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State Space Model Reference Adaptive Control for a Class of Second-Degree Fractional Order Systems with Different Eigenvalues

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Highlights

- This paper addresses the state space MRAC design for the class of fractional order systems (FOSs).
- The addressed FOS's have all their poles different.
- An extension of adaptive control with fractional order dynamics is designed for second-degree FOSs.
- The stability analysis is performed and the control efficiency is illustrated with a simulation example.

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Abstract

This study proposes an adaptive control synthesis for a class of second-degree fractional order systems with different eigenvalues in the state-space domain. The proposed fractional order adaptive controller is a generalization of the MRAC controller for the class of scalar fractional order systems. In order to control the fractional order plant, an adaptive state space feedback controller is applied based on the error between the system output and a chosen reference model using a fractional adaptation law to make the fractional order plant track the fractional order reference model. We show that the resulting adaptive regulator is able to stabilize the fractional order second degree system with a satisfying performance. A simulation example illustrating these performance properties is provided along with a comparison with a fractional order sliding mode control (FOSMC) to demonstrate the superiority of the proposed control scheme.

1. INTRODUCTION

Fractional Order Model Reference Adaptive Control (FOMRAC) became a leading research area in the field of automatic control engineering since its first apparition in the literature which was pioneered by the teams of Winagre et al. [1] and Ladaci et al. [2, 3]. Many design configurations and various applications of FOMRAC have been developed since by a continuously growing research community [4].

Engineers soon realized that integer-order linear models often didn't correspond to reality in the field. As a result, there was a move towards more complex adaptive MRAC control structures, often requiring a minimum of process information.

The most widely used methods are based on the input-output model approach proposed by Narendra and Annaswamy in 1989 [5], but for many systems such as power systems and robot manipulators, all state information can be measured or observed in real time. As a result, much work has focused on the state-space MRAC approach, which can be very efficient and more accurate, and which easily allows the application of Lyapunov's stability theorem for its analysis [6].Moreover, by introducing fractal-order operators for process modeling and/or closed-loop control, we have become able to ensure the stability of certain complex systems while achieving better temporal performance and robustness.

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The great interest of the control engineering community for FOMRAC could be justified by the following four facts:

- Fractional order models are able to describe physical systems better than the classical 'integer-order' models.
- Introducing fractional order dynamics in the closed-loop control system, can improve the system's behaviour,
- The inherent properties of fractional-order systems, including a fairly wide gain margin and the memory effect, make the control system more robust against disturbances and noise than integer-order systems.
- FOMRAC configuration is still a very simple scheme and easy to implement compared to other adaptive controllers.

So, compared with conventional integer-order MRAC, the FOMRAC control scheme not only improves the performance of the controlled system, increasing its robustness against external noise and disturbances that are often associated with processes operating in hostile environments, but also enables a wider class of systems with more complex or higher-order models to be tackled [7, 8].

A multitude of research works have proposed FOMRAC-based adaptive control configurations;Some examples of the research work performed in this area are: robust fractional adaptive control based on MRAC structure [9], FOMRAC for scalar systems in state space domain [10], indirect MRAC for a class of linear FOSs [11], MRAC for FOSs using discrete-time approximation methods [12], fractional order composite MRAC for MIMO systems [13,14], fractional order tube MRAC applied to fractional order linear systems [15], direct FOMRAC for a class of fractional order commensurate linear systems [16] and [17], optimal fractional order Lyapunov based MRAC [18], …etc.

A number of these aforementioned design proposals for adaptive controllers were done in the state space domain, which justifies the growing number of publications dealing with this topic [19-21].Additionally, numerous applications have used FOMRAC-based to achieve their control, e.g., blood pressure supervision for postoperative patients [22], the control of a tank's level in [23], and of a magnetic levitation system in [24], adaptive control for anaesthesia [25], an electrical vehicle cruise adaptive control [26], adaptive control strategy for piezo-actuated active vibration isolation systems [27], control of an active suspension system [28], MRAC configuration for robust fuzzy control for uncertain FOSs [29],MRAC control of an F15 aircraft pitch angular motion [30], …etc.

Many researchers are currently investigating the use of reference model control for systems represented by a fractional-order model. Various sub-classes of this type of model can represent many industrial or technological processes.

A fractional model reference adaptive control design is proposed in [31] to deal with fractional order scalar systems. In [8] FOMRAC adaptive control is applied to the class of fractional-order multivariable system with parameter uncertainty while in [32] the adaptive control design addresses the class of systems whose orders are switched among a fractional value in the interval (0,1) and 1 at certain time instants. In [33], the class of fractional order systems with matched uncertainty was addressed and in [34], the 'composite' MRAC control was extended to the class of multivariable fractional-order systems with arbitrary relative degree. More recently, the authors in [35] addressed the problem of constrained performance FOMRAC control design for a class of fractional order transfer functions with unknown parameters.

In this paper, a direct FOMRAC synthesis is introduced to deal with the class of second degree non-integer order systems represented in the state space domain with a state matrix having different eigenvalues. The resulting fractional adaptive controller is a generalization of the controller introduced in [10] for the class of scalar FOSs.

In particular, the proposed FOMRAC scheme is designed to control fractional-order systems described in state space by a state vector of order $n > 1$. To the best of our knowledge, this is the first time that a FOMRAC adaptive controller has been proposed in the literature for this class of fractional-order systems.

Besides, the processes modelized by Equation (8) present greater complexity than those most often studied in the literature. For this reason, in this study we consider only the subclass of second-order systems whose state matrix has distinct eigenvalues.

