Universal Journal of Mathematics and Applications, 6 (2) (2023) 76-85
Research paper

UJMA

Universal Journal of Mathematics and Applications

Journal Homepage: www.dergipark.gov.tr/ujma
ISSN 2619-9653
DOI: https://doi.org/10.32323/ujma.1282831

Characterizations of Matrix and Compact Operators on BK Spaces

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Article Info

Keywords: *Absolute summability, Euler matrix, Hausdorff measures of noncompactness, Matrix transformations, Operator norm, Sequence spaces*
2010 AMS: *40C05, 40F05, 46A45*
Received: *13 April 2023*
Accepted: *30 June 2023*
Available online: *30 June 2023*

Abstract

In the present paper, by estimating operator norms, we give some characterizations of infinite matrix classes $\left(\left|E_{\mu}^{r}\right|_{q},\Lambda\right)$ and $\left(\left|E_{\mu}^{r}\right|_{\infty},\Lambda\right)$, where the absolute spaces $\left|E_{\mu}^{r}\right|_{q}$, $\left|E_{\mu}^{r}\right|_{\infty}$ have been recently studied by Gökçe and Sarıgöl [1] and Λ is one of the well-known spaces $c_0, c, l_\infty, l_q (q \geq 1)$. Also, we obtain necessary and sufficient conditions for each matrix in these classes to be compact establishing their identities or estimates for the Hausdorff measures of noncompactness.

1. Introduction

The summability theory is one of the most important field in mathematics specially analysis, applied mathematics, engineering sciences, quantum mechanics and probability theory, therefore, it has been chosen as the subject of study by many authors. The theory of sequence space, which is one of the main topics of the summability theory, is mainly about generalizing the concepts of convergence-divergence for sequences and series. In this context, the primary aim is to assign a limit value for non-convergent sequences or series by using a transformation given by the most general linear mappings of infinite matrices. So, several studies can be traced in the literature dealing with characterization of matrix transformation between special sequence spaces. To mention few of them are [2–6]. On the one hand, from a different perspective, using the notion of absolute summability, a lot of new spaces of series summable by the absolute summability methods have studied and introduced by authors (see [1, 7–14]). In recent paper [1], the infinite matrix classes $\left(\left|E_{\mu}^{r}\right|,\left|E_{\mu}^{r}\right|_{q}\right)$ and $\left(\left|E_{\mu}^{r}\right|_{q},\left|E_{\mu}^{r}\right|\right)$ have been investigated. In the present paper, the matrix classes $\left(\left|E_{\mu}^{r}\right|_{q},\Lambda\right)$ and $\left(\left|E_{\mu}^{r}\right|_{\infty},\Lambda\right)$ have been characterized with operator norms, where $1 \leq q < \infty$ and $\Lambda \in \left\{c, c_0, l_\infty, l_q\right\}$. Besides, establishing their identities or estimates for the Hausdorff measures of noncompactness, the necessary and sufficient conditions for each matrix in these classes to be compact have been investigated .

A linear subspaces of ω, the set of all sequences of complex numbers, is called a *sequence space*. Let Δ, Γ be any subspaces of ω and $U = (u_{nv})$ be any infinite matrix of complex components. The transform of a sequence $\delta = (\delta_v) \in \omega$ is the sequence $U(\delta)$ deduced by the usual matrix product and its terms are written as

$$U_n(\delta) = \sum_{j=0}^{\infty} u_{nj}\delta_j,$$

provided that the series converges for all $n \geq 0$. Then, U is called a matrix mapping from the space Δ into the another spaces Γ, if the sequence $U(\delta)$ exists and $U(\delta) \in \Gamma$ for all $\delta \in \Delta$. The collection, containing all such infinite matrices, is denoted by (Δ, Γ).
A triangle matrix U is given as $u_{nn} \neq 0$ for all n and $u_{nj} = 0$ for $n > j$.
The concept of domain of an infinite matrix U in the Δ is described by

$$\Lambda_U = \{\delta = (\delta_n) \in \omega : U(\delta) \in \Delta\} \tag{1.1}$$

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Cite as: F. Gökçe, *Characterizations of Matrix and Compact Operators on BK Spaces*, Univers. J. Math. Appl., 6(2) (2023), 76-85.

and also the β-dual of the sequence space Λ is given by the set

$$\Delta^{\beta}=\left\{y:\sum_{v=0}^{\infty}y_v\delta_v \text{ converges for all } \delta\in\Delta\right\}.$$

If $\Delta\subset\omega$ is a Frechet space that is a complete locally convex linear metric space, on which all coordinate functionals $r_n(\delta)=\delta_n$ are continuous for all n, then it is said to be an FK space; an FK space whose metric is given by a norm is called a BK space.
BK-spaces have a significant role in summability theory. For instance, the matrix operators between BK-spaces are continuous and when Δ is a BK-space, the matrix domain Δ_U is also a BK-space, and also its norm is given by

$$\|\delta\|_{\Delta_U}=\|U(\delta)\|_{\Delta}.$$

A BK-space $\Lambda\supset\phi$ is said to have AK property if, for all sequence $\delta=(\delta_v)\in\Delta$, there is a unique representation $\delta=\sum_{v=0}^{\infty}\delta_v e^{(v)}$ where $(e^{(v)})$ is the sequence whose only nonzero term is 1 in v-th place for $v\geq 0$ and ϕ is the set of all finite sequences. For example, while the space l_∞ does not have AK property, the sequence space l_q has AK property in respect to its natural norm where $q\geq 1$.

