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Research Article

Thermodynamic and Economic Optimization of Plate-Fin Heat Exchangers Using the Grasshopper Optimization Algorithm

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Abstract

The present study demonstrates the successful application of the grasshopper optimization algorithm (GOA) to the thermodynamic and economic modeling and optimization of cross-flow plate-fin heat exchangers with offset strip fins. To this end, the $\varepsilon-\mathrm{NTU}$ method was played to determine the efficiency and pressure drop. Seven parameters, namely the exchanger length at hot and cold sides, number of hot-side layers, fin frequency, fin-strip length, fin thickness, and fin height, constitute the design parameters for the optimization of the heat exchanger. The efficiency of the heat exchanger, the entropy generation, and the total annual system cost were considered the objective functions. Accordingly, the optimization of each objective function was investigated separately. The efficiency and accuracy of the proposed algorithm were validated using two examples from the literature. Comparison between the obtained results and those in the previous studies indicates that GOA performed better in minimizing total annual cost and entropy generation and maximizing efficiency.

Keywords: *Optimization, plate-fin heat exchanger, grasshopper optimization algorithm, total annual cost.*

1. Introduction

In recent years, numerous modeling and optimization methods have been implemented in the design of different thermal systems.

A heat exchanger is a device used to recover thermal energy between two or more fluids maintained at different temperatures. The application of various types of heat exchangers is not limited to the chemical industry. They are important also in the food, mineral, power plant, electronic, air conditioning, and automotive industries in addition to household and other areas [1]. Plate-fin heat exchangers are a type of heat exchanger that is widely utilized in various industries due to their good heat transfer efficiency, compactness, and high reliability at high-volume and multi-flow applications [2]. Offset strip fins are among the most common fins used in these heat exchangers. Fig. 1 displays this type of fin. Offset strip fins possess more heat transfer efficiency than pin fins. Also, they are stronger and more reliable than perforated fins [3].

Numerous studies have addressed the optimal design of plate-fin heat exchangers as a result of their widespread application. The design of a plate-fin heat exchanger is a sophisticated process involving many geometric and operational parameters and trial and error methods to meet the thermal energy demand. The hot and cold side lengths, fin height, fin frequency, fin length, fin thickness, and the number of flow channels are the most significant parameters in the design of plate-fin heat exchangers. In these techniques, the operational and geometric parameters are selected in such a way as to meet specific goals in terms of outlet temperature, thermal load, and pressure drop.

In the recent decade, extensive research has been conducted on optimizing plate-fin heat exchangers via metaheuristic algorithms, such as genetic algorithm, particle swarm optimization, differential evolution, simulated annealing algorithm, imperialist competitive algorithm, artificial bee colony algorithm, biogeography-based optimization, and firefly algorithm. This research aimed at minimizing the total annual cost [4-8], maximizing thermal performance [9], minimizing pressure drop [10], increasing the heat transfer rate [11], minimizing the number of entropy generation units [12], minimizing the volume and weight of the plate-fin heat exchanger [13], minimizing the heat transfer surface [14], optimizing the number of flow layers [15], and optimizing the Fanning and Colburn factors [16]. Numerous researchers have attempted the multi-objective optimization of plate-fin heat exchangers given the differences in the mentioned objective [17, 18].

The present study employed the GOA to optimize a plate-fin heat exchanger and to minimize the total annual cost and entropy generation and maximize the efficiency, which is proportional to the total heat transfer surface area, pressure drop, and operating cost. The main objectives of this work are the optimization of the parameters affecting plate-fin heat exchangers, namely the fin height H at the hot and cold sides, fin thickness t at the hot and cold sides, fin frequency n, number of channels($N_c = N_h + 1$), heat exchanger length L, and fin length l, in order to reduce the total annual cost and entropy generation and increase the heat exchanger efficiency in addition to demonstrating the performance of GOA in this optimization.

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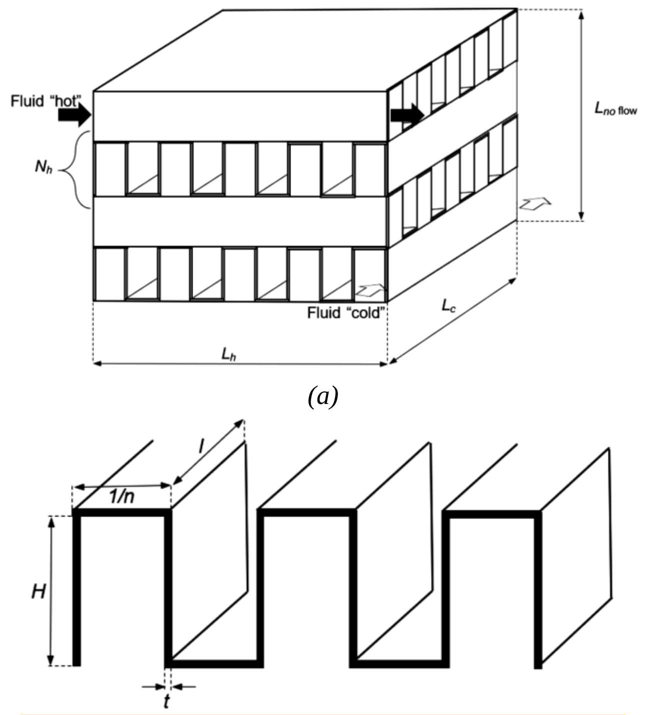

(a)

(b)

Figure 1. (a) View of the plate-fin heat exchanger and (b) the offset strip fin[10].

