Mathematical Modelling and Numerical Simulation with Applications, 2023, 3(1), 33–57

https://dergipark.org.tr/en/pub/mmnsa
ISSN Online: 2791-8564 / Open Access
https://doi.org/10.53391/mmnsa.1273982

RESEARCH PAPER

Malaria and cholera co–dynamic model analysis furnished with fractional-order differential equations

Livinus L. Iwa [1,‡,*], **Ugochukwu K. Nwajeri** [1,‡], **Anne O. Atede** [1,‡], **Augustine B. Panle** [1,‡] **and Kenneth U. Egeonu** [1,‡]

[1]Department of Mathematics, Federal University of Technology, Owerri, Nigeria

*Corresponding Author
‡iwa.livinus@gmail.com (Livinus L. Iwa); ugochukwu.nwajeri@futo.edu.ng (Ugochukwu K. Nwajeri); anoja4god@gmail.com (Anne O. Atede); panle.augustine@futo.edu.ng (Augustine B. Panle); kennethegeonu91@gmail.com (Kenneth U. Egeonu)

Abstract

This paper presents malaria and cholera co–dynamics under Caputo–Fabrizio derivative of order $\alpha \in (0,1)$ varied with some notable parameters in the fractional system. The fractional order system comprises ten compartments divided into human and vector classes. The human population is exposed to obnoxious diseases such as malaria and cholera which can lead to an untimely death if proper care is not taken. As a result, we present the qualitative analysis of the fractional order system where the existence and uniqueness of the solution using the well-known Banach and Schauder fixed point theorems. The numerical solution of the system is achieved through the famous iterative Atangana–Baleanu fractional order Adams–Bashforth scheme. The numerical algorithm obtained from the scheme is used for graphic simulation for different fractional orders $\alpha \in (0,1)$. The figures produced using various fractional orders show total convergence and stability as time increases. It is also evident that stability and convergence are achieved as the fractional orders tend to 1. The actual behavior of the fractional co–dynamical system of the diseases is established also in the numerical simulation.

Keywords: Malaria disease; cholera disease; fractional derivative; stability; co-infection

AMS 2020 Classification: 34C60; 92C42; 92D30; 92D25

1 Introduction

The disease malaria caused by harmful parasites and transmitted from infected female **anopheles** mosquitoes to humans through contagious bites is a serious life-threatening disease and one of the most common deadly diseases in the world. This disease has caused millions of life all over the world, especially in tropical and sub-Saharan Africa [1], where the parasites can survive freely in the host. The world malaria report of 2021, shows that in the year 2020, an estimated 627,000

➤ **Received:** 30.01.2023 ➤ **Revised:** 20.03.2023 ➤ **Accepted:** 24.03.2023 ➤ **Published:** 31.03.2023

deaths occurred out of estimated 241 million cases of malaria reported worldwide. Among these number of reported cases of malaria, there were 95 percent recorded cases in Africa with 94 percent deaths which include 80 percent of children under the age of five [2]. Most of the deaths recorded occurred during a latent period of 10–15 days with symptoms of severe fever, headaches, and loss of appetite for food or drinks. Suspected cases of malaria symptoms are confirmed through parasite–based diagnosis testing. The WHO recommends the use of insecticide-treated nets (ITNs), indoor residual spraying (IRS) and draining of stagnant waters to prevent transmission of malaria. On the other hand, the disease cholera is an acute diarrhoeal infection caused by ingestion of food and water contaminated with the bacterium Vibrio cholera [3, 4]. An estimated 1.3 to 4 million cases of cholera are reported globally each year and 21,000 to 143,000 deaths are recorded annually to cholera. Inadequate access to clean water and properly sanitized water are normally linked to the transmission of cholera.

The number of reported cases of cholera has continued to grow in recent years. In 2020, 323,369 cases were recorded with 857 deaths from 24 countries amidst limitations in surveillance systems and the fear of trade and tourism. The co-infection of these diseases is prevalent in the sense that the parasite which transmits these diseases are associated with contaminated water and the environment. It is widely known that mosquitoes are the real agents of malaria which breed from stagnant and contaminated water. In another hand, the contaminated water where these mosquitoes breed is the main source of cholera transmission. Both these diseases are treatable through clinical means but can be harmful to vital organs if proper care and diagnosis are found wanting.

Mathematical models have been used extensively over the years as relevant tools in understanding the dynamics of disease transmission and policy-making with regard control mechanism of diseases. For instance, Ross [5] first formulated the malaria transmission models. In his paper, he focused on malaria prevention and showed that the mosquito population should decrease to a certain threshold for malaria to be eradicated. Okosun et al. [6] formulated a mathematical model for malaria–cholera co-infection for the purpose of investigating the synergy between malaria and cholera in the face of treatment. Other important contribution includes Egeonu et al. [7] who proposed a co-infection model for two–strains of malaria and cholera with optimal control. Mandal et al. [8] proposed a hierarchical structure of a range of deterministic models of different levels of complexity and the evolution of modelling strategies to describe malaria incidence by including the critical features of host–vector–parasite interactions.

Oke et al. [9] proposed a mathematical model of malaria disease with a control strategy where prevention through bed nets, treatment, and insecticide were considered. In their paper, it was demonstrated that the use of treatments and treated bed nets should be taken into account when scarce resources arise while combining the two gives maximum results to malaria control. Osman and Adu [10] analyzed two sections in their mathematical model; the SEIR and the SEIR-SEI models. They showed that malaria may be controlled through the use of active malaria drugs, insecticides, and mosquito-treated nets. Tilahun et al. [11] proposed a stochastic and deterministic mathematical model of cholera disease dynamics with direct transmission. Hintsa and Kahsay [12] proposed the analysis of cholera epidemic control using mathematical models. In their findings, they showed that the introduction of preventive measures for contracting the disease reduces the basic reproduction number to below one as against the opposite where the reproduction number is greater than one. This suggests that cholera disease can be controlled and eliminated from the community if susceptible and recovered individuals comply with the preventive measures. However, a few studies have been carried out on the formulation and application of fractional order differential equations of malaria models. To the best of our knowledge, no work has been carried out on the analysis of co–dynamic model of malaria and cholera via Caputo–Fabrizio

fractional order differential equation. Only recently, the authors in [13] used Caputo–Fabrizio fractional order derivative to model HPV and Syphilis diseases. Omame et al. [14] proposed a fractional order model for dual variants of COVID-19 and HIV co-infection via another derivative that has a non–singular kernel, Atangana–Baleanu fractional order derivative. Nwajeri et al. [15] proposed the analysis of HPV and CT co-infection model using a fractional order derivative. Fractional order derivatives have made a tremendous contribution to the field of epidemiological modelling and have huge developments [16–25]. It is evident nowadays that fractional differential equations are efficient tools and very useful and effective in numerous fields of science and engineering such as biology, finance, rheology, electro–chemistry, chemical physics, etc. These wonderful applications of fractional differential equations to physical problems are due to their natural relation to the system with memory which is a common feature of many phenomena [26–31].

The aim of this work is to analyze vividly the co–dynamism of malaria and cholera via fractional order derivative. In particular, we formulate a fractional order model of ten compartments which depicts the two diseases' interaction within a population. The model is based on the paper of Omame et al. [32] where Atangana–Baleanu fractional order derivative to model the impact of SARS-CoV-2 infection on the dynamics of dengue and HIV. Moreover, we describe and analyze the results using Caputo–Fabrizio fractional derivative and Caputo fractional derivative.

The rest of the paper is arranged as follows; the fractional order model is formulated in Section 2 via Caputo–Fabrizio fractional derivative. In Section 3, we present in nutshell, some definitions of fractional order differential equations. Qualitative analysis of the fractional system takes the centre stage in Section 4 where the positive invariant region of the system, and basic reproduction number are presented. We also present the existence and uniqueness of the solution using the fixed point theorem. In Section 5, we developed the numerical solution of the fractional system with the aid of the Atangana-Baleanu technique. In Section 6, graphical figures and their biological discussions and results were presented after rigorous MATLAB simulations. Finally, we draw the necessary conclusion of our manuscript in Section 7.

2 Model formulation

The model consists of different compartments of human and vector populations. The human population is divided into the following compartments: susceptible humans $\mathcal{S}_{HB}$, infected individuals with malaria $\mathcal{I}_{MAL}$, recovered individuals from malaria disease $\mathcal{R}_{MAL}$, individuals infected with cholera $\mathcal{I}_{CHO}$, recovered individuals from cholera disease $\mathcal{R}_{CHO}$, individuals co-infected with malaria and cholera $\mathcal{I}_{MAC}$ and recovered individuals from malaria and cholera $\mathcal{R}_{MAC}$. On another note, we have bacterial population denoted by $\mathcal{B}_{CHO}$ and the vector population which is divided into two compartments, namely, the susceptible vectors $\mathcal{S}_{VEC}(t)$ and infected vectors $\mathcal{I}_{VEC}(t)$. The total population of humans and vectors at the time t are given by $\mathcal{N}_{HB}(t) = \mathcal{S}_{HB}(t) + \mathcal{I}_{MAL}(t) + \mathcal{R}_{MAL}(t) + \mathcal{I}_{CHO}(t) + \mathcal{R}_{CHO}(t) + \mathcal{I}_{MAC}(t) + \mathcal{R}_{MAC}(t)$ and $\mathcal{N}_{VEC}(t) = \mathcal{S}_{VEC}(t) + \mathcal{I}_{VEC}(t)$, respectively. The parameter β_{MAL} denotes the probability of humans infected with malaria while μ_{HB} denotes the natural mortality rate from the human population. The quantity $\frac{\beta_{MAL} b \mathcal{I}_{VEC}}{N_{HB}}$ represents the rate $\mathcal{S}_{HB}$ are exposed to the infected vectors and moved to $\mathcal{I}_{MAL}$ and and $\mathcal{I}_{MAC}$ compartments while the quantity $\frac{\mathcal{B}_{CHO} q_{CHO}}{\kappa + \mathcal{B}_{CHO}} \mathcal{S}_{HB}$ denotes the rate $\mathcal{S}_{HB}$ contracts cholera through bacteria and moved to $\mathcal{I}_{CHO}$ and $\mathcal{I}_{MAC}$ classes. The remaining parameters are properly defined in Table 1.

Table 1. Description of parameters in model (1)

Parameter	Description	Value	References
Λ_{HB}	Recruitment rate of humans	$100day^{-1}$	[33]
β_{MAL}	Human probability of infection with malaria	0.181 (dimensionless)	[34]
q_{CHO}	Bacterial contact rate for human	$0.05day^{-1}$	[6]
β_{CHO}	Human probability of infection with cholera	0.005	Assumed
μ_{HB}	Natural death rate for humans	$\frac{1}{(70\times360)}day^{-1}$	[7]
r	Per capita bacteria reproduction rate	0.5	Assumed
b	Average vectors bitting rate	$0.5day^{-1}$	[35]
ω_{MAL}	Waning rate of malaria	$\frac{1}{(60\times360)}day^{-1}$	[33]
ω_{CHO}	Waning rate of cholera	$0.001day^{-1}$	[34]
ω_{mc}	Malaria-Cholera co-infection waning rate	$0.001-0.02day^{-1}$	[6]
δ_{MAL}	Malaria induced death rate	$0.05-0.1day^{-1}$	[6, 36]
γ_{MAL}	Recovery rate from malaria	0.25	[37]
δ_{CHO}	cholera induced death rate	$0.0002day^{-1}$	Assumed
γ_{CHO}	Recovery rate from cholera	$0.07day^{-1}$	[34]
β_{VEC}	Vector probability of infection with malaria	0.181 (dimensionless)	[34]
δ_{MAC}	Death caused by malaria-cholera co-infection	$0.05day^{-1}$	Assumed
γ_{MAC}	Recovery rate for malaria and cholera	$0.4day^{-1}$	Assumed
ξ_{MP1}, ξ_{MP2}	Modification parameters	0.6 (dimensionless)	Assumed
μ_{DBR}	Natural death rate for bacterial	0.123	[38]
μ_{VEC}	Natural death rate for vectors	$\frac{1}{15}, 0.143day^{-1}$	[33]
Λ_{VEC}	Recruitment rate for vectors	$1000day^{-1}$	[33]