The remainder of this paper is organized as follows: section 2 presents the main definitions and concepts of fractional order systems. In section 3, the main control problem is detailed whereas section 4 presents the proposed state space FOMRAC scheme. In section 5 a numerical simulation example is given and discussed. Finally, some concluding remarks are given in section 6.

2. BASIC CONCEPTS FOR FRACTIONAL ORDER SYSTEMS

2.1. Definitions

The most popular definitions for fractional order derivation and integration are named Riemann-Liouville, Caputo and Grünwald-Letnikov [36].

The Riemann-Liouville (RL) fractional order integral of order $\eta > 0$ for the function $x(t)$ is defined as

$$ {}_{RL}I_{t_0}^{\eta} x(t) = \frac{1}{\Gamma(\eta)} \int_{t_0}^{t} (t-\tau)^{\eta-1} x(\tau) d\tau \,, \tag{1}$$

whereas the definition of RL fractional order derivative of order $\eta > 0$ for the function $x(t)$ is given by

$$ \begin{aligned} {}_{RL}D_{t_0}^{\eta} x(t) &= D^n I^{n-\eta} x(t) \,, \\ &= \frac{d^n}{dt^n}\left[\frac{1}{\Gamma(n-\eta)} \int_{t_0}^{t} (t-\tau)^{n-\eta-1} x(\tau) d\tau\right], \end{aligned} \tag{2}$$

where $\Gamma(.)$ represents the Euler's gamma function, such that $(n-1 < \eta < n, n \in N)$.

The definition of the fractional order Caputo(C) derivative of order $\eta > 0$ is given by:

$$ {}_{C}D_{t_0}^{\eta} x(t) = \frac{1}{\Gamma(n-\eta)} \int_{t_0}^{t} (t-\tau)^{n-\eta-1} x^{(n)}(\tau) d\tau \,, \tag{3}$$

where η is a real number verifying: $n-1 < \eta < n$.

The definition of Grünwald-Letnikov (GL) is presented as:

$$ {}_{GL}D_{t_0}^{\eta} x(t) = \lim_{h\to 0} \sum_{j=0}^{k} \omega_j^{(\eta)} x(kh - jh) \,, \tag{4}$$

where h is the sampling time, whereas the coefficients $\omega_j^{(\eta)}$ are computed as follows:

$$ \omega_j^{(\eta)} = \frac{(-1)^j \Gamma(\eta+1)}{\Gamma(j+1)\Gamma(\eta-j+1)}, \text{ j=0, 1, ..., k.} \tag{5}$$

This last definition is also used as a numerical approximation technique for the fractional order integral and derivative by taking the time $t = kh$.

2.2. Fractional-Order Systems Representation

2.2.1. Fractional-order transfer functions and matrices

Many natural dynamic systems have behavior that can be modeled by differential equations with fractional-order derivatives. By applying the Laplace transform to such equations and assuming zero initial conditions we obtain transfer functions with non-integer powers of the complex Laplace variable $'s'$.
A general form of this function is given below:

$$ F(s) = \frac{\sum_{i=1}^{M} b_i s^{q_i}}{\sum_{i=1}^{N} a_j s^{p_j}}, \quad M, N \in \mathbb{N}^*, a_j, b_i, p_j, q_i \in \mathcal{R}. \tag{6}$$

If the fractional-order process is MIMO (Multiple Input – Multiple Output) its m inputs and its n outputs will be linked by an $n \times m$ transfer matrix whose elements are similar to Equation (6).

2.2.2. Bode ideal transfer function

In order to illustrate some interesting properties of fractional-order models, let us consider the ideal transfer function proposed by Bode in his works about amplifying feedbacks which used as reference model in many fractional-order control schemes [37].

This function has the form:

$$L(s) = \left(\frac{s}{\omega_c}\right)^{\alpha} \quad (7)$$

where $\omega_c$is the desired cut frequency and$\alpha$is the ideal characteristic slope of the gain. The phase margin is constant and equal to $\phi_m = \pi(1 + \frac{\alpha}{2})$for all the gain values whereas the gain margin$A_m$is infinite. The bode diagram of this function is represented in Figure 1.

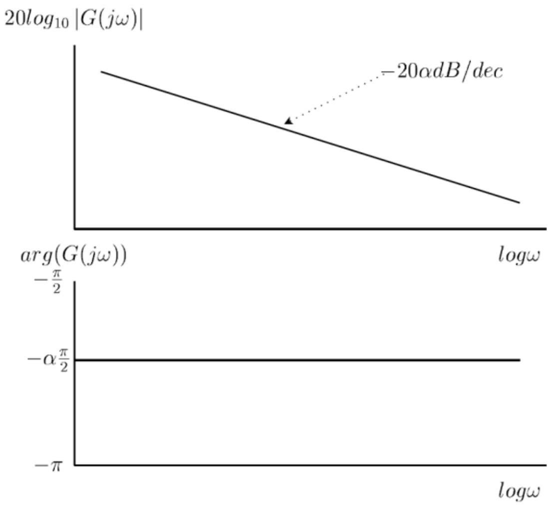

Figure 1. *Bode diagram of the ideal transfer function*

2.2.3. Fractional-order state representation

Fractional-order systems can also be represented in state space [38]:

$$\begin{aligned} D_{t_0}^{\alpha}x &= Ax + Bu \\ y &= Cx + Du \end{aligned} \quad (8)$$

where u is the input vector, x the state vector and y the system output vector. The state space representation of integer order systems is a particular case of Equation (8) with $\alpha = 1$.

