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Solution of the Cauchy Problem in Fourier Series for a Linear RC Circuit with a Resistor and Two Capacitors

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Abstract

As it is known, solutions obtained in terms of Fourier series are widely used for more detailed analysis of mathematical models defined for different engineering problems. In this study, for the mathematical analysis of transient events in active RC based linear electronic circuits, the Cauchy problem is defined and solved in terms of Fourier series. For this reason, first of all, the mathematical model needed for the mathematical definition of the investigated problem was created. Based on the defined mathematical model, a quadratic differential equation for the linear RC electric circuit is obtained and the voltage and current changes are analysed for simple special cases using the Fourier series. As a result, mathematical modelling of transient processes occurring in linear RC circuits and mathematical problems defined as the Cauchy problem for such circuits have been solved analytically. It is thought that the results of the research will contribute theoretically and practically to the solution of the problems that arise in the study and design automation of different electronic circuits containing nonlinear circuit elements.

Keywords: electronics, differential equations, Fourier series, mathematical model, RC circuit

Bir Direnç ve İki Kapasitörlü Lineer Bir RC Devresi için Cauchy Probleminin Fourier Serisi Cinsinden Çözümü

Öz

Bilindiği üzere, değişik mühendislik problemleri için tanımlanan matematiksel modellerin daha ayrıntılı analizi için Fourier serisi cinsinden elde edilen çözümler yaygın olarak kullanılmaktadır. Bu çalışmada, aktif RC tabanlı lineer elektronik devrelerde gerçekleşen transient olayların matematiksel analizi için Cauchy problemi tanımlanmış ve Fourier serisi cinsinden çözülmüştür. Bu nedenle öncelikle, incelenen problemin matematiksel tanımı için ihtiyaç duyulan matematiksel model oluşturulmuştur. Tanımlanan matematiksel modele dayalı olarak lineer RC elektrik devresi için ikinci dereceden bir diferansiyel denklem elde edilmiş ve Fourier serisi kullanılarak basit özel durumlar için gerilim ve akım değişimleri analiz edilmiştir. Sonuç olarak, lineer RC devrelerinde gerçekleşen geçici süreçlerin matematiksel modellenmesi ile bu tür devreler için Cauchy problemi biçiminde tanımlanmış matematiksel problemler analitik çözülmüştür. Elde edilen araştırma sonuçlarının, doğrusal olmayan devre elemanları içeren değişik elektronik devrelerin çalışma ve tasarım otomasyonunda ortaya çıkan problemlerin çözümüne teorik ve pratik katkı sağlayacağı düşünülmektedir.

Anahtar Kelimeler: elektronik, diferansiyel denklemler, Fourier serileri, matematiksel model, RC devre

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Introduction

In today's conditions, the production of information transmission systems, computers and other electronic equipment has become one of the priority areas of scientific research. Electrical signals whose physical magnitude is current or voltage are used in information transmission systems. At the same time, information transfer tools include highly complex electronic circuits (Chua & NG, 1979; Rabiner, 1971; Sun & Fidler, 1997). For this reason, the quality requirements of the designed electronic devices are increasing and every device developed must provide the necessary accuracy (Bruton, 1980; Chua & Lin, 1975; Manthe et al., 2003; Sanchez-Lopez et al., 2011; Soliman & Saad, 2010). Apart from these, it is known that the calculations encountered in the design of electronic circuits have a very complex structure. This situation has made it necessary not to use modern computational facilities. Also, circuit-based methods of prototyping have proven to be ineffective, making it impossible to study integrated circuits that are traditionally assembled from discrete elements (Yerzhan & Langheinrich, 1985). Thus, the complexity and responsibility of the electronic circuits being developed made it necessary to abandon the old traditional methods (Dumitriu et al., 2007; Sanchez-Lopez, 2013; Sanzhez-Lopez et al., 2011).