Motivated by the numerous advantages of fractional order operators as already stated, we hereby state the fractional order co–dynamic model under the Caputo–Fabrizio derivative as

$$
\begin{aligned}
{}^{CF}\mathcal{D}_t^{\alpha}\mathcal{S}_{HB}(t) &= \Lambda_{HB} - \frac{\beta_{MAL} b \mathcal{I}_{VEC}}{N_{HB}}\mathcal{S}_{HB} - \frac{\mathcal{B}_{CHO} q_{CHO}}{\kappa + \mathcal{B}_{CHO}}\mathcal{S}_{HB} - \mu_{HB}\mathcal{S}_{HB} + \omega_{MAL}\mathcal{R}_{MAL} \\
&\quad + \omega_{CHO}\mathcal{R}_{CHO} + \omega_{mc}\mathcal{R}_{MAC}, \\
{}^{CF}\mathcal{D}_t^{\alpha}\mathcal{I}_{MAL}(t) &= \beta_{MAL} b \frac{\mathcal{I}_{VEC}}{\mathcal{N}_{HB}}(\mathcal{S}_{HB} + \mathcal{R}_{CHO}) + \frac{\mathcal{B}_{CHO} q_{CHO}}{\kappa + \mathcal{B}_{CHO}}\mathcal{I}_{MAL} - (\mu_{HB} + \delta_{MAL} + \gamma_{MAL})\mathcal{I}_{MAL}, \\
{}^{CF}\mathcal{D}_t^{\alpha}\mathcal{R}_{MAL}(t) &= \gamma_{MAL}\mathcal{I}_{MAL} - (\mu_{MAL} + \omega_{MAL})\mathcal{R}_{MAL} - \frac{\mathcal{B}_{CHO} q_{CHO}}{\kappa + \mathcal{B}_{CHO}}\mathcal{R}_{MAL}, \\
{}^{CF}\mathcal{D}_t^{\alpha}\mathcal{I}_{CHO}(t) &= \frac{\beta_{CHO} q_{CHO}}{\kappa + \mathcal{B}_{CHO}}(\mathcal{S}_{HB} + \mathcal{R}_{MAL}) - (\delta_{CHO} + \mu_{HB} + \gamma_{CHO})\mathcal{I}_{CHO} - \frac{\beta_{VEC} b \mathcal{I}_{VEC}}{\mathcal{N}_{HB}}\mathcal{I}_{CHO}, \\
{}^{CF}\mathcal{D}_t^{\alpha}\mathcal{R}_{CHO}(t) &= \gamma_{CHO}\mathcal{I}_{CHO} - (\mu_{HB} + \omega_{CHO})\mathcal{R}_{CHO} - \frac{\beta_{MAL} b \mathcal{I}_{VEC}}{\mathcal{N}_{HB}}\mathcal{R}_{CHO}, \\
{}^{CF}\mathcal{D}_t^{\alpha}\mathcal{I}_{MAC}(t) &= \frac{\mathcal{B}_{CHO} q_{CHO}}{\kappa + \mathcal{B}_{CHO}}\mathcal{I}_{MAL} + \frac{\beta_{MAL} b \mathcal{I}_{VEC}}{\mathcal{N}_{HB}}\mathcal{I}_{CHO} - (\mu_{HB} + \delta_{MAC} + \gamma_{MAC})\mathcal{I}_{MAC}, \\
{}^{CF}\mathcal{D}_t^{\alpha}\mathcal{R}_{MAC}(t) &= \gamma_{MAC}\mathcal{I}_{CHO} - (\mu_{HB} + \omega_{mc})\mathcal{R}_{MAC}, \\
{}^{CF}\mathcal{D}_t^{\alpha}\mathcal{B}_{CHO}(t) &= r\mathcal{B}_{CHO}\left(1 - \frac{\mathcal{B}_{CHO}}{\kappa}\right) + \xi_{MP1}\mathcal{I}_{CHO} + \xi_{MP2}\mathcal{I}_{MAC} - \mu_{DBR}\mathcal{B}_{CHO}, \\
{}^{CF}\mathcal{D}_t^{\alpha}\mathcal{S}_{VEC}(t) &= \Lambda_{VEC} - \frac{\beta_{VEC} b (\mathcal{I}_{MAL} + \mathcal{I}_{MAC})}{\mathcal{N}_{HB}}\mathcal{S}_{VEC} - \mu_{VEC}\mathcal{S}_{VEC}, \\
{}^{CF}\mathcal{D}_t^{\alpha}\mathcal{I}_{VEC}(t) &= \frac{\beta_{VEC} b (\mathcal{I}_{MAL} + \mathcal{I}_{MAC})}{\mathcal{N}_{HB}}\mathcal{S}_{VEC} - \mu_{VEC}\mathcal{I}_{VEC},
\end{aligned}
\tag{1}
$$

which corresponds to the following initial conditions

$$\begin{cases} \mathcal{S}_{HB}(0) = \mathcal{S}_{HB(0)} \geq 0, \quad \mathcal{I}_{MAL}(0) = \mathcal{I}_{MAL(0)} \geq 0, \quad \mathcal{R}_{MAL}(0) = \mathcal{R}_{MAL(0)} \geq 0, \\ \mathcal{I}_{CHO}(0) = \mathcal{I}_{CHO(0)} \geq 0, \quad \mathcal{R}_{CHO}(0) = \mathcal{R}_{CHO(0)} \geq 0, \quad \mathcal{I}_{MAC}(0) = \mathcal{I}_{MAC(0)} \geq 0, \\ \mathcal{R}_{MAC}(0) = \mathcal{R}_{MAC(0)} \geq 0, \quad \mathcal{B}_{CHO}(0) = \mathcal{B}_{CHO(0)} \geq 0, \quad \mathcal{S}_{VEC}(0) = \mathcal{S}_{VEC(0)} \geq 0, \\ \mathcal{I}_{VEC}(0) = \mathcal{I}_{VEC(0)} \geq 0. \end{cases} \tag{2}$$

3 Preliminaries

This section presents several important properties and definitions of Caputo-Fabrizio derivative in the Caputo sense which will aid the analysis of the manuscript.

Definition 1 ([39]) *The Caputo–Fabrizio fractional derivative of order α for the function $\mathcal{K} \in H^1([0,b],\mathbb{R}^+)$ where $b > 0$ is given by*

$$^{CF}\mathcal{D}_t^{\alpha}\mathcal{K}(t) = \frac{(2-\alpha)\mathcal{U}(\alpha)}{2(1-\alpha)}\int_0^t \exp\left(\frac{-\alpha(t-\xi)}{1-\alpha}\right)\mathcal{K}'(\zeta)d\zeta, \qquad 0 < \alpha \leq 1, \qquad t \geq 0. \tag{3}$$

Definition 2 ([40]) *The Caputo–Fabrizio fractional integral order α for the function $\mathcal{K} \in H^1([0,b],\mathbb{R}^+)$ where $b > 0$ is given by*

$$^{CF}I_t^{\alpha}\mathcal{K}(t) = \frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)}\mathcal{K}(t) + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)}\int_0^t \mathcal{K}(\zeta)d\zeta, \qquad 0 < \alpha \leq 1, \qquad t \geq 0. \tag{4}$$

Definition 3 ([22]) *The Atangana–Baleanu fractional derivative in the Caputo sense of order α for the function $\mathcal{K} \in H^1([0,b],\mathbb{R}^+)$, where $b > 0$, is given by*

$$^{ABC}\mathcal{D}_t^{\alpha}\mathcal{K}(t) = \frac{\mathcal{U}(\alpha)}{(1-\alpha)}\int_0^t \mathcal{E}_{\alpha}\left(\frac{-\alpha(t-\zeta)}{1-\alpha}\right)\mathcal{K}'(\zeta)d\zeta, \qquad 0 < \alpha \leq 1, \qquad t \geq 0.$$

Definition 4 ([39]) *The Atangana–Baleanu fractional integral of order α for the function $\mathcal{K} \in H^1([0,b],\mathbb{R}^+)$, where $b > 0$, is given by*

$$^{AB}I_t^{\alpha}\mathcal{K}(t) = \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{K}(t) + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\mathcal{K}(\zeta)d\zeta, \qquad 0 < \alpha \leq 1, \qquad t \geq 0, \tag{5}$$

where $\mathcal{U}(\alpha)$ denotes the normalization function and

$$\mathcal{E}_{\alpha}(d) = \sum_{k=0}^{\infty}\frac{d^k}{\Gamma(\alpha k+1)}, \quad \Re(\alpha) > 0.$$

Lemma 1 ([39]) *The solution for the following problem with $\alpha \in (0,1]$ is given as:*

$$\begin{aligned} ^{CF}\mathcal{D}_t^{\alpha}\mathcal{W}(t) &= \mathcal{V}(t), \\ \mathcal{W}(0) &= \mathcal{W}_0, \end{aligned} \tag{6}$$

that is assumed to be equivalent to the following fractional Volterra integral equation

$$\mathcal{W}(t) = \mathcal{W}_0 + \frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)}\left(\mathcal{V}(t)-\mathcal{V}(0)\right) + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)}\int_0^t \mathcal{W}(\zeta)d\zeta, \qquad t \geq 0. \tag{7}$$

Lemma 2 ([40]) *The Laplace transform of Caputo-Fabrizio fractional derivative in the Caputo sense of order $\alpha \in (0,1]$ for the function $\mathcal{K}(t)$ is given by*

$$\mathcal{L}\{D_t^\alpha \mathcal{K}(t), s\} = \frac{(2-\alpha)\mathcal{U}(\alpha)}{2}\frac{s\mathcal{L}\{\mathcal{K}(t)\}-\mathcal{K}(0)}{s+\alpha(1-s)}, \qquad s \geq 0.$$

4 Qualitative analysis of the constructed model

In this section, we present carefully the analysis of the fractional order co–dynamic model of malaria and cholera (1) where the positivity of the solution, basic reproduction number via next-generation matrix method, existence and uniqueness of the solution are presented.

Positivity

Lemma 3 *The region $\mathcal{D} = \mathcal{D}_h \cup \mathcal{D}_b \cup \mathcal{D}_v \subset \mathcal{R}_+^7 \times \mathcal{R}_+ \times \mathcal{R}_+^2$ is non-negatively invariant for the model (1) with initial conditions in $\mathcal{R}_+^{10}$, where*

$$\begin{aligned}\mathcal{D}_h = \Big\{ &(\mathcal{S}_{HB}, \mathcal{I}_{MAL}, \mathcal{R}_{MAL}, \mathcal{I}_{CHO}, \mathcal{R}_{CHO}, \mathcal{I}_{MAC}, \mathcal{R}_{MAC}) : \mathcal{S}_{HB} + \mathcal{I}_{MAL} \\ &+\mathcal{R}_{MAL} + \mathcal{I}_{CHO} + \mathcal{R}_{CHO} + \mathcal{I}_{MAC} + \mathcal{R}_{MAC} < \frac{\Lambda_{HB}}{\mu_{HB}}\Big\},\end{aligned}$$

$$\mathcal{D}_b = \left\{\mathcal{B}_{CHO} : \mathcal{B}_{CHO} \leq \frac{(\rho_1+\rho_2)\,\Lambda_{HB}}{\mu_{HB}\mu_{DBR}}\right\},$$

$$\mathcal{D}_v = \left\{(\mathcal{S}_{VEC}, \mathcal{I}_{VEC}) : \mathcal{S}_{VEC} + \mathcal{I}_{VEC} \leq \frac{\Lambda_{VEC}}{\mu_{VEC}}\right\}.$$

Proof Adding all the equations corresponding to the human components of the system (1) gives

$${}^{\mathcal{CF}}\mathcal{D}_t^\alpha \mathcal{N}_{HB}(t) = \Lambda_{HB} - \mu_{HB}\mathcal{N}_{HB}(t) - \left[\delta_{MAL}\mathcal{I}_{MAL} + \delta_{CHO}\mathcal{I}_{CHO} + \delta_{MAC}\mathcal{I}_{MAC}\right], \tag{8}$$

so that from (8), we have

$${}^{\mathcal{CF}}\mathcal{D}_t^\alpha \mathcal{N}_{HB}(t) \leq \Lambda_{HB} - \mu_{HB}\mathcal{N}_{HB}(t).$$

Applying the Laplace transform of the Caputo-Fabrizio derivative on the above inequality, and

simplifying, we obtain

$$\begin{aligned}\mathcal{N}_{HB}(t) \leq & \frac{\Lambda_{HB}}{\mu_{HB}} - \frac{\Lambda_{HB}(2\alpha-\varrho_1)}{[1+\mu_{HB}(1-\alpha)]\varrho_1}e^{-\varrho_1 t} - \frac{\mathcal{N}_{HB}(0)}{(1-\alpha)[1+\mu_{HB}(1-\alpha)](\varrho_1-\varrho_2)}e^{-\varrho_1 t} \\ & + \frac{\mathcal{N}_{HB}(0)}{(1-\alpha)[1+\mu_{HB}(1-\alpha)](\varrho_1-\varrho_2)}e^{-\varrho_2 t},\end{aligned}$$

where $\varrho_1 = \frac{\mu_{HB}\alpha}{1+\mu_{HB}(1-\alpha)}$, $\varrho_2 = \frac{\alpha}{(1-\alpha)}$. Thus, the total population of humans, $\mathcal{N}_{HB}(t) \leq \frac{\Lambda_{HB}}{\mu_{HB}}$ as $t \to \infty$. Similarly, the total population of vectors $\mathcal{N}_{VEC}(t) \leq \frac{\Lambda_{VEC}}{\mu_{VEC}}$ and total bacteria population $\mathcal{B}_{CHO}(t) \leq \frac{K(r-\mu_{DBR})}{\mu_{DBR}}$. This shows that malaria and cholera fractional order model (1) is bounded and has a solution in $\mathcal{D}_h$, $\mathcal{D}_b$ and $\mathcal{D}_v$, respectively. Hence, for fractional malaria and cholera co–dynamic model, $\mathcal{D}_h$, $\mathcal{D}_b$ and $\mathcal{D}_v$ are positively invariant regions and thus the proof. ■

Basic reproduction number

The disease-free equilibrium (DFE) of the fractional order malaria and cholera co–dynamic model (1) achieved by setting the right-hand side of the equations of the model to zero is given by

$$\begin{aligned}\mathcal{T}_0 &= (\mathcal{S}^*_{HB}, \mathcal{I}^*_{MAL}, \mathcal{R}^*_{MAL}, \mathcal{I}^*_{CHO}, \mathcal{R}^*_{CHO}, \mathcal{I}^*_{MAC}, \mathcal{R}^*_{MAC}, \mathcal{B}^*_{CHO}, \mathcal{S}^*_{VEC}, \mathcal{I}^*_{VEC}) \\ &= \left(\frac{\Lambda_{HB}}{\mu_{HB}}, 0, 0, 0, 0, 0, 0, 0, \frac{\Lambda_{VEC}}{\mu_{VEC}}, 0\right).\end{aligned} \tag{9}$$

Using the similar approach in [41], we obtain the basic reproduction number as follows

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & \beta_{MAL} b \\ 0 & 0 & 0 & \frac{q_{CHO}\mathcal{S}^*_{HB}}{\kappa} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{\beta_{VEC}\mathcal{S}^*_{VEC}}{\mathcal{N}^*_{HB}} & 0 & \frac{\beta_{VEC}\mathcal{S}^*_{VEC}}{\mathcal{N}^*_{HB}} & 0 & 0 \end{pmatrix},$$

$$V = \begin{pmatrix} \mathcal{A}_1 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{A}_2 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{A}_3 & 0 & 0 \\ 0 & -\rho_1 & -\rho_2 & \mu & 0 \\ 0 & 0 & 0 & 0 & \mu \end{pmatrix},$$

where

$$\mathcal{A}_1 = \mu_{HB} + \delta_{MAL} + \gamma_{MAL}, \quad \mathcal{A}_2 = \delta_{CHO} + \mu_{HB} + \gamma_{CHO}, \quad \mathcal{A}_3 = \mu_{HB} + \delta_{MAC} + \gamma_{MAC}.$$

After elementary algebra, we obtain the basic reproduction number $\mathcal{R}_0 = \max\{\mathcal{R}_{0P}, \mathcal{R}_{0T}\}$, where

$$\mathcal{R}_{0P} = \sqrt{\frac{b\beta_{MAL}\beta_{VEC} b \mathcal{S}_{VEC}}{N^*_h \mathcal{A}_1 \mu_{VEC}}}, \qquad \mathcal{R}_{0T} = \frac{\rho_1 q_{CHO} S^*_h}{\kappa \mu \mathcal{A}_2}.$$

Theorem 1 *The DFE, $\mathcal{T}_0$, of the fractional Malaria and Cholera model (1) is locally asymptotically stable*

(LAS) if $\mathcal{R}_0 < 1$, *and unstable if* $\mathcal{R}_0 > 1$.