Theorem 2.1. [38] The state space representation Equation (8) corresponds to the transfer matrix,

$$y = [C(s^{\alpha}I - A)^{-1}B + D]u \quad (9)$$

with the assumption that all initial conditions are zero.

2.3. Mathematical Tools for Stability Analysis

We recall here, some important results on the stability of FOSs.

Lemma 2.2. (Meyer–Kalman–Yakubovich) [39]

Consider a stable matrix A, vectors B, C and a scalar $d \geq 0$, then: If

$$G(s) = d + C^T(sI - A)^{-1}B \tag{10}$$

is strictly positive real (SPR), then for any given $H = H^T > 0$, there exist a scalar $r > 0$, a vector z and a matrix $P = P^T > 0$ such that:

$$\begin{aligned} A^T P + PA &= -zz^T - rH \\ PB - C &= \mp z\sqrt{2d}. \end{aligned} \tag{11}$$

Now, if we consider a non-integer order nonlinear time-varying system of the form:

$${}_{C}D_{t_0}^{\mu} f(t) = f(x,t) \tag{12}$$

where t represents the time and $\mu \in (0,1)$ with Caputo derivative. The following important result for the stability analysis of the fractional order system depicted byEquation (12) is obtained.

Definition 2.3. [39] We say that a continuous function φ: [0, t) ⟶ [0,∞) is a class-K function if it is strictly increasing and $\varphi(0) = 0$.

Theorem 2.4. [40] If we consider a non-autonomous FOSof the form Equation (12), with an equilibrium point at $x = 0$. If a Lyapunov function $V(t, x(t))$and the class −K functionsφ_i $(i = 1,2,3)$exist and satisfy:

$$\varphi_1(\|\xi\|) \leq V(t, \xi(t)) \leq \varphi_2(\|\xi\|) \tag{13}$$

$${}_{C}D_{t_0}^{\mu} V(t, \xi(t)) = -\varphi_3(\|\xi\|) \tag{14}$$

with $\mu \in (0,1)$, then the FOS Equation (12) is asymptotically stable.

We recall also the following lemma,

Lemma 2.5. [40] Suppose that $x(t) \in \Re^n$ *is a* derivable function, with $Q = Q^T \geq 0 \in \Re^{n\times n}$ then, for any time instant t_0

$$\frac{1}{2}\, {}_{C}D_{t_0}^{\mu} \varphi^T(t) Q \varphi(t) \leq \varphi(t) Q \frac{1}{2}\, {}_{C}D_{t_0}^{\mu} \varphi(t) \tag{15}$$

3. PROBLEM STATEMENT

We consider the fractional order system described by the following standard state space controllable form:

$$\begin{aligned} D^{\alpha} x &= Ax + Bu, \\ y &= Cx \end{aligned} \tag{16}$$

where α is the fractional order lying between $(0, 1)$, y is the plant output, u is the input, and $A \in \Re^{n\times n}$, $B \in \Re^{1\times n}$and $C \in \Re^{n\times 1}$are constant plant parameters that are assumed to be unknown, and are given as follow:

$$A=\begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \ddots & \ddots & \cdots & 0 \\ 0 & & \dots & 0 & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-1} \end{bmatrix}, B=\begin{bmatrix} 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad C=[1\ 0 \dots 0] \tag{17}$$

Hypothesis 3.1.

The state matrix A has eigenvalues that are all different.

This Hypothesis means that it is possible to decouple the system directly to a diagonal (real or complex) one by making use of congruence or strict equivalence transformations to A [41].

A chosen reference model specified by a fractional state space equation is given by:

$$D^{\alpha} x_m = A_r x_m + B_r r(t), \quad y_m = C_r x_m \tag{18}$$

where $A_m \in \Re^{n\times n}$, $B_m \in \Re^{n\times 1}$ and $C_m \in \Re^{1\times n}$ are constant parameters and $r(t)$ is a bounded external reference signal.

We define the tracking error

$$e = y - y_m. \tag{19}$$

We propose an adaptive feedback control scheme using a fractional adaptation law to make the fractional order plant Equation (16) track the fractional order reference model Equation (18), as follows

$$u = K_x x + K_r r \tag{20}$$

where $K_x \in \Re^{1\times n}$ and $K_r \in \Re$ are the adaptive gains.

4. FRACTIONAL-ORDER ADAPTIVE CONTROL SCHEME

We propose the following adaptations laws for the control gain matrix K update:

$$D^{\alpha} K = -\gamma B^T P e x_a^T \tag{21}$$

where $K = [K_x K_r]$, $x_a = \begin{bmatrix} x \\ r \end{bmatrix}$, and γ, a positive scalar, is the adaptation gain. The overall adaptive controller configuration is represented in Figure 2.

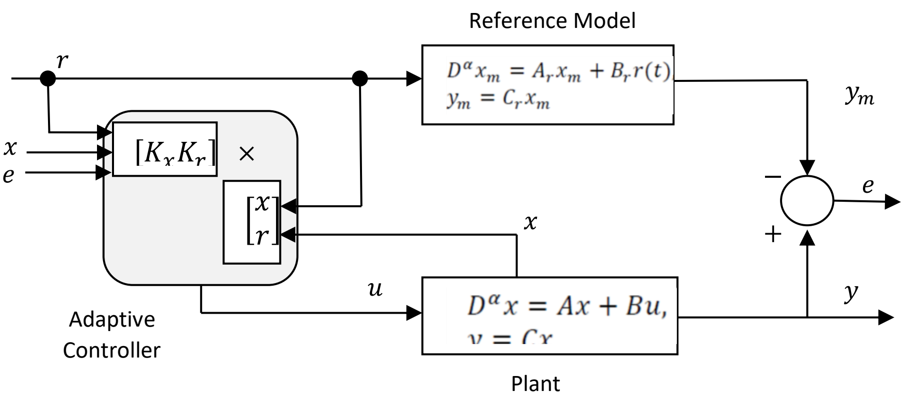

Figure 2. *Proposed adaptive control configuration*

4.1. Stability Result

Our main result can be stated in the following theorem.