Let Δ and Γ be two Banach spaces. The set of all continuous linear operators from Δ into Γ is represented by $\mathscr{B}(\Delta,\Gamma)$ and, for $U\in\mathscr{B}(\Delta,\Gamma)$, the norm of U is stated by

$$\|U\|=\sup_{\delta\in S_\Delta}\|U(\delta)\|_{\Gamma}.$$

If $y\in\omega$ and $\Delta\supset\phi$ is a BK-space, then

$$\|y\|^{*}_{\Delta}=\sup_{\delta\in S_\Delta}\left|\sum_{k=0}^{\infty}y_k\delta_k\right|,$$

and it is finite for $y\in\Delta^{\beta}$. Here, S'_{Δ} is the unit sphere in Δ.
Throughout this study, $\mu=(\mu_n)$ is any sequence of positive real numbers, $U=(u_{nj})$ be an infinite matrix of complex components for all $n,j\geq 0$ and q^* is the conjugate of q, that is $1/q+1/q^*=1$ for $q>1$, and $1/q^*=0$ for $q=1$.
Let $\sum\delta_k$ be an infinite series with partial sums s_n, and (μ_n) be a sequence of positive terms. The series $\sum\delta_v$ is said to be summable $|U,\mu_n|_q$, $1\leq q<\infty$, if (see [15])

$$\sum_{n=0}^{\infty}\mu_n^{q-1}|U_n(s)-U_{n-1}(s)|^q<\infty, \tag{1.2}$$

where $U_{-1}(s)=0$.
Point out that the method includes certain well known methods. For instance, for Cesàro matrix with $\mu_n=n$ and the weighted mean matrix, it reduces to the absolute Cesàro summability due to Flett [7] and the absolute weighted summability given by Sulaiman [6], respectively. For more applications, we refer readers to ([1, 8–10, 12]).
Also, if we choose the Euler matrix $E^r=(e^r_{ni})$ instead of U, the summability $|U,\mu_n|_q$ is reduced to the absolute Euler summability $|E^r,\mu_n|_q$ of order r. Here the terms of the matrix $E^r=(e^r_{ni})$ is given by

$$e^r_{ni}=\begin{cases}\binom{n}{i}(1-r)^{n-i}r^i, & 0\leq i\leq n\\ 0, & i>n\end{cases}$$

for all $n,i\geq 0$ and $0<r\leq 1$, [1].
The spaces of all series summable by the methods $|E^r,\mu_n|_q$, $1\leq q<\infty$, and $|E^r,\mu_n|_\infty$ have recently been introduced by Gökçe and Sarıgöl [1] as follows:

$$\left|E^r_\mu\right|_q=\left\{\delta=(\delta_v):\sum_{n=1}^{\infty}|T^r_n(q)(\delta)|^q<\infty\right\}$$

$$\left|E^r_{\mu,q}\right|_\infty=\left\{\delta=(\delta_v):\sup_n|T^r_n(q)(\delta)|<\infty\right\}$$

where $T^r_0(q)(\delta)=\delta_0$ and

$$T^r_n(q)(\delta)=\mu_n^{1/q^*}\sum_{i=1}^{n}\binom{n-1}{i-1}(1-r)^{n-i}r^i\delta_i. \tag{1.3}$$

Also, with the notation of domain, we can state $\left|E^r_\mu\right|_q=(l_q)_{T^r(q)}$ and $\left|E^r_{\mu,q}\right|_\infty=(l_\infty)_{T^r(q)}$, if we define the matrix $T^r(q)=(t^r_{nj}(q))$ by

$$t^r_{ni}(q)=\begin{cases}\mu_n^{1/q^*}\binom{n-1}{i-1}(1-r)^{n-i}r^i, & 1\leq i\leq n\\ 0, & i>n.\end{cases}$$

The inverse transformation of $T^r_n(q)$ can be written as

$$\delta_n=\sum_{i=1}^{n}\mu_i^{-1/q^*}\binom{n-1}{i-1}(r-1)^{n-i}r^{-n}T^r_i(q)(\delta), \tag{1.4}$$

[1].
Now, we list some known lemmas:

Lemma 1.1 ([1]). *Let $1 \leq q < \infty$. The spaces $\left|E^r_\mu\right|_q$ and $\left|E^r_{\mu,q}\right|_\infty$ are BK-spaces with the norms $\|\delta\|_{\left|E^r_\mu\right|_q} = \|T^r(q)(\delta)\|_{l_q}$ and $\|\delta\|_{\left|E^r_{\mu,q}\right|_\infty} = \|T^r(q)(\delta)\|_\infty$. Also, these are linearly isomorphic to the space l_q and l_∞, respectively.*

Lemma 1.2 ([16]). *The following statements hold:*

1. $U \in (l,c)$ *iff (i)* $\lim_n u_{nj}$ *exists for all* $j \geq 0$, *(ii)* $\sup_{n,j} |u_{nj}| < \infty$,
   $U \in (l,l_\infty)$ *iff (ii) holds.*
2. *If* $1 < q < \infty$, *then,* $U \in (l_q,c)$ *if and only if* (i) *holds,* $(iii) \sup_n \sum_{j=0}^{\infty} |u_{nj}|^{q^*} < \infty$,
   $U \in (l_q,l_\infty)$ *iff* (iii) *holds.*
3. $U \in (l,c_0)$ *iff (iv)* $\lim_n u_{nj} = 0$ *for all* $j \geq 0$, *(ii) hold.*
4. *If* $1 < q < \infty$, *then,* $U \in (l_q,c_0)$ *iff* (iii) *and* (iv) *hold.*
5. $U \in (l_\infty,c)$ *iff (i) holds, (v)* $\sum_{j=0}^{\infty} |u_{nj}| < \infty$ *uniformly in n,*
   $U \in (l_\infty,l_\infty)$ *iff (vi)* $\sup_n \sum_{j=0}^{\infty} |u_{nj}| < \infty$.
6. $U \in (l_\infty,c_0)$ *iff (vii)* $\lim_n \sum_{j=0}^{\infty} |u_{nj}| = 0$.
7. *If* $1 \leq p < \infty$, *then* $U \in (l_\infty,l_q)$ *iff (viii)* $\sup_K \sum_{n=0}^{\infty} \left|\sum_{k\in K} u_{nj}\right|^q < \infty$.

