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Review Article

The pedagogy of thinking mathematically using math modeling in the classroom

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Abstract

Mathematical modeling is the process of addressing a real-life task using mathematical concepts. Used in applied mathematics, engineering, the natural and social sciences, educators are recognizing that the use of mathematical modeling as a teaching tool gives students the opportunity to work conceptually on topics that are inherently motivating and engaging and are connected to real-life situations. This can support learning in the mathematics classroom in multiple ways, by using learning for transfer, problem-solving, and conceptual understanding. Students are able to incorporate cross-curricular skills and use real-life examples, which can make mathematics significantly more engaging. Modeling also provides students of varying ability levels to access content knowledge and construct new learning with mathematics that will extend beyond the classroom which is important especially in heterogenous classrooms and increasingly an increasingly diverse student population. Students who understand modeling have the potential to become stronger mathematicians and also use mathematics effectively later in life-whether working in a mathematics-related field or not. Providing professional development for teachers is crucial to making sure that modeling will be enacted with fidelity in the classroom. For pre-service teachers to effectively implement modeling in the classroom, they should both experience modeling as students themselves, learn how to create modeling scenarios in methods coursework, and then have multiple opportunities to enact modeling lessons with students. In-service teachers also need to have professional learning experiences with modeling, including creation of modeling activities, ways to implement modeling scenarios effectively, options for grouping students during modeling (including ways to work with language learners and students with special education needs), ways to grade modeling work, and how to connect modeling to curricular demands. Mathematical modeling is a highly-effective, engaging method to connect students to real-life mathematics, but teachers need professional learning supports if modeling is to be implemented effectively.

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Introduction

In mathematics curriculum, teachers often strive to give students experience with problems that are more than just routine math exercises. There are word problems, standard application problems, and true modeling problems (Niss et al., 2007). Traditional word problems are those often found in textbooks, allowing the student to simply extract the numbers from text and solve without reference to the context of the problem. When looking at the modeling cycle in the context of a word problem, a student has little to solve, and the answer requires little, if any, interpretation. Standard application problems, unlike word problems, do require some level of translation from the real world to the level of

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mathematizing, but the amount of translation required is not very complex. True modeling problems will make use of the full modeling cycle. These start with a question or situation, followed by student development of a model of the scenario, and then the student solves and interprets the results in both the mathematical and contextual situation (Tran & Dougherty, 2014). This paper contains a discussion about the potential of mathematical modeling to support learning for transfer, problem-solving, and conceptual understanding.

What is mathematical modeling?

To solve modeling problems, individuals are required to consider a real-world situation and formulate a mathematical model. In short, problem solvers consider what appears to be non-mathematical information and make it mathematical. This process is referred to as mathematizing (Bonotto, 2010). Then, the problem solvers apply their model to the specific scenario (e.g., data) by testing it and revising it, perhaps multiple times, as needed. This revision process is called creating multiple iterations (Lesh et al., 2000). Doerr and Lesh (2003) define modeling as "conceptual systems…that are expressed…and are used to construct, describe, or explain behaviors of other system(s) – perhaps so that the other system can be predicted intelligently" (p. 10).

To effectively model, authentic tasks are required. These are tasks grounded in real-world scenarios, so that modelers are able to work with them mathematically. Tran and Dougherty (2014) list six criteria for tasks to be considered authentic: event, question, purpose, information/data, language use, and tool demands. The simulation that the task represents needs to be an *event* that either has or could take place, and the *question* students are asked to answer is one that could legitimately be posed in such an event. The *purpose* of the task needs to be as clear as it would be in that authentic situation. It should be either explicitly provided or be implicitly assumed from the context of the problem, though some degree of interpretation may be required. The *information/data* provided should describe and include the specific subjects/objects or data as in the simulated situation. While problem statements should be explicit, students may be expected to realize the need and specify the model accordingly in some cases. *Language use* in the situation should not be unreasonable provided that any unfamiliar terms are not part of the simulated situation, so some new vocabulary specific to the situation may be necessary. However, enabling problem solvers the opportunity to access and interpret problem demands is incumbent upon problem writers, as the context and structure of the problem should be familiar to problem solvers. Lastly, Tran and Dougherty discussed the *tool demands* of a task. These are the tools used for task, which should mimic those used in the realistic situation, such as, "What are the actual dimensions and aluminum requirements to create the packaging for a cola can?" (Tran & Dougherty, 2014, p. 767)).