Theorem 4.1. Consider the FOS of Equation (16), and the reference model Equation (17) then the MRAC controller given by Equation (20) and Equation (21) stabilizes the closed loop system and all the error signal Equation (19) converge to zero.

4.2. Stability Analysis and Proof

To prove Theorem 4.1, we will have to need some important results. First, when considering the scalar system case, that is $n = 1$.

Theorem 4.2. (Scalar system, $n = 1$) [10] Consider the FOS given by Equation (16), and the reference model given by Equation (17) with $n = 1$, then the MRAC controller given by Equation (20) and Equation (21) stabilizes the closed loop system and the error signal Equation (19) converge to zero.

Proof of Theorem 4.2. See ref. [10].

Now let us consider the fractional order system of Equation (16), and the reference model of Equation (17) with $n = 2$.

Since the system verifies Hypothesis 3.1, then the matrix P for which the columns are the eigenvectors of the state matrix A is invertible and we have the following: By putting,

$$z = Px. \tag{22}$$

And using the dynamics of x given in Equation (16), the dynamics of z can be written as follows:

$$\begin{aligned} D^{\alpha}z &= PAP^{-1}z + PBu, \\ y &= CP^{-1}z. \end{aligned} \tag{23}$$

And if we decompose the state vector$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$, the control action u into two scalar controls u_1 and u_2, and defining the command vector PB adequately we can rewrite the system Equation (16) into two subsystems:

$$\begin{aligned} \Sigma_1: D^{\alpha}z_1 &= \check{A}_1 z_1 + \check{B}_1 u_1, \\ y_1 &= \check{C}_1 z_1 \end{aligned} \tag{24}$$

$$\begin{aligned} \Sigma_2: D^{\alpha}z_2 &= \check{A}_2 z_2 + \check{B}_2 u_2, \\ y_2 &= \check{C}_2 z_2. \end{aligned} \tag{25}$$

Using two FOMRAC controllers of the forms Equation (16) and Equation (17) to the subsystems Σ_1 Equation (24) and Σ_2 Equation (25) respectively,

$$\begin{aligned} u_i &= k_{zi} z_i + k_r r \\ D^{\alpha}K &= -\gamma B^T P e z_a^T \end{aligned} \tag{26}$$

where $K = [k_z k_r]$, $z_a = \begin{bmatrix} z_i \\ r \end{bmatrix}$, and $i \in [1, 2]$.

Then from Theorem 4.2, both the closed loop systems are stabilized, and thus the original system Equation (16).

5. SIMULATION EXAMPLE AND DISCUSSION

We will proceed now with a numerical simulation example for the application of the proposed FOMRAC regulator strategy to a plant with a fractional order model given by Equation (16), and then we'll compare it to a fractional-order FOPID controller which is optimized using the Particle Swarm Optimization (PSO) algorithm. Simulations are implemented on Matlab Simulink.
The plant model is given by,

$$D^{\alpha}x = \begin{bmatrix} 0 & 1 \\ -1 & 0.5 \end{bmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \quad (27)$$

and let us choose a reference model as,

$$D^{\alpha}x_m = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x_m + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r. \quad (28)$$

5.1. Application of the Proposed FOMRAC Scheme

Let us apply the proposed MRAC algorithm with $B^T P = [3.4 \;\; 4]^T$and different values of the fractional-order$\alpha \in \Re^+$. Simulations results are illustrated in Figures 1 to 5.

We made a comparison between the conventional MRAC and the fractional one while changing α from 0.8 to 1.9 by computing the quadratic error criterion J_α,

$$J_\alpha = \int_0^T e^2(t)dt \quad (29)$$

where T is the simulation time window.

The variation of the quadratic error criterion J_α versus $\alpha \in [0.8 \;\; 1.9]$ is given in Figure 3. The conventional MRAC is a special case of the proposed fractional order adaptive controller when $\alpha = 1$.