Based on the proposed method, a complete computer program has been developed in MATLAB Version 2017a for the design of plate-fin heat exchangers of the proposed algorithm. GOA has not been used so far to optimize plate-fin heat exchangers.

2. Mathematical Model

Fig.1 shows a view of a cross-flow plate-fin heat exchanger with offset strip fins of a rectangular cross section. In the analysis, the variation in the physical properties of the fluid with temperature is ignored and both fluids are assumed to behave as an ideal gas in order to simplify the equations. The rest of the assumptions are as follows [19]:

- The number of fin layers on the cold side is assumed to be one more than that on the hot side in order to minimize heat dissipation.
- The heat exchanger operates under stable conditions.
- The coefficient of heat transfer and the distribution are considered constant and uniform.
- The thickness of the plates is considered insignificant, and the thermal resistance and longitudinal heat transfer of the walls are assumed negligible.
- Fouling is negligibly small for a gas-to-gas heat exchanger. Hence, it is neglected.

The present study used the $\varepsilon - NTU$ method to analyze the heat exchanger sizing during optimization since the outlet fluid temperature was unknown. For a cross-flow heat exchanger with two unmixed flows, the rate of heat transfer is expressed as follows [19]:

$$Q = \varepsilon C_{min}(T_{h1} - T_{c1}) = m_h c_{ph}(T_{h1} - T_{h2}) = m_c c_{pc}(T_{c2} - T_{c1}) \quad (1)$$

where ε is the effectiveness of the heat exchanger, Cmin represents the minimum heat duty, and T_{h1} and T_{c1} denote the inlet temperature of the hot and cold fluids, respectively. For the efficiency [16],

$$\varepsilon = 1 - e^{\left(\left(\frac{1}{C_r}\right) NTU^{0.22}\left(e^{\left(-C_r NTU^{0.78}\right)} - 1\right)\right)} \quad (2)$$

In this equation, $C_r = \frac{C_{min}}{C_{max}}$, and the number of units of transfer units NTU is determined from Eq. (3) [16]:

$$\frac{1}{NTU} = C_{min}\left(\frac{Aff_h}{j_h Cp_h Pr_h^{-0.667} \dot{m}_h A_h} + \frac{Aff_c}{j_c Cp_c Pr_c^{-0.667} \dot{m}_c A_c}\right) \quad (3)$$

Moreover, Aff is the free flow area, A denotes the heat transfer surface area, j is the Colburn factor, Cp represents the heat duty, and Pr is the Prandtl number (Pr) for each of the cold and hot flows. For the free flow surface area for the plate-fin heat exchanger geometry [19]

$$Aff_h = (H_h - t_h)(1 - n_h t_h) L_c N_h \quad (4)$$

$$\mathrm{Aff_c} = (\mathrm{H_c} - \mathrm{t_c})(1 - \mathrm{n_c t_c}) \mathrm{L_h N_c} \quad (5)$$

where H, t, n, and L represent the fin height, fin thickness, fin frequency, and heat exchanger length, respectively, and N is the number of channels for each flow and is equal to $N_c = N_h + 1$ according to assumptions. Similarly, assuming identical geometry on both sides of the heat exchanger, the heat transfer surface areas are computed as follows [19].

$$A_h = L_h L_c N_h \left(1 + \left(2n_h(H_h - t_h)\right)\right) \quad (6)$$

$$A_c = L_h L_c N_c \left(1 + \left(2n_c(H_c - t_c)\right)\right) \quad (7)$$

Therefore, the total heat transfer surface area of the heat exchanger is expressed as follows:

$$A_{tot} = A_h + A_c \quad (8)$$

There exist numerous equations for evaluating the Colburn factor and the friction factor of offset strip fins. The equations by Manglik and Bergles [20] were used to calculate these factors:

$$j = 0.6522(Re)^{-0.5403}(\alpha)^{-0.1541}(\delta)^{0.1499}(\gamma)^{-0.0677}[1 + 5.3 \times 10^{-5}(Re)^{1.34}(\alpha)^{0.504}(\delta)^{0.456}(\gamma)^{-1.055}]^{0.1} \quad (9)$$

$$f = 9.6243(Re)^{-0.7422}(\alpha)^{-0.1856}(\delta)^{0.3053}(\gamma)^{-0.2659}[1 + 7.7 \times 10^{-7}(Re)^{4.429}(\alpha)^{0.920}(\delta)^{3.767}(\gamma)^{0.236}]^{0.1} \quad (10)$$

where Re is the Reynolds number (Re), $\alpha = {}^{s}/_{(H-t)}$, $\delta = {}^{t}/_{l}$, $\gamma = {}^{t}/_{s}$, and $s = \left({}^{1}/_{n}\right) - t$ denotes the inter-fin distance for the hot and cold flows. These equations hold for the ranges $120 < Re < 10^4$, $0.134 < \alpha < 0.997$, $0.134 < \delta < 0.997$, and $0.041 < \gamma < 0.121$ [20]. The equations for the Colburn and Fanning factors have 20% accuracy compared to the experimental results in the laminar, transient, and turbulence flow regimes. Therefore, there is no need for flow regime description for a given set of operational conditions, and these equations can be useful in most applications [20]. Re is calculated as follows:

$$Re = \frac{Gdh}{\mu} \tag{11}$$

where $G = \frac{\dot{m}}{Aff}$ represents the mass flux of the flow. The hydraulic radius dh for calculating Re is defined as follows [19]:

$$dh = \frac{4s(H-t)l}{2\left(sl+(H-t)l+t(H-t)\right)+ts} \tag{12}$$

In addition, the viscous pressure drop for both hot and cold fluids is obtained as follows [19]:

$$\Delta P = \frac{2fLG^2}{\rho dh} \tag{13}$$

Considering the Colburn factor, the coefficient of heat transfer is expressed as follows [19]:

$$h = jC_P GPr^{-\frac{2}{3}} \tag{14}$$

For the no-flow length [18]:

$$L_{no\ flow} = H - 2t_P + N_h(2H + 2t_P) \tag{15}$$

Based on Bejan's method [19], the entropy generation method is determined from the temperature and pressure as follows:

$$\dot{S} = \dot{m}_h \left[c_{ph} ln\left(\frac{T_{h2}}{T_{h1}}\right) - R_h ln\left(\frac{P_{h2}}{P_{h1}}\right)\right] + \dot{m}_c \left[c_{pc} ln\left(\frac{T_{c2}}{T_{c1}}\right) - R_c ln\left(\frac{P_{c2}}{P_{c1}}\right)\right] \tag{16}$$

where T_{h2}, T_{c2}, P_{h2}, and P_{c2} are the outlet temperatures and pressures of the hot and cold flows, respectively, and can be determined based on the efficiency of the heat exchanger [16]:

$$\varepsilon = \frac{c_h(T_{h1}-T_{h2})}{c_{min}(T_{h1}-T_{c1})} = \frac{c_c(T_{c2}-T_{c1})}{c_{min}(T_{h1}-T_{c1})} \tag{17}$$

Accordingly,

$$T_{h2} = T_{h1} - \varepsilon \frac{c_{min}}{c_h}(T_{h1} - T_{c1}) \tag{18}$$

$$T_{c2} = T_{c1} + \varepsilon \frac{c_{min}}{c_c}(T_{h1} - T_{c1}) \tag{19}$$

Also, for the outlet fluid pressures,

$$P_{h.2} = P_{h.1} - \Delta P_h \tag{20}$$

$$P_{c.2} = P_{c.1} - \Delta P_c \tag{21}$$

3. Optimization Technique

The minimization or maximization of a specific objective function is called optimization. The optimization process is applicable in various fields of science. To solve the optimization problem, different steps must be done. In the first step, the parameters of the problem must be determined. Based on the nature of the parameters, the problem is divided into two categories, discrete or continuous. In the second step, the restrictions that must be applied on the parameters are identified. In the third step, the purpose of the given problem should be examined. In this case, optimization problems are classified into single-objective and multi-objective problems. Finally, based on the type of known parameters, constraints, and number of objectives, a suitable optimizer should be selected to solve the problem in question.

Meta-heuristic optimization algorithms are of great interest in engineering applications; Because they: (i) rely on relatively simple concepts and their implementation is easy; (ii) do not require variable information; (iii) they can bypass the desired local state; (iv) They can be used in a wide range of problems in different fields.

Meta-heuristic algorithms inspired by nature solve optimization problems by imitating biological and physical phenomena. They can be divided into three main categories: evolutionary based, physics based and swarm based methods. Evolutionary methods are inspired by the laws of natural evolution. The search process starts with a randomly generated community and evolves in subsequent generations. The strength of these methods is that the best members always combine with each other to form the next generation. This causes the population to be optimized during the next generations. The most famous evolutionary method is the genetic algorithm (GA)[21], which simulates Darwinian evolution. Physics-based methods imitate the physical laws in the world. The most famous algorithms are: simulated refrigeration (SA)[22], gravitational search algorithm (GSA)[23], central force optimization (CFO)[24] and curved space optimization (CSO)[25]. The third group of methods inspired by nature includes swarm-based techniques that imitate the social behavior of groups of animals. The most famous algorithm is particle swarm optimization, which was first created by Kennedy and Eberhart[26].

Among the stochastic optimization methods, population-based algorithms inspired by nature are the most popular. These methods mimic the problem solving methods found in nature that are often used by living organisms. Surviving is the main goal for all creatures in nature. To achieve this goal, living organisms are evolving and adapting themselves in various ways. In general, crowding-based algorithms have advantages over evolution-based algorithms. For example, swarm-based algorithms preserve information about the search space in subsequent iterations, while evolutionary-based algorithms lose all information as soon as a new population is formed. These methods usually include fewer operators compared to evolutionary approaches (selection, intersection, mutation, elitism, etc.) and hence their implementation is easier.

GOA was employed to solve the heat exchanger optimization problem. Grasshoppers are known as agricultural pests with eleven thousand species found in nature. As seen in Fig.2, a grasshopper moves through egg, nymph, and adult phases in its life cycle.