Electrical circuits may consist of simple or complex systems for a specific purpose. The basic laws that can solve these systems using old traditional methods are also the same. Therefore, the qualitative theory of dynamical systems should provide the basis for a more accurate necessary approach (Teor, 1984). From this point of view, computerized analysis of transient processes occurring in electrical circuits has become quite common. Therefore, it is necessary to determine the mathematical models needed for the transient processes in line with the targeted problems. It should be known that the mathematical model of the problem depends on its physical formulation, symbolic analysis and the properties of the circuit elements. At this point, symbolic analysis is a powerful tool that can model the behaviour of a circuit with symbolic parameters. At the same time, symbolic expressions not only give a better idea of the behaviour of the circuit, but can also be used in synthesis and optimization procedures (Sanchez-Lopez, 2011). In this modelling, the mathematical models of the processes occurring in various linear RC circuits are examined and the analytical solution of the transient processes is calculated.

In the transient occurring in a linear RC circuit containing two capacitors and a resistor, the changes in the determined component of the solution at the initial moment are determined. Thus, the mathematical definition of the problem was made with the help of mathematical modelling method. Special cases were also determined by analysing the solution obtained from the mathematical model. As a result, an analytical solution is found by obtaining mathematical models of transient processes in a linear electronic circuit.

Material and Method

Transition Processes in Linear Circuits

In electrical circuits, during various interactions that lead to a change in their mode of operation, i.e., under the action of various kinds of switching equipment, for example, switches for turning on or off the source or receiver of energy, when the circuit breaks, etc., transient processes occur. Among these effects, an important role is played by harmonic oscillations, which are widely used for the transmission of signals and electrical energy. The study of the mode of harmonic oscillations is also important in those cases when non-harmonic effects are considered and expansions of the function describing the effect signal into Fourier series are used.

The physical reason for the occurrence of transient processes in circuits is the presence of inductive and capacitive elements in them in the corresponding equivalent circuits. This is explained by the fact that the energy of the magnetic and electric fields formed on the components during the transition process in the circuit cannot suddenly change. It is known that for the analysis and design of electronic circuits, it is necessary to develop a mathematical model of the possible processes that may occur in these circuits. Therefore, the temporal process in a linear circuit is described by linear differential equations. In most cases, there is a mathematical problem associated with the Cauchy problem for a system of ordinary differential equations. For this reason, various analytical methods are used to solve constant parameter linear differential equations. These methods are used to calculate transient processes occurring in electrical circuits. The most common of these methods is the classical method, which has a physical nature and is used to calculate simple circuits. At the same time, the operator method and the Fourier integral method are used to simplify the computation of complex circuits. The purpose of this work is to implement the Fourier integral method for the equivalent electrical circuit shown in Figure 1, when the source is a complex function.

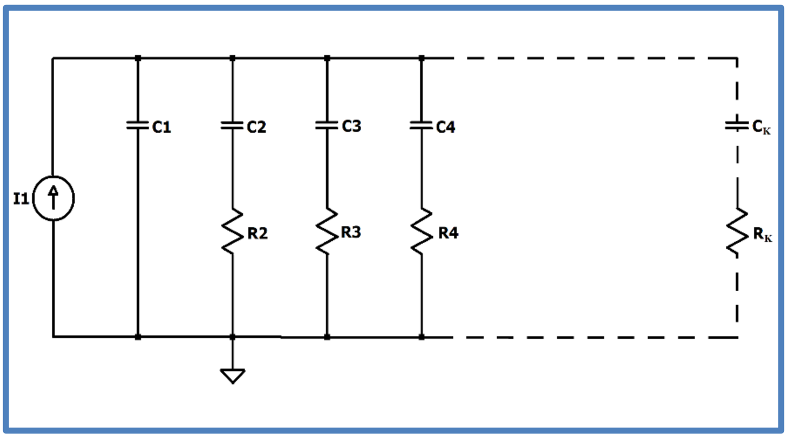

Figure 1. Linear RC Circuit with n Capacitors

The processes occurring in electronic circuits can be described using mathematical relationships between the fundamental parameters of the circuit. These parameters are mainly currents, voltages and properties of circuit elements. It is also known that these equations are arranged according to Kirchhoff's laws. The number of these equations depends on the number of energy storage elements of the electric and magnetic fields in the circuit under consideration. In this work, capacitors act as a storage device for electrical energy. Starting from the laws of Kirchhoff and commutation (second commutation law $V_C(0^-) = V_C(0^+)$) to construct the circuit's state equations, the following well-known formulas can be written for the electrical circuit under consideration (Figure 1):