Proof The Jacobian matrix of the malaria and cholera fractional order model (1) evaluated at the disease-free equilibrium, $\mathcal{T}_0$ is given by:

$$J(\mathcal{T}_0) = \begin{pmatrix} -\mu_{HB} & 0 & \omega_{MAL} & 0 & \omega_{CHO} & 0 & \omega_{mc} & -\frac{q_{CHO}\mathcal{S}^*_{HB}}{N^*_{HB}} & 0 & -\beta_{MAL}b \\ 0 & -\mathcal{A}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{MAL}b \\ 0 & \gamma_{MAL} & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathcal{A}_2 & 0 & 0 & 0 & \frac{q_{CHO}\mathcal{S}^*_{HB}}{\kappa} & 0 & 0 \\ 0 & 0 & 0 & \gamma_{CHO} & -Q_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mathcal{A}_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_{MAC} & -Q_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_1 & 0 & \rho_2 & 0 & -\mu_{DBR} & 0 & 0 \\ 0 & -\theta & 0 & 0 & 0 & -\theta & 0 & 0 & -\mu_{VEC} & 0 \\ 0 & \theta & 0 & 0 & 0 & \theta & 0 & 0 & 0 & -\mu_{VEC} \end{pmatrix},$$

where

$$\mathcal{A}_1 = \mu_{HB} + \delta_{MAL} + \gamma_{MAL}, \quad \mathcal{A}_2 = \delta_{CHO} + \mu_{HB} + \gamma_{CHO}, \quad \mathcal{A}_3 = \mu_{HB} + \delta_{MAC} + \gamma_{MAC},$$
$$Q_1 = \mu + \omega_{MAL}, Q_2 = \mu + \omega_{CHO}, Q_3 = \mu + \omega_{mc}, \theta = \frac{\beta_{VEC}bS^*_v}{N^*_h}.$$

Thus the eigenvalues are as follows $\lambda_1 = -Q_1$, $\lambda_2 = -Q_2$, $\lambda_3 = -Q_3$, $\lambda_4 = -\mathcal{A}_3$, $\lambda_5 = -\mu_{HB}$, $\lambda_6 = -\mu_{VEC}$ and the following characteristic equations given below

$$\left(-b\theta\beta_{MAL}\left(\lambda + \mathcal{A}_1\right)\left(\lambda + \mu_{VEC}\right)\right)\left(1 - \mathcal{R}_{0P}\right) = 0,$$

and

$$\left(\left(\lambda + \mathcal{A}_2\right)\left(\lambda + \mu_{DBR}\right)\frac{\rho_1 q_{CHO}S^*_h}{\kappa}\right)\left(1 - \mathcal{R}_{0T}\right) = 0.$$

By the Routh-Hurwitz criterion, the above equations will possess negative real roots if and only if $\mathcal{R}_{0P} < 1$ and $\mathcal{R}_{0T} < 1$, respectively. Hence, the DFE, $\mathcal{T}_0$ is locally asymptotically stable if $\mathcal{R}_0 = \max\{\mathcal{R}_{0P}, \mathcal{R}_{0T}\} < 1$. ■

Existence and uniqueness of solution

Here, we present the existence and uniqueness of the formulated fractional order system (1) using fixed point theorems. For this purpose, we rewrite malaria and cholera fractional co–dynamic model as

$$^{CF}\mathcal{D}^{\alpha}\mathcal{S}_{HB}(t) = \mathcal{Q}_1\left(t, \mathcal{S}_{HB}, \mathcal{I}_{MAL}, \mathcal{R}_{MAL}, \mathcal{I}_{CHO}, \mathcal{R}_{CHO}, \mathcal{I}_{MAC}, \mathcal{R}_{MAC}, \mathcal{B}_{CHO}, \mathcal{S}_{VEC}, \mathcal{I}_{VEC}\right),$$
$$^{CF}\mathcal{D}^{\alpha}\mathcal{I}_{MAL}(t) = \mathcal{Q}_2\left(t, \mathcal{S}_{HB}, \mathcal{I}_{MAL}, \mathcal{R}_{MAL}, \mathcal{I}_{CHO}, \mathcal{R}_{CHO}, \mathcal{I}_{MAC}, \mathcal{R}_{MAC}, \mathcal{B}_{CHO}, \mathcal{S}_{VEC}, \mathcal{I}_{VEC}\right),$$

$$
\begin{aligned}
{}^{CF}\mathcal{D}^{\alpha}\mathcal{R}_{MAL}(t) &= \mathcal{Q}_3\left(t,\mathcal{S}_{HB},\mathcal{I}_{MAL},\mathcal{R}_{MAL},\mathcal{I}_{CHO},\mathcal{R}_{CHO},\mathcal{I}_{MAC},\mathcal{R}_{MAC},\mathcal{B}_{CHO},\mathcal{S}_{VEC},\mathcal{I}_{VEC}\right),\\
{}^{CF}\mathcal{D}^{\alpha}\mathcal{I}_{CHO}(t) &= \mathcal{Q}_4\left(t,\mathcal{S}_{HB},\mathcal{I}_{MAL},\mathcal{R}_{MAL},\mathcal{I}_{CHO},\mathcal{R}_{CHO},\mathcal{I}_{MAC},\mathcal{R}_{MAC},\mathcal{B}_{CHO},\mathcal{S}_{VEC},\mathcal{I}_{VEC}\right),\\
{}^{CF}\mathcal{D}^{\alpha}\mathcal{R}_{CHO}(t) &= \mathcal{Q}_5\left(t,\mathcal{S}_{HB},\mathcal{I}_{MAL},\mathcal{R}_{MAL},\mathcal{I}_{CHO},\mathcal{R}_{CHO},\mathcal{I}_{MAC},\mathcal{R}_{MAC},\mathcal{B}_{CHO},\mathcal{S}_{VEC},\mathcal{I}_{VEC}\right),\\
{}^{CF}\mathcal{D}^{\alpha}\mathcal{I}_{MAC}(t) &= \mathcal{Q}_6\left(t,\mathcal{S}_{HB},\mathcal{I}_{MAL},\mathcal{R}_{MAL},\mathcal{I}_{CHO},\mathcal{R}_{CHO},\mathcal{I}_{MAC},\mathcal{R}_{MAC},\mathcal{B}_{CHO},\mathcal{S}_{VEC},\mathcal{I}_{VEC}\right),\\
{}^{CF}\mathcal{D}^{\alpha}\mathcal{R}_{MAC}(t) &= \mathcal{Q}_7\left(t,\mathcal{S}_{HB},\mathcal{I}_{MAL},\mathcal{R}_{MAL},\mathcal{I}_{CHO},\mathcal{R}_{CHO},\mathcal{I}_{MAC},\mathcal{R}_{MAC},\mathcal{B}_{CHO},\mathcal{S}_{VEC},\mathcal{I}_{VEC}\right),\\
{}^{CF}\mathcal{D}^{\alpha}\mathcal{B}_{CHO}(t) &= \mathcal{Q}_8\left(t,\mathcal{S}_{HB},\mathcal{I}_{MAL},\mathcal{R}_{MAL},\mathcal{I}_{CHO},\mathcal{R}_{CHO},\mathcal{I}_{MAC},\mathcal{R}_{MAC},\mathcal{B}_{CHO},\mathcal{S}_{VEC},\mathcal{I}_{VEC}\right),\\
{}^{CF}\mathcal{D}^{\alpha}\mathcal{S}_{VEC}(t) &= \mathcal{Q}_9\left(t,\mathcal{S}_{HB},\mathcal{I}_{MAL},\mathcal{R}_{MAL},\mathcal{I}_{CHO},\mathcal{R}_{CHO},\mathcal{I}_{MAC},\mathcal{R}_{MAC},\mathcal{B}_{CHO},\mathcal{S}_{VEC},\mathcal{I}_{VEC}\right),\\
{}^{CF}\mathcal{D}^{\alpha}\mathcal{I}_{VEC}(t) &= \mathcal{Q}_{10}\left(t,\mathcal{S}_{HB},\mathcal{I}_{MAL},\mathcal{R}_{MAL},\mathcal{I}_{CHO},\mathcal{R}_{CHO},\mathcal{I}_{MAC},\mathcal{R}_{MAC},\mathcal{B}_{CHO},\mathcal{S}_{VEC},\mathcal{I}_{VEC}\right),
\end{aligned}
\tag{10}
$$

where $\mathcal{Q}_j = \mathcal{Q}_j\left(t,\mathcal{S}_{HB},\mathcal{I}_{MAL},\mathcal{R}_{MAL},\mathcal{I}_{CHO},\mathcal{R}_{CHO},\mathcal{I}_{MAC},\mathcal{R}_{MAC},\mathcal{B}_{CHO},\mathcal{S}_{VEC},\mathcal{I}_{VEC}\right)$, for $j = 1,2,\ldots,10$ is given by

$$
\begin{cases}
\mathcal{Q}_1 = \Lambda_{HB} - \frac{\beta_{MAL} b \mathcal{I}_{VEC}}{N_h}\mathcal{S}_{HB} - \frac{\mathcal{B}_{CHO}\alpha}{\kappa+\mathcal{B}_{CHO}}\mathcal{S}_{HB} - \mu_{HB}\mathcal{S}_{HB} + \omega_{MAL}\mathcal{R}_{MAL} + \omega_{CHO}\mathcal{R}_{CHO} + \omega_{mc}\mathcal{R}_{MAC},\\
\mathcal{Q}_2 = \beta_{MAL} b \frac{\mathcal{I}_{VEC}}{\mathcal{N}_{HB}}(\mathcal{S}_{HB} + \mathcal{R}_{CHO}) - (\mu_{HB} + \delta_{MAL} + \gamma_{MAL})\mathcal{I}_{MAL} + \frac{\mathcal{B}_{CHO} q_{CHO}}{\kappa+\mathcal{B}_{CHO}}\mathcal{I}_{MAL},\\
\mathcal{Q}_3 = \gamma_{MAL}\mathcal{I}_{MAL} - (\mu + \omega_{MAL})\mathcal{R}_{MAL} - \frac{\mathcal{B}_{CHO}\alpha}{\kappa+\mathcal{B}_{CHO}}\mathcal{R}_{MAL},\\
\mathcal{Q}_4 = \frac{\beta_c \alpha}{\kappa+\mathcal{B}_{CHO}}(\mathcal{S}_{HB} + \mathcal{R}_{MAL}) - (\delta_{CHO} + \mu_{HB} + \gamma_{CHO})\mathcal{I}_{CHO} - \frac{\beta_{VEC} b \mathcal{I}_{VEC}}{\mathcal{N}_{HB}}\mathcal{I}_{CHO},\\
\mathcal{Q}_5 = \gamma_{CHO}\mathcal{I}_{CHO} - (\mu_{HB} + \omega_{CHO})\mathcal{R}_{CHO} - \frac{\beta_{MAL} b \mathcal{I}_{VEC}}{\mathcal{N}_{HB}}\mathcal{R}_{CHO},\\
\mathcal{Q}_6 = \frac{\mathcal{B}_{CHO}\alpha}{\kappa+\mathcal{B}_{CHO}}\mathcal{I}_{MAL} + \frac{\beta_{MAL} b \mathcal{I}_{VEC}}{\mathcal{N}_{HB}}\mathcal{I}_{CHO} - (\mu_{HB} + \delta_{MAC} + \gamma_{MAC})\mathcal{I}_{MAC},\\
\mathcal{Q}_7 = \gamma_{MAC}\mathcal{I}_{CHO} - (\mu_{HB} + \omega_{mc})\mathcal{R}_{MAC},\\
\mathcal{Q}_8 = r\mathcal{B}_{CHO}\left(1 - \frac{\mathcal{B}_{CHO}}{K}\right) + \xi_{MP1}\mathcal{I}_{CHO} + \xi_{MP2}\mathcal{I}_{MAC} - \mu_{DBR}\mathcal{B}_{CHO},\\
\mathcal{Q}_9 = \Lambda_{VEC} - \frac{\beta_{VEC} b(\mathcal{I}_{MAL}+\mathcal{I}_{MAC})}{\mathcal{N}_{HB}}\mathcal{S}_{VEC} - \mu_{VEC}\mathcal{S}_{VEC},\\
\mathcal{Q}_{10} = \frac{\beta_{VEC} b(\mathcal{I}_{MAL}+\mathcal{I}_{MAC})}{\mathcal{N}_{HB}}\mathcal{S}_{VEC} - \mu_{VEC}\mathcal{I}_{VEC}.
\end{cases}
\tag{11}
$$