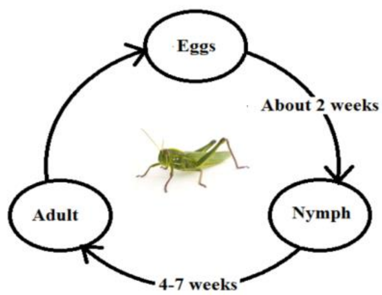

Figure 2. Life cycle of grasshoppers [27].

GOA is a metaheuristic optimization technique and is categorized as a swarm intelligence algorithm based on an initial population similar to particle swarm optimization (PSO). The mathematical model used to simulate grasshopper behavior is as follows [27]:

$$X_i = S_i + G_i + A_i \tag{22}$$

In the above equation, X_i, S_i, G_i, and A_i are the position, social interaction, gravity force, and wind advection, respectively, of the ith grasshopper. To create random behavior, Eq. (22) can be expressed as follows [27]:

$$X_i = r_1 S_i + r_2 G_i + r_3 A_i \tag{23}$$

where the coefficients r1, r2, and r3 are random numbers between zero and one. Social interaction represents one of the principal concepts in the behavior of grasshoppers and is expressed as follows [27]:

$$S_i = \sum_{\substack{i=1 \\ j\neq i}}^{N} s\left(d_{ij}\right)\ \hat{d}_{ij} \tag{24}$$

In this equation, d_{ij} is the distance between the ith and jth grasshopper and is calculated as $d_{ij} = |X_j - X_i|$, and $\hat{d}_{ij} = \frac{(X_j - X_i)}{d_{ij}}$ is a unit vector connecting the ith to the jth grasshopper is a function determining the social interaction strength and expresses the effect on the social interaction (attraction and repulsion). It is calculated as follows [27]:

$$s(r) = f e^{\frac{-r}{l}} - e^{-r} \tag{25}$$

where f represents the intensity of attraction, and l is the attractive length scale. Changes in these parameters lead to social behaviors in artificial grasshoppers and significantly change the comfort, attraction, and repulsion zones. The gravity force in Eq. (22) is calculated as follows [27]:

$$G_i = -g\hat{e}_g \tag{26}$$

In this equation, g is the gravitational constant, and $\hat{e}_g$ denotes a unit vector toward the center of the Earth. For the direction of wind advection [27]

$$A_i = u\ \hat{e}_\omega \tag{27}$$

where u is the drift constant, and $\hat{e}_\omega$ denotes a unit vector in the wind direction. Accordingly, Eq. (22) can be expanded as follows [27]:

$$X_i = \sum_{\substack{j=1 \\ j\neq i}}^{N} s\left(|X_j - X_i|\right) \frac{(X_j - X_i)}{d_{ij}} - g\hat{e}_g + u\ \hat{e}_\omega \tag{28}$$

where N represents the number of grasshoppers. Eq (28) is able to simulate a swarm of grasshoppers in 2D, 3D, and hyper dimensional spaces. Given that the grasshoppers reach the comfort zone rapidly and do not converge to a point, this model cannot be directly used to solve optimization algorithms. For this reason, the practical model of Eq. (28) is presented as follows [27]:

$$X_i^d = c\left(\sum_{\substack{j=1 \\ j\neq i}}^{N} c\,\frac{ub_d - lb_d}{2}\, s\left(X_j^d - X_i^d|\right) \frac{(X_j - X_i)}{d_{ij}}\right) + \hat{T}_d \tag{29}$$

lb_d and ub_d represent the upper and lower bounds in the dth dimension, and $\hat{T}_d$ is the value of the best solution in the dth dimension up to a given iteration. c is a decreasing constant and maintains a balance between exploitation and exploration. At first, since the first term in Eq. (29) (exploration term) must be given more weight, c are large. They are gradually reduced and guided toward the best solution. The factor c is updated according to the following equation [27]:

$$c = c_{max} - l\frac{c_{max} - c_{min}}{L} \tag{30}$$

where, in this study, the highest value c_{max} equals 1, and the lowest value c_{min} equals 0.00001. Moreover, l is the current iteration number, and L denotes the maximum number of iterations in the algorithm. Fig. 3 depicts the steps of GOA, and Fig. 4 represents the block diagram of this algorithm [27].

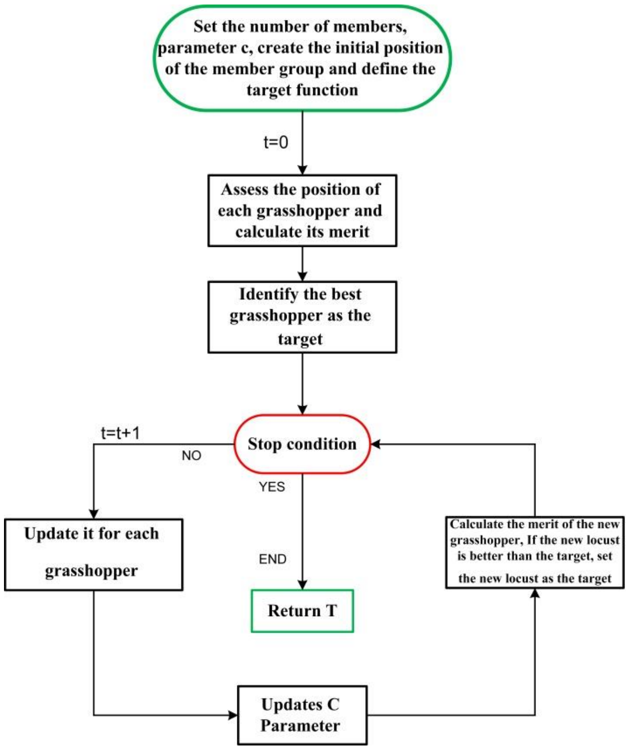

Figure 3. Flowchart of GOA [27].