$$\begin{cases} C_1 \dfrac{dV_{C_1}}{dt} = \sum_{k=2}^{n} \dfrac{V_{C_j} - V_{C_1}}{R_j} + i(t) \\ C_j \dfrac{dV_{C_k}}{dt} = \dfrac{V_{C_1} - V_{C_j}}{R_j} \quad , \qquad j = 2,3,\dots,n \\ i_k = C_k \cdot \dfrac{dV_{C_k}}{dt} \,, \qquad k = 1,2,\dots,n \end{cases} \tag{1}$$

$$V_{C_k}(t)|_{t=0} = 0, \qquad k = 1,2,\dots,n \tag{2}$$

Thus, the Cauchy problem for differential equations (1) with initial conditions (2) compiled for this scheme in the general formulation can be formulated as follows:

Let a system of n differential equations with n sought-for functions in normal form (1) and a system of initial conditions (2) be given,

i. If the point $M_0(t_0, x_{10}, \dots, x_{n0})$ is $(n+1)$- inside the dimensional domain of variables $\Omega(t, x_1, \dots, x_n)$ in which $lim_{M \to M_0} f(M) = f(M_0)$, then there exists a particular solution of system (1) that satisfies the conditions (2)

ii. If, in addition, in this region $\Omega(t, x_1, \ldots, x_n)$ the functions f_i- have continuous partial derivatives $(f_i)'_{x_j}(M)$, $\quad (i, j = 1,2, \ldots, n)$ then this particular solution is unique.

For simplicity, the described Cauchy problem is primarily solved a linear RC circuit with two capacitors. For this special case, the solution of the system of linear equations defined by (1), satisfying the initial condition defined by (2), can be written in terms of circuit parameters and source $i(t)$ function as follows:

$$V_{C_2}(t) = \frac{1}{R_2 C_2}\int_0^t f(x) e^{-\left(\frac{C_1+C_2}{R_2 C_1 C_2}\right)x} dx = \frac{1}{\tau_2}\int_0^t f(x) e^{-\alpha x} dx \tag{3a}$$

$$V_{C_1}(t) = V_{C_2}(t) + \left[f(t) e^{-\left(\frac{C_1+C_2}{R_2 C_1 C_2}\right)t}\right] = V_{C_2}(t) + [f(t) e^{-\alpha t}] \tag{3b}$$

Here,

$$f(t) = \frac{1}{C_1}\int_0^t i(x) e^{\left(\frac{C_1+C_2}{R_2 C_1 C_2}\right)x} dx = \frac{1}{C_1}\int_0^t i(x) e^{\alpha x} dx \tag{4}$$

In these formulas, the notation $\propto = \frac{C_1+C_2}{R_2 C_1 C_2} = \frac{1}{\tau_2} + \frac{1}{\tau_1}$ is made, and τ_1 and τ_2 represent the time constants of the capacitors C_1 and C_2 in the circuit. As it can be seen, the solution of the Cauchy problem defined for the investigated special case depends on the source function $i(t)$.

If all discontinuity points of the function $i(t)$ are discontinuity points of the first kind and have no more than a finite number of local extrema on the segment $[0,\ T]$, then the Dirichlet conditions are satisfied. Therefore, according to the Dirichlet theorem, $i(t)$ can be expanded into a Fourier series (Ionkin, 1976; Polyanin, 2006).

$$i(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k \cos\left(\frac{2k\pi t}{T}\right) + b_k \sin\left(\frac{2k\pi t}{T}\right)\right) \tag{5}$$

The coefficients a_0, a_k and b_k contained in this expression are determined by using the formulas in (6), respectively.