Using the above illustration, malaria and cholera fractional order co–dynamic model (1) can be written as

$$
\begin{cases}
{}^{CF}\mathcal{D}_t^{\alpha}\Omega_{MC}(t) = \mathcal{K}(t,\Omega_{MC}(t)), \qquad 0 < \alpha \leq 1, \qquad t \in \mathcal{J} = [0,T],\\
\Omega_{MC}(0) = \Omega_{MC0} \geq 0,
\end{cases}
\tag{12}
$$

where $\mathcal{K} : \mathcal{J} \times \mathbb{R} \to \mathbb{R}$ is continuous and

$$
\begin{cases}
\Omega_{MC}(t) = \left(\mathcal{S}_{HB},\mathcal{I}_{MAL},\mathcal{R}_{MAL},\mathcal{I}_{CHO},\mathcal{R}_{CHO},\mathcal{I}_{MAC},\mathcal{R}_{MAC},\mathcal{B}_{CHO},\mathcal{S}_{VEC},\mathcal{I}_{VEC}\right)^T,\\
\Omega_{MC0} = \left(S_{h0}, I_{m0}, R_{m0}, I_{c0}, R_{c0}, I_{MC0}, R_{MC0}, B_{c0}, S_{v0}, I_{v0}\right)^T,\\
\mathcal{K}(t,\Omega_{MC}(t)) = \mathcal{Q}_j\left(t,\mathcal{S}_{HB},\mathcal{I}_{MAL},\mathcal{R}_{MAL},\mathcal{I}_{CHO},\mathcal{R}_{CHO},\mathcal{I}_{MAC},\mathcal{R}_{MAC},\mathcal{B}_{CHO},\mathcal{S}_{VEC},\mathcal{I}_{VEC}\right)^T,
\end{cases}
\tag{13}
$$

where $j = 1,2,3,\ldots 10$, and $(.)^T$ denotes the transpose of the vector. Using Lemma (1), the initial value problem (12) is equivalent to the following fractional order Volterra integral equation

$$
\Omega_{MC}(t) = \Omega_{MC0} + \frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)}\left(\mathcal{K}(t,\Omega_{MC}(t)) - \mathcal{K}_0\right) + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)}\int_0^t \mathcal{K}(\zeta,\Omega_{MC}(\zeta))d\zeta, \tag{14}
$$

where $t \geq 0$.
Furthermore, let us define $C(\mathcal{J}, \mathbb{R})$ as the Banach space of continuous functions from $\mathcal{J} = [0, T]$ into $\mathbb{R}$ endowed with the Chebyshev norm

$$\|\Omega_{MC}\|_{\infty} := \sup_{t \in \mathcal{J}} \{|\Omega_{MC}(t)|\}, \qquad \mathcal{J} = [0, T].$$

Theorem 2 *Assume that* ***(B1):*** *There exists a Lipschitz constant* $\mathcal{L}_{MC} > 0$, *such that*

$$|\mathcal{K}(t, \Omega_{MC1}(t)) - \mathcal{K}(t, \Omega_{MC2}(t))| \leq \mathcal{L}_{MC} |\Omega_{MC1}(t) - \Omega_{MC2}(t)|, \quad t \in \mathcal{J} = [0, T], \quad \Omega_{MC1}, \Omega_{MC2} \in \mathbb{R},$$

then if

$$\mathcal{L}_{MC} \left(\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} + \frac{2\alpha T_{\max}}{(2-\alpha)\mathcal{U}(\alpha)} \right) < 1, \tag{15}$$

the initial value problem (1) *has a unique solution on* $\mathcal{J} = [0, T]$.

Proof Consider the transformed initial value problem (12) plugged into a fixed point quantity under the operation

$$\Phi : C^m(\mathcal{J}, \mathbb{R}) \to C^m(\mathcal{J}, \mathbb{R}),$$

with the corresponding definition as follows,

$$\begin{aligned} \Phi\Omega_{MC}(t) &= \Omega_{MC0} + \frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} \left(\mathcal{K}(t, \Omega_{MC}(t)) - \mathcal{K}(0, \Omega_{MC}(0))\right) \\ &\quad + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)} \int_0^t \mathcal{K}(\zeta, \Omega_{MC}(\zeta)) d\zeta. \end{aligned} \tag{16}$$

Next, we apply the Banach contraction principle to prove that the quantity Φ has a unique fixed point. In that case, we let the two solutions $\Omega_{MC1}(t), \Omega_{MC2}(t) \in C^m(\mathcal{J}, \mathbb{R})$, where $\mathcal{J} = [0, T]$, then we have:

$$\begin{aligned} |\Phi\Omega_{MC1}(t) - \Phi\Omega_{MC2}(t)| &\leq \frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} |\mathcal{K}(t, \Omega_{MC1}(t)) - \mathcal{K}(t, \Omega_{MC2}(t))| \\ &+ \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)} \int_0^t |\mathcal{K}(\zeta, \Omega_{MC1}(\zeta)) - \mathcal{K}(\zeta, \Omega_{MC2}(\zeta))| \, d\zeta \\ &\leq \mathcal{L}_{MC} \left(\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} + \frac{2\alpha T_{\max}}{(2-\alpha)\mathcal{U}(\alpha)} \right) |\Omega_{MC1}(t) - \Omega_{MC2}(t)|_{\infty}. \end{aligned}$$

Thus,

$$\|\Phi\Omega_{MC1}(t) - \Phi\Omega_{MC2}(t)\|_{\infty} \leq \mathcal{L}_{MC} \left(\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} + \frac{2\alpha T_{\max}}{(2-\alpha)\mathcal{U}(\alpha)} \right) |\Omega_{MC1}(t) - \Omega_{MC2}(t)|_{\infty}.$$

Applying (15), we see that the operator Φ is a contraction and hence possesses a fixed point, and hence, (1) has a unique solution. ■

Theorem 3 *Assume that*
(B2): *The function* $\mathcal{K} : \mathcal{J} \times \mathbb{R} \to \mathbb{R}$ *is totally continuous.*
(B3): *There exists a constant* $\mathcal{G}_{MC} > 0$ *such that*

$$|\mathcal{K}(t, \Omega_{MC}(t))| \leq \mathcal{G}_{MC}, \quad \text{for all } t \in \mathcal{J} = [0, T], \quad \Omega_{MC} \in \mathbb{R},$$

then the malaria and cholera fractional order system (1) *with its corresponding initial condition* (2) *has at least one solution on* $\mathcal{J} = [0, T]$.

Proof Following a similar process in [15], we apply Schauder's fixed point theorem to illustrate that Φ, defined by (16) possesses a fixed point. Thus consider the following steps;
STEP (1): The operator $\Phi : C^m(\mathcal{J}, \mathbb{R}) \to C^m(\mathcal{J}, \mathbb{R})$ is totally continuous. Define the function Ω_{MC} as a sequence $\left\{\Omega_{MC(m)}\right\}$ such that $\Omega_{MC(m)} \to \Omega_{MC}$ in $C(\mathcal{J}, \mathbb{R})$. Thus, for each $t \in \mathcal{J} = [0, T]$, we have

$$\begin{aligned}
&\left|\Phi\Omega_{MC(m)}(t) - \Phi\Omega_{MC}(t)\right| \\
= &\left|\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)}\left(\mathcal{K}(t, \Omega_{MC(m)}(t)) - \mathcal{K}(0, \Omega_{MC}(0))\right) + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)}\int_0^t \mathcal{K}(\zeta, \Omega_{MC(m)}(\zeta))d\zeta\right. \\
&\left. - \left(\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)}\left(\mathcal{K}(t, \Omega_{MC}(t)) - \mathcal{K}(0, \Omega_{MC}(0))\right) + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)}\int_0^t \mathcal{K}(\zeta, \Omega_{MC}(\zeta))d\zeta\right)\right| \\
\leq &\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)}\left|\mathcal{K}(t, \Omega_{MC(m)}(t)) - \mathcal{K}(t, \Omega_{MC}(t))\right| \\
&+ \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)}\int_0^t \left|\mathcal{K}(\zeta, \Omega_{MC(m)}(\zeta)) - \mathcal{K}(\zeta, \Omega_{MC}(\zeta))\right| d\zeta \\
\leq &\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)}\sup_{t\in\mathcal{J}}\left|\mathcal{K}(\zeta, \Omega_{MC(m)}(\zeta)) - \mathcal{K}(t, \Omega_{MC}(t))\right| + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)} \\
\times &\int_0^t \sup_{t\in\mathcal{J}}\left|\mathcal{K}(\zeta, \Omega_{MC(m)}(\zeta)) - \mathcal{K}(\zeta, \Omega_{MC}(\zeta))\right| d\zeta,
\end{aligned}$$

such that

$$\left|\Phi\Omega_{MC(m)}(t) - \Phi\Omega_{MC}(t)\right| \leq \left\|\mathcal{K}(., \Omega_{MC(m)}(.)) - \mathcal{K}(., \Omega_{MC}(.))\right\|_\infty \left(\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} + \frac{2\alpha T_{\max}}{(2-\alpha)\mathcal{U}(\alpha)}\right).$$

Since $\mathcal{K}$ is continuous, clearly,

$$\left\|\Phi\Omega_{MC(m)}(t) - \Phi\Omega_{MC}(t)\right\|_\infty \leq \left(\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} + \frac{2\alpha T_{\max}}{(2-\alpha)\mathcal{U}(\alpha)}\right)\left\|\mathcal{K}(., \Omega_{MC(m)}(.)) - \mathcal{K}(., \Omega_{MC}(.))\right\|_\infty.$$

Hence, $\left\|\Phi\Omega_{MC(m)}(t) - \Phi\Omega_{MC}(t)\right\|_\infty \to 0$ as $p \to \infty$.

STEP (2): Φ maps a "bounded set into another bounded" set in $C(\mathcal{J}, \mathbb{R})$.
Therefore, for every real number $\bar{k} > 0$, it can be shown that there exists an associated real number $\chi > o$ such that

$$\mathcal{B}_{\bar{k}} = \left\{\Omega_{MC} \in C(\mathcal{J}.\mathbb{R}) : \|\Omega_{MC}\|_\infty \leq \bar{k}\right\}, \quad \|\Phi\Omega_{MC}\|_\infty \leq \chi, \quad \forall \Omega_{MC} \in \mathcal{B}_{\bar{k}}.$$

Thus $\forall t \in \mathcal{J} = [0, T]$, from (16) and **(B3)** that

$$\begin{aligned} |\Phi\Omega_{MC}(t)| &\leq \frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} |\mathcal{K}(t, \Omega_{MC}(t))| + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)} \int_0^t |\mathcal{K}(\zeta, \Omega_{MC}(\zeta))| \, d\zeta \\ &\leq \mathcal{G}_{MC} \left(\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} + \frac{2\alpha T_{\max}}{(2-\alpha)\mathcal{U}(\alpha)} \right). \end{aligned}$$

Thus $\|\Phi\Omega_{MC}\|_\infty \leq \chi$, where

$$\chi = \mathcal{G}_{MC} \left(\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} + \frac{2\alpha T_{\max}}{(2-\alpha)\mathcal{U}(\alpha)} \right).$$

STEP (3): The operator $\Phi : C^m(\mathcal{J}, \mathbb{R}) \to C^m(\mathcal{J}, \mathbb{R})$ maps a bounded set into equi-continuous set in $C(\mathcal{J}, \mathbb{R})$. Let $t_1, t_2 \in \mathcal{J} = [0, T]$ and $t_2 > t_1$ and let $\mathcal{B}_{\bar{k}}$ be bounded set of $C(\mathcal{J}, \mathbb{R})$ as defined above and $\Omega_{MC} \in \mathcal{B}_{\bar{k}}$, then using (16), **(B3)** and triangle inequality, we have

$$\begin{aligned} |\Phi\Omega_{MC}(t_2) - \Phi\Omega_{MC}(t_1)| &= \left| \frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} (\mathcal{K}(t_2, \Omega_{MC}(t_2))) + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)} \int_0^{t_2} \mathcal{K}(\zeta, \Omega_{MC}(\zeta)) d\zeta \right. \\ &\quad \left. - \left(\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} (\mathcal{K}(t_1, \Omega_{MC}(t_1))) + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)} \int_0^{t_1} \mathcal{K}(\zeta, \Omega_{MC}(\zeta)) d\zeta \right) \right| \\ &= \left| \frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} (\mathcal{K}(t_2, \Omega_{MC}(t_2)) - \mathcal{K}(t_1, \Omega_{MC}(t_1))) + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)} \right. \\ &\quad \left. \times \left(\int_0^{t_2} \mathcal{K}(\zeta, \Omega_{MC(m)}(\zeta)) - \int_0^{t_2} \mathcal{K}(\zeta, \Omega_{MC}(\zeta)) \right) d\zeta \right| \\ &\leq \frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} (|\mathcal{K}(t_2, \Omega_{MC}(t_2)) - \mathcal{K}(t_1, \Omega_{MC}(t_1))|) \\ &\quad + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)} \int_{t_1}^{t_2} |\mathcal{K}(\zeta, \Omega_{MC}(\zeta))| \, d\zeta \\ &\leq \frac{2(1-\alpha)(2\mathcal{G}_{MC})}{(2-\alpha)\mathcal{U}(\alpha)} + \frac{2\alpha\mathcal{G}_{MC}}{(2-\alpha)\mathcal{U}(\alpha)} \int_{t_1}^{t_2} d\zeta \\ &= \mathcal{G}_{MC} \left(\frac{4(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} + \frac{2\alpha(t_2 - t_1)}{(2-\alpha)\mathcal{U}(\alpha)} \right). \end{aligned}$$

The right-hand side of the above inequality tends to zero as $t_1 \to t_2$. Thus, from **STEPS** (1) to (2) and also recalling the Arzela-Ascoli's theorem, the operator $\Phi : C^m(\mathcal{J}, \mathbb{R}) \to C^m(\mathcal{J}, \mathbb{R})$ is totally continuous.