Lemma 1.3 ([17]). *Let $1 \leq q < \infty$. Then, $U \in (l,l_q)$ iff*

$$\|U\|_{(l,l_q)} = \sup_j \left\{ \sum_{n=0}^{\infty} |u_{nj}|^q \right\}^{1/p} < \infty.$$

Lemma 1.4 ([16]). *Let $1 < q < \infty$. Then, $U \in (l_q,l)$ iff*

$$\|U\|_{(l_q,l)} = \sup_{N\in\mathfrak{T}} \left\{ \sum_{j=0}^{\infty} \left| \sum_{n\in N} u_{nj} \right|^{q^*} \right\}^{1/q^*} < \infty$$

where $\mathfrak{T}$ stands for the collection of all finite subsets of $\mathbb{N}$.

It is difficult to apply Lemma 1.4 in applications. The following lemma presents to us an equivalent applicable norm.

Lemma 1.5 ([18]). *Let $1 < q < \infty$. Then, $U \in (l_q,l)$ iff*

$$\|U\|^{'}_{(l_q,l)} = \left\{ \sum_{j=0}^{\infty} \left(\sum_{n=0}^{\infty} |u_{nj}| \right)^{q^*} \right\}^{1/q^*} < \infty.$$

Since $\|U\|_{(l_q,l)} \leq \|U\|^{'}_{(l_q,l)} \leq 4\|U\|_{(l_q,l)}$, there exists $\zeta \in [1,4]$ such that $\|U\|^{'}_{(l_q,l)} = \zeta \|U\|_{(l_q,l)}$.

Using the Hausdorff measure of noncompactness χ introduced in [19], characterizations of compact operators on great number of sequence spaces are investigated by many researchers. For instance, to characterize the class of compact operators on several spaces, the Hausdorff measure of noncompactness method have been used by Malkowsky and Rakocevic in [20], Mursaleen and Noman in [21, 22], (see also [1, 23–26]).
Let (Δ, d) be a metric space and Q be a bounded subset of Δ. Then, χ and the number

$$\chi(Q) = \inf\{\varepsilon > 0 : Q \text{ has a finite } \varepsilon-\text{net in } \Delta\}$$

are called the Hausdorff measure of noncompactness and the Hausdorff measure of noncompactness of Q, respectively.
Suppose that S is a linear operator between the Banach spaces Δ and Γ such that $S : \Delta \to \Gamma$. Then, it is said that S is compact if its domain is all of Δ and, for every bounded sequence (δ_n) in Δ, the sequence $(S(\delta_n))$ has a convergent subsequence in Γ.

Lemma 1.6 ([27]). *Let $Q \subset \Delta$ be a bounded set where Δ is one of the normed spaces c_0 or l_q for $1 \leq q < \infty$. If $R_r : \Delta \to \Delta$ is the operator defined by $R_r(y) = (y_0, y_1, ... y_r, 0, 0, ...)$ for all $y \in \Delta$, then*

$$\chi(Q) = \lim_{r\to\infty} \left(\sup_{\delta\in Q} \|(I - R_r)(y)\| \right).$$

Let χ_1, χ_2 be Hausdorff measures on Δ and Γ. If $S(Q)$ is a bounded subset of Γ and there exists $M > 0$ such that $\chi_2(S(Q)) \leq M\chi_1(Q)$ for each bounded subset Q of Δ, then the linear operator $S : \Delta \to \Gamma$ is called (χ_1, χ_2)- bounded. If an operator S is (χ_1, χ_2)- bounded, then the number

$$\|S\|_{(\chi_1,\chi_2)} = \inf\{M > 0 : \chi_2(S(Q)) \leq M\chi_1(Q) \text{ for all bounded sets } Q \subset \Delta\}$$

is called the (χ_1, χ_2)-measure noncompactness of L. Also, in case of $\chi_1 = \chi_2 = \chi$, it is written by $\|S\|_{(\chi,\chi)} = \|S\|_\chi$.

Lemma 1.7 ([28]). *$L \in \mathscr{B}(\Delta,\Gamma)$ and $S'_{\delta} = \{\delta \in \Delta : \|\delta\| \leq 1\}$ be the unit ball in Δ. Then,*

$$\|S\|_{\chi} = \chi\left(S\left(S'_{\delta}\right)\right)$$

and

$$S \text{ is compact } \Leftrightarrow \|S\|_{\chi} = 0.$$

Lemma 1.8 ([29]). *Let $T = (t_{nv})$ be an infinite triangle matrix, Δ be a normed sequence space and χ_T and χ stand for the Hausdorff measures of noncompactness on M_{Δ_T} and M_{Δ}, the collections of all bounded sets in Δ_T and Δ, respectively. Then, $\chi_T(Q) = \chi(T(Q))$ for each $Q \in M_{\Delta_T}$.*

Lemma 1.9 ([22]). *Let $\Delta = l_\infty$ or $\Delta \supset \phi$ be any BK-space with AK property. If $U \in (\Delta, c)$, then*

$$\lim_{n\to\infty} u_{nk} = \lambda_k \text{ exists for all } k,$$

$$\lambda = (\lambda_k) \in \Delta^{\beta},$$

$$\sup_n \|U_n - \lambda\|_X^* < \infty,$$

$$\lim_{n\to\infty} U_n(\delta) = \sum_{k=0}^{\infty} \lambda_k \delta_k \text{ for each } \delta = (\delta_k) \in \Delta.$$

Lemma 1.10 ([22]). *Let $\Delta \supset \phi$ be a BK-space. Then,*
(a) If $U \in (\Delta, c_0)$, then

$$\|L_U\|_{\chi} = \lim_{r\to\infty}\left(\sup_{n>i} \|U_n\|_{\Delta}^*\right).$$