Step1: Initialize the parameters of algorithm ;
Step2: Produce the population of grasshopper randomly ;
Step3: Assess the position of each grasshopper and calculate its merit ;
Step4: Identify the best grasshopper as the target ;
Step5: Repeat Steps 6 to 12 until the stop condition is established ;
Step6: Repeat steps 7 to 11 for each grasshopper ;
Step7: $c = c_{max} - l\frac{c_{max} - c_{min}}{L}$ *;*
Step8: Update the value of c ;
Step9: Update it for each grasshopper ;
Step10: Calculate the merit of the new grasshopper ;
Step11: If the new grasshopper's merit is better than the target, set the new grasshopper as the target ;
Step12: If the stop condition is not met, go to step 5, otherwise go to end ;
Step13: End

Figure 4. Block diagram of GOA [27].

Hence, based on the presented flowchart, first, the decision parameters and the corresponding region are specified according to Table 1. Subsequently, the parameters related to GOA are defined. In the present study, the stop criterion is the number of iterations, which is different in different study cases. 18 grasshopper groups were considered for the design of the plate-fin heat exchangers based on the 6 design variables. c is a decreasing constant and maintains a balance between exploitation and exploration. As mentioned previously, for the present work, the highest value c_{max} is equal to 1, while the lowest value c_{min} is equal to 0.00001.

In the next step, the position and cost function value are randomly specified for all the grasshoppers. This equation states that the next position of a grasshopper is determined by its present position, target position, and the positions of all other grasshoppers. The first term in this equation is the current position of the grasshopper based on those of the other grasshoppers. In other words, the positions of all the grasshoppers are considered for defining the positions of the search agents around the target. In short, the first part of the equation simulates the interaction between grasshoppers in nature, while the second part simulates their tendency to move toward the source.

In PSO, there are two vectors for each particle: the position vector and the velocity vector. The positions of the particles are updated according to their current positions, best personal experience, and best global experience. However, in GOA, there is only a position vector for each search agent, which is optimized based on its current position, best global solution, and the positions of other grasshoppers. This means that all the search agents are involved in determining the next position of every search agent. Based on Eq. (30), c is updated in the first step of each iteration. The first adaptive parameter, c, in Eq. (29) decreases the displacements of the grasshoppers around the target. In other words, this parameter strikes a balance between exploration and exploitation around the target and reduces the search space around the target with the aim of increasing the number of iterations in the algorithm. C is a decreasing coefficient to shrink the comfort zone, repulsion zone, and attraction zone.

In the next step, the objective function $\hat{T}_d$ is updated after all the members are evaluated. Based on previous discussions, the mathematical model of the algorithm requires the grasshoppers to converge gradually to the target over the iterations. In the actual search space, there is no target, and the global optimal position, i.e., the main target, is unknown. Thus, one target is found for the grasshoppers in each optimization step. This helps GOA to store the most promising target in the search space in each iteration and to require the grasshoppers to move toward this target. This is done with the hope of finding a better and more accurate target as the best approximation of the global optimal in the search space.

4. Objective Function and Design Parameters

The first step in optimization is to introduce the objective function and the problem. Identifying the objective functions and detecting their dependence on different variables is one of the most important optimization steps.

The present study addresses the single-objective optimization of the heat exchanger and seeks three objectives. The first objective is economic and aims to minimize the total annual cost, the second objective aims to reduce the number of entropy generation units, and the third objective aims to increase the efficiency based on Eq. (2). For the cost objective function, the sum of the operational and capital (fixed) costs is considered the annual cost. The capital cost is related to the heat transfer surface area, while the operating cost is related to the electricity cost of the compressors. Cost estimation is performed in the same manner as in [15]:

$$C_{in} = A_f . C_A . A_{tot}^{n_1} \tag{31}$$

$$C_{op} = \frac{k_{el}\tau}{\eta}\left[\frac{\Delta P_h m_h}{\rho_h} + \frac{\Delta P_c m_c}{\rho_c}\right] \tag{32}$$

$$TAC = C_{in} + C_{op} \tag{33}$$

where A_f is the annual cost factor and is determined as follows:

$$A_f = \frac{r}{1-(1+r)^{-y}} \tag{34}$$

All the factors corresponding to the calculation of the total cost are presented in Table 1 [15]. Simplifying Eq. (16)-(21), one may write the following for the entropy growth rate as the objective function

$$N_s = \frac{C_h}{C_{max}}\left[ln\left(1 - \varepsilon\frac{C_{min}}{C_h}\left(1 - \frac{T_{c1}}{T_{h1}}\right)\right) - \frac{R_h}{C_{Ph}} ln\left(1 - \frac{\Delta P_h}{P_{h1}}\right)\right] + \frac{C_c}{C_{max}}\left[ln\left(1 + \varepsilon\frac{C_{min}}{C_c}\left(\frac{T_{h1}}{T_{c1}} - 1\right)\right) - \frac{R_c}{C_{Pc}} ln\left(1 - \frac{\Delta P_c}{P_{c1}}\right)\right] \tag{35}$$

Table 1. Cost parameters for the plate-fin heat exchanger [15].