STEP (4): The boundedness of priori: Let

$$\xi = \{\Omega_{MC} \in C(\mathcal{J}, \mathbb{R}) : \Omega_{MC} = \Lambda\Phi\Omega_{MC}\},$$

for some $\Lambda \in (0, 1)$. We show that set ξ is bounded. Let $\Omega_{MC} \in \xi$, then $\Omega_{MC} = \Lambda\Phi\Omega_{MC}$ for some $\Lambda \in (0, 1)$. Thus for all $t \in \mathcal{J}$, we have that

$$\begin{aligned} \Omega_{MC} &= \Lambda\Phi\Omega_{MC} \\ &= \Lambda \left(\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} (\mathcal{K}(t_2, \Omega_{MC}(t_2))) + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)} \int_0^{t_2} \mathcal{K}(\zeta, \Omega_{MC}(\zeta)) d\zeta \right). \end{aligned}$$

Using **(B3)** and **STEP (2)** we get

$$\begin{aligned}|\Omega_{MC}| &= \Lambda\left(\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)}|\mathcal{K}(t_2,\Omega_{MC}(t_2))| + \frac{2\alpha}{(2-\alpha)\mathcal{U}(\alpha)}\int_0^{t_2}|\mathcal{K}(\zeta,\Omega_{MC}(\zeta))|\,d\zeta\right)\\ &\leq \mathcal{G}\left(\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} + \frac{2\alpha T_{\max}}{(2-\alpha)\mathcal{U}(\alpha)}\right).\end{aligned}$$

Thus, for every $t \in \mathcal{J} = [0, T]$,

$$\|\Omega_{MC}\|_\infty \leq \mathcal{G}\left(\frac{2(1-\alpha)}{(2-\alpha)\mathcal{U}(\alpha)} + \frac{2\alpha T_{\max}}{(2-\alpha)\mathcal{U}(\alpha)}\right) := \chi, \qquad \chi \in \mathbb{R},$$

which is the boundedness of set ξ. Applying Schauder's fixed point theorem, the operator Φ possesses a unique fixed point which is the solution of the IVP (12). Thus malaria and cholera co–dynamic fractional order system (1) solution exists. ■

5 Numerical scheme

In this section, we apply a numerical algorithm to the proposed malaria and cholera fractional order system to obtain the numerical solution of the proposed system. The numerical scheme proposed and proved by Toufik and Atangana [42] which has a tremendous convergence property is applied to approximate the fractional order malaria and cholera system. Using the initial condition (2) and the Atangana–Baleanu integral (5), we obtain the following Atangana–Baleanu fractional Volterra equation of the system (1) as;

$$\begin{aligned}
\mathcal{S}_{HB}(t) - \mathcal{S}_{HB}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_1\left(t, \mathcal{S}_{HB}(t)\right) + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\mathcal{Q}_1\left(\zeta, \mathcal{S}_{HB}(\zeta)\right)d\zeta,\\
\mathcal{I}_{MAL}(t) - \mathcal{I}_{MAL}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_2\left(t, \mathcal{I}_{MAL}(t)\right) + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\mathcal{Q}_2\left(\zeta, \mathcal{I}_{MAL}(\zeta)\right)d\zeta,\\
\mathcal{R}_{MAL}(t) - \mathcal{R}_{MAL}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_3\left(t, \mathcal{R}_{MAL}(t)\right) + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\mathcal{Q}_3\left(\zeta, \mathcal{R}_{MAL}(\zeta)\right)d\zeta,\\
\mathcal{I}_{CHO}(t) - \mathcal{I}_{CHO}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_4\left(t, \mathcal{I}_{CHO}(t)\right) + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\mathcal{Q}_4\left(\zeta, \mathcal{I}_{CHO}(\zeta)\right)d\zeta,\\
\mathcal{R}_{CHO}(t) - \mathcal{R}_{CHO}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_5\left(t, \mathcal{R}_{CHO}(t)\right) + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\mathcal{Q}_5\left(\zeta, \mathcal{R}_{CHO}(\zeta)\right)d\zeta,\\
\mathcal{I}_{MAC}(t) - \mathcal{I}_{MAC}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_6\left(t, \mathcal{I}_{MAC}(t)\right) + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\mathcal{Q}_6\left(\zeta, \mathcal{I}_{MAC}(\zeta)\right)d\zeta,\\
\mathcal{R}_{MAC}(t) - \mathcal{R}_{MAC}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_7\left(t, \mathcal{R}_{MAC}(t)\right) + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\mathcal{Q}_7\left(\zeta, \mathcal{R}_{MAC}(\zeta)\right)d\zeta,\\
\mathcal{B}_{CHO}(t) - \mathcal{B}_{CHO}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_8\left(t, \mathcal{B}_{CHO}(t)\right) + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\mathcal{Q}_8\left(\zeta, \mathcal{B}_{CHO}(\zeta)\right)d\zeta,\\
\mathcal{S}_{VEC}(t) - \mathcal{S}_{VEC}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_9\left(t, \mathcal{S}_{VEC}(t)\right) + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\mathcal{Q}_9\left(\zeta, \mathcal{S}_{VEC}(\zeta)\right)d\zeta,\\
\mathcal{I}_{VEC}(t) - \mathcal{I}_{VEC}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_{10}\left(t, \mathcal{I}_{VEC}(t)\right) + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\mathcal{Q}_{10}\left(\zeta, \mathcal{I}_{VEC}(\zeta)\right)d\zeta.
\end{aligned} \tag{17}$$

Next, setting $t = t_{m+1}$ for $m = 0, 1, 2, \ldots$ into the above equation (17), we get

$$\begin{aligned}
\mathcal{S}_{HB}(t_{m+1}) - \mathcal{S}_{HB}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_1\left(t_m, \mathcal{S}_{HB}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\int_0^{t_{k+1}}(t_{m+1}-\zeta)^{\alpha-1}\mathcal{Q}_1\left(\zeta, \mathcal{S}_{HB}(\zeta)\right)d\zeta, \\
\mathcal{I}_{MAL}(t_{m+1}) - \mathcal{I}_{MAL}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_2\left(t_m, \mathcal{I}_{MAL}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\int_0^{t_{k+1}}(t_{m+1}-\zeta)^{\alpha-1}\mathcal{Q}_2\left(\zeta, \mathcal{I}_{MAL}(\zeta)\right)d\zeta, \\
\mathcal{R}_{MAL}(t_{m+1}) - \mathcal{R}_{MAL}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_3\left(t_m, \mathcal{R}_{MAL}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\int_0^{t_{k+1}}(t_{m+1}-\zeta)^{\alpha-1}\mathcal{Q}_3\left(\zeta, \mathcal{R}_{MAL}(\zeta)\right)d\zeta, \\
\mathcal{I}_{CHO}(t_{m+1}) - \mathcal{I}_{CHO}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_4\left(t_m, \mathcal{I}_{CHO}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\int_0^{t_{k+1}}(t_{m+1}-\zeta)^{\alpha-1}\mathcal{Q}_4\left(\zeta, \mathcal{I}_{CHO}(\zeta)\right)d\zeta, \\
\mathcal{R}_{CHO}(t_{m+1}) - \mathcal{R}_{CHO}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_5\left(t_m, \mathcal{R}_{CHO}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\int_0^{t_{k+1}}(V-\zeta)^{\alpha-1}\mathcal{Q}_5\left(\zeta, \mathcal{R}_{CHO}(\zeta)\right)d\zeta, \\
\mathcal{I}_{MAC}(t_{m+1}) - \mathcal{I}_{MAC}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_6\left(t_m, \mathcal{I}_{MAC}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\int_0^{t_{k+1}}(t_{m+1}-\zeta)^{\alpha-1}\mathcal{Q}_6\left(\zeta, \mathcal{I}_{MAC}(\zeta)\right)d\zeta, \\
\mathcal{R}_{MAC}(t_{m+1}) - \mathcal{R}_{MAC}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_7\left(t_m, \mathcal{R}_{MAC}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\int_0^{t_{k+1}}(t_{m+1}-\zeta)^{\alpha-1}\mathcal{Q}_7\left(\zeta, \mathcal{R}_{MAC}(\zeta)\right)d\zeta, \\
\mathcal{B}_{CHO}(t_{m+1}) - \mathcal{B}_{CHO}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_8\left(t_m, \mathcal{B}_{CHO}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\int_0^{t_{k+1}}(t_{m+1}-\zeta)^{\alpha-1}\mathcal{Q}_8\left(\zeta, \mathcal{B}_{CHO}(\zeta)\right)d\zeta, \\
\mathcal{S}_{VEC}(t_{m+1}) - \mathcal{S}_{VEC}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_9\left(t_m, \mathcal{S}_{VEC}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\int_0^{t_{k+1}}(t_{m+1}-\zeta)^{\alpha-1}\mathcal{Q}_9\left(\zeta, \mathcal{S}_{VEC}(\zeta)\right)d\zeta, \\
\mathcal{I}_{VEC}(t_{m+1}) - \mathcal{I}_{VEC}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_{10}\left(t_m, \mathcal{I}_{VEC}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\int_0^{t_{k+1}}(t_{m+1}-\zeta)^{\alpha-1}\mathcal{Q}_{10}\left(\zeta, \mathcal{I}_{VEC}(\zeta)\right)d\zeta.
\end{aligned}$$

By two–point Lagrange interpolation polynomial, we approximate $\mathcal{Q}_1(\zeta, \mathcal{S}_{HB}(\zeta))$, $\mathcal{Q}_2(\zeta, \mathcal{I}_{MAL}(\zeta))$, $\mathcal{Q}_3(\zeta, \mathcal{R}_{MAL}(\zeta))$, $\mathcal{Q}_4(\zeta, \mathcal{I}_{CHO}(\zeta))$, $\mathcal{Q}_5(\zeta, \mathcal{R}_{CHO}(\zeta))$, $\mathcal{Q}_6(\zeta, \mathcal{I}_{MAC}(\zeta))$, $\mathcal{Q}_7(\zeta, \mathcal{R}_{MAC}(\zeta))$, $\mathcal{Q}_8(\zeta, \mathcal{B}_{CHO}(\zeta))$, $\mathcal{Q}_9(\zeta, \mathcal{S}_{VEC}(\zeta))$, $\mathcal{Q}_{10}(\zeta, \mathcal{I}_{VEC}(\zeta))$ in (18) on the interval $[t_k, t_{k+1}]$ and get

$$
\begin{aligned}
\mathcal{Q}_1(\zeta, \mathcal{S}_{HB}(\zeta)) &\approx \frac{\mathcal{Q}_1(t_k, \mathcal{S}_{HB}(t_k))}{h}(t - t_{k-1}) + \frac{\mathcal{Q}_1(t_{k-1}, \mathcal{S}_{HB}(t_{k-1}))}{h}(t - t_k),\\
\mathcal{Q}_2(\zeta, \mathcal{I}_{MAL}(\zeta)) &\approx \frac{\mathcal{Q}_2(t_k, \mathcal{I}_{MAL}(t_k))}{h}(t - t_{k-1}) + \frac{\mathcal{Q}_2(t_{k-1}, \mathcal{I}_{MAL}(t_{k-1}))}{h}(t - t_k),\\
\mathcal{Q}_3(\zeta, \mathcal{R}_{MAL}(\zeta)) &\approx \frac{\mathcal{Q}_3(t_k, \mathcal{R}_{MAL}(t_k))}{h}(t - t_{k-1}) + \frac{\mathcal{Q}_3(t_{k-1}, \mathcal{R}_{MAL}(t_{k-1}))}{h}(t - t_k),\\
\mathcal{Q}_4(\zeta, \mathcal{I}_{CHO}(\zeta)) &\approx \frac{\mathcal{Q}_4(t_k, \mathcal{I}_{CHO}(t_k))}{h}(t - t_{k-1}) + \frac{\mathcal{Q}_4(t_{k-1}, \mathcal{I}_{CHO}(t_{k-1}))}{h}(t - t_k),\\
\mathcal{Q}_5(\zeta, \mathcal{R}_{CHO}(\zeta)) &\approx \frac{\mathcal{Q}_5(t_k, \mathcal{R}_{CHO}(t_k))}{h}(t - t_{k-1}) + \frac{\mathcal{Q}_5(t_{k-1}, \mathcal{R}_{CHO}(t_{k-1}))}{h}(t - t_k),\\
\mathcal{Q}_6(\zeta, \mathcal{I}_{MAC}(\zeta)) &\approx \frac{\mathcal{Q}_6(t_k, \mathcal{I}_{MAC}(t_k))}{h}(t - t_{k-1}) + \frac{\mathcal{Q}_6(t_{k-1}, \mathcal{I}_{MAC}(t_{k-1}))}{h}(t - t_k),\\
\mathcal{Q}_7(\zeta, \mathcal{R}_{MAC}(\zeta)) &\approx \frac{\mathcal{Q}_7(t_k, \mathcal{R}_{MAC}(t_k))}{h}(t - t_{k-1}) + \frac{\mathcal{Q}_7(t_{k-1}, \mathcal{R}_{MAC}(t_{k-1}))}{h}(t - t_k),\\
\mathcal{Q}_8(\zeta, \mathcal{B}_{CHO}(\zeta)) &\approx \frac{\mathcal{Q}_8(t_k, \mathcal{B}_{CHO}(t_k))}{h}(t - t_{k-1}) + \frac{\mathcal{Q}_8(t_{k-1}, \mathcal{B}_{CHO}(t_{k-1}))}{h}(t - t_k),\\
\mathcal{Q}_9(\zeta, \mathcal{S}_{VEC}(\zeta)) &\approx \frac{\mathcal{Q}_9(t_k, \mathcal{S}_{VEC}(t_k))}{h}(t - t_{k-1}) + \frac{\mathcal{Q}_9(t_{k-1}, \mathcal{S}_{VEC}(t_{k-1}))}{h}(t - t_k),\\
\mathcal{Q}_{10}(\zeta, \mathcal{I}_{VEC}(\zeta)) &\approx \frac{\mathcal{Q}_{10}(t_k, \mathcal{I}_{VEC}(t_k))}{h}(t - t_{k-1}) + \frac{\mathcal{Q}_{10}(t_{k-1}, \mathcal{I}_{VEC}(t_{k-1}))}{h}(t - t_k),
\end{aligned}
\tag{18}
$$