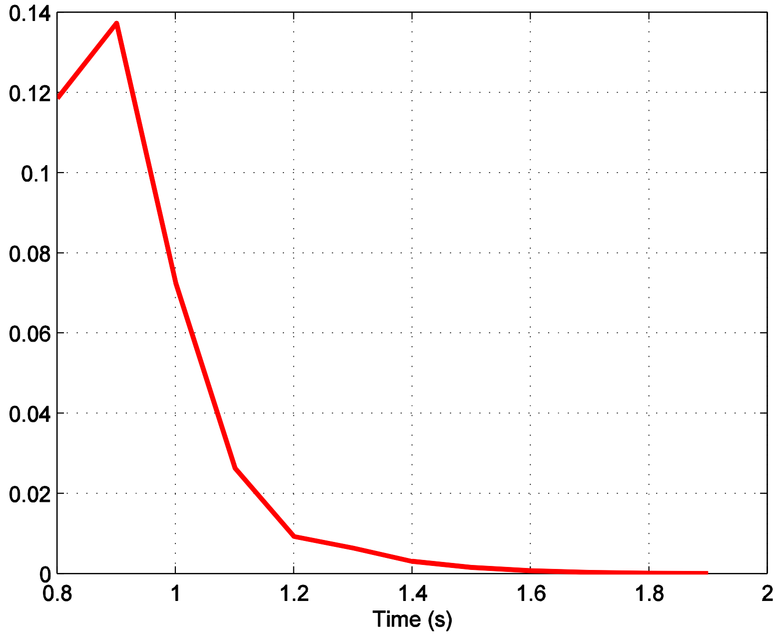

Figure 3. *Quadratic error criterion J_α for $\alpha = 0.8$ to 1.9*

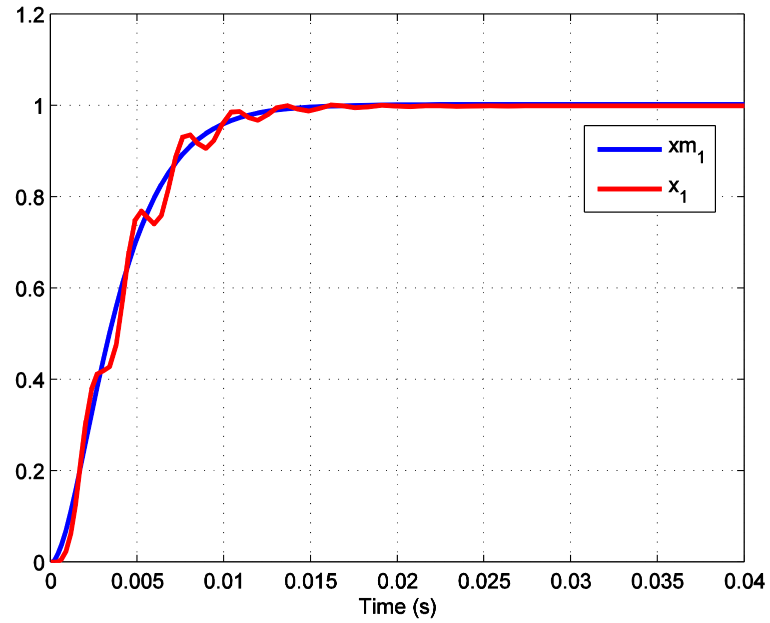

Figure 4. *State* x_1 *behavior for* $\alpha = 1.8$

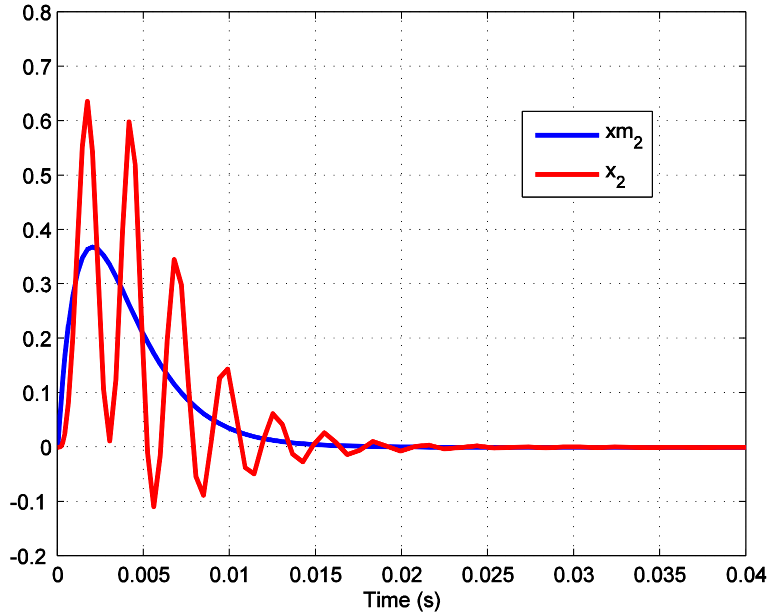

Figure 5. *State* x_2 *behavior for* $\alpha = 1.8$

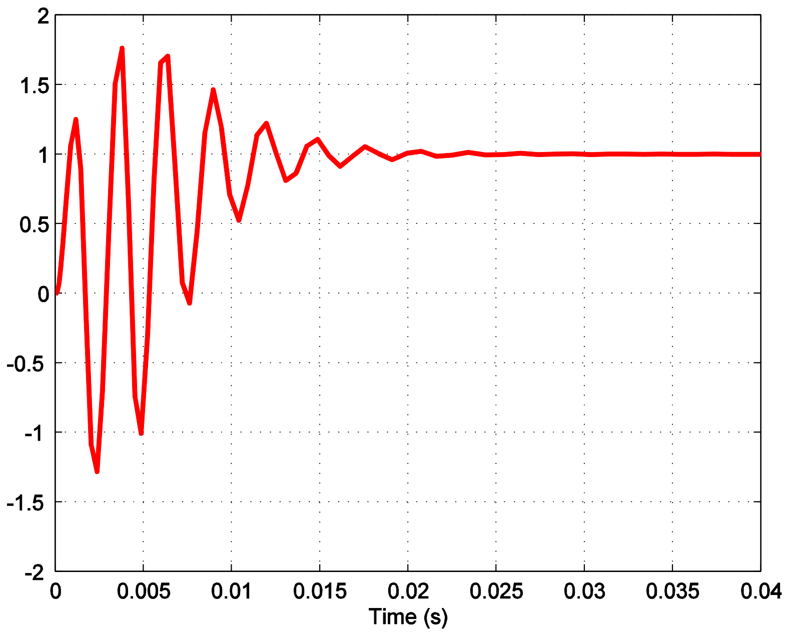

Figure 6. *Control signal* u *for* $\alpha = 1.8$

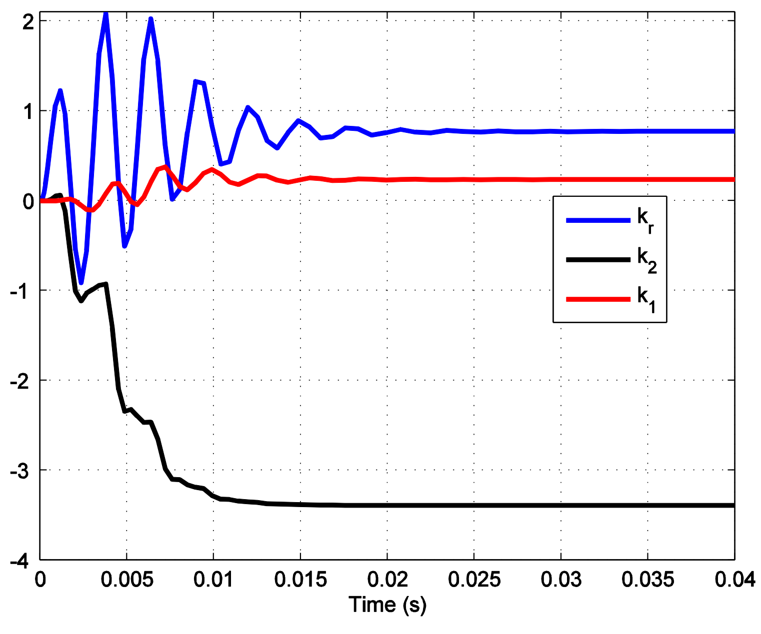

Figure 7. *Control law gains evolution*

Figures 4 and 5 illustrate the system's states x_1 and x_2 for $\alpha = 1.8$ (for which the quadratic criterion $J_\alpha$is minimal). The control signal u is depicted in Figure 6 whereas the control law's gains Equation (20), K_x and K_r are given in Figure 7.

From this simulation results, it is seen that all the variables (states and gains) converge rapidly to their final values. The system's states follow the reference model states with precision. We notice that the time response is very short $\tau_r < 0.02\ sec$.

The oscillations in the transitory phase are mainly due to the unstable dynamics of the open loop system Equation (27) and to the aggressiveness of the fractional-order adaptive controller, which makes it rapidly converge to the desired set point.

From the results shown in Figures 4 and 5, we can draw the conclusion that the states of the system Equation (22) controlled by the proposed adaptive regulator track the reference signal $x_m(t)$with remarkable performance under the control signal $u(t)$, and the tracking error $e(t)$. The simulation illustrates the efficiency of the state space FOMRAC adaptive control scheme for stabilizing the considered class of fractional order systems Equation (16).