Parameters for total cost	
Cost per unit arrea, $C_A\left[{}^{\$}/_{m^2}\right]$	90
Hourrs of operation, $\tau[hour]$	5000
Electricity price, $k_{el}\left[{}^{\$}/_{MWh}\right]$	20
Compressor efficiency, η	60%
Exponent of non linear increase with area increase, n_1	0.6
Depreciation time, $y[year]$	10
Inflation rate, r	0.1

Based on the relationship between the objective function and the other equations (dependence on the surface area and pressure drop on the cold and hot sides), they can be considered as follows:

$$Cost\ Function = f(H\ .\ t\ .\ n\ .\ N\ .\ L\ .\ l\ ) \tag{36}$$

As can be observed, the objective function is a function of fin height, fin thickness, fin frequency, fin length, number of flow channels, and heat exchanger length and cannot be solved analytically. In other words, the objective function is not differentiable (i.e., does not have a closed-form solution); hence, metaheuristic algorithms must be used to approach the optimal solution. Metaheuristic algorithms explore using trial and error. One of their main features is approaching the optimal solution in the search space by managing the search process.

5. Decision Variables and Constraints

The decision variables of the present study are the design variables of the plate-fin heat exchanger, namely the cold- and hot-side fin height H, the cold- and hot-side fin thickness t, the fin frequency n, the number of channels for each flow $N_c = N_h + 1$, the heat exchanger length L, and the fin length l. These variables are displayed in Fig. 1 and presented in Table 2 according to [15].

Table 2. Range of variation of the design variables [15].

Parameter	Lower	Upper
Hot side flow length, $[m]$	0.1	1
Cold side flow length, $[m]$	0.1	1
Fin height, $[m]$	0.002	0.01
Fin thickness, $[m]$	0.0001	0.0002
Fin frequency	100	1000
Fin offset length, $[m]$	0.001	0.01
Number of hot side layers	1	200

6. Case Studies

Two case studies were used from the literature to examine the applicability of the proposed algorithm. The first was adopted from Shah et al. [3] and the second from Kakac [19]. The first case study involves a cross-flow gas-air heat exchanger with a heat duty of 1069.8 kW, which was designed separately for minimizing the entropy generation unit and the total annual cost. The other performance specifications and the flow thermophysical properties are shown in Table 3. The second case study involves a cross-flow gas-air heat exchanger with a heat duty of 3300 kW, which was designed separately for minimizing the entropy generation unit and maximizing the efficiency.

Table 3. Performance specifications of the case studies.

Parameters	Case Study A[3]		Case Study B[19]	
	Hot Side	Cold Side	Hot Side	Cold Side
Mass flow rate, $[^{kg}/_{s}]$	1.66	2	25.4	25
Inlet temperature, [°C]	900	200	460	300
Inlet pressure, $[kPa]$	160	200	100	900
Specific heat, $[^{J}/_{kg.K}]$	1122	1073	1060	1060
Density, ρ $[^{kg}/_{m^3}]$	0.6296	0.9638	0.54	4.86
Dynamic viscosity, μ$[^{kg}/_{ms}]$	4.01E-5	3.36E-5	3.2E-5	3.2E-5
Prandtl number, Pr	0.731	0.694	0.69	0.69
Maximum pressure drop, ΔP $[kPa]$	9.5	8	7.5	4.5

7. Results and Discussion

7.1 Minimization of Entropy Generation in the First Case Study

For the first case, a preheater cross-flow heat exchanger with exhaust gases as the hot fluid and air as the cold fluid (both single-pass) was considered. In fact, the air entering the furnace is heated by the exhaust gases discharged to the environment, after which the air and the gases exit the heat exchanger and higher and lower temperatures, respectively. The air exiting the heat exchanger constitutes the furnace inlet. The fin type used for the heat exchanger is rectangular, and the heat exchanger is made of aluminum. Based on Eq. (35), heat transfer and pressure drop generate entropy. The entropy generation minimization results are displayed in Table 4.

Fig. 5 displays the graph of entropy convergence as the objective function. A significant reduction was observed in the objective function after 20 iterations. The changes in the objective function became relatively small after about 80 iterations. The minimum entropy generation by the plate-fin heat exchanger appeared after 180 iterations.

Table 4. Optimal entropy generation results in the first case study.

