6.832 | 4.732 |
| vol- => entr+ | 31.219(*) | 12.167 | 6.982 | 4.290 |
| vol- => entr- | 45.208(*) | 11.291 | 6.219 | 4.231 |
| rr +=>vol+ | 3.764 | 10.874 | 6.326 | 4.248 |
| rr +=>vol- | 5.043 | 12.215 | 7.884 | 6.981 |
| rr -=>vol+ | 1.028 | 12.183 | 6.994 | 5.875 |
| rr -=>vol- | 1.432 | 11.764 | 7.162 | 6.231 |
| vol+ => rr+ | 35.278(*) | 10.342 | 6.442 | 4.743 |
| vol+ => rr- | 42.237(*) | 12.453 | 7.349 | 5.658 |
| vol- => rr+ | 46.286(*) | 12.893 | 6.238 | 4.673 |
| vol- => rr- | 39.587(*) | 12.752 | 7.872 | 5.125 |
| det +=>vol+ | 3.543 | 11.592 | 7.816 | 5.827 |
| det +=>vol- | 2.445 | 12.986 | 8.438 | 6.521 |
| det -=>vol+ | 3.091 | 11.446 | 7.171 | 6.392 |
| det -=>vol- | 2.854 | 12.659 | 8.215 | 6.383 |
| vol+ => det+ | 23.842(*) | 11.832 | 6.212 | 5.408 |
| vol+ => det- | 36.128(*) | 11.109 | 6.649 | 5.787 |
| vol- => det+ | 43.485(*) | 12.954 | 8.221 | 6.734 |
| vol- => det- | 44.228(*) | 11.184 | 7.843 | 5.543 |
| lam +=>vol+ | 1.556 | 11.265 | 6.978 | 4.874 |
| lam+=>vol- | 1.754 | 13.129 | 8.508 | 6.548 |
| lam-=>vol+ | 2.386 | 12.328 | 8.761 | 6.109 |
| lam-=>vol- | 1.326 | 13.673 | 8.265 | 6.439 |
| vol+ => lam+ | 22.452(*) | 11.912 | 6.867 | 4.381 |
| vol+ => lam- | 35.945(*) | 12.769 | 6.743 | 4.214 |
| vol- => lam+ | 39.281(*) | 14.679 | 9.389 | 7.222 |
| vol- => lam- | 28.328(*) | 12.769 | 8.325 | 6.927 |

According to this result, it is not possible to make early decisions about volatility using chaos data. However, chaos data was used directly in the research and it was not allowed to clearly reveal negative and positive shocks. In order to eliminate this problem, new time series were created by taking the logarithmic differences of the chaos data. The new time series created are presented in Figures 7,8,9,10.

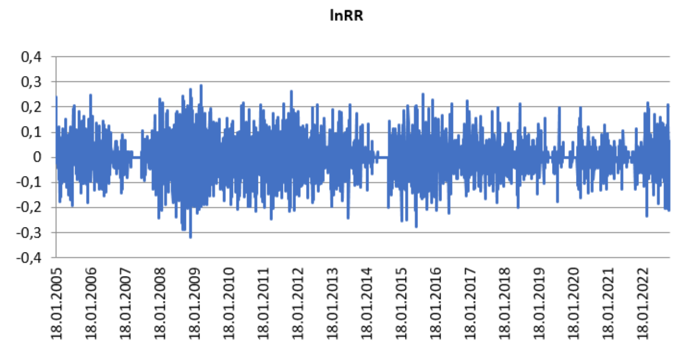


**Figure 7** $\ln RR = \ln(RR_n / RR_{n-1})$

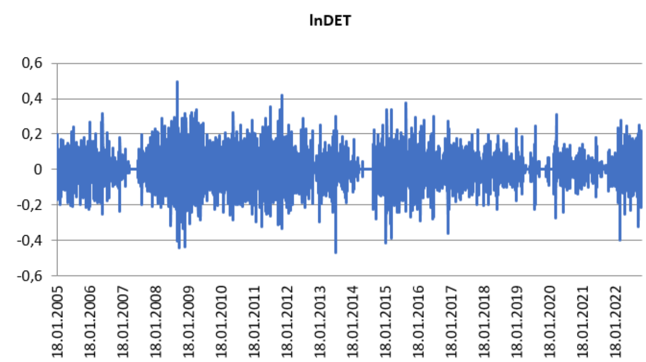


**Figure 8** $\ln DET = \ln(DET_n / DET_{n-1})$

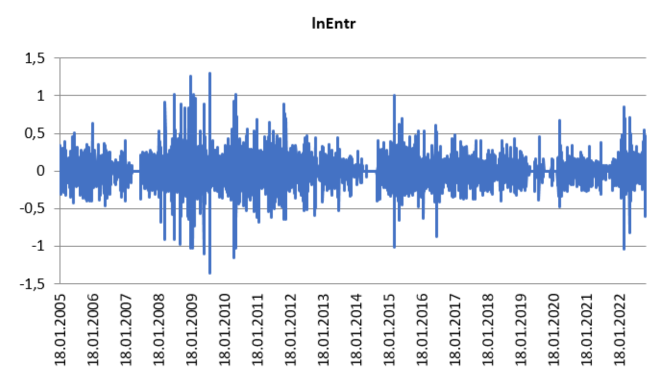


**Figure 9** $\ln Entr = \ln(Entr_n / Entr_{n-1})$

With the new variables created, the Hatemi-J asymmetric causality analysis procedures were repeated and the new analysis results are presented in tables below. While the negative and positive differences in the logarithmic difference of the RR data, which gives the repetition percentage of the time series, were determined as the cause of the positive shocks of volatility, causality towards