Figure 7 shows that the adaptation gain $K = [K_x\ K_r]$ stabilizes quickly at final values, as expected in [42].

5.2. Comparison with a FOSMC Controller

In order to demonstrate the effectiveness of this fractional-order adaptive control scheme, we will compare it with the results obtained with a fractional-order sliding mode controller (FOSMC)designed on the basis of the power rate reaching law approach used to reduce the chattering phenomena in the control signals [43, 44]. In particular, we consider the fractional order version of this SMC approach proposed by Efe in [45].

The siding surface is defined as follows:

$$s(t) = ae(t) + D^{\alpha}e(t) \tag{30}$$

with $e = r - x_1$ and α is the fractional order of the system Equation (27).

This leads to,

$$\mathrm{D}^{\alpha}\mathrm{e} = \mathrm{D}^{\alpha}\mathrm{r} - \mathrm{D}^{\alpha}\mathrm{x}_1 \tag{31}$$

Considering a step reference signal r, we obtain

$$\mathrm{D}^{\alpha}\mathrm{e} = -\mathrm{D}^{\alpha}\mathrm{x}_1 = -\mathrm{x}_2. \tag{32}$$

From Equation (30), we get

$$\mathrm{s} = \mathrm{ar} - \mathrm{ax}_1 - \mathrm{x}_2. \tag{33}$$

We define

$$\mathrm{D}^{\alpha}\mathrm{s} = -\mathrm{ks}^{\gamma} \tag{34}$$

where γ is a positive real number.

Taking the fractional order derivative of the sliding surface Equation (33), we obtain

$$\begin{aligned} D^{\alpha}s &= -aD^{\alpha}x_1 - D^{\alpha}x_2 \\ &= -ax_2 + x_1 - 0.5x_2 - u = -ks^{\gamma}. \end{aligned} \tag{35}$$

So, the control law is given by,

$$u(t) = -(a + 0.5)x_2 + x_1 + ks^{\gamma} \tag{36}$$

The stability analysis of this command can be found in [44, 45].