(b) If the space Δ has AK or $\Delta = l_\infty$ and $U \in (\Delta, c)$, then

$$\frac{1}{2}\lim_{i\to\infty}\left(\sup_{n\geq i}\|U_n - \lambda\|_{\Delta}^*\right) \leq \|S_U\|_{\chi} \leq \lim_{i\to\infty}\left(\sup_{n\geq i}\|U_n - \lambda\|_{\Delta}^*\right)$$

where $\lambda = (\lambda_k)$ defined by $\lambda_k = \lim_{n\to\infty} u_{nk}$, for all $n \in \mathbb{N}$.
(c) If $U \in (\Delta, l_\infty)$, then

$$0 \leq \|S_U\|_{\chi} \leq \lim_{i\to\infty}\left(\sup_{n>i}\|U_n\|_{\Delta}^*\right).$$

2. Matrix and Compact Operators on the Spaces $\left|E_\mu^r\right|_q$ and $\left|E_{\mu,q}^r\right|_\infty$

In this part of the study, firstly, by computing operator norms we obtain some characterizations of infinite matrix classes $\left(\left|E_\mu^r\right|_q, \Lambda\right)$ and $\left(\left|E_\mu^r\right|_\infty, \Lambda\right)$, where Λ is one of the spaces c, c_0, l_∞, l_q and $1 \leq q < \infty$. Moreover, we search the necessary and sufficient conditions for each matrix in these classes to be compact establishing their estimates or identities for the Hausdorff measures of noncompactness.

Lemma 2.1. *Let $1 \leq q < \infty$. Then,*
(i) If $u = (u_v) \in \left\{\left|E_\mu^r\right|_q\right\}^{\beta}$, then, $\tilde{u}^{(q)} = (\tilde{u}_v^{(q)}) \in l_{q^*}$ for all $\delta \in \left|E_\mu^r\right|_q$
(ii) If $u = (u_v) \in \left\{\left|E_\mu^r\right|\right\}^{\beta}$, then, $\tilde{u}^{(1)} = (\tilde{u}_v^{(1)}) \in l_\infty$ for all $\delta \in \left|E_\mu^r\right|$
(iii) If $u = (u_v) \in \left\{\left|E_{\mu,q}^r\right|_\infty\right\}^{\beta}$, then, $\tilde{u}^{(q)} = (\tilde{u}_v^{(q)}) \in l$ for all $\delta \in \left|E_{\mu,q}^r\right|_\infty$
and the equality

$$\sum_{v=0}^{\infty} u_v \delta v = \sum_{v=0}^{\infty} \tilde{u}_v^{(q)} y_v \tag{2.1}$$

holds, where $y = T^r(q)(\delta)$ is $T^r(q)$-transformation sequence of the sequence $\delta = (\delta_v)$ and

$$\tilde{u}_v^{(q)} = \mu_v^{-1/q^*} \sum_{n=v}^{\infty} \binom{n-1}{v-1} (r-1)^{n-v} r^{-n} u_n, \tilde{u}_0^{(q)} = u_0.$$

Proof. (i) Let $u = (u_v) \in \left\{\left|E_\mu^r\right|_q\right\}^{\beta}$. Considering (1.4) the equation (2.1) is obtained immediately. Also, it follows from Theorem 1.29 in [30] that $\tilde{u}^{(q)} \in l_{q^*}$ whenever $u \in \left\{\left|E_\mu^r\right|_q\right\}^{\beta}$.
As (ii) and (iii) can be proved with similar lines, these parts are left to reader. □

Lemma 2.2. *Let $1<q<\infty$. Then, we have $\|u\|^*_{\left|E^r_\mu\right|_q}=\left\|\tilde{u}^{(q)}\right\|_{l_{q^*}}$ for all $u\in\left\{\left|E^r_\mu\right|_q\right\}^\beta$, $\|u\|^*_{\left|E^r_\mu\right|}=\left\|\tilde{u}^{(1)}\right\|_{l_\infty}$ for all $u\in\left\{\left|E^r_\mu\right|\right\}^\beta$ and $\|u\|^*_{\left|E^r_{\mu,q}\right|_\infty}=\left\|\tilde{u}^{(q)}\right\|_{l}$ for all $u\in\left\{\left|E^r_{\mu,q}\right|_\infty\right\}^\beta$.*

Proof. Take $u\in\left\{\left|E^r_\mu\right|_q\right\}^\beta$. Since $l_q^\beta=l_{q^*}$, we get $\tilde{u}^{(q)}\in l_{q^*}$. Also, it follows from Theorem 1.29 in [30] and Lemma 2.1 that

$$\|u\|^*_{\left|E^r_\mu\right|_q}=\sup_{\delta\in S_{\left|E^r_\mu\right|_q}}\left|\sum_{v=0}^\infty u_v\delta_v\right|=\sup_{y\in S_{l_q}}\left|\sum_{v=0}^\infty \tilde{u}^{(q)}_v y_v\right|=\left\|\tilde{u}^{(q)}\right\|^*_{l_q}=\left\|\tilde{u}^{(q)}\right\|_{l_{q^*}}.$$

For $u\in\left\{\left|E^r_\mu\right|\right\}^\beta$ and $u\in\left\{\left|E^r_{\mu,q}\right|_\infty\right\}^\beta$, the proofs are similar, so the proofs are omitted. □

Theorem 2.3. *Let $1\le q<\infty$. Further, let $W=(w_{nj})$ be a matrix satisfying*

$$w_{nj}=\mu_n^{1/q^*}\sum_{i=1}^n\binom{n-1}{i-1}(1-r)^{n-i}r^iu_{ij}. \tag{2.2}$$

Then, $U\in\left(\Delta,\left|E^r_\mu\right|_q\right)$ equals to $W\in\left(\Delta,l_q\right)$, and $U\in\left(\Delta,\left|E^r_{\mu,q}\right|_\infty\right)$ if and only if $W\in(\Delta,l_\infty)$.