so that system (18) becomes

$$
\begin{aligned}
\mathcal{S}_{HB}(t_{m+1}) - \mathcal{S}_{HB}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_1(t_m, \mathcal{S}_{HB}(t_m))\\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\left(\frac{\mathcal{Q}_1(t_k, \mathcal{S}_{HB}(t_k))}{h}\mathcal{P}_{k-1,\alpha} + \frac{\mathcal{Q}_1(t_{k-1}, \mathcal{S}_{HB}(t_{k-1}))}{h}\mathcal{P}_{k,\alpha}\right),\\
\mathcal{I}_{MAL}(t_{m+1}) - \mathcal{I}_{MAL}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_2(t_m, \mathcal{I}_{MAL}(t_m))\\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\left(\frac{\mathcal{Q}_2(t_k, \mathcal{I}_{MAL}(t_k))}{h}\mathcal{P}_{k-1,\alpha} + \frac{\mathcal{Q}_2(t_{k-1}, \mathcal{I}_{MAL}(t_{k-1}))}{h}\mathcal{P}_{k,\alpha}\right),\\
\mathcal{R}_{MAL}(t_{m+1}) - \mathcal{R}_{MAL}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_3(t_m, \mathcal{R}_{MAL}(t_m))\\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\left(\frac{\mathcal{Q}_3(t_k, \mathcal{R}_{MAL}(t_k))}{h}\mathcal{P}_{k-1,\alpha} + \frac{\mathcal{Q}_3(t_{k-1}, \mathcal{R}_{MAL}(t_{k-1}))}{h}\mathcal{P}_{k,\alpha}\right),\\
\mathcal{I}_{CHO}(t_{m+1}) - \mathcal{I}_{CHO}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_4(t_m, \mathcal{I}_{CHO}(t_m))\\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\left(\frac{\mathcal{Q}_4(t_k, \mathcal{I}_{CHO}(t_k))}{h}\mathcal{P}_{k-1,\alpha} + \frac{\mathcal{Q}_4(t_{k-1}, \mathcal{I}_{CHO}(t_{k-1}))}{h}\mathcal{P}_{k,\alpha}\right),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_{CHO}(t_{m+1}) - \mathcal{R}_{CHO}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_5\left(t_m, \mathcal{R}_{CHO}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\left(\frac{\mathcal{Q}_5\left(t_k, \mathcal{R}_{CHO}(t_k)\right)}{h}\mathcal{P}_{k-1,\alpha} + \frac{\mathcal{Q}_5\left(t_{k-1}, \mathcal{R}_{CHO}(t_{k-1})\right)}{h}\mathcal{P}_{k,\alpha}\right), \\
\mathcal{I}_{MAC}(t_{m+1}) - \mathcal{I}_{MAC}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_6\left(t_m, \mathcal{I}_{MAC}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\left(\frac{\mathcal{Q}_6\left(t_k, \mathcal{I}_{MAC}(t_k)\right)}{h}\mathcal{P}_{k-1,\alpha} + \frac{\mathcal{Q}_6\left(t_{k-1}, \mathcal{I}_{MAC}(t_{k-1})\right)}{h}\mathcal{P}_{k,\alpha}\right), \\
\mathcal{R}_{MAC}(t_{m+1}) - \mathcal{R}_{MAC}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_7\left(t_m, \mathcal{R}_{MAC}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\left(\frac{\mathcal{Q}_1\left(t_k, \mathcal{R}_{MAC}(t_k)\right)}{h}\mathcal{P}_{k-1,\alpha} + \frac{\mathcal{Q}_7\left(t_{k-1}, \mathcal{R}_{MAC}(t_{k-1})\right)}{h}\mathcal{P}_{k,\alpha}\right), \\
\mathcal{B}_{CHO}(t_{m+1}) - \mathcal{B}_{CHO}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_8\left(t_m, \mathcal{B}_{CHO}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\left(\frac{\mathcal{Q}_8\left(t_k, \mathcal{B}_{CHO}(t_k)\right)}{h}\mathcal{P}_{k-1,\alpha} + \frac{\mathcal{Q}_8\left(t_{k-1}, \mathcal{B}_{CHO}(t_{k-1})\right)}{h}\mathcal{P}_{k,\alpha}\right), \\
\mathcal{S}_{VEC}(t_{m+1}) - \mathcal{S}_{VEC}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_9\left(t_m, \mathcal{S}_{VEC}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\left(\frac{\mathcal{Q}_9\left(t_k, \mathcal{S}_{VEC}(t_k)\right)}{h}\mathcal{P}_{k-1,\alpha} + \frac{\mathcal{Q}_9\left(t_{k-1}, \mathcal{S}_{VEC}(t_{k-1})\right)}{h}\mathcal{P}_{k,\alpha}\right), \\
\mathcal{I}_{VEC}(t_{m+1}) - \mathcal{I}_{VEC}(0) &= \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_{10}\left(t_m, \mathcal{I}_{VEC}(t_m)\right) \\
&\quad + \frac{\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha)}\sum_{k=0}^{m}\left(\frac{\mathcal{Q}_{10}\left(t_k, \mathcal{I}_{VEC}(t_k)\right)}{h}\mathcal{P}_{k-1,\alpha} + \frac{\mathcal{Q}_{10}\left(t_{k-1}, \mathcal{I}_{VEC}(t_{k-1})\right)}{h}\mathcal{P}_{k,\alpha}\right),
\end{aligned}
\tag{19}
$$

where and using $t_k = kh$, we obtain

$$
\begin{aligned}
\mathcal{P}_{k-1,\alpha} &= \int_{t_k}^{t_{k+1}} (t - t_{k-1})(t_{m+1} - t)^{\alpha-1} dt \\
&= \frac{h^{\alpha+1}}{\alpha(\alpha+1)}\Big[(m+1-k)^{\alpha}(m-k+2+\alpha) - (m-k)^{\alpha}(m-k+2+2\alpha)\Big], \\
\mathcal{P}_{k,\alpha} &= \int_{t_k}^{t_{k+1}} (t - t_k)(t_{m+1} - t)^{\alpha-1} dt = \frac{h^{\alpha+1}}{\alpha(\alpha+1)}\Big[(m+1-k)^{\alpha+1} - (m-k)^{\alpha}(m-k+1+\alpha)\Big].
\end{aligned}
\tag{20}
$$

Substituting (20) into (19) gives

$$
\begin{aligned}
\mathcal{S}_{HB}(t_{m+1}) &= \mathcal{S}_{HB}(0) + \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_1\left(t_m, \mathcal{S}_{HB}(t_m)\right) + \frac{\alpha h^{\alpha}}{\mathcal{U}(\alpha)\Gamma(\alpha+2)} \\
&\quad \times \sum_{k=0}^{m}\Bigg[\mathcal{Q}_1\left(t_k, \mathcal{S}_{HB}(t_k)\right)\Big((m+1-k)^{\alpha}(m-k+2+\alpha) - (m-k)^{\alpha}(m-k+2+2\alpha)\Big) \\
&\quad + \mathcal{Q}_1\left(t_{k-1}, \mathcal{S}_{HB}(t_{k-1})\right)\Big((m+1-k)^{\alpha+1} - (m-k)^{\alpha}(m-k+1+\alpha)\Big)\Bigg].
\end{aligned}
$$

$$\mathcal{I}_{MAL}(t_{m+1}) = \mathcal{I}_{MAL}(0) + \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_2\left(t_m, \mathcal{I}_{MAL}(t_m)\right) + \frac{\alpha h^\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha+2)}$$
$$\times \sum_{k=0}^{m}\left[\mathcal{Q}_2\left(t_k, \mathcal{I}_{MAL}(t_k)\right)\left((m+1-k)^\alpha(m-k+2+\alpha) - (m-k)^\alpha(m-k+2+2\alpha)\right)\right.$$
$$\left. + \mathcal{Q}_2\left(t_{k-1}, \mathcal{I}_{MAL}(t_{k-1})\right)\left((m+1-k)^{\alpha+1} - (m-k)^\alpha(m-k+1+\alpha)\right)\right].$$

$$\mathcal{R}_{MAL}(t_{m+1}) = \mathcal{R}_{MAL}(0) + \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_3\left(t_m, \mathcal{R}_{MAL}(t_m)\right) + \frac{\alpha h^\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha+2)}$$
$$\times \sum_{k=0}^{m}\left[\mathcal{Q}_3\left(t_k, \mathcal{R}_{MAL}(t_k)\right)\left((m+1-k)^\alpha(m-k+2+\alpha) - (m-k)^\alpha(m-k+2+2\alpha)\right)\right.$$
$$\left. + \mathcal{Q}_3\left(t_{k-1}, \mathcal{R}_{MAL}(t_{k-1})\right)\left((m+1-k)^{\alpha+1} - (m-k)^\alpha(m-k+1+\alpha)\right)\right].$$

$$\mathcal{I}_{CHO}(t_{m+1}) = \mathcal{I}_{CHO}(0) + \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_4\left(t_m, \mathcal{I}_{CHO}(t_m)\right) + \frac{\alpha h^\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha+2)}$$
$$\times \sum_{k=0}^{m}\left[\mathcal{Q}_4\left(t_k, \mathcal{I}_{CHO}(t_k)\right)\left((m+1-k)^\alpha(m-k+2+\alpha) - (m-k)^\alpha(m-k+2+2\alpha)\right)\right.$$
$$\left. + \mathcal{Q}_4\left(t_{k-1}, \mathcal{I}_{CHO}(t_{k-1})\right)\left((m+1-k)^{\alpha+1} - (m-k)^\alpha(m-k+1+\alpha)\right)\right].$$

$$\mathcal{R}_{CHO}(t_{m+1}) = \mathcal{R}_{CHO}(0) + \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_5\left(t_m, \mathcal{R}_{CHO}(t_m)\right) + \frac{\alpha h^\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha+2)}$$
$$\times \sum_{k=0}^{m}\left[\mathcal{Q}_5\left(t_k, \mathcal{R}_{CHO}(t_k)\right)\left((m+1-k)^\alpha(m-k+2+\alpha) - (m-k)^\alpha(m-k+2+2\alpha)\right)\right.$$
$$\left. + \mathcal{Q}_5\left(t_{k-1}, \mathcal{R}_{CHO}(t_{k-1})\right)\left((m+1-k)^{\alpha+1} - (m-k)^\alpha(m-k+1+\alpha)\right)\right].$$

$$\mathcal{I}_{MAC}(t_{m+1}) = \mathcal{I}_{MAC}(0) + \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_6\left(t_m, \mathcal{I}_{MAC}(t_m)\right) + \frac{\alpha h^\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha+2)}$$
$$\times \sum_{k=0}^{m}\left[\mathcal{Q}_6\left(t_k, \mathcal{I}_{MAC}(t_k)\right)\left((m+1-k)^\alpha(m-k+2+\alpha) - (m-k)^\alpha(m-k+2+2\alpha)\right)\right.$$
$$\left. + \mathcal{Q}_6\left(t_{k-1}, \mathcal{I}_{MAC}(t_{k-1})\right)\left((m+1-k)^{\alpha+1} - (m-k)^\alpha(m-k+1+\alpha)\right)\right].$$

$$\mathcal{R}_{MAC}(t_{m+1}) = \mathcal{R}_{MAC}(0) + \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_7\left(t_m, \mathcal{R}_{MAC}(t_m)\right) + \frac{\alpha h^\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha+2)}$$
$$\times \sum_{k=0}^{m}\left[\mathcal{Q}_7\left(t_k, \mathcal{R}_{MAC}(t_k)\right)\left((m+1-k)^\alpha(m-k+2+\alpha) - (m-k)^\alpha(m-k+2+2\alpha)\right)\right.$$
$$\left. + \mathcal{Q}_7\left(t_{k-1}, \mathcal{R}_{MAC}(t_{k-1})\right)\left((m+1-k)^{\alpha+1} - (m-k)^\alpha(m-k+1+\alpha)\right)\right].$$

$$\mathcal{B}_{CHO}(t_{m+1}) = \mathcal{B}_{CHO}(0) + \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_8\left(t_m, \mathcal{B}_{CHO}(t_m)\right) + \frac{\alpha h^\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha+2)}$$
$$\times \sum_{k=0}^{m}\left[\mathcal{Q}_8\left(t_k, \mathcal{B}_{CHO}(t_k)\right)\left((m+1-k)^\alpha(m-k+2+\alpha) - (m-k)^\alpha(m-k+2+2\alpha)\right)\right.$$
$$\left. + \mathcal{Q}_8\left(t_{k-1}, \mathcal{B}_{CHO}(t_{k-1})\right)\left((m+1-k)^{\alpha+1} - (m-k)^\alpha(m-k+1+\alpha)\right)\right].$$

$$\mathcal{S}_{VEC}(t_{m+1}) = \mathcal{S}_{VEC}(0) + \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_9\left(t_m, \mathcal{S}_{VEC}(t_m)\right) + \frac{\alpha h^\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha+2)}$$
$$\times \sum_{k=0}^{m}\left[\mathcal{Q}_9\left(t_k, \mathcal{S}_{VEC}(t_k)\right)\left((m+1-k)^\alpha(m-k+2+\alpha) - (m-k)^\alpha(m-k+2+2\alpha)\right)\right.$$
$$\left. + \mathcal{Q}_9\left(t_{k-1}, \mathcal{S}_{VEC}(t_{k-1})\right)\left((m+1-k)^{\alpha+1} - (m-k)^\alpha(m-k+1+\alpha)\right)\right].$$

$$\mathcal{I}_{VEC}(t_{m+1}) = \mathcal{I}_{VEC}(0) + \frac{(1-\alpha)}{\mathcal{U}(\alpha)}\mathcal{Q}_{10}\left(t_m, \mathcal{I}_{VEC}(t_m)\right) + \frac{\alpha h^\alpha}{\mathcal{U}(\alpha)\Gamma(\alpha+2)}$$
$$\times \sum_{k=0}^{m}\left[\mathcal{Q}_{10}\left(t_k, \mathcal{I}_{VEC}(t_k)\right)\left((m+1-k)^\alpha(m-k+2+\alpha) - (m-k)^\alpha(m-k+2+2\alpha)\right)\right.$$
$$\left. + \mathcal{Q}_{10}\left(t_{k-1}, \mathcal{I}_{VEC}(t_{k-1})\right)\left((m+1-k)^{\alpha+1} - (m-k)^\alpha(m-k+1+\alpha)\right)\right].$$

6 Results and discussion

We present the results of the subject matter with the aid of the above numerical scheme for the numerical solution of the proposed malaria and cholera system. For this purpose, we will adopt the data in Table 1 and support it with the following initial conditions for each compartment $\mathcal{S}_{HB}(0) = 10000$, $\mathcal{I}_{MAL}(0) = 2000$, $\mathcal{R}_{MAL}(0) = 300$, $\mathcal{I}_{CHO}(0) = 400$, $\mathcal{R}_{CHO}(0) = 300$, $\mathcal{I}_{MAC}(0) = 200$, $\mathcal{R}_{MAC}(0) = 100$, $\mathcal{B}_{CHO}(0) = 2000$, $\mathcal{S}_{VEC}(0) = 2000$, $\mathcal{I}_{VEC}(0) = 2000$ and the fractional orders 0.50, 0.60, 0.70, 0.80, 0.90 to plot all the ten classes in malaria and cholera dynamic system. The dynamic behavior of all the compartments is shown in Figs. 1–10. The susceptible populations to these diseases represented in Fig. 1 increase mildly until infected by the diseases and transfer to other compartments within the system (1).