Parameters	Preliminary design[3]	ICA[28]	BA[11]	FOA[10]	GOA
L_h , $[m]$	0.3	1	0.997	0.9	0.998
L_c , $[m]$	0.3	0.88	0.94	1	0.9975
H , $[mm]$	2.49	5	8.33	8.6	2.51
n , $[^{fin}/_{m}]$	782	240	25702	256.2	987
t, $[mm]$	0.1	0.19	0.166	0.1	0.19981
l, $[mm]$	3.18	9.6	9.51	7.2	3.27
N_h	167	77	56	53	181
ΔP_h , $[kPa]$	9.34	1.23	0.741	0.656	9.4235
ΔP_c , $[kPa]$	6.9	0.67	0.46	0.589	6.5563
$L_{no\ flow}$, $[m]$	1	0.87	0.997	0.967	0.983
ε	-	0.821	0.826	0.827	0.9573
N_s	0.1576	0.137	0.134	0.133	0.1297

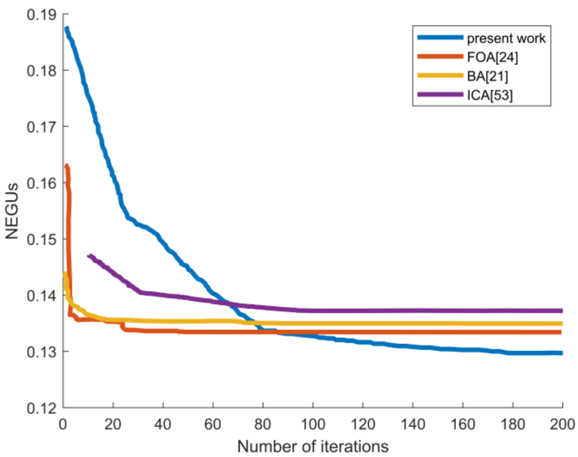

Figure 5. Graph of entropy convergence as the objective function in the first case study.

As seen in the figure, a 17% reduction in entropy generation was achieved by the optimization method compared to the initial design. It was observed that the fin frequency increased and almost reached its maximum value in minimizing the design entropy.

7.2 Minimization of the Total Cost in the First Case Study

The minimization results of the total annual cost for the first case study are displayed in Table 5. Moreover, Fig. 6 displays the graph of the general cost as the objective function. A significant reduction in the objective function was obtained at the beginning of the evaluation (after 10 iterations). Furthermore, the objective function stopped changing after 90 iterations.

7.3 Minimization of Entropy Generation in the Second Case Study

The aim of entropy optimization in heat exchangers is to achieve minimum dissipation of the available energy. A heat exchanger operates based on the temperature difference between two fluids flowing in adjacent channels.

The flowing of the two fluids causes a pressure drop in the channels. The principal mechanisms of exergy destruction, or entropy generation, are heat transfer and pressure drop, which are inevitable in heat exchangers. These devices operate in such a way that a decrease in one leads to an increase in the other.

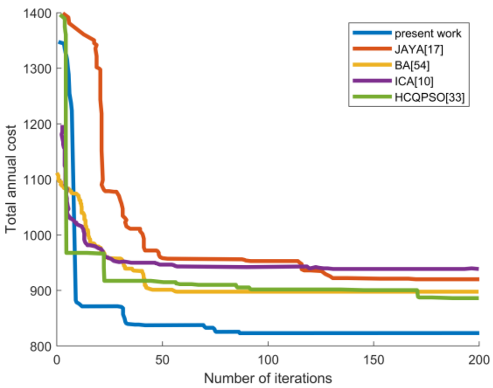

Figure 6. Convergence graph of the total annual cost as the objective function in the first case study.

Table 5. Optimal total cost results in the first case study.

Parameters	BBO [29]	Jaya[6]	BA [30]	HCQPSO[15]	GOA
L_h , $[m]$	0.793	0.84281	0.756	0.6	0.678
L_c , $[m]$	1	1	0.934	0.64	0.784
H , $[mm]$	10	10	9.84	9.06	8.56
n , $[{fin}/{m}]$	218	198.08	227.73	299.66	243
t , $[mm]$	0.2	0.19881	0.192	0.155	0.192
l , $[mm]$	7	4.9359	7.77	9.2	8.95
N_h	74	71	72	64	86
ΔP_h , $[kPa]$	0.269	0.30019	0.298	0.582	0.3052
ΔP_c , $[kPa]$	0.325	0.32212	0.343	0.48	0.3027
$L_{no\ flow}$, $[m]$	-	-	1.499	1.23	1.5
ε	0.820544	0.82055	-	-	0.82862
Investment cost, $[{\$}/{year}]$	692.99	672.34	647.1	464.89	584.45
Operation cost, $[{\$}/{year}]$	230.6	243.32	250.56	423.10	238.8
Total annual cost	923.59	915.66	897.65	888	823.25

In this section, the objective is to find the optimal ratio of these two mechanisms in heat exchangers such that entropy generation during the process is minimized. To this end, factors such as the heat transfer surface area, heat exchanger dimensions, flow rates of the two fluids, heat transfer surface distribution along the heat exchanger, and pressure drop in the heat exchanger are influential. The entropy generation minimization results for the second case study are displayed in Table 6.

7.4 Efficiency Increase in the Second Case Study

The effectiveness of a heat exchanger ε is a measure of its performance and is defined as the ratio of the actual to the ideal heat transfer. In other words, ε expresses the effective thermodynamic performance specifications of the heat exchanger. This parameter is a function of the flow configuration, the ratio of heat capacities, and the number of heat transfer units. The results of maximizing the effectiveness in the second case study are shown in Table 7.

Table 6. Optimal entropy generation results in the second case study.