$$\begin{cases} a_0 = \frac{2}{T}\int_0^T i(t) dt \\ a_k = \frac{2}{T}\int_0^T i(t) cos(k\omega t) dt \\ b_k = \frac{2}{T}\int_0^T i(t) sin(k\omega t) dt \end{cases} \tag{6}$$

In subsequent calculations, it is necessary to know the integral values obtained as a result of rearrangement of expression (4) by considering expression (5).

$$f(t) = \frac{1}{C_1}\left[A_0 + \sum_{k=1}^{\infty} A_k + \sum_{k=1}^{\infty} B_k\right] \tag{7}$$

Here,

$$A_0 = \frac{a_0}{2}\int_0^t e^{\alpha x}dx = \frac{a_0(e^{\alpha x}-1)}{2\alpha} \tag{8a}$$

$$A_k = a_k\int_0^t e^{\alpha x}cos(k\omega x)dx = \frac{a_k}{\alpha^2+(k\omega)^2}\{e^{\alpha t}[\alpha\cos(k\omega t)+k\omega\sin(k\omega t)]-\alpha\} \tag{8b}$$

$$B_k = b_k\int_0^t e^{\alpha x}sin(k\omega x)dx = \frac{b_k}{\alpha^2+(k\omega)^2}\{e^{\alpha t}[\alpha\sin(k\omega t)-k\omega\cos(k\omega t)]+k\omega\} \tag{8c}$$

Thus, we can determine the desired functions $V_{C_1}(t)$ and $V_{C_2}(t)$. To do this, we first need to calculate the integral using the expression (8a-8c):

$$V_{C_2}(t) = \frac{1}{\tau_2 C_1}\int_0^t e^{-\alpha x}\left[A_0(x)+\sum_{k=1}^{\infty}A_k(x)+B_k(x)\right]dx \tag{9}$$

Substituting these formulas in equation (9) and calculating the integrals, we obtain the function $V_{C_2}(t)$ in the following form.

$$\begin{aligned} V_{C_2}(t) = \frac{1}{\tau_2 C_1}\Bigg\{&\frac{a_0}{2\alpha}\left(t+\frac{(e^{-\alpha t}-1)}{\alpha}\right) \\ &+\sum_{k=1}^{\infty}\frac{a_k}{\alpha^2+(k\omega)^2}\left[\frac{\alpha}{k\omega}sin(k\omega t)-cos(k\omega t)+e^{-\alpha t}+1\right] \\ &+\sum_{k=1}^{\infty}\frac{b_k}{\alpha^2+(k\omega)^2}\left[\frac{\alpha}{k\omega}(1-cos(k\omega t))-sin(k\omega t)+\frac{k\omega}{\alpha}(1-e^{-\alpha t})\right]\Bigg\} \end{aligned} \tag{10}$$

Then, using the $V_{C_2}(t)$ function we determined, the $V_{C_1}(t)$ function is determined as follows, starting from the expression (3b):

$$\begin{aligned} V_{C_1}(t) = V_{C_2}(t) + R_2C_2\frac{dV_{C_2}}{dt} &= V_{C_2} \\ &+\frac{1}{C_1}\Bigg\{\frac{a_0}{2\alpha}(1-e^{-\alpha t}) \\ &+\sum_{k=1}^{\infty}\frac{a_k}{\alpha^2+(k\omega)^2}[\alpha\, cos(k\omega t)-k\omega\, sin(k\omega t)-\alpha e^{-\alpha t}] \\ &+\sum_{k=1}^{\infty}\frac{b_k}{\alpha^2+(k\omega)^2}[\alpha\, sin(k\omega t)-k\omega\, cos(k\omega t)+k\omega e^{-\alpha t}]\Bigg\} \end{aligned} \tag{11}$$

Thus, the expressions (10) and (11) that allow to determine the desired functions $V_{C_1}(t)$ and $V_{C_2}(t)$ depending on the $i(t)$ function were determined with the help of the Fourier series. In the next step, $i_1(t)$ and $i_2(t)$ functions, which define the change of currents in the circuit depending on time t, are determined. For this purpose, the following formulas are used.