The mild increase of susceptible populations without interaction with the diseases occurs as the fractional order tends to one. Fig. 2 shows the infection with malaria caused a transitional increase with time in the population, then decreases and stabilizes after precautionary measures were applied. A considerable response within the infected class as individuals respond to treatment and necessary malaria control measures can be seen in Fig. 3.

As individuals recover from malaria and are infected by cholera, it can be seen from Fig. 4 that a sharp decrease occurs as the fractional order increases. As individuals recover from cholera after being infected through the necessary, an increase is recorded as depicted in Fig. 5. It can be seen in Figs. 6 and 8 that there is a decrease within the population as co-infection of malaria and cholera occurs.

The presence of the bacteria compartment facilitates this decrease as fractional order decreases and converges to a certain point as time increases. The increase is seen in Fig. 7. The numerous numbers of susceptible vectors show a correlation with the fractional order in Fig. 9. An increase in the fractional order increases the number of susceptible vectors. After a stable population of about a hundred individuals for the first three days, a decrease in the population of infectious vectors is noticed when the fractional order increases as shown in Fig. 10. This explains the level of infection in the vector population. Also, infection with cholera reduces more population of individuals compared to infection with malaria within the same time interval.

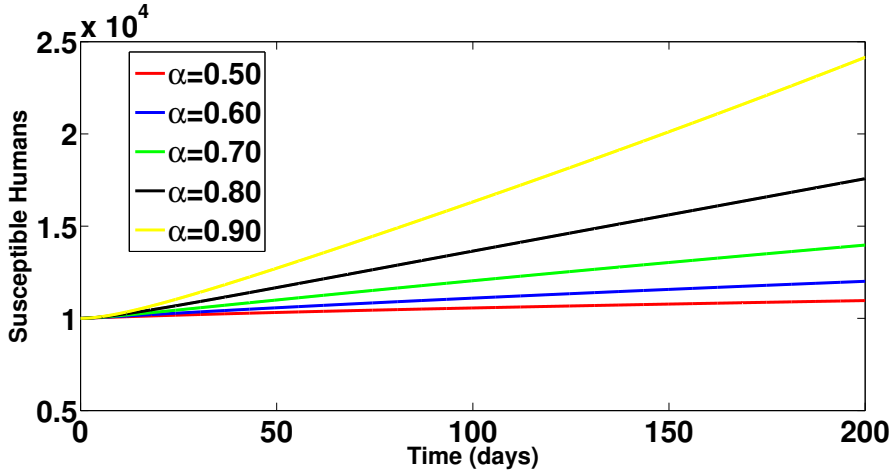

Figure 1. Simulations of the total number of $\mathcal{S}_{HB}(t)$ at different values of α on the interval of $(0, 1)$ with the stated initial data.

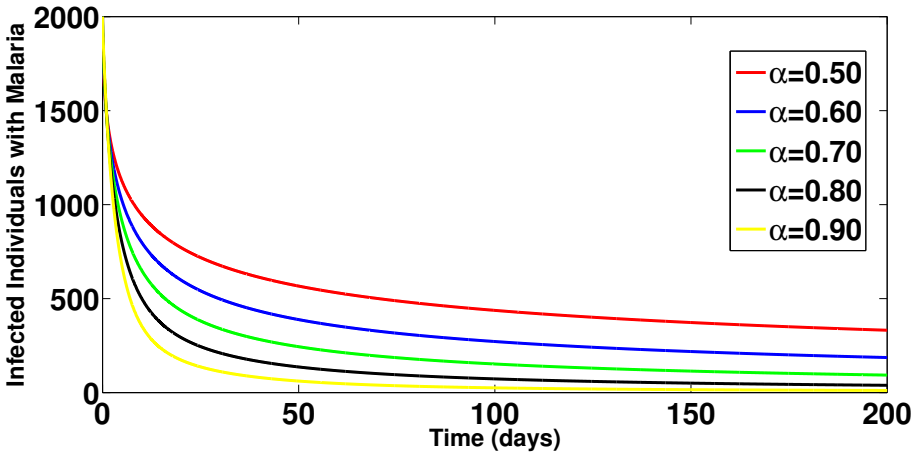

Figure 2. Simulations of the total number of $\mathcal{I}_{MAL}(t)$ at different values of α on the interval of $(0, 1)$ with the stated initial data.

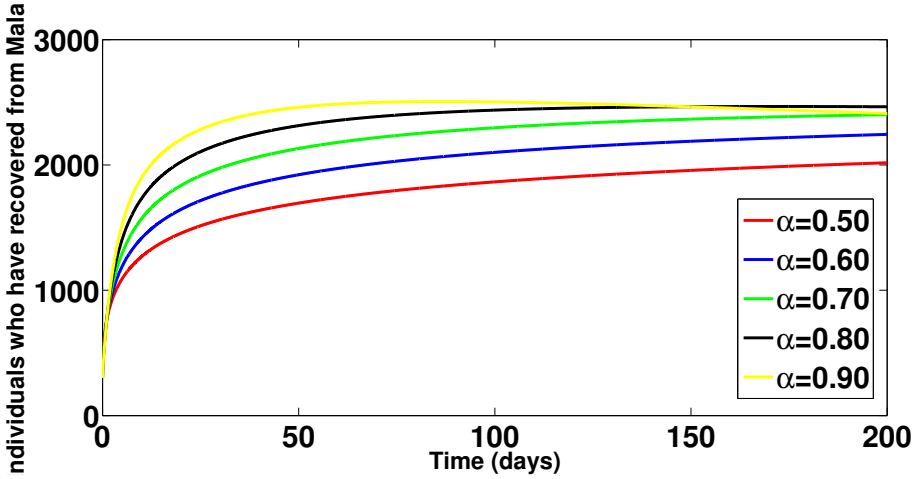

Figure 3. Simulations of the total number of $\mathcal{R}_{MAL}(t)$ at different values of α on the interval of $(0, 1)$ with the stated initial data.

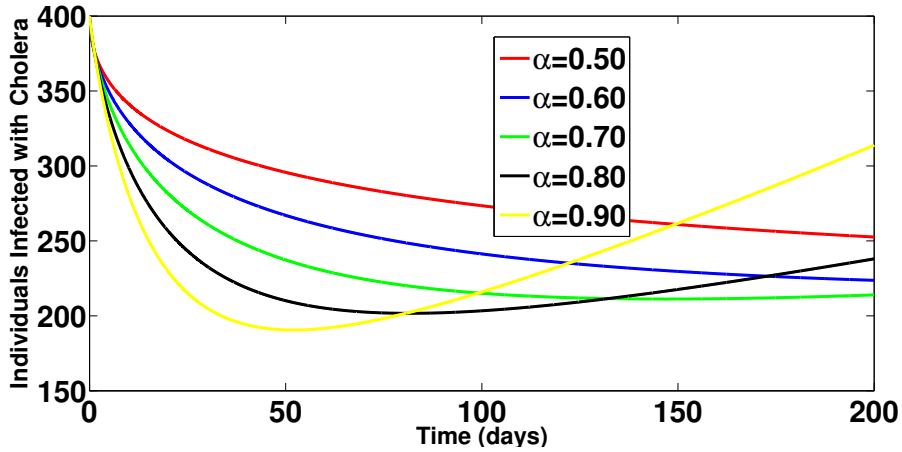

Figure 4. Simulations of the total number of $\mathcal{I}_{CHO}(t)$ at different values of α on the interval of $(0, 1)$ with the stated initial data.

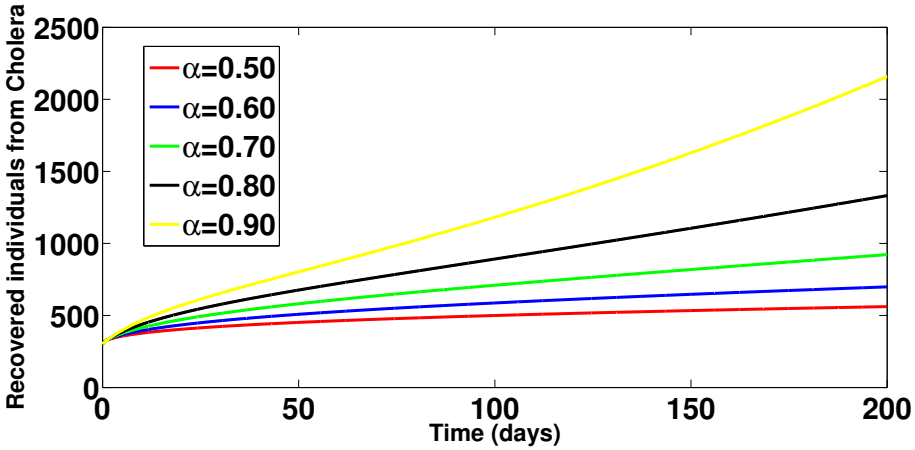

Figure 5. Simulations of the total number of $\mathcal{R}_{CHO}(t)$ at different values of α on the interval of $(0, 1)$ with the stated initial data.

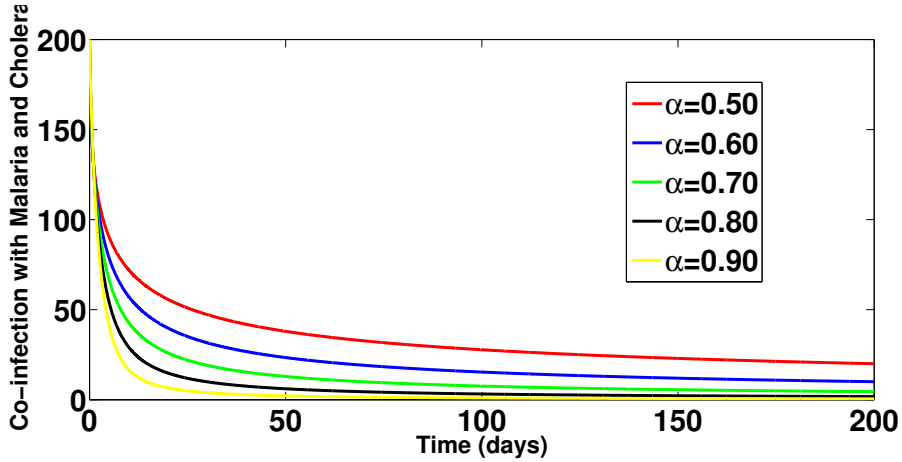

Figure 6. Simulations of the total number of $\mathcal{I}_{MAC}(t)$ at different values of α on the interval of $(0, 1)$ with the stated initial data.

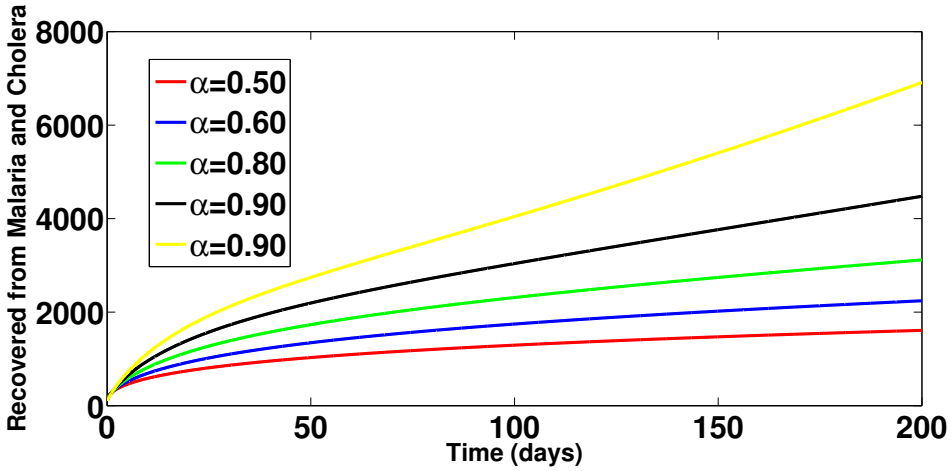

Figure 7. Simulations of the total number of $\mathcal{R}_{MAC}(t)$ at different values of α on the interval of $(0,1)$ with the stated initial data.

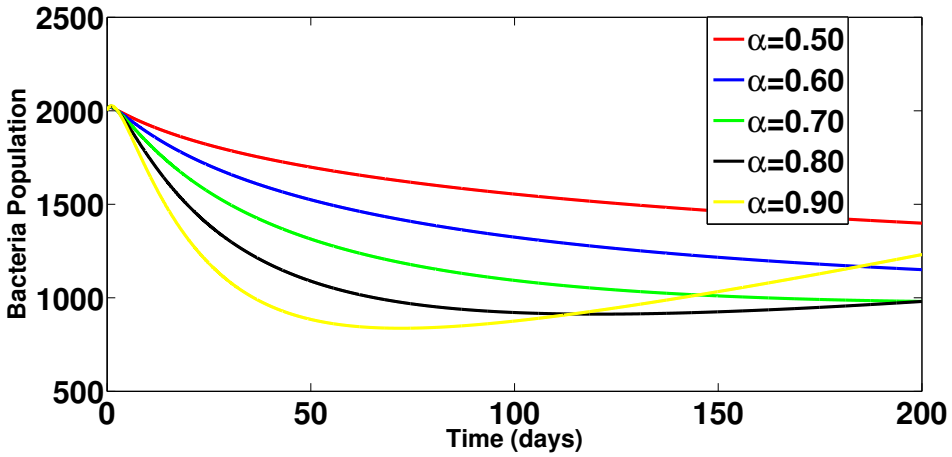

Figure 8. Simulations of the total number of $\mathcal{B}_{CHO}(t)$ at different values of α on the interval of $(0,1)$ with the stated initial data.