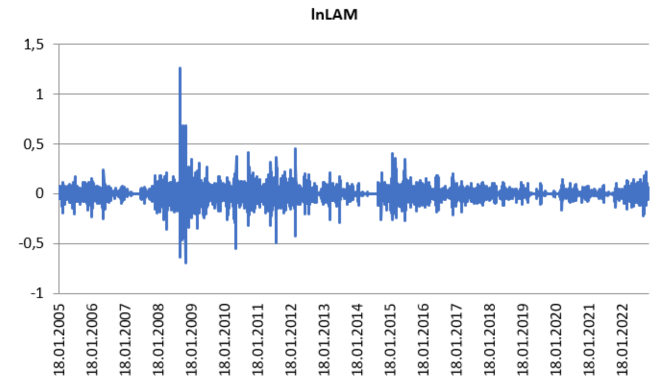


**Figure 10** $\ln LAM = \ln(LAM_n / LAM_{n-1})$

the negative shocks of volatility of the same variable could not be determined.

The negative and positive shocks of the €/$ exchange rate volatility are the cause of both the negative and positive shocks of the LnRR variable. Based on these determinations, it is accepted that the LnRR variable can be used as a leading indicator for positive shocks of €/$ exchange rate volatility,and additionally that all shocks in the €/$ exchange rate volatility are the cause of the LnRR variable makes it difficult to accept the LnRR variable as a leading indicator in any case.

Depending on these determinations, LnRR data can be used as a leading indicator that €/$ exchange rate volatility will increase when sudden and large shocks are detected, as in the crises experienced in 2008 and 2020. However, this determination does not reduce the importance of the LnRR variable, if it is followed as an index, it can be added to the literature as a very important leading indicator in terms of increasing the limited data diversity in risk management.

It has been understood that the logarithmic difference of LnDET, which is the predictive variable in chaos data, does not affect the negative or positive shocks of volatility. On the contrary, it is understood that both negative and positive shocks of volatility affect the negative and positive shocks of LnDET. Depending on these determinations, it is not possible to use the LnDET variable as a leading indicator. However, it can be used to understand and interpret the general situation in the time series.

When the asymmetric causality analysis results between the logarithmic difference value of the entropy data, which is the indicator of the irregularity in the time series, and the €/$ exchange rate volatility are examined, it is understood that the LnRR, that is, the repetition data, shows consistent results with the logarithmic difference. Here, the hypothesis that the negative and positive shocks of the LnEntr data is the cause of the negative shock of the €/$ exchange rate volatility is accepted.

The negative and positive shocks of the €/$ exchange rate volatility are the cause of both the negative and positive shocks of the LnEntr data. In this case, it is understood that the result obtained with LnRR data is the opposite for LnEntr, and this result should be considered quite consistent. Because while RR is a measure of regular repetitions, Entr is defined as a measure of irregularity and they give opposite results. With this study, it has been accepted that it is possible to use sudden large shocks in the logarithmic difference of entropy as a leading indicator that volatility will decrease.

**Table 4** Causality Results Between LnRR and Exchange Rate Volatility

| | Wald İst. | %1 | %5 | %10 |
|---|---|---|---|---|
| Lnrr +=>vol+ | 10.199 (**) | 11.863 | 7.538 | 5.728 |
| Lnrr +=>vol- | 4.390 | 12.005 | 7.994 | 6.241 |
| Lnrr -=>vol+ | 918.028(*) | 9.243 | 5.928 | 4.605 |
| Lnrr -=>vol- | 1.156 | 11.884 | 7.817 | 6.191 |
| vol+ => Lnrr+ | 79.618(*) | 10.587 | 6.213 | 4.698 |
| vol+ => Lnrr- | 44.174(*) | 12.149 | 7.733 | 5.909 |
| vol- => Lnrr+ | 86.936(*) | 10.593 | 6.101 | 4.535 |
| vol- => Lnrr- | 49.521(*) | 10.902 | 6.638 | 5.008 |

**Table 5** Causality Results Between LnDET and Exchange Rate Volatility

| | Wald İst. | %1 | %5 | %10 |
|---|---|---|---|---|
| Lndet +=>vol+ | 2.815 | 11.828 | 7.535 | 5.836 |
| Lndet +=>vol- | 0.020 | 12.273 | 8.434 | 6.626 |
| Lndet -=>vol+ | 3.464 | 11.961 | 7.814 | 6.224 |
| Lndet -=>vol- | 0.324 | 12.726 | 8.791 | 6.863 |
| vol+ => Lndet+ | 47.638(*) | 11.868 | 6.892 | 5.114 |
| vol+ => Lndet- | 76.446(*) | 12.090 | 7.456 | 5.577 |
| vol- => Lndet+ | 26.934(*) | 12.779 | 8.612 | 6.800 |
| vol- => Lndet- | 37.082(*) | 11.311 | 7.550 | 5.895 |