$$\begin{cases} i_2(t) = C_2\dfrac{dV_{C_2}}{dt} \\ i_1(t) = i(t) - C_2\dfrac{dV_{C_2}}{dt} \end{cases} \tag{12}$$

As can be seen, after determining the first derivative of the $V_{C_2}(t)$ function, we obtain the following equation to determine the $i_2(t)$ function.

$$i_2(t) = \frac{1}{R_2C_1}\left\{\frac{a_0}{2\alpha}(1-e^{-\alpha t}) + \sum_{k=1}^{\infty}\frac{a_k}{\alpha^2+(k\omega)^2}[\alpha\, cos(k\omega t) - k\omega\, sin(k\omega t) - \alpha e^{-\alpha t}] + \sum_{k=1}^{\infty}\frac{b_k}{\alpha^2+(k\omega)^2}[\alpha\, sin(k\omega t) - k\omega\, cos(k\omega t) + k\omega e^{-\alpha t}]\right\} \quad (13)$$

Thus, by substituting the determined expression (13) in the second equation in the expression (12), the $i_1(t)$ function is also determined.

As a result, the solution of the Cauchy problem for the above-defined two-capacitor linear RC electric circuit is performed in terms of Fourier series, and formulas that allow determining the functions $V_{C_1}(t)$, $V_{C_2}(t)$, $i_1(t)$ and $i_2(t)$ are derived.

Along with these operations, it is necessary to take into account special cases when the current source function $i(t)$ acting at the input of the circuit has certain types. In practice, the following two cases often occur.

Special Case where Current Is a Periodic Function

Let the current $i(t)$ be variable and use the following periodic function to define it:

$$i(t) = I_m sin(\omega t) \quad (14)$$

where A is the amplitude and ω is the angular frequency. This periodic function is written in the formulas $V_{C_1}(t)$ and $V_{C_2}(t)$ obtained in fourier terms.

$$\int_0^t e^{\alpha x} sin(\omega x)\, dx = \frac{e^{\alpha t}}{\alpha^2+\omega^2}[\alpha sin(\omega t) + \omega(1 - cos(\omega t))] \quad (15)$$

Also, the functions $V_{C_1}(t)$ and $V_{C_2}(t)$ are restated considering equation (15).

$$V_{C_2}(t) = \frac{I_m}{R_2C_2C_1(\alpha^2+\omega^2)}\left[\frac{\omega}{\alpha}(1-e^{-\alpha t}) - sin(\omega t) + \frac{\alpha}{\omega}(1 - cos(\omega t))\right] \quad (16)$$

$$V_{C_1}(t) = V_{C_2}(t) + \frac{I_m\alpha}{C_1(\alpha^2+\omega^2)}\left[sin(\omega t) - \frac{\omega}{\alpha}(e^{-\alpha t} - cos(\omega t))\right] \quad (17)$$

In addition to these expressions, new expressions of $i_1(t)$ and $i_2(t)$ functions are determined using these formulas.

$$i_2(t) = \frac{I_m\alpha}{R_2C_2C_1(\alpha^2+\omega^2)}\left[sin(\omega t) + \frac{\omega}{\alpha}\ [e^{-\alpha t} - cos(\omega t)]\right] \quad (18)$$

$$i_1(t) = i(t) - i_2(t) \quad (19)$$

The obtained analytical solution of the mathematical problem defined for the case under study makes it possible to perform a numerical analysis for various values of the parameters describing a particular electrical circuit.