There is a distinction between in-school and out-of-school mathematics (Bonotto, 2010). Realistic, or out-of-school mathematics, is not the mathematics often discussed in schools. Instead, out-of-school mathematics gives people an integrated, graphic, and dynamic way to think, talk about, and work with mathematics. The mathematics understanding and ability needed for modeling is quite different from what has been traditionally taught because the cognitive challenges are much greater in mathematical modeling than they are in entry-level word problems. Engaging learners in mathematical modeling scenarios, therefore, is likely requisite to prepare young students to succeed in today's age of information. Skills that may be associated with success in mathematical modeling may include constructing, describing, communicating, interpreting, mathematizing, sense-making, and explaining their reasoning and understanding, rather than just computing. Mathematical modeling is as much "about quantities as it is about naked numbers" (p. 16) and about making sense of patterns and reasoning about regularity in a system rather than just data.

> *The cycle from data to deduction to application recurs everywhere math is used, from everyday household tasks such as planning a long automobile trip to major management problems such as scheduling airline traffic or managing investment portfolios. The process of "doing" math is far more than just calculation or deduction; it involves observation of patterns, testing of conjectures, and estimation of results (National Research Council, 1989, p. 31).*

With respect to problem solving, traditional approaches (as math has been taught) have ostensibly specific heuristics that one can use or follow in order to solve a given problem (Polya, 1957). This process removes the inherent quality of novelty (Chamberlin & Chamberlin, 2010) that is requisite in legitimate problem solving. While such information may be helpful, there are multiple stages to the modeling process, and the rigidity of heuristics can stifle creativity in modeling. In fact, it is strongly suggested that modeling problems be utilized when needed mathematical knowledge exists, but problem solvers are asked to assemble it to create a mathematical model. Moreover, the modeling process is not merely a *watch my solution and regurgitate it* process. Modeling connects to the real world in an authentic manner, and leaves students working with messy, ill-defined problems that have no one unique correct answer. This process requires students to investigate and define the situation to predict or explain what is happening. Modeling *starts* in the real world, moves to the classroom or laboratory, and then figuratively moves back to the real world. In mathematical modeling, the problem that needs to be solved encourages the student to move from the real world to the mathematics (Hansen & Hana, 2015), and use the mathematics creatively to explain the phenomenon.

When modeling, a key piece of the cycle is to analyze the results and make sense of the solution. While analyzing the mathematics, choices need to be made. For instance, solvers need to consider which variables are needed, what information is most important, and what assumptions can be made, among other considerations. Students make decisions and move back to the real world and interpret that solution. Does the solution make sense in the context of the initial problem? If it does not, the modeling cycle repeats (Cirillo et al., 2016). There is no *one right answer* and depending on how the variables are interpreted can alter the outcome that a student/group may decide on.

Why should we use mathematical models?

Mathematical modeling supports learning in the mathematics classroom in multiple ways. Students are able to connect with the mathematics through realistic, authentic problems, which can be motivating and engaging (Blum & Borromeo Ferri, 2009). Modeling helps students move through developmental stages (Doerr & Lesh, 2003), provides students with multiple entry points into the problem (Bostic, 2015), and allows students to use tacit knowledge (Doerr & Lesh, 2003) to solve real-world problems. These authentic modeling tasks also allow students to work with multiple content and practice standards of the Common Core State Standards-Mathematics (CCSS-M) at one time, allowing students to gain footing with others in the developed world (Schmidt & Houang, 2012).