Proof. Let take $\lambda\in\Delta$. Then, considering (2.2) it can be written that

$$\sum_{j=0}^\infty w_{nj}\delta_j=\mu_n^{1/q^*}\sum_{v=1}^n\binom{n-1}{v-1}(1-r)^{n-v}r^v\sum_{j=0}^\infty u_{jv}\delta_j,$$

which implies that $W_n(\delta)=T^r_n(q)(U(\delta))$. This shows that $U_n(\delta)\in\left|E^r_\mu\right|_q$ when $\delta\in\Delta$ if and only if $W(\delta)\in l_q$ when $\delta\in\Delta$, which completes the first part of the proof of the theorem.
The remaining part of the proof is omitted, as it is similar. □

Theorem 2.4. *Assume that $1\le q<\infty$ and Δ is arbitrary sequence space. Then, $U\in\left(\left|E^r_\mu\right|_q,\Delta\right)$ if and only if for all $n\ge0$*

$$V^{(n)}\in\left(l_q,c\right)\ \text{and}\ \tilde{U}^{(q)}\in\left(l_q,\Delta\right),$$

$U\in\left(\left|E^r_{\mu,q}\right|_\infty,\Delta\right)$ if and only if for all $n\ge0$

$$V^{(n)}\in\left(l_\infty,c\right)\ \text{and}\ \tilde{U}^{(q)}\in\left(l_\infty,\Delta\right).$$

Here the matrices $\tilde{U}$ and $V^{(n)}$ are described as

$$\tilde{u}^{(q)}_{nk}=\mu_k^{-1/p^*}\sum_{v=k}^\infty\binom{v-1}{k-1}(r-1)^{v-k}r^{-v}u_{nv}$$

and

$$v^{(n)}_{mk}=\begin{cases}u_{n0}, & k=0\\ \mu_k^{-1/q^*}\sum\limits_{v=k}^m\binom{v-1}{k-1}(r-1)^{v-k}r^{-v}u_{nv}, & 1\le k\le m\\ 0, & k>m.\end{cases}$$

Proof. We only demonstrate for $U\in\left(\left|E^r_\mu\right|_q,\Delta\right)$ to avoid repetition. Assume that $U\in\left(\left|E^r_\mu\right|_q,\Delta\right)$. Given $\delta\in\left|E^r_\mu\right|_q$. Since $\left|E^r_\mu\right|_q=\left(l_q\right)_{T^{(r)}(q)}$, it follows from (1.4) that, for $n,m\ge0$,

$$\sum_{k=0}^m u_{nk}\delta_k=\sum_{k=0}^m v^{(n)}_{mk}y_k. \tag{2.3}$$

So, we get that, for all $\delta\in\left|E^r_\mu\right|_q$, $U\delta$ is well defined iff $V^{(n)}\in\left(l_q,c\right)$. Also, letting $m\to\infty$, gives (2.3) that $U\delta=\tilde{U}^{(q)}y$. Since $U\delta\in\Delta$, $\tilde{U}^{(q)}y$ is also in Δ, and so $\tilde{U}\in\left(l_q,\Delta\right)$.

On the contrary, let $V^{(n)}\in\left(l_q,c\right)$ and $\tilde{U}^{(q)}\in\left(l_q,\Delta\right)$. Then, by (2.3), we have $U_n\in\left\{\left|E^r_\mu\right|_q\right\}^\beta$ for all n, which gives that $U\delta$ exists. Also, by $\tilde{U}^{(q)}\in\left(l_q,\Delta\right)$ and (2.3), by letting $m\to\infty$, we get $U\in\left(\left|E^r_\mu\right|_q,\Delta\right)$. □

We present the following tables and conditions:

From To	c	c_0	l_∞	l	$l_p(p>1)$
$\left\|E_\mu^r\right\|_q$	1,3,12,14	2,3,12,14	3,12,14	4,12,14	–
$\left\|E_\mu^r\right\|$	1,6,11,14	2,6,11,14	6,11,14	5,11,14	5,11,14
$\left\|E_{\mu,q}^r\right\|_\infty$	1,7,13,14	8,13,14	10,13,14	9,13,14	9,13,14

Table 1: From Absolute Euler spaces to $\{l_\infty, c_0, c, l, l_p\}$

From To	c_s	b_s
$\left\|E_\mu^r\right\|_q$	1, 3,12,14	3,12,14
$\left\|E_\mu^r\right\|$	1,6,11,14	6,11,14
$\left\|E_{\mu,q}^r\right\|_\infty$	1,7,13,14	10,13,14

Table 2: From Absolute Euler spaces to $\{c_s, b_s\}$

1. $\lim_{n\to\infty} \tilde{u}_{nj}^{(q)}$ exists for all $j\in\mathbb{N}$
2. $\lim_{n\to\infty} \tilde{u}_{nj}^{(q)} = 0$ for all $j\in\mathbb{N}$
3. $\sup_n \sum_{j=0}^{\infty} \left|\tilde{u}_{nv}^{(q)}\right|^{q^*} < \infty$
4. $\sup_N \sum_v \left|\sum_{n\in N} \tilde{u}_{nj}^{(q)}\right|^{q^*} < \infty$
5. $\sup_j \sum_n \left|\tilde{u}_{nj}^{(q)}\right|^{p} < \infty, (1\le p<\infty)$
6. $\sup_{n,j} \left|\tilde{u}_{nj}^{(q)}\right| < \infty$
7. $\sum_{j=0}^{\infty} |\tilde{u}_{nj}^{q}| < \infty$ uniformly in n
8. $\lim_n \sum_{j=0}^{\infty} |\tilde{u}_{nj}^{q}| = 0$
9. $\sup_K \sum_{n=0}^{\infty} \left|\sum_{k\in K}^{\infty} \tilde{u}_{nj}^{q}\right|^{p} < \infty, (1\le p<\infty)$
10. $\sup_n \sum_{j=0}^{\infty} |\tilde{u}_{nj}^{q}| < \infty$
11. $\sup_{m,j} \left|v_{mj}^{(n)}\right| < \infty$
12. $\sup_m \sum_{j=0}^{\infty} \left|v_{mj}^{(n)}\right|^{q^*} < \infty$
13. $\sum_{j=0}^{\infty} |v_{mj}^{(n)}| < \infty$ uniformly in m
14. $\lim_{m\to\infty} v_{mj}^{(n)}$ exists for all $j,n\in\mathbb{N}$

We obtain following by Theorem 2.4.