Analysis of the results corresponding to the problem solving;

For the examined electrical circuit, the results of the numerical calculation are given for the values of the parameters ($C_1 = 5\mu F,\ C_2 = 2{,}5\ \mu F\ ;\ R = 10k\Omega\ ;\ f = 150Hz$) included in the circuit. For this case, $I_m = 1A\ ;\ i(t) = sin(\omega t) = sin(2\pi f t)$

$$V_{C_2}(t) \cong 1.41.10^2\left[1 - e^{-60t} - \frac{1}{5\pi} sin(300\pi t) + \frac{1}{25\pi^2}(1 - cos(300\pi t))\right] \quad (20a)$$

$$V_{C_1}(t) \cong V_{C_2}(t) + 13.47[sin(300\pi t) + 5\pi(cos(300\pi t) - e^{-60t})] \quad (20b)$$

$$i_2(t) \cong 5.387.10^2[sin(300\pi t) + 5\pi(e^{-60t} - cos(300\pi t))] \quad (20c)$$

$$i_1(t) \cong sin(300\pi t) - i_2(t) \quad (20d)$$

Calculations are performed using a step of $h = 0.001$ for a time interval of $0 < t < 0.15$. Based on these expressions, the voltage variation graphs on the capacitors C_1 and C_2 are given in Figure 2.

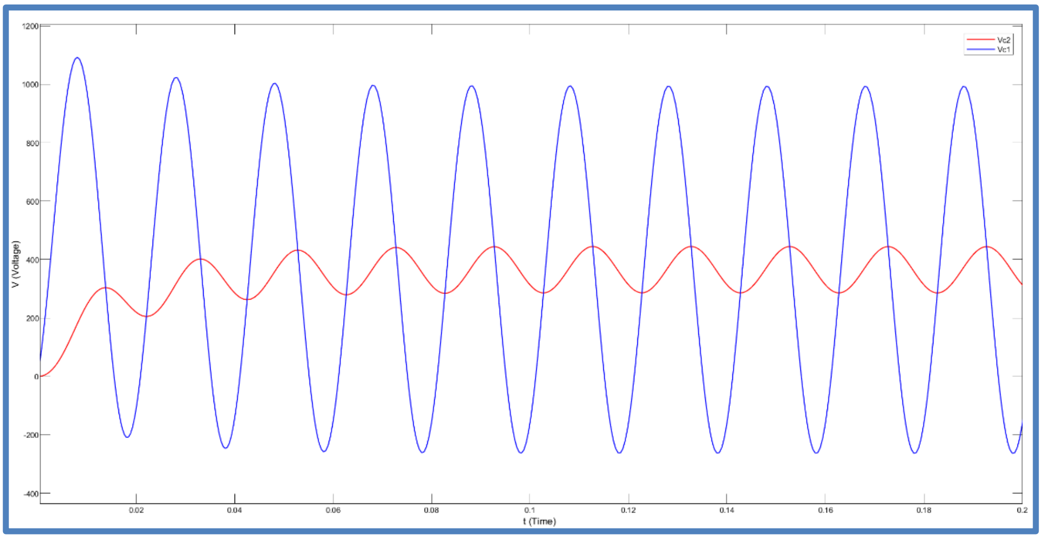

Figure 2. The Variation of Voltage Value $V_{C_1}(t)$ and $V_{C_2}(t)$ on Capacitors C_1 and C_2 When $i(t) = I_m.sin\omega t$

Accordingly, the variation of currents $i_1(t)$ and $i_2(t)$ with respect to time is given in Figure 3.