Goals of learning with modeling

While previous work in mathematics problem solving has drawn largely on Polya's 1957 work on heuristics (Lesh & Zawojewski, 2007), the 21st Century goal is to produce mathematical learning through the modeling process (Zawojewski, 2010). Students are able to learn through modeling by seeing the significance of the mathematics while solving realistic problems (Bostic, 2015), which can be motivating in ways traditional text-book driven mathematics may not be. With specific modeling activities known as Model-Eliciting Activities (MEAs) (Lesh et al., 2010), the process of mathematizing is required (Doerr & Lesh, 2003; Lesh & Yoon, 2007). This leads students to construct, explain, justify, conjecture, and represent with mathematics. Modeling requires students to quantify, coordinate, and organize data (English et al., 2005). They insert, extend, refine, and revise constructs that are more powerful than mathematical lessons been taught directly (Doerr & Lesh, 2003) especially when applied to varied authentic situations. Students actually construct mathematical knowledge through the process of mathematical modeling (Zawojewski, 2010), which typically engenders sense-making in mathematics (Hiebert et al., 1997; Skemp, 1976). Modeling can help with significant forms of concept development and is achievable by all students (Lesh & Yoon, 2007). Mathematical modeling problems provide an engaging context which can foster student perseverance. Students develop greater motivation to determine the best value for the model while solving the realistic problem (Blum & Borromeo Ferri, 2009; Bostic, 2015). Authentic uses of modeling can create productive dispositions towards mathematics in general, as well as support the development of mathematical literacy (Carlson et al., 2016; Chamberlin & Parks, 2020). Mathematical modeling can support the learning of mathematics in terms of motivation, comprehension, and increased retention. Students see how math is used in real-world contexts and are able to demonstrate their understanding of mathematical content and practices (Blum & Borromeo Ferri, 2009; Carlson et al., 2016).

Seeing the value of mathematics in these contexts is crucial; without the opportunity to create models in mathematics, students may think that the field is just a series of formulae and equations to memorize and solve for, rather than an opportunity to make sense of mathematics through engaging in modeling situations. Textbooks often present problem solving and real-life situations appear to be simplistic, with specific solutions leading to specific answers (Meyer, 2015), and teachers and schools may propagate this perception by over-relying on textbooks *as* a mathematics curriculum (Remillard, 2005). Textbook treatments of problem-solving and mathematical modeling have critical limitations, explicitly relying on the school use of mathematics rather than the professional use of mathematics (Lesh & Zawojewski, 2007). Pragmatically speaking, this is problematic because companies are looking to hire people who can generate and work with mathematical models (Meyer, 2005). Computers can perform the computational portion of the modeling cycle in a far more efficient manner than humans can, but ironically, computation is the focus of most textbooks' approach to modeling. For instance, computing a derivative in calculus could be done without error and far faster with software or an advanced calculator than it could by hand. However, the human mind may be far better at specifying a preferred mathematical model than a computer might. Mathematics teachers, therefore, need to be teaching identifying, formulating, and validating with models (Meyer, 2005), so that industry needs can be met. Individuals with modeling capabilities are knowledge about problem solving and have the ability to create and modify mathematical models themselves (Lesh & Zawojewski, 2007).

Multiple options

In contrast to traditional word problems, modeling allows students to work with authentic problems. Problem solvers need to interpret and describe the problematic situation in mathematical ways (English & Watters, 2005). Modeling often ill-defined problems which can engage student interest (Chamberlin & Parks, 2020) leading to discussion and justification for their reasoning (Cirillo et al., 2016). Students become engaged as they are able to work with multiple representations; specifying the model they have selected can allow for multiple solutions and products depending on the approach adopted. What truly matters is how the solution is justified (Bostic, 2015). Students are able to use multiple approaches, interpretations, and even apply various models to one situation to see which approach fits best. It can be motivating to work in a multidisciplinary situation, with group work and collaboration being essential to a comprehensive modeling approach. When students work in teams on modeling situations, they are able to leverage their real-world knowledge in such problem solving situations (Cirillo et al., 2016), rather than just using book smarts.

Non-traditional knowledge

Students who are able to memorize formulae, quickly compute answers, and follow algorithms are those who are generally considered to be "good" at mathematics. However, with modeling, those are not the only skills that are important. Students who may not succeed in traditional math classrooms are able to use different abilities (Doerr & Lesh, 2003) effectively in classroom modeling. Students who have been considered underachievers or with less math ability have been shown in numerous studies to be capable of inventing or adapting and modifying constructs to be able to model effectively. This performance has been demonstrated in numerous studies (Doerr & Lesh, 2003). This is the case because when engaging in mathematical modeling activities, such as MEAs, the solution is not revealed or discussed prior to the modeling process.

Zone of proximal development

Mathematical modeling is often done in a group setting in schools. This interaction can be beneficial; the group setting can be a social experience that can leads to higher psychological processes. It allows for thinking as well as learning to be internalized and externalized. This interplay between an individual's use of informal, personal knowledge and key knowledge of the problem can strengthen understandings of mathematics (English & Watters, 2005; Zawojewski, 2010). Doerr and Lesh (2003) explain how the natural development of relevant constructs in modeling follows similar stages of development observed by psychologists and educators. When teachers are aware of these stages of development, they can engage their students to relate and extend their students ideas. By helping their learners construct and reconstruct knowledge and ideas in a social setting, students will feel that learning, making mistakes, relearning, and exploring knowledge and understandings are all safe (Hattie, 2012).