The FOSMC simulation results were performed using the following parameters' values:

$$k = 10,\ a = 5,\ \gamma = 1.4\ . \tag{37}$$

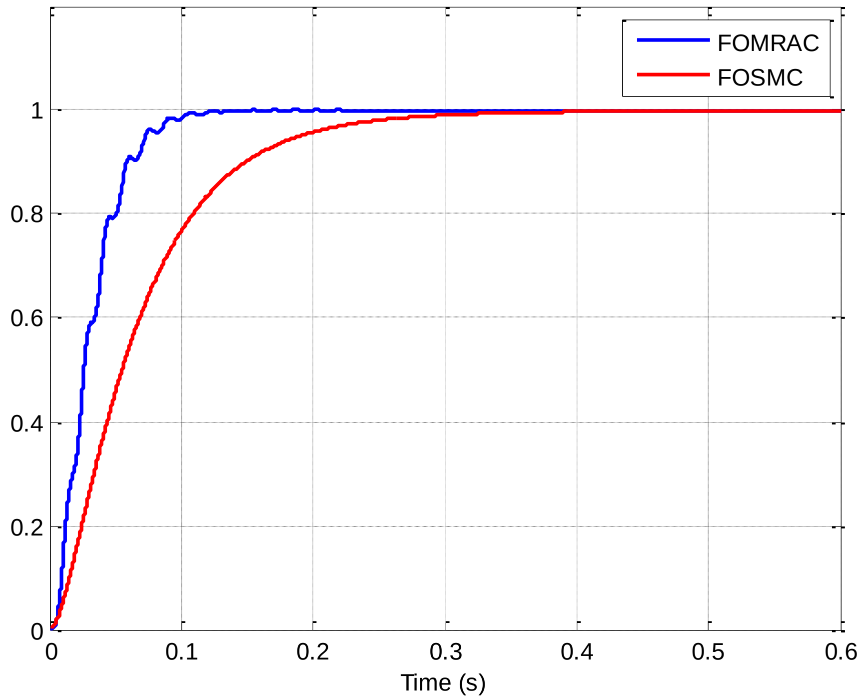

Figure 8. *Comparative response of FOMRAC and FOSMC controllers*

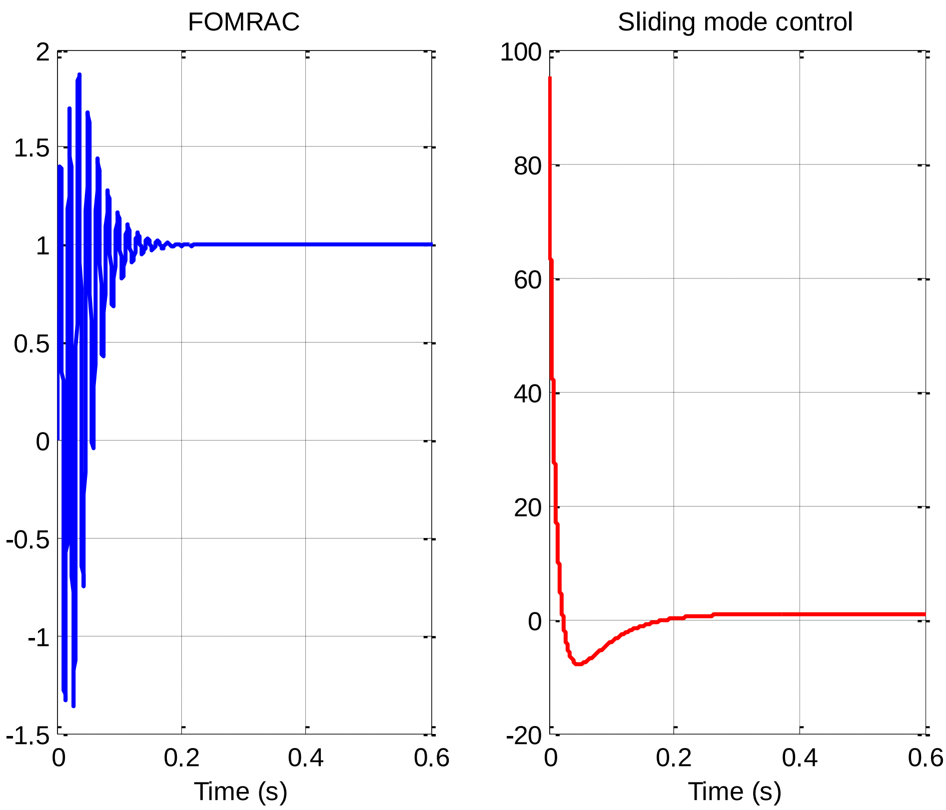

Figure 9. *Control signals of FOMRAC and FOSMC controllers*

The comparative responses are given in Figures 8 and 9. They illustrate that the response of the proposed adaptive controller is far superior to that obtained with a FOSMC regulator. The latter exhibits a slower response time than the FOMRAC.

It appears also from Figure 9 that the control effort of FOSMC controller presents a huge initial overshoot comparatively to the control signal of the proposed FOMRAC controller. This validates the superiority of the proposed fractional adaptive controller as confirmed also recently in [46].

5.3. Comparison Study with an Uncertain Fractional-order System

In this experiment, uncertainty is introduced into the system state matrix Equation (27) in order to test and compare the robustness of the proposed FOMRAC control scheme with that of a FOSMC control. We obtain the following uncertain model,

$$D^{\alpha}x = \begin{pmatrix} 0 & 1 \\ -1 & \delta \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \tag{38}$$

where δ is a random number such that $\delta \in [0, 1]$.

Applying the proposed FOMRAC controller and the FOSMC given by Equation (36) with the parameters of Equation (37) we obtain the comparative output responses and control efforts illustrated in Figure 10 and Figure 11, respectively.