**Table 6** Causality Results Between LnEntr and Exchange Rate Volatility

| | Wald İst. | %1 | %5 | %10 |
|---|---|---|---|---|
| Lnentr +=>vol+ | 2.193 | 14.229 | 8.078 | 5.959 |
| Lnentr +=>vol- | 217.272(*) | 12.897 | 7.890 | 5.802 |
| Lnentr -=>vol+ | 3.022 | 12.611 | 8.011 | 6.227 |
| Lnentr -=>vol- | 216.543(*) | 11.855 | 7.562 | 5.653 |
| Vol+ => Lnen+ | 37.221(*) | 14.161 | 7.053 | 5.106 |
| Vol+ => Lnen- | 58.123(*) | 11.390 | 6.620 | 4.855 |
| vol- => Lnen+ | 41.256(*) | 12.000 | 7.539 | 5.670 |
| vol- => Lnen- | 41.256(*) | 12.000 | 7.539 | 5.670 |

**Table 7** Causality Results Between LnLAM and Exchange Rate Volatility

| | Wald İst. | %1 | %5 | %10 |
|---|---|---|---|---|
| Lnlam +=>vol+ | 198.185(*) | 9.802 | 5.921 | 4.478 |
| Lnlam+=>vol- | 3.277 | 13.418 | 8.090 | 6.259 |
| Lnlam-=>vol+ | 2.825 | 15.565 | 8.400 | 6.078 |
| Lnlam-=>vol- | 1.181 | 13.579 | 8.250 | 6.198 |
| vol+ => Lnlam+ | 36.330(*) | 13.907 | 6.780 | 4.901 |
| vol+ => Lnlam- | 56.945(*) | 12.240 | 6.667 | 4.884 |
| vol- => Lnlam+ | 39.281(*) | 14.679 | 9.389 | 7.222 |
| vol- => Lnlam- | 56.272(*) | 12.954 | 8.229 | 6.183 |

*Notes: (*)There is causality with 1% margin of error. (**)There is causality with 5% margin of error. (***)There is causality with 10% margin of error.*

According to the results of the causality inquiry that is made between the logarithmic difference values of the LAM data, which is an indicator of a stable structure in time series, and the €/$ exchange rate volatility; The positive shocks of the LnLAM data are determined to be the cause of only the positive shocks of the €/$ exchange rate volatility, while the negative shock of the LnLAM data is not the cause of the negative and positive shocks of the volatility. With this finding, it was concluded that positive shocks in LnLAM data can be used as an important indicator that volatility will increase.

## CONCLUSION

When the literature in the field of RQA is examined, it has been seen that the chaotic structures on the time series are interpreted by calculating static chaos data by dividing the time series into certain sub-time periods. By using the chaos data converted from static structure to dynamic structure with windowed RQA, it was possible to create time series and limited number of studies on this subject could be reached. From these studies;

Soloviev et al. (2020) tested whether the chaos data would be a leading indicator by detecting structural breaks on the daily data of the US, German and French stock markets. According to the results of the research, it has been suggested that DET, LAM and Entr data can be used as crisis leading indicators.

Piskun and Piskun (2011) produced windowed RQA dynamic chaos data during financial crisis periods in different countries and argued that LAM data could be the leading indicator of crises by detecting structural breaks on these data.

The path followed in the research conducted in the field of finance with windowed RQA is generally to determine the structural break times of the time series in order to determine the relationship between the generated chaos data and volatility. In our study, whether the chaos data will be a leading indicator for volatility was investigated by Hatemi-J asymmetric causality analysis. In this respect, this study reveals an innovation for the finance literature. Our study has proven that RR, Entr and LAM data can be leading indicators of volatility, consistent with other studies. However, it has also been proven in the study that using logarithmic differences instead of using these data directly will give better results.

As a result of econometric analysis, it was determined that the negative and positive shocks of the lnRR value were the cause of the positive shocks of the €/$ exchange rate volatility, and the negative and positive shocks of the lnEntr value were the cause of the negative shocks of the €/$ exchange rate volatility. In addition, it has been determined that the positive shock of the lnLAM value is the cause of the positive shock of the €/$ exchange rate volatility. It has also been determined that both negative and positive shocks of the €/$ exchange rate volatility are the cause of the negative and positive shocks of all chaos data. According to this result, it is concluded that the €/$ exchange rate volatility affects the chaos data under normal conditions, while the chaos data has the ability to affect the €/$ exchange rate volatility in extreme cases (Financial Crises). This interpretation of opinion does not eliminate the importance of chaos data in volatility analysis. It indices to be derived from chaos data (especially LnRR, LnEntr and LnLAM) have increased their importance in order to provide new opportunities for analysis for stakeholders working in the field of volatility detection and risk management.

### Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

### Availability of data and material

Not applicable.

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