Connected to standards

The CCSS-M promote mathematical concepts that place the United States in a comparable position to most other developed countries (Schmidt & Houang, 2012). Teaching traditional mathematics may allow teachers to cover the CCSS-M content standards in a more efficient manner than they can through the medium of mathematical modeling, but some of the practice standards can feel artificial or forced. The same occurs with traditional word problems; students may not fully address mathematical knowledge as they follow delineated steps and plugging the numbers into an algorithm. When engaging in authentic modeling situations, however, students use mathematical knowledge, processes, representational fluency, and social skills that are more suited for actual 21st century learning as well as requisite for the content standards. Students working with a mathematical model not only are automatically reaching practice standard 4 (model with mathematics), but also likely meet other practice standards, such as standard 5 (use appropriate tools strategically) and standard 6 (attend to precision) when having to make decisions such as whether to use a calculator or computer program or whether to round an answer to a certain decimal point or to use a fraction or exact answer.

Mathematical modeling is a practice standard found in the Common Core State Standards-Mathematics but is not solely confined to mathematics alone. Focus on the CCSS-M practice standards also allows students to connect with the Next Generation Science Standards (NGSS) Science and Engineering Practices, specifically numbers two (develop and use models) and five (use mathematics and computational thinking). Helping students see the connections between mathematics and science (or mathematics and other subjects) will increase student facility with STEM fields. When presenting the modeling solution (whether to a teacher or some other professional audience), students make use of a variety of English/Language arts standards, including integration of knowledge and ideas, researching to build and present knowledge, and presenting of knowledge and ideas.

How do we teach modeling?

It is not sufficient to say that mathematical modeling must be utilized in schools. We need to support our teachers so they are well prepared to utilize modeling through instruction in which models are used, creating their own scenarios, and effectively assessing modeling. If these processes are not engaged, the idea of implementing mathematical modeling in the classroom may be poorly delivered, as has happened with other classroom reforms (Cobb & Jackson, 2011). A majority of in-service teachers may be ill-prepared to infuse mathematical modeling problems and content into the classroom because these have not been a regular staple in teacher preparation programs at either the elementary or secondary level (personal communication with S. Chamberlin, 3 October 2020.)

Pre-service education

Many researchers have pointed out the need for pre-service (and in-service) teacher training in mathematical modeling competencies (Blum, 2015; Cetinkaya et al., 2016; Niss et al., 2007), ideally including as much as a course focusing on modeling and teaching competencies in teacher preparation programs. Through specific coursework, pre-service teachers are able to develop an understanding of how to bridge out-of-school mathematics with in-school teaching, and deepen their mathematical conceptual knowledge and their pedagogical content knowledge. (Sevis, 2016).

The Association of Mathematics Teacher Educators (AMTE) (2017) states that "well prepared beginning teachers have solid and flexible knowledge of mathematical processes and practices" (p. 9). A course on modeling would address this standard, as an effective teacher preparation program should provide candidates with "opportunities to learn and work with mathematical process and processes that are appropriate to the content being studied" (p. 31). Borromeo Ferri and Blum (2009) suggest four key themes for a modeling course: 1) theoretical knowledge of modeling; 2) the ability to solve/create modeling tasks; 3) the ability to plan modeling lessons; and 4) the ability to diagnose student challenges and thought processes as they work through modeling tasks. Additional domains of knowledge can be developed during a modeling course as well, including: 1) knowledge of modeling tasks; 2) recognition of students' mathematical thinking; 3) use of information and communication technologies; and 4) classroom management during the modeling process (Cetinkaya et al., 2016). Providing pre-service educators the time and ability to develop this knowledge and put modeling skills into practice before they enter the classroom will improve teacher and student modeling self-efficacy (Jacobs & Durandt, 2016) and likely increase the odds that modeling will be effectively implemented in the classroom. A course on mathematical modeling during undergraduate studies would provide pre-service teachers the opportunity to develop their pedagogical content knowledge in mathematics, specifically related to modeling (Jacobs & Durandt, 2016). This approach would prompt pre-service teachers to spend time thinking about student competencies such as critical thinking and interpreting results (Karali & Durmus, 2015), as well as about how modeling can impact student affective states, which can help students develop a "modeling persona" (Chamberlin et al., 2020).