Parameters	PSO [31]	GA [32]	BA [30]	GOA
L_h , $[m]$	0.925	0.994	0.995	1.216
L_c , $[m]$	0.996	0.887	0.995	2.297
H , $[mm]$	9.98	9.43	9.99	9.85
n , $[{fin}/{m}]$	0.1	534.9	405.69	412
t$[mm]$	0.1	534.9	405.69	412
l$[mm]$	9.8	63	9.998	9.54
N_h	10	8	10	76
ΔP_h , $[kPa]$	3.331	5.287	1.75	7.408
ΔP_c , $[kPa]$	1.834	2.216	1.143	4.236
$L_{no\ flow}$, $[m]$	0.214	0.169	0.218	1.5297
ε	0.8327	0.8277	0.832	0.7886
N_s	0.053028	0.063332	0.052886	0.040045

Table 7. Optimal efficiency results in the second case study.

Parameters	Preliminary design [3]	BA [30]	GOA
L_h , $[m]$	0.9	1.654	1.5892
L_c , $[m]$	1.8	2.99	2.9973
H , $[mm]$	5.7	5.72	7.88
n , $[{fin}/{m}]$	500	388.9	473.92
t$[mm]$	0.15	0.169	0.1202
l$[mm]$	6	8.57	9.9638
N_h	149	165	124.47
ΔP_h , $[kPa]$	15	7.5	5.4505
ΔP_c , $[kPa]$	10	3.38	2.4361
$L_{no\ flow}$, $[m]$	1.79	1.99	1.9992
ε	0.778	0.83	0.85574

8. Conclusion

This study presents the successful application of a new algorithm for the optimal design of plate fin heat exchangers. This algorithm is used in most thermal engineering problems that consist of several discrete and continuous variables and a large amount of discontinuity in the objective function.

This algorithm can be employed in most thermal engineering problems involving a large number of discrete and continuous variables. Identifying the objective functions and their dependence on various variables is among the most important optimization steps. Based on applications, seven design parameters were considered to be the optimization variables. Moreover, the $\varepsilon - NTU$ method was utilized for the thermal analysis of the plate-fin heat exchanger. Two case studies were adopted from the literature to validate the accuracy of this algorithm. The results for the total annual cost, entropy generation, and efficiency objective functions indicated the superior performance of GOA compared to the original design and the higher accuracy of GOA compared to other algorithms in converging to the optimal solutions over a given number of iterations. The following conclusions may be drawn from the results:

- The findings demonstrate that the results attained from the GOA are better than the preliminary design considering the respected objective function.
- Grasshoppers effectively explore the promising regions in a given search space.

- Grasshoppers encounter large variations in the initial optimization steps, which helps them search the space more thoroughly.
- Grasshoppers tend to move locally in the final optimization step, which allows them to exploit the search space.
- GOA increases the merit of the members, indicating that this algorithm can effectively improve the merit of the initial random population.
- The target merit increased over iterations, demonstrating that the global optimal approximation becomes more accurate in proportion to the number of iterations.
- As a result, in order to optimally design of heat exchangers by using meta-heuristic algorithms with regard to searching for promising areas, exploiting the entire search space and increasing the competence of members, this method can be considered as a suitable method.

Nomenclature

A	heat exchanger surface area, $[m^2]$
A_f	annual cost factor
A_{ff}	free flow area, $[m^2]$
C	heat capacity rate, $\left[W/_{k}\right]$
C_A	cost per unit arrea, $\left[\$/_{m^2}\right]$
C_p	specific heat, $\left[J/_{kg.K}\right]$
C_r	$\frac{C_{min}}{C_{max}}$
C_{op}	operational cost
C_{in}	capital cost
d_h	hydraulic diameter, $[m]$
f	fanning friction factor
$f(x)$	objective function
G	mass flux velocity, $\left[kg/_{m^2 s}\right]$
GOA	grasshopper optimization algorithm
h	convective heat transfer coefficient, $\left[W/_{m^2 k}\right]$
H	height of fin, $[m]$
j	Colburn factor
K_{le}	electricity price
l	interrupted length of serrated fin, $[m]$
l_f	lance length of the fin, $[m]$
L	heat exchanger length, $[m]$
m	mass flow rate (kg/s)
n	fin frequency, $\left[fin/_{m}\right]$
n_1	exponent of non linear increase with area increase
N_c , N_h	number of fin layers for fluid c and h
N_s	number of entropy generation units (EGU)
NTU	number of transfer units
P	pressure, $[kPa]$
Pr	Prandtl number
Q	heat duty, $[W]$
r	inflation rate
R	specific gas constant, $\left[J/_{kg.K}\right]$
Re	Reynolds number
s	fin spacing, $[m]$
$\dot{S}$	rate of entropy generation, $\left[W/_{K}\right]$
t	fin thickness, $[m]$
T	Temperature, $[K]$
$T_{h,c}$	outlet and inlet temperatures of the hot and cold flows, $[K]$
TAC	cost objective function
U	overall heat transfer coefficient, $\left[W/_{m^2 k}\right]$

Greek symbols

μ	viscosity
ρ	density
ε	effectiveness
Δp	Pressure drop
ΔS	entropy difference, $\left[W/_{kgK}\right]$
τ	hourrs of operation
γ	$t/_{s}$
η	compressor efficiency
α	$s/_{(H-t)}$
δ	$t/_{l}$

Subscripts

c , h	fluid *cold* and hot
1	inlet
2	outlet
max	maximum
min	minimum

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