Theorem 2.5. *Let $1<p,q<\infty$. Then, Table 1 presents us the necessary and sufficient conditions for $U\in(\eta,\Lambda)$, where η is one of absolute Euler spaces and $\Lambda\in\{c,c_0,l_\infty,l,l_p\}$.*

Take the matrices $T_1=(t_{nj}^1)$ and $T_2=(t_{nj}^2)$ as

$$t_{nj}^1 = \begin{cases} 1, & 0\le j\le n \\ 0, & j>n \end{cases}$$

and

$$t_{nj}^2 = \begin{cases} 1, & n=j \\ -1, & n=j+1 \\ 0, & \textit{otherwise}. \end{cases}$$

Then, since $b_s = \{l_\infty\}_{T_1}$, $c_s = \{c\}_{T_1}$ and $bv_q = \{l_q\}_{T_2}$, characterization of the matrix classes (η, Θ) can be obtained immediately as follows, where $\Theta \in \{c_s, b_s, bv_q\}$ and η is one of the any absolute Euler spaces.

Corollary 2.6. *Let's take $u(n,j) = \sum_{i=0}^{n} u_{ij}$ instead of u_{nj} in the matrices $V^{(n)} = (v_{mv}^{(n)})$ and $\tilde{U}^{(p)} = (\tilde{u}_{nv}^{(p)})$ for all $n, j \geq 0$. Then, Table 2 presents us the necessary and sufficient conditions for $U \in (\eta, \Theta)$, where $\Theta \in \{c_s, b_s\}$ and η is one of the absolute Euler spaces.*

Corollary 2.7. *Put $b_{nj} = u_{nj} - u_{n+1,j}$ instead of u_{nj} in the matrices $V^{(n)}$ and $\tilde{U}^{(q)}$ for all $n, j \geq 0$. Then,*

$$U \in \left(\left|E_\mu^r\right|, bv_p\right) \text{ iff the conditions } 5, 11, 14 \text{ hold},$$

$$U \in \left(\left|E_{\mu,q}^r\right|_\infty, bv_p\right) \text{ iff the conditions } 9, 13, 14 \text{ hold}.$$

Theorem 2.8. *(i) Let $1 < q < \infty$ and $\Lambda \in \{c_0, c, l_\infty\}$. Then,*

$$U \in \left(\left|E_\mu^r\right|_q, \Lambda\right) \Rightarrow \|S_U\| = \sup_n \left\|\tilde{U}_n^{(q)}\right\|_{l_{q^*}} = \sup_n \left(\sum_{v=0}^{\infty} \left|\tilde{u}_{nv}^{(q)}\right|^{q^*}\right)^{1/q^*}$$

$$U \in \left(\left|E_\mu^r\right|, \Lambda\right) \Rightarrow \|S_U\| = \sup_n \left\|\tilde{U}_n^{(1)}\right\|_{l_\infty} = \sup_{n,v} \left|\tilde{u}_{nv}^{(1)}\right|$$

$$U \in \left(\left|E_{\mu,}^r\right|_\infty, \Lambda\right) \Rightarrow \|S_U\| = \sup_n \left\|\tilde{U}_n^{(q)}\right\|_l = \sup_n \sum_{v=0}^{\infty} \left|\tilde{u}_{nv}^{(q)}\right|.$$

(ii) Let $1 < q < \infty$. Then, there exists $\zeta \in [1,4]$ such that

$$U \in \left(\left|E_\mu^r\right|_q, l\right) \Rightarrow \|S_U\| = \frac{1}{\zeta} \left\|\tilde{U}^{(q)}\right\|'_{(l_q, l)} = \frac{1}{\zeta} \left\{\sum_{v=0}^{\infty} \left(\sum_{n=0}^{\infty} \left|\tilde{u}_{nv}^{(q)}\right|\right)^{q^*}\right\}^{1/q^*},$$

$$U \in \left(\left|E_\mu^r\right|, l_q\right) \Rightarrow \|S_U\| = \left\|\tilde{U}^{(1)}\right\|_{(l, l_q)} = \sup_v \left\{\sum_{n=0}^{\infty} \left|\tilde{u}_{nv}^{(1)}\right|^q\right\}^{\frac{1}{q}},$$

$$U \in \left(\left|E_\mu^r\right|, l\right) \Rightarrow \|S_U\| = \left\|\tilde{U}_n^{(1)}\right\|_{(l,l)} = \sup_v \sum_{n=0}^{\infty} \left|\tilde{u}_{nv}^{(1)}\right|,$$

$$U \in \left(\left|E_{\mu,q}^r\right|_\infty, l_q\right) \Rightarrow \|S_U\| = \left\|\tilde{U}^{(q)}\right\|_{(l_\infty, l_q)},$$

$$U \in \left(\left|E_{\mu,q}^r\right|_\infty, l\right) \Rightarrow \|S_U\| = \left\|\tilde{U}^{(q)}\right\|_{(l,l)}.$$

Proof. The theorem can be easily proved by using Lemma 1.3, Lemma 1.5, Lemma 2.2 and Theorem 1.23 in [30], so it have left to reader. □

Theorem 2.9. *Let $1 < q < \infty$.*

(a) If $U \in \left(\left|E_\mu^r\right|_q, c_0\right)$, then

$$\|S_U\|_\chi = \lim_{r\to\infty} \sup_{n>r} \left\|\tilde{U}_n^{(q)}\right\|_{l_{q^*}} = \lim_{r\to\infty} \sup_{n>r} \left(\sum_{v=0}^{\infty} \left|\tilde{u}_{nv}^{(q)}\right|^{q^*}\right)^{1/q^*},$$

and

L_U is compact iff $\lim_{r\to\infty} \sup_{n>r} \sum_{v=0}^{\infty} \left|\tilde{u}_{nv}^{(q)}\right|^{q^*} = 0$.