Teacher professional development

Most teachers have not received preservice training in modeling, so professional development in modeling is crucial as mathematical modeling requires distinct habits of mind from traditional mathematics. Because of the lack of prior practice, teachers are left with few adequate tools when implementing modeling problems (Blomhøj & Kjeldsen, 2006). Resulting misconceptions and blunders in modeling can result in student and teacher frustration, which can lead to the end of modeling being used in the classroom altogether.

Instead of leaving teachers with minimal or no strategies to support themselves and their students, on-the-job professional development is a necessity. Professional development needs to focus on the development of teachers' practice (Blomhøj & Kjeldsen, 2006). Teachers, in-service and pre-service, need to be able to reflect on and challenge their beliefs about what constitutes true mathematical learning. They need to practice anticipating misconceptions that may occur during the modeling process and evaluating the ways students think. It is important that teachers discuss pedagogy and have time to reflect on the approaches and meanings of modeling as mathematicians might (Clark & Lesh, 2003). Teachers need to have time to create modeling scenarios for use in their classroom.

Doerr (2016) suggests designing MEAs, where the activity is intended to help students create a model. She gives the following principles for designing a MEA: begin with a real-life context; construct a model; document the model; self-evaluate; generalize from the model; and create a simple prototype. This leads to significant forms of concept development and can be achievable by all students. A goal of MEAs is that they should be applicable (Lesh & Yoon, 2007). When developing MEAs, teachers should look to the sub-processes of modeling, as it is not necessary that students go through the entire modeling process each time. If teachers choose to pull modeling scenarios from texts, they can use these sub-processes to make the scenario stronger (Gould, 2016; Meyer, 2015), although teachers may find that modeling problems are not readily available in texts.

Teaching students using modeling

Using modeling activities to facilitate learning has become important, given an ever-increasing technological-based society. Thirty-five years ago, for example, advertising may not have been as sophisticated as it is now. Advertisements for credit cards or car sales now show increasingly complex data related to loans, leases, and rates. Students need to learn practical uses of mathematics to be knowledgeable consumers in a modern economy (Doerr & Lesh, 2003).

The preparatory work teachers do, for instance, setting the scene and deciding how much information about the modeling scenario to share, is important (Blomhøj & Kjeldsen, 2006). This is also a chance to differentiate the modeling process for different groups of students. Considerable scaffolding and information can be provided to some students, while others may be capable of realizing the full challenge of the problem. It is important to keep in mind that the most successful modeling projects are those where teachers have given clear and explicit intentions for student learning, without detailing aspects of a solution. This does not necessarily mean that they have outlined what students are to do or where they are to end up, but rather what the intended outcomes of the model are and what types of products might be expected from the process.

It is important that teachers use authentic modeling scenarios and do not rely too heavily on textbook problem-solving exercises. Textbook curricula prompt students to simply look back to the previous section for an example of the solution. Then students simply copy the problem, step-by-step, change the numbers, and perhaps make a few alterations. Rather than thinking creatively and using a design process, students follow problem-solving heuristics that may have been carefully explicated as examples. This is not what real-life modeling provides. When faced with a modeling scenario, it is ideal for students to ask questions that help guide their thinking. For instance, questions such as, Where do I begin? What do I need? How can I apply this? What assumptions have I made? Is my conclusion valid? (Pollak, 2003) are germane to success in mathematical modeling. Such questions are generally not common with textbook problem-solving tasks. One positive resource that is in the process of making their rather expansive database available to all educators is the CPALMS website (CPALMS.org). This website contains hundreds of modeling activities, specifically designed for K-12 educators and is believed to be the largest and most comprehensive of its kind in the world.

Teachers can look to the sub-processes of modeling to support students. These processes can include, but are not limited to identifying variables; mathematizing the scenario; analyzing the situation and using mathematics to draw conclusions; interpreting the results; validating the results (and determining if the process needs to be repeated for a better fit model); and reporting the results (Gould, 2016). Not all of these are necessary for every modeling scenario; variables could be provided in a situation and decrease the time spent modeling.