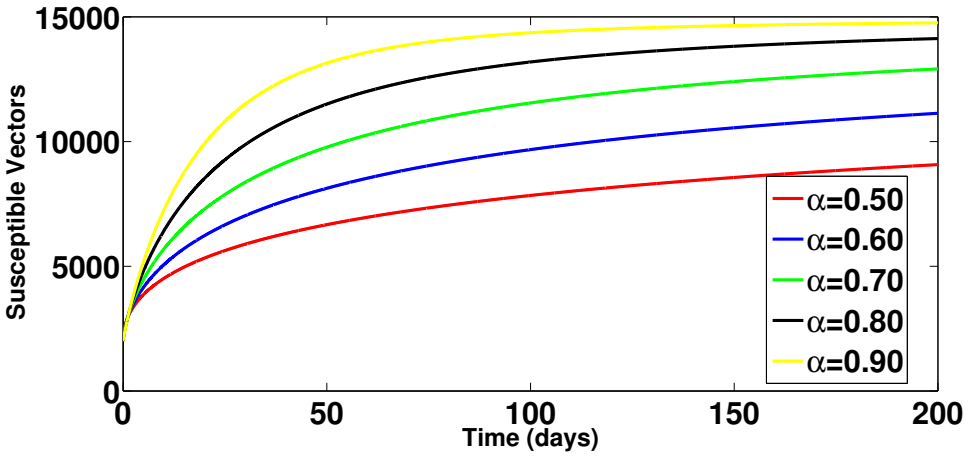

Figure 9. Simulations of the total number of $\mathcal{S}_{VEC}(t)$ at different values of α on the interval of $(0,1)$ with the stated initial data.

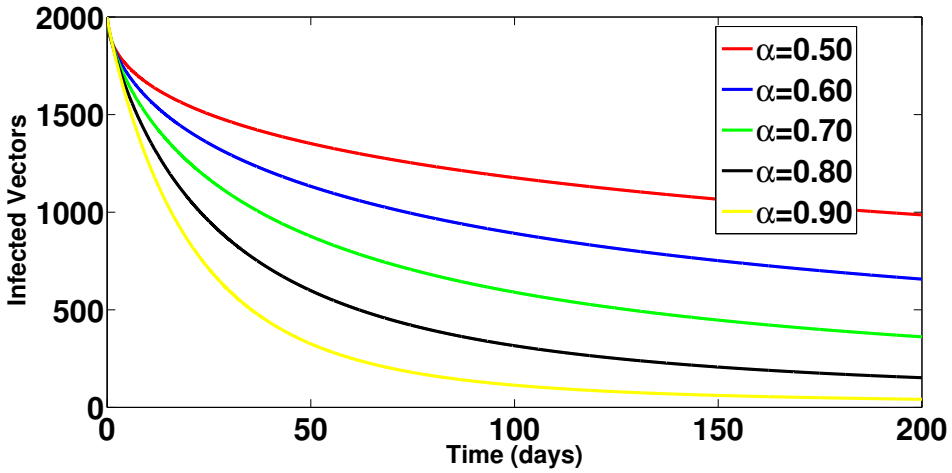

Figure 10. Simulations of the total number of $\mathcal{I}_{VEC}(t)$ at different values of α on the interval of $(0, 1)$ with the stated initial data.

7 Conclusion

In this paper, we investigated the fractional co-infection model of malaria and cholera in detail. We also established the existence and uniqueness of the solution using Banach and Schauder's fixed point theorems. The positivity and boundedness of the fractional system solution are stated and proved by using Mittag–Leffler function. The basic reproduction number R_0 is computed using the next-generation matrix method and it reveals that malaria–cholera model is locally asymptotically stable when $R_0 < 1$. Several simulations on the model were performed numerically and we obtained various graphical results that align with the theoretical result obtained. Further results revealed that infection with cholera reduces more population of individuals compared to infection with malaria during the same time interval. For the future research interests of this work, we recommend consideration of control measures and other fractional derivatives for this purpose.

Declarations

Consent for publication

Not applicable.

Conflicts of interest

The authors declare that they have no conflict of interest.

Funding

Not applicable.

Author's contributions

L.L.I.: Conceptualization, Software, Formal Analysis, Writing - Original draft, U.K.N.: Methodology, Supervision, Writing - Review and Editing, A.O.A.: Formal Analysis, Validation, A.B.P.: Formal Analysis, Validation, K.U.E.: Visualization, Data Curation, All authors discussed the results and contributed to the final manuscript.

Acknowledgements

Not applicable.

References

[1] Birhanie, M., Tessema, B., Ferede, G., Endris, M., & Enawgaw, B. Malaria, typhoid fever, and their coinfection among febrile patients at a rural health center in Northwest Ethiopia: a cross-sectional study. *Advances in Medicine*, 531074, (2014). [CrossRef]

[2] World malaria report 2019. https://www.who.int/malaria/publications/world-malaria-report-2019/en/.(2019), Meeting Report, Access date: 25th November 2022.

[3] World Health Organization, www.who.int/news-room, Access date: 30th March 2022.

[4] Centres for Disease Control and Prevention (CDC). Cholera - Vibrio cholera infection, (2020). https://www.cdc.gov/cholera/general/index.html. Access date: 12th August 2022.

[5] Ross, S. *The Prevention of Malaria* Dutton: New York, NY, USA, (1911).

[6] Okosun, K.O. & Makinde O.D. A co-infection model of malaria and cholera diseases with optimal control. *Mathematical Biosciences*, 258, 19-32, (2014). [CrossRef]

[7] Egeonu, K.U., Omame, A., & Inyama, S.C. A co-infection model for two-strain malaria and cholera with optimal control. *International Journal of Dynamics and Control*, 9, 1612–1632, (2021). [CrossRef]

[8] Mandal, S., Sarkar, R.R., & Sinha, S. Mathematical models of malaria-a review. *Malaria Journal*, 10, 202, (2011). [CrossRef]

[9] Oke, S.I., Ojo, M.M., Adeniyi, M.O., & Matadi, M.B. Mathematical modeling of malaria disease with control strategy. *Communication in Mathematical Biology and Neuroscience*, (2020). [CrossRef]

[10] Osman, M.A.E., Adu, I.K., Simple mathematical model for malaria transmission. *Journal of Advances in Mathematics and Computer Science*, 25(6), 1-24, (2017). [CrossRef]

[11] Tilahun, G.T., Woldegerima, W.A., & Wondifraw, A. Stochastic and deterministic mathematical model of cholera disease dynamics with direct transmission. *Advances in Difference Equation*, 2020, (2020). [CrossRef]

[12] Hntsa, K.H., & Kahsay, B.N. Analysis of cholera epidemic controlling using mathematical modeling. *International Journal of Mathematics and Mathematical Sciences*, 2020, 1-13, (2020). [CrossRef]

[13] Nwajeri, U.K., Panle, A.B., Omame, A., Obi M.C., & Onyenegecha, C.P. On the fractional order model for HPV and Syphilis using non-singular kernel. *Results in Physics*, 37, 105463, (2022). [CrossRef]

[14] Omame, A., Isah, M.E., Abbas, M., Abdel-Aty, A.H, & Onyenegecha, C.P. A fractional order model for Dual Variants of COVID-19 and HIV co-infection via Atangana-Baleanu derivative. *Alexandria Engineering Journal*, 61(12), 9715-9731, (2022). [CrossRef]

[15] Nwajeri, U.K., Omame, A., & Onyenegecha, C.P. Analysis of a fractional order model for HPV and CT co-infection. *Results in Physics*, 28, 104643, (2021). [CrossRef]

[16] Ogunrinde, R.B., Nwajeri, U.K., Fadugba, S.E., Ogunrinde, R.R., & Oshinubi, K.I. Dynamic model of COVID-19 and citizens reaction using fractional derivative. *Alexandria Engineering Journal*, 60(2), 2001-2012, (2021). [CrossRef]

[17] Ahmed, I., Baba, I.A., Yusuf, A., Kumam, P., & Kumam, W. Analysis of Caputo fractional-

order model for COVID-19 with lockdown. *Advances in Difference Equations*, 394, (2020). [CrossRef]

[18] Almeida, R., Cruz, A.M.C.B., Martins, N., & Monteiro, M.T.T. An epidemiological MSEIR model described by the Caputo fractional derivative. *International Journal of Dynamics and Control*, 7, 776-784, (2019). [CrossRef]

[19] Karaji, P.T., & Nyamoradi, N. Analysis of a fractional SIR model with general incidence function. *Applied Mathematics Letters*, 108, 106499, (2020). [CrossRef]

[20] Lin, W. Global existence theory and chaos control of fractional differential equations. *Journal of Mathematical Analysis and Applications*, 332(1), 709-726, (2007). [CrossRef]

[21] Tuan, N.H., Mohammadi, H., & Rezapour, S. A mathematical model for COVID-19 transmission by using the Caputo fractional derivative. *Chaos, Solitons & Fractals*, 140, 110107, (2020). [CrossRef]

[22] Alrabaiah, H., Ur-Rahman, M., Mahariq, I., Bushnaq, S., & Arfan, M. Fractional order analysis of HBV and HCV co-infection under ABC derivative. *Fractals*, 30(01), 2240036, (2022). [CrossRef]

[23] Wei-Yun, S., Yu-Ming, C., Ur-Rahman, M., Mahariq, I., & Zeb, A. Mathematical analysis of HBV and HCV co-infection model nonsingular fractional order derivative. *Results in Physics*, 28, 104582, (2021). [CrossRef]

[24] Arafa, A.A.M., Rida, S.Z, & Khalil, M. A fractional-order model of HIV infection with drug therapy effect. *Journal of the Egyptian Mathematical Society*,22(3), 538-543, (2014). [CrossRef]

[25] Baleanu, D., Jajarmi, A., Sajjadi, S.S., & Mozyrska, D. A new fractional model and optimal control of a tumor-immune surveillance with non-singular derivative operator. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 29(8), 083127, (2019). [CrossRef]

[26] Liu, X., Arfan, M., Ur Rahman, M., & Fatima, B. Analysis of SIQR type mathematical model under Atangana-Baleanu fractional differential operator. *Computer Methods in Biomechanics and Biomedical Engineering*, 26(1), 98-112, (2022). [CrossRef]

[27] Losada, J., & Nieto, J.J. Properties of a new fractional derivative without singular kernel. *Progress in Fractional Differentiation and Application*, 1(2), 87-92, (2015). [CrossRef]

[28] Özköse, F., Şenel, M.T., & Habbireeh, R. Fractional-order mathematical modelling of cancer cells-cancer stem cells-immune system interaction with chemotherapy. *Mathematical Modelling and Numerical Simulation with Applications*, 1(2), 67-83, (2021). [CrossRef]

[29] Omame, A., Abbas, M., & Onyenegecha, C.P. A fractional-order model for COVID-19 and tuberculosis co-infection using Atangana-Baleanu derivative. *Chaos Solitons & Fractals*, 153, 111486, (2021). [CrossRef]

[30] Uçar, E., Uçar, S., Evirgen, F., & Özdemir, N. A fractional SAIDR model in the frame of Atangana-Baleanu derivative. *Fractal and Fractional*, (2021). [CrossRef]

[31] Uçar, S. Analysis of a basic SEIRA model with Atangana-Baleanu derivative. *AIMS Mathematics*, (2020). [CrossRef]

[32] Omame, A., Abbas, M., & Abdel-Aty, A.H. Assessing the impact of SARS-CoV-2 infection on the dynamics of dengue and HIV via fractional derivatives. *Chaos Solitons & Fractals*, 162, 112427, (2022). [CrossRef]

[33] Blayneh, K.W., Cao, Y., & Kwon, H.D. Optimal control of vector-borne disease: treatment and prevention. *Discrete and Continuous Dynamical Systems B*, 11(3), 587-611, (2009). [CrossRef]

[34] Ishikawa, H., Ishii, A., Nagai, N., Ohmae, H., Harada, M., Suguri, S., & Leafasia, J. A mathematical model for the transmission of Plasmodium vivax malaria. *Parasitology International*, 52(1), 81-93, (2003). [CrossRef]

[35] Aron, J.L., & May, R.M. The population dynamics of malaria. In: *Anderson RM(ed) Population dynamics of infectious diseases* (pp. 139-179). London: Chapman and Hall, (1982). [CrossRef]

[36] Smith, R.J., & Hove-Musekwa, S.D. Determining effective spraying periods to control malaria via indoor residual spraying in Sub-Saharan Africa. *Journal of Applied Mathematics and Decision Sciences*, 745463, (2008). [CrossRef]

[37] Buonomo, B. Analysis of a malaria model with mosquito host choice and bed-net control. *International Journal of Biomathematics*, 8(6), 1550077, (2015). [CrossRef]

[38] Nielan, R.L.M., Schaefer, E., Gaff, H., Fister, K.R., & Lenhart, S. Modeling optimal control intervention strategies for cholera. *Bulletin of Mathematical Biology*, 72, 2004-2018, (2010). [CrossRef]

[39] Owolabi, K.M., & Atangana, A. *Numerical Methods for Fractional Differentiation*. Singapore: Springer Nature, (2019). [CrossRef]

[40] Thabet, S.T.M., Abdo, M.S., & Shah, K. Theoretical and numerical analysis for transmission dynamics of COVID-19 mathematical model involving Caputo–Fabrizio derivative. *Advances in Differential Equations*, 184, (2021). [CrossRef]

[41] Van den Driessche, P., & Watmough, J. Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical Biosciences*, 180(1-2), 29-48, (2002). [CrossRef]

[42] Toufik, M., & Atangana, A. New numerical approximation of fractional derivative with non-local and non-singular kernel: application to chaotic models. *The European Physical Journal Plus*, 132, 444, (2017). [CrossRef]

Mathematical Modelling and Numerical Simulation with Applications (MMNSA)
(https://dergipark.org.tr/en/pub/mmnsa)

How to cite this article: Iwa, L.L., Nwajeri, U.K., Atede, A.O., Panle, A.B. & Egeonu, K.U. (2023). Malaria and cholera co-dynamic model analysis furnished with fractional-order differential equations. *Mathematical Modelling and Numerical Simulation with Applications*, 3(1), 33-57. https://doi.org/10.53391/mmnsa.1273982