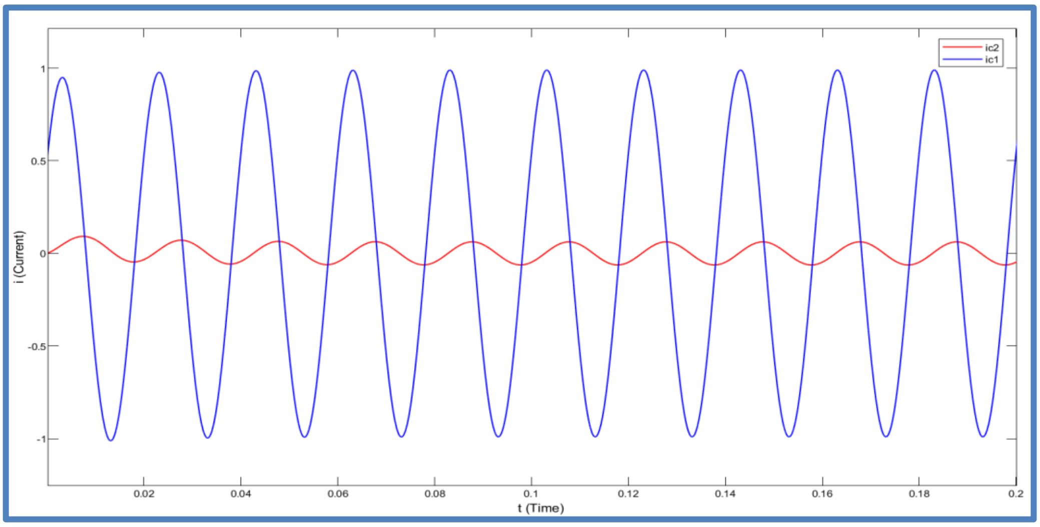

Figure 3. The Variation of Current Value $i_1(t)$ and $i_2(t)$ When $i(t) = I_m sin\omega t$

Special Case Where Current is a Unit Step Function

In this case, it is accepted that the current $i(t)$ is a signal in the form of a unit step applied to the circuit and is the cause of the transient event in the electrical circuit under consideration.

$$i(t) = \begin{cases} I_m & if \quad 0 < t < \frac{T}{2} \\ 0 & if \quad \frac{T}{2} < t < T \end{cases} \tag{21}$$

Extending this $i(t)$ function to a Fourier series gives the following expression.

$$i(t) = \frac{I_m}{2} + \frac{2I_m}{\pi}\left(\frac{sin\omega t}{1} + \frac{sin(3\omega t)}{3} + \frac{sin(5\omega t)}{5} + \cdots\right) \tag{22}$$

The coefficients of this series (22) are determined from the formulas:

$$a_0 = I_m \ ; \ a_k = 0 \ ; \ b_k = \frac{I_m}{k\pi}[1-(-1)^k], \qquad k = 1,2,3,\ldots \tag{23}$$

In this particular case, the desired functions are obtained as follows.

$$V_{C_2}(t) = \frac{1}{\tau_2 C_2} + \left\{\frac{I_m}{2\alpha}\left(t + \frac{1}{\alpha}e^{-\alpha t} - \frac{1}{\alpha}\right) + \frac{I_m T^2 \alpha}{\pi}\sum_{k=1}^{\infty}\frac{[1-(-1)^k]}{k[(2k\pi)^2 + (\alpha T)^2]}\left[1 - cos\left(\frac{2\pi kt}{T}\right) - \frac{1}{\alpha}sin\left(\frac{2k\pi t}{T}\right) + \frac{2k\pi}{\alpha^2 T}(1 - e^{-\alpha t})\right]\right\} \tag{24a}$$

$$V_{C_1}(t) = V_{C_2}(t) + \frac{1}{C_1} + \left\{\frac{I_m}{2\alpha}(1 - e^{-\alpha t}) + \frac{I_m T^2 \alpha}{\pi}\sum_{k=1}^{\infty}\frac{[1-(-1)^k]}{k[(2k\pi)^2 + (\alpha T)^2]}\left[\sin\left(\frac{2\pi kt}{T}\right) - \frac{2k\pi}{\alpha T}\left(\cos\left(\frac{2k\pi t}{T}\right) - e^{-\alpha t}\right)\right]\right\} \tag{24b}$$

$$i_2(t) = \frac{C_2}{\tau_2 C_1} + \left\{\frac{I_m}{2\alpha}(1 - e^{-\alpha t}) + \frac{I_m T^2 \alpha}{\pi}\sum_{k=1}^{\infty}\frac{[1-(-1)^k]}{k[(2k\pi)^2 + (\alpha T)^2]}\left[\frac{2\pi k}{T}sin(\frac{2k\pi t}{T}) - \frac{2k\pi}{\alpha T}cos\left(\frac{2k\pi t}{T}\right) + \frac{2k\pi}{\alpha T}(e^{-\alpha t})\right]\right\} \tag{24c}$$

$$i_1(t) = \frac{I_m}{2} + \frac{2I_m}