(b) If $U \in \left(\left|E_\mu^r\right|_q, c\right)$, then

$$\frac{1}{2} \lim_{r\to\infty} \sup_{n>r} \left(\sum_{v=0}^{\infty} \left|\tilde{u}_{nv}^{(q)} - \tilde{u}_v\right|^{q^*}\right)^{1/q^*} \leq \|S_U\|_\chi \leq \lim_{r\to\infty} \sup_{n>r} \left(\sum_{v=0}^{\infty} \left|\tilde{u}_{nv}^{(q)} - \tilde{u}_v\right|^{q^*}\right)^{1/q^*}$$

and

S_U is compact iff $\lim_{r\to\infty} \sup_{n>r} \sum_{v=0}^{\infty} \left|\tilde{u}_{nv}^{(q)} - \tilde{u}_v\right|^{q^*} = 0$, where $\tilde{u}_v = \lim_{n\to\infty} \tilde{u}_{nv}$, for all $n \in \mathbb{N}$.

(c) If $U \in \left(\left|E_\mu^r\right|_q, l_\infty\right)$, then

$$0 \leq \|S_U\|_\chi \leq \lim_{r\to\infty} \sup_{n>r} \left(\sum_{v=0}^{\infty} \left|\tilde{u}_{nv}^{(q)}\right|^{q^*}\right)^{1/q^*},$$

and

if $\lim_{r\to\infty} \sup_{n>r} \sum_{v=0}^{\infty} \left|\tilde{u}_{nv}^{(q)}\right|^{q^*} = 0$, S_U is compact.

Proof. (a) Let $U \in \left(\left|E_\mu^r\right|_q, c_0\right)$. Then, the series $\sum_{n=0}^{\infty} u_{nv}\lambda_v$ converges for all $\lambda \in \left|E_\mu^r\right|_q$, or, equivalently $U_n = \{u_{nv}\}_{v=0}^{\infty} \in \left\{\left|E_\mu^r\right|_q\right\}^\beta$. So, it follows from Lemma 2.2 that $\|U_n\|^*_{|E_\mu^r|_q} = \left\|\tilde{U}_n\right\|_{l_{q^*}}$. Also, by Lemma 1.10 (a), we have

$$\|S_U\|_\chi = \lim_{r\to\infty}\sup_{n>r}\left\|\tilde{U}_n\right\|_{l_{q^*}}.$$

Hence, the compactness of S_U is immediate by Lemma 1.7, which completes the proof of (a).
(b) Let take the unit sphere $S'_{|E_\mu^r|_q}$ in $\left|E_\mu^r\right|_q$. From Lemma 1.7 it follows that

$$\|S_U\|_\chi = \chi(U(S'_{|E_\mu^r|_q})).$$

Further, since $\left|E_\mu^r\right|_q \cong l_q$, $U \in \left(\left|E_\mu^r\right|_q, c\right)$ equals to $\tilde{U} \in (l_q, c)$, and

$$\|S_U\|_\chi = \chi(U(S'_{|E_\mu^r|_q})) = \chi(\tilde{U}(T(S'_{|E_\mu^r|_q}))) = \|S_{\tilde{U}}\|_\chi.$$

which implies, by Lemma 1.10 (b),

$$\frac{1}{2}\lim_{r\to\infty}\left(\sup_{n\geq r}\left\|\tilde{U}_n-\tilde{u}\right\|^*_{l_q}\right) \leq \|L_U\|_\chi \leq \lim_{r\to\infty}\left(\sup_{n\geq r}\left\|\tilde{U}_n-\tilde{u}\right\|^*_{l_q}\right), \tag{2.4}$$

where $\tilde{u}_k = \lim_{n\to\infty}\tilde{u}_{nk}$, for all $k \geq 0$.
Considering Theorem 1.29 in [30], it can be easily written that $\left\|\tilde{U}_n-\tilde{u}\right\|^*_{l_q} = \left\|\tilde{U}_n-\tilde{u}\right\|_{l_{p^*}}$. The last equality and (2.4) complete the first part of the proof of (b). Also, the compactness of S_U is concluded by Lemma 1.7.
(c) can be proved by similar way, so it is omitted. □

By following the above lines,the proof of the following theorems also can be obtained immediately. Therefore, we just give the statement of the theorems.

Theorem 2.10. *(a) If $U \in \left(\left|E_\mu^r\right|, c_0\right)$. Then*

$$\|S_U\|_\chi = \lim_{r\to\infty}\sup_{n>r}\left\|\tilde{U}_n^{(1)}\right\|_{l_\infty} = \lim_{r\to\infty}\sup_{n>r}\sup_v\left|\tilde{u}_{nv}^{(1)}\right|,$$

and
S_U is compact iff $\lim_{r\to\infty}\sup_{n>r}\sup_v\left|\tilde{u}_{nv}^{(1)}\right| = 0.$
(b) If $U \in \left(\left|E_\mu^r\right|, c\right)$, then

$$\frac{1}{2}\lim_{r\to\infty}\sup_{n>r}\sup_v\left|\tilde{u}_{nv}^{(1)}-\tilde{u}_v\right| \leq \|S_U\|_\chi \leq \lim_{r\to\infty}\sup_{n>r}\sup_v\left|\tilde{u}_{nv}^{(1)}-\tilde{u}_v\right|$$

and
S_U is compact iff $\lim_{r\to\infty}\sup_{n>r}\sup_v\left|\tilde{u}_{nv}^{(1)}-\tilde{u}_v\right| = 0$
where $\tilde{u}_v = \lim_{n\to\infty}\tilde{u}_{nv}$, for all $v \in \mathbb{N}$.
(c) If $U \in \left(\left|E_\mu^r\right|, l_\infty\right)$, then

$$0 \leq \|S_U\|_\chi \leq \lim_{r\to\infty}\sup_{n>r}\sup_v\left|\tilde{u}_{nv}^{(1)}\right|,$$

and
S_U is compact if $\lim_{r\to\infty}\sup_{n>r}\sup_v\left|\tilde{u}_{nv}^{(1)}\right| = 0.$

