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Analysis and Control of Chaos in Permanent Magnet Synchronous Motor

Abdallah Moussa YAYA*¹ , Yılmaz UYAROĞLU¹ 

Abstract

This paper explores the chaotic dynamics exhibited by a Permanent Magnet Synchronous Motors (PMSM) through an analysis of Lyapunov exponents and equilibrium points. Subsequently, the study focuses on controlling the motor's chaotic behavior under specific parameter conditions using a straightforward controller. The approach employed in this paper involves utilizing a single-state feedback controller as the resolution method. The derived control law enables the stabilization of the motor's state around a reference state, even in the presence of parameter uncertainties, thereby preventing chaotic behavior. To illustrate the proposed method, numerical simulations were conducted in MATLAB, showcasing the practical application of this approach. The simulation results demonstrate the success of the controller used.

Keywords: Chaos, chaotic system, chaos control, PMSM

1. INTRODUCTION

Permanent Magnet Synchronous Motors (PMSM) are extensively used in various industries due to their many advantages, such as robustness, high power density, compactness and low maintenance cost [1, 2]. Over the years, several control techniques have been applied to PMSM. These include feedback control methods [3–5], adaptive fuzzy control [6–8], simple sliding mode adaptive control [7], [9], adaptive neural sliding mode control [10], optimal Lyapunov exponents' placement [11], passive control [12], impulsive control [13, 14] and finite-time stability theory [15].

Chaos is a prevalent phenomenon observed in various systems, including circuits [16], power grid [17], fluid dynamics [18, 19], thermodynamics [20, 21]. The chaos theory, also known as chaotic dynamics or nonlinear dynamics, is a scientific concept that studies complex systems and their behavior. It suggests that seemingly random and unpredictable patterns can emerge from deterministic systems due to their extreme sensitivity to initial conditions. This theory explores the notion that small changes in the initial conditions of a system can lead to significantly different outcomes over time, making long-term prediction and control difficult. Chaos theory can be observed in various fields, such as physics, mathematics, biology, economics, and even PMSM.

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This paper is centered around addressing the speed control challenge of a PMSM operating in a chaotic regime while taking into account parametric uncertainties. Chaos in dynamic systems refers to a complex and highly unpredictable behavior that, despite being deterministic, can be significantly influenced by minute changes in initial conditions, making it difficult to predict or control.

The application of controllers in chaotic systems aims to regulate and manage their behavior. In the context of Permanent Magnet Synchronous Motors with controlled feedback, chaos control offers several advantages concerning performance and system reliability. These motors may exhibit chaotic behavior under specific operating conditions, leading to undesirable vibration and noise. By implementing chaos control, these chaotic oscillations can be limited, resulting in enhanced comfort and motor durability. Additionally, controlling chaos helps minimize energy losses, leading to improved motor efficiency and energy savings.

The presence of chaos can complicate the prediction and control of motor behavior. Implementing chaos control ensures better system stability, facilitating controller design and ensuring a more reliable motor operation. This approach helps prevent unpredictable and non-linear motor behavior, thereby enabling the development of accurate and efficient control strategies that enhance overall system performance. Moreover, chaos control reduces vibration, noise, and stress, thereby prolonging the motor's lifespan and reducing maintenance costs. In critical systems such as aeronautic, space, industrial, and medical applications, where stability and reliability are paramount, chaos control becomes particularly relevant for ensuring proper engine operation.

The study adopts an approach involving two single-state feedback controllers. The single-state feedback control method is known for its simplicity, conciseness, and ease of

implementation. The paper is structured into three main parts. Firstly, it presents the problem formulation, where the modeling of the PMSM and its behavioral characterization are discussed. In the second part, the adopted approach is introduced, along with an explanation of how it is applied to synthesize a control law for the PMSM. Finally, the paper concludes by presenting simulation results that validate the proposed method.

2. MATERIAL AND METHOD

2.1. Equations of PMSM

The equation system of PMSM is defined by the following set of equations [22]:

$$\begin{cases} \dot{\omega} = a(i_q - \omega) + \check{T}_L + \varepsilon i_d i_q \\ \dot{i}_q = -i_q - i_d \omega + b\omega + \bar{u}_q \\ \dot{i}_d = -i_d + i_q \omega + \bar{u}_d \end{cases} \quad (1)$$

where :

$$\varepsilon = \frac{\rho b L_q^2 k^2 (L_q - L_d)}{J R_s^2}$$

i_d and i_q represent the components of the stator current along the d-q axes, and ω the angular velocity of the motor. The components of the stator voltage with respect to the dq frame are denoted by \bar{u}_d and \bar{u}_q and the external torque \check{T}_L . a and b are the characteristic parameters of the system, ρ is the air density, R_s is the stator resistance, J is the rotor moment of inertia. We suppose that after a certain operating time all external voltages reset each other ($\bar{u}_d = \bar{u}_q = \check{T}_L = 0$) and the system goes into autonomous mode ; in this situation we will have :

$$\begin{cases} \dot{\omega} = a(i_q - \omega) + \varepsilon i_d i_q \\ \dot{i}_q = -i_q - i_d \omega + b\omega \\ \dot{i}_d = -i_d + i_q \omega \end{cases} \quad (2)$$

Let $\omega = x$, $i_q = y$ and $i_d = z$. Then the equation becomes :

$$\begin{cases} \dot{x} = a(y - x) + \varepsilon zy \\ \dot{y} = -y - zx + bx \\ \dot{z} = -z + xy \end{cases} \quad (3)$$

2.2. Chaotic Characteristic Analysis for the PMSM

System (3) shows chaotic behavior for certain values of a and b parameters [23]. To observe the chaotic behavior of PMSM, we kept the parameter b constant as b=20 and then changed the parameter a between 5 and 100.

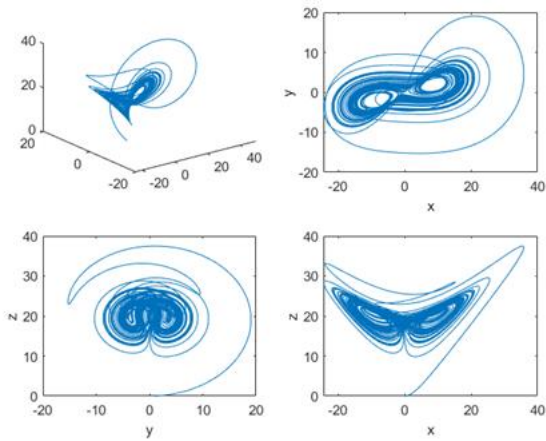


Figure 1 Phase portraits of the PMSM system attractor for a=5.45 ; b=20 ; x₀=0; y₀=1 ve z₀=0

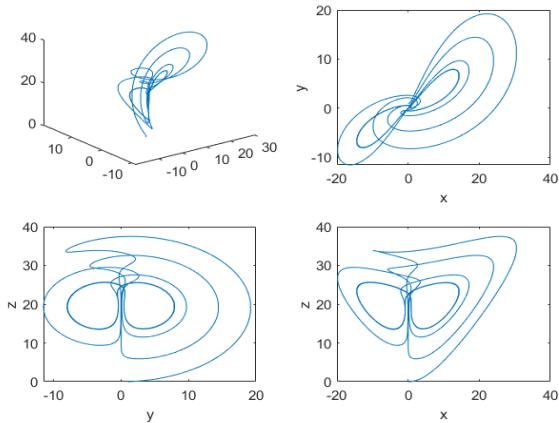


Figure 2 Phase portraits of the PMSM system attractor for a=20 ; b=20 ; x₀=0; y₀=1 ve z₀=0

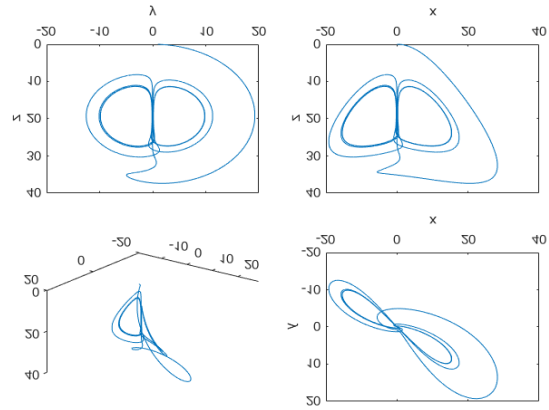


Figure 3 Phase portraits of the PMSM system attractor for a=30 ; b=20; x₀=0; y₀=1 ve z₀=0

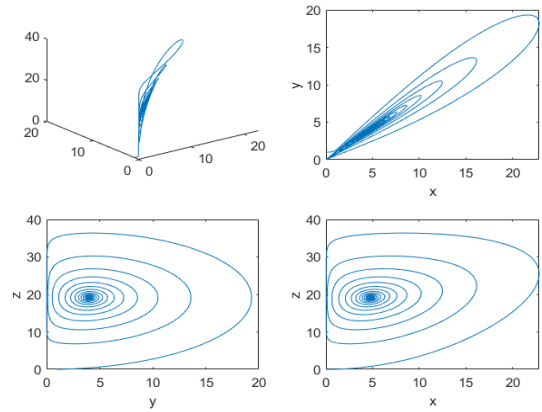


Figure 4 Phase portraits of the PMSM system attractor for a=100 ; b=20 ; x₀=0; y₀=1 ve z₀=0

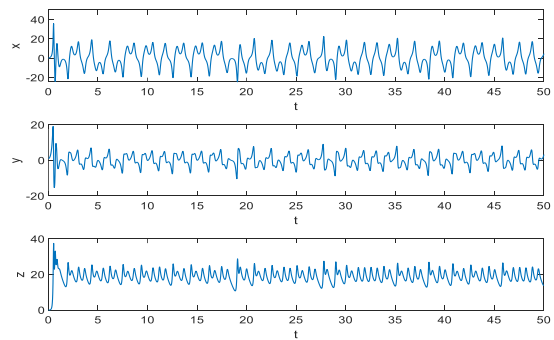


Figure 5 Time series of x, y and z for a=5.45 ; b=20; x₀=0; y₀=1 ve z₀=0

2.3. Equilibrium Points and Lyapunov Exponents

To obtain the equilibrium points of system (3), we must solve the following system :

$$\begin{cases} 0 = a(y - x) + \varepsilon zy \\ 0 = -y - zx + bx \\ 0 = -z + xy \end{cases} \quad (4)$$

After equation (4) is solved, there are 3 equilibrium points of system (3). One (0,0,0) is locally stable, the other two $(b-1+\sqrt{b-1} + \sqrt{b-1})$ and $(b-1-\sqrt{b-1} - \sqrt{b-1})$, locally unstable [23].

The Jacobian matrix of the system is defined as:

$$J = \begin{bmatrix} -a & a+z & y \\ b-z & -1 & -x \\ y & x & -1 \end{bmatrix} \quad (5)$$

The Lyapunov exponent of the system (3) for $a=5.45$, $b=20$ parameter values and initial conditions $x_0=0$; $y_0=1$ ve $z_0=0$ is shown in Figure 6.

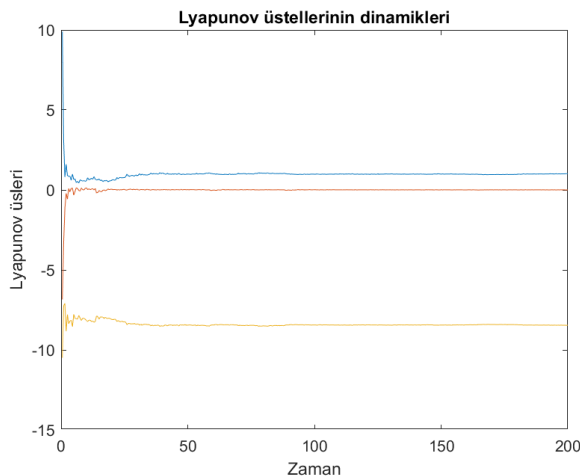


Figure 6 Lyapunov exponent of the PMSM system

3. CHAOS CONTROL OF PMSM USING A SINGLE-STATE FEEDBACK CONTROLLER

Feedback control is a powerful technique used to control chaos in permanent magnet synchronous machines (PMSMs). Chaos refers to the unpredictable and erratic behavior exhibited by the system, which can lead to undesirable performance and instability. By employing feedback control,

the chaotic dynamics of the PMSM can be effectively tamed and brought under control.

In PMSMs, feedback control is typically applied to regulate the motor's speed, torque, or position. The control system continuously measures the actual output of the motor and compares it with the desired reference value. Based on this error signal, the control algorithm calculates the appropriate control action to be applied to the motor. In the context of chaos control, the control algorithm is designed to introduce small perturbations or corrections to the system's state variables, such as the rotor position or current. These perturbations disrupt the chaotic behavior and guide the system towards a stable and desired operating point.

Consider an equilibrium point denoted by (x^*, y^*, z^*) for the given system. The objective of this investigation is to devise a control strategy that eradicates chaotic behavior and ensures stability of the system (3) in the vicinity of the equilibrium point. To achieve this, we employed specifically designed controllers based on the mathematical principles of Lyapunov's spherical asymptotic stability method [24].

3.1. The First Controller

The first control approach in this study focuses on regulating the speed (ω) of the motor. Ensuring chaos control in the motor's speed is crucial to achieve safe, stable, and predictable motor operation. In this context, chaos refers to unpredictable and undesired variations in the engine speed, which can have adverse effects on its overall performance. By precisely controlling the speed, the motor can be maintained within its optimal operating range, leading to maximum performance.

Chaotic variations in engine speed can result in undesirable vibrations and oscillations, potentially causing mechanical component damage and premature wear. By implementing precise speed control, these oscillations can be minimized, thereby

improving system stability and extending the motor's lifespan.

Moreover, a motor operating in a chaotic manner can consume energy inefficiently. Optimal speed control helps minimize energy losses and enhances the overall efficiency of the motor. In critical applications, such as transportation systems or industrial machinery, maintaining the motor speed within specific limits is essential to ensure safe operations. Chaos control plays a crucial role in keeping the speed within preset ranges, thereby helping to prevent dangerous situations or catastrophic failures.

$u_1 = -ay - \varepsilon zy$ controller is added to equation (3):

$$\begin{cases} \dot{x} = a(y - x) + \varepsilon zy + u_1 \\ \dot{y} = -y - zx + bz \\ \dot{z} = -z + xy \end{cases} \quad (6)$$

The substitution of controller expression u_1 in the system (6) would be:

$$\begin{cases} \dot{x} = -ax \\ \dot{y} = -y - zx + bz \\ \dot{z} = -z + xy \end{cases} \quad (7)$$

The solution of the equation is $x(t)=x(0)e^{-at}$ and thus $\lim_{t \rightarrow \infty} x(t) = 0$. System (7) can be reduced as follows :

$$\begin{cases} \dot{y} = -y + bz \\ \dot{z} = -z \end{cases} \quad (8)$$

From this equation, we can get: $z(t)=z(0)e^{-t}$ then $\lim_{t \rightarrow \infty} z(t) = 0$. System (8) can be reduced as follows :

$$\dot{y} = -y \quad (9)$$

The solution to the equation is $y(t)=y(0)e^{-t}$, which is $\lim_{t \rightarrow \infty} y(t) = 0$.

$u_1 = -ay - \varepsilon zy$ controller can control the chaotic behavior found in a PMSM. To prove this, we simulated the equation on Matlab

with $x_0=0; y_0=1$ ve $z_0=0$ as initial values and $a= 5.45$ and $b= 20 ; \varepsilon=1$. The Fig. 7 show us that chaos is under control.

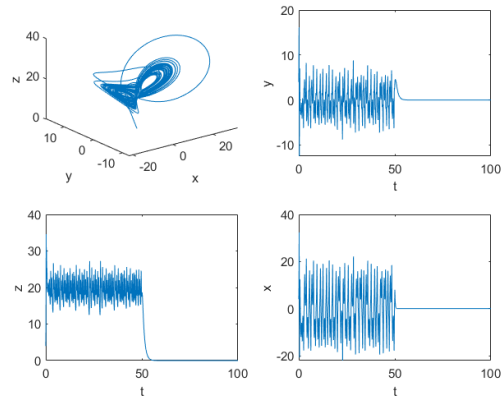


Figure 7 Time series of x, y and z after activation of u_1 at $t=50s$

3.2. The Second Controller

The second control approach in this study focuses on regulating the intensity (iq) of the motor. Implementing chaos control in the motor's intensity is crucial to ensure reliable and efficient motor operation.

The electrical intensity that powers the motor is directly linked to its operation and energy consumption. If the motor operates at an uncontrolled intensity, it poses the risk of overloading or overheating, potentially causing damage to the motor and surrounding components. Current control is essential to maintain safe operating conditions and prevent hazardous situations.

Precise regulation of the motor's intensity allows for the optimization of its energy consumption. Motors running at excessive currents can lead to energy wastage, whereas accurate control can reduce energy loss and enhance the overall system efficiency.

Furthermore, motors subjected to excessive currents are prone to premature wear. By controlling the amperage, the motor's internal components can be protected from excessive stress, leading to an extended useful life of the motor. This controlled approach to intensity helps ensure the motor operates within safe limits, promoting reliability and efficiency.

So $u_2 = zx-bz$ controller is added to equation (3):

$$\begin{cases} \dot{x} = a(y - x) + \varepsilon zy \\ \dot{y} = -y - zx + bz + u_2 \\ \dot{z} = -z + xy \end{cases} \quad (10)$$

The substitution of controller expression U_2 in the system (10) would be :

$$\begin{cases} \dot{x} = a(y - x) + \varepsilon zy \\ \dot{y} = -y \\ \dot{z} = -z + xy \end{cases} \quad (11)$$

The solution of the equation is $y(t)=y(0)e^{-t}$ and thus $\lim_{t \rightarrow \infty} y(t) = 0$. System (11) can be reduced as follows:

$$\begin{cases} \dot{x} = -ax \\ \dot{z} = -z \end{cases} \quad (12)$$

From this equation, we can get: $z(t)=z(0)e^{-t}$ then $\lim_{t \rightarrow \infty} z(t) = 0$.

$u_2 = zx-bz$ controller can control the chaotic behavior found in a PMSM. To prove this, we simulated the equation on Matlab using with $x_0=0; y_0=1$ ve $z_0=0$ as initial values and $a=5.45$ and $b=20$ $\varepsilon=1$. The Fig. 7 show us that chaos is under control.

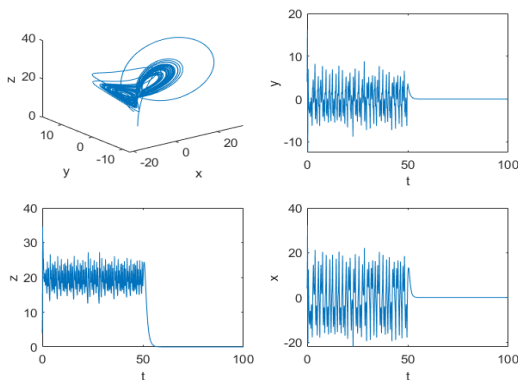


Figure 8 Time series of x, y and z after activation of u_2 at $t=50s$

4. SIMULATION RESULTS

Numerical simulations carried out using MATLAB with a calculation step of 0.01 second for the system (3) played an essential

role in the validation of the proposed approach. Using MATLAB's ode45 algorithm, the simulations made it possible to simulate and study the behavior of the system under controlled conditions.

Figures 7 and 8 present the results of the simulations, showing the evolution of the states of the system deviation vector. The nominal values used for the parameters were $a = 5.45$, $b = 20$ and $\varepsilon = 1$. The command was executed at $t=50$ seconds, which allowed analysis of the system response after the controller was applied.

Initial conditions for all simulations were set to $x_0=0$, $y_0=1$, and $z_0=0$, ensuring consistent comparison of controller performance. These initial conditions determine the state of the system when the controller is activated.

The results of the simulations indicate that the first controller produced a result of $x = -0.07$ at $t=51.013$ seconds. On the other hand, the second controller provided the same result at $t=50.934$ seconds. These results show that both controllers were able to stabilize the system successfully, reducing the deviation to a value close to zero.

However, it is important to note that the second controller was slightly faster than the first. This may be due to differences in the algorithms or tuning parameters used by the controllers. The difference in response time between the two controllers can be attributed to their specific design and implementation.

It should be emphasized that controller performance may vary depending on initial conditions, parameter values, and the specific context of the application. It is therefore important to analyze the results in the overall context of the proposed approach and to consider other performance criteria such as stability, accuracy and robustness of the system.

5. CONCLUSION

In conclusion, the utilization of feedback control techniques for chaos control in permanent magnet synchronous motors (PMSMs) has been demonstrated to be a promising approach. By employing the MATLAB algorithm ode45 and conducting numerical simulations, the efficiency of the proposed controller has been effectively showcased.

Chaos, a complex and unpredictable behavior exhibited by nonlinear systems like PMSMs, can have detrimental effects on their performance and stability. However, through the implementation of feedback control, it is possible to mitigate the chaotic behavior and restore system stability.

The conducted numerical simulations on MATLAB have highlighted the effectiveness of the proposed controller. The ode45 algorithm, a robust and widely used numerical integration technique, has proven its ability to accurately capture the dynamic behavior of the PMSM system under chaotic conditions. By applying the proposed control strategy, chaos control has been successfully achieved, leading to improved system performance and stability.

The results obtained from the simulations serve as evidence of the efficiency of the feedback control approach. The proposed controller not only mitigates chaos but also enables precise regulation and tracking of the motor's behavior. This has significant implications for various applications, where maintaining control over PMSMs is crucial, such as robotics, electric vehicles, and industrial automation.

It is worth noting that the success of the proposed controller lies in its ability to leverage feedback information, allowing for real-time adjustments and corrections. This enables the system to respond swiftly to disturbances and deviations, ensuring a stable and controlled operation of the PMSM.

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The Declaration of Conflict of Interest/Common Interest

No conflict of interest or common interest has been declared by the authors.

Authors' Contribution

The authors contributed equally to the study.

The Declaration of Ethics Committee Approval

This study does not require ethics committee permission or any special permission.

The Declaration of Research and Publication Ethics

The authors of the paper declare that they comply with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that they do not make any falsification on the data collected. In addition, they declare that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

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