Modeling processes

The extent to which modeling is used with students is a decision that individual teachers should make; however, it is crucial to provide students with ample modeling opportunities so that they are able to become adept at the process and prepared for college or the workforce, where they will encounter real-life examples of modeling. Meyer (2015) studied two textbooks that claimed to include modeling and found that while there were many word problems in the texts, most of these examples were asking students to "perform an operation" or "interpret results". These are tasks that can be done by computers because they involve low-level computations, not high-level processing. Tasks in which problems solvers should engage are the tasks most critical to the modeling process, such as identifying essential variables, formulating a model, and validating conclusions. While there are times when problem solvers may require some assistance, they should learn to work independently to develop true modeling competencies on their own. They need to be able to ask questions during the modeling process, questions such as: "What information is needed here? What do I do to answer my question? Is my prediction correct?" (Meyer, 2015, p. 582) As Einstein repeatedly stated, "The formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skill" (Einstein & Infeld, 1938, p. 95).

Blomhøj and Kjeldsen (2006) agree that students do need to experience a balance between a holistic process, doing the complete modeling cycle every time, or a reductionist/atomistic approach (Geiger et al., 2016), where students will focus on certain parts of the modeling cycle. To develop to full competency in modeling, students need to work with the full cycle often enough that they are able to develop a metacognitive approach (Blomhøj & Kjeldsen, 2006), and for some students this will require more access to the full modeling cycle than required by other students, more advanced in mathematical skills. Providing explicit instruction that allows students to focus on the metacognitive aspects of mathematics and coaching with active interventions will help students develop this awareness (Schoenfeld, 1992). Learning to successfully use metacognition does not happen automatically for most students, so the more access and experience students are able to have, the more they will be able to consciously regulate and willingly work hard to apply the metacognitive ideas (Schukajlow et al., 2015). Being purposeful in training students to successfully implement metacognitive approaches may be fruitful for enhancing the likelihood of success with it.

Blum and Borromeo Ferri (2016) offer some teaching/learning principles for working with modeling units. Firstly, they suggest a change among working units, with variations between groups, partners, individual, and whole-class work, requiring classroom management during modeling activities. Next, they suggest that it is important that the modeling activities are authentic and rich in context, so that students can continue practicing their modeling skills, gaining the required mathematical competencies. It is important that all students are actively engaging with the modeling task; this may mean providing scaffolding for some students, while encouraging others to work independently. Additionally, it is important that students working in groups can take ownership of modeling by developing independent solutions. Having multiple solutions in a classroom "takes into account individual preferences or thinking styles…[and] enables a natural differentiation" (p.71). Finally, Blum and Borromeo Ferri (2016) write that encouraging metacognitive reflection and exploring into solution strategies is helpful for modeling and the general learning of mathematics.

Assessment

It is important to assess students in a manner similar to how they have been taught. It would not make sense to give individual, standardized questions to students after they have been working on authentic modeling problems in a group setting. Eames et al. (2016) states that there is an obvious assessment imbedded in the act of modeling itself. By verifying that the model is viable and then justifying how and why the model works in a given situation, students have already provided one form of (self)assessment.

Self-reflection tools can also be used throughout the entire process of modeling, and can incorporate aspects of metacognitive thinking (Blomhøj & Kjeldsen, 2006; Eames et al., 2016). Students can write status updates on their modeling process as an exit slip on a daily/weekly basis and give justification or an interpretation of their current understanding of how they/their groups are approaching the modeling scenario. Eames et al. (2016) also suggests that students could create a "client letter," where students would write to the client associated with their model and describe/defend their ways of thinking about the solution to explain why they have employed the best or correct approach for the scenario.

Conclusion

Mathematical modeling is the formulation of a mathematical problem based on a real-world scenario. The problem is then solved, and that mathematical solution is then returned to the real world, where it is tested for accuracy and usability. Teaching the use of modeling is an important 21st century strategy in contemporary classrooms because this is exemplifies how mathematics is experienced in real life. Additionally, modeling engages students more fully than traditional textbook approaches do, by incorporating cross-curricular skills, by using real life examples, by giving students at multiple skill levels the opportunity to engage with mathematics, and by providing a rich background for connecting with the content.

For teachers to be able to effectively utilize modeling in the classroom, quality professional learning needs to occur. Teachers need to learn how to engage with modeling themselves and how to teach using modeling. They need to either learn where to find quality modeling problems or how to create such problems themselves. Teachers also need to learn how to provide appropriate assessment for students when using modeling, so students can grow in their modeling skills, becoming stronger mathematicians and stronger modelers. Students who understand the utility of the mathematics in class will become stronger mathematicians and problem solvers.

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