Theorem 2.11. *Let $1 < q < \infty$.*
(a) If $U \in \left(\left|E_{\mu,q}^r\right|_\infty, c_0\right)$, then

$$\|S_U\|_\chi = \lim_{r\to\infty}\sup_{n>r}\left\|\tilde{U}_n^{(q)}\right\|_l = \lim_{r\to\infty}\sup_{n>r}\sum_{v=0}^{\infty}\left|\tilde{u}_{nv}^{(q)}\right|,$$

and
S_U is compact iff $\lim_{r\to\infty}\sup_{n>r}\sum_{v=0}^{\infty}\left|\tilde{u}_{nv}^{(q)}\right| = 0.$
(b) If $U \in \left(\left|E_{\mu,q}^r\right|_\infty, c\right)$, then

$$\frac{1}{2}\lim_{r\to\infty}\sup_{n>r}\sum_{v=0}^{\infty}\left|\tilde{u}_{nv}^{(q)}-\tilde{u}_v\right| \leq \|S_U\|_\chi \leq \lim_{r\to\infty}\sup_{n>r}\sum_{v=0}^{\infty}\left|\tilde{u}_{nv}^{(q)}-\tilde{u}_v\right|$$

and
S_U *is compact iff* $\lim_{r\to\infty}\sup_{n>r}\sum_{v=0}^{\infty}\left|\tilde{u}_{nv}^{(q)}-\tilde{u}_v\right|=0$
where $\tilde{u}_v=\lim_{n\to\infty}\tilde{u}_{nv}$, *for all* $v\in\mathbb{N}$.
(c) *If* $U\in\left(\left|E_{\mu,q}^r\right|_\infty, l_\infty\right)$, *then*

$$0\le\|S_U\|_\chi\le\lim_{r\to\infty}\sup_{n>r}\sum_{v=0}^{\infty}\left|\tilde{u}_{nv}^{(q)}\right|,$$

and
S_U *is compact if* $\lim_{r\to\infty}\sup_{n>r}\sum_{v=0}^{\infty}\left|\tilde{u}_{nv}^{(q)}\right|=0.$

Theorem 2.12. (a) *If* $U\in\left(\left|E_\mu^r\right|, l_q\right)$, $1\le q<\infty$, *then*

$$\|S_U\|_\chi=\lim_{r\to\infty}\left\{\sup_v\left(\sum_{n=r+1}^{\infty}\left|\tilde{u}_{nv}^{(1)}\right|^q\right)^{1/q}\right\},$$

and
S_U *is compact iff* $\lim_{r\to\infty}\sup_v\sum_{n=r+1}^{\infty}\left|\tilde{u}_{nv}^{(1)}\right|^q=0.$
(b) *If* $U\in\left(\left|E_\mu^r\right|_q, l\right)$, $1<q<\infty$, *then there exists* $\zeta\in[1,4]$ *such that*

$$\|S_U\|_\chi=\frac{1}{\zeta}\lim_{r\to\infty}\left\{\sum_{v=0}^{\infty}\left(\sum_{n=r+1}^{\infty}\left|\tilde{u}_{nv}^{(q)}\right|\right)^{q^*}\right\}^{1/q},$$

and
S_U *is compact iff* $\lim_{r\to\infty}\sum_{v=0}^{\infty}\left(\sum_{n=r+1}\left|\tilde{u}_{nv}^{(q)}\right|\right)^{q^*}=0.$

3. Conclusion

One of the most important subjects in summability theory is the theory of sequence spaces which concerns with the generalization of the concept of convergence for series and sequences. In this sense, the primary aim is to assign a limit value for divergent sequences or series by using transformation which is given by the most general linear mappings of infinite special matrices. So, there has been a large literature, concerned with characterizing completely all matrices which transform one given sequence space into another. Besides this, the literature has been also grown up in terms of the studies of many sequence spaces defined as domain of special matrices and related matrix operators (see, for instance, [1–4, 6–12]). For a recent paper [1], the infinite matrix classes $\left(\left|E_\mu^r\right|, \left|E_\mu^r\right|_q\right)$ and $\left(\left|E_\mu^r\right|_q, \left|E_\mu^r\right|\right)$ have been introduced. In this study, estimating the operator norms, the classes $\left(\left|E_\mu^r\right|_q, \Lambda\right)$ and $\left(\left|E_\mu^r\right|_\infty, \Lambda\right)$ have been characterized where $1\le q<\infty$. Also, in case Λ is one of the spaces c_0, c, l_∞, l_q, the necessary and sufficient conditions for each matrix in these classes to be compact have been obtained and certain identities or estimates for the Hausdorff measures of noncompactness have been established.

Article Information

Acknowledgements: The authors would like to express their sincere thanks to the editor and the anonymous reviewers for their helpful comments and suggestions.

Author's contributions: The article has a single author. The author has read and approved the final manuscript.

Conflict of Interest Disclosure: No potential conflict of interest was declared by the author.

Supporting/Supporting Organizations: No grants were received from any public, private or non-profit organizations for this research.

Ethical Approval and Participant Consent: It is declared that during the preparation process of this study, scientific and ethical principles were followed and all the studies benefited from are stated in the bibliography.

Plagiarism Statement: This article was scanned by the plagiarism program. No plagiarism detected.

Availability of data and materials: Not applicable.

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