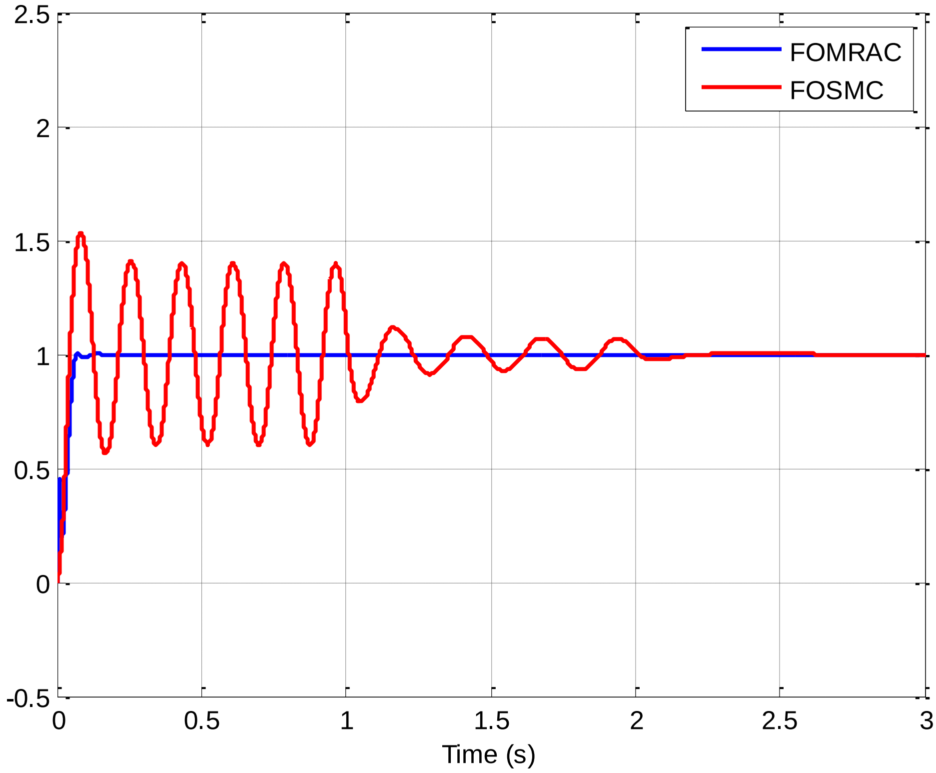

Figure 10. *Comparative response with a FOSMC controller for an uncertain model*

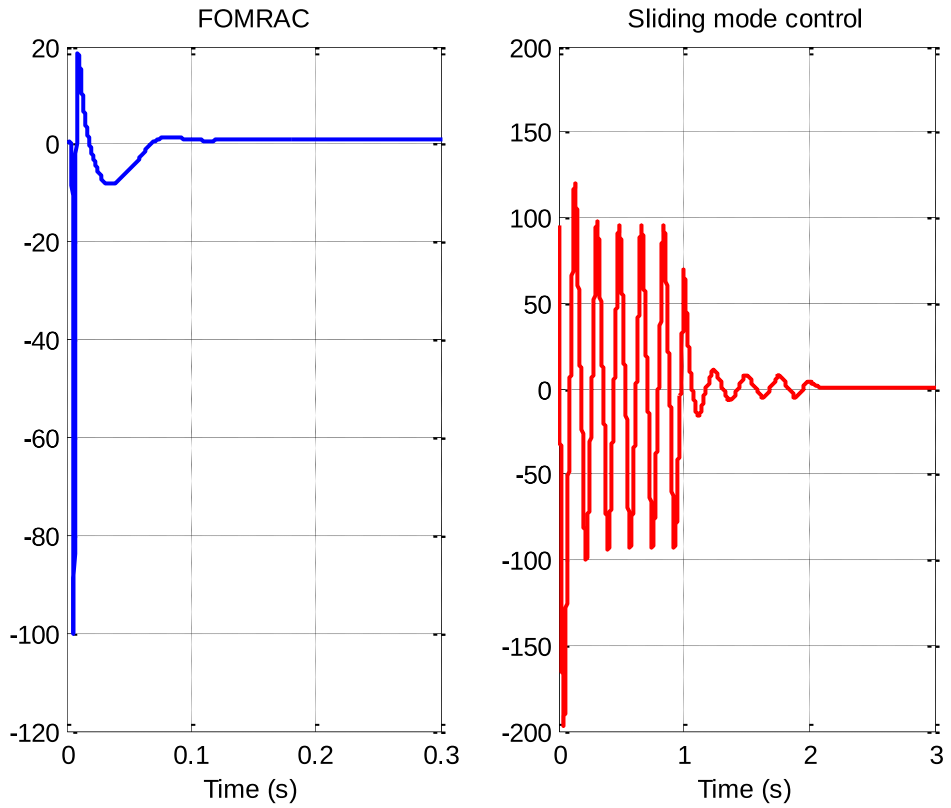

Figure 11. *Comparative control efforts with a FOSMC controller for an uncertain model*

It can be seen that the response of the proposed controller is better than that of the PID controller in terms of response time and transient behavior, as shown in Figure 10.Figure 11 also clearly shows that the effort required by the FOMRAC controller is smaller and less oscillatory than that of the FOSMC controller.

6. CONCLUSION

In this study, a model reference adaptive controller for a class of FOSs of second degree with different poles has been proposed. The proposed FOMRAC adaptive controller is a generalization of the MRAC controller for the class of scalar FOSs. In order to control the fractional order plant, an adaptive state space feedback controller is applied based on the error between the system output and the chosen reference model. We showed that the FOS can be stabilized using the proposed fractional order state space feedback controller which is based on the MRAC configuration for a second-degree system. A numerical example illustrating the effectiveness of this generalized control solution was also provided and its superiority to FOSMC control illustrated even in presence of model uncertainties.

Future research work will include experimental application of the proposed FOMRAC control scheme and validation of the obtained theoretical and simulation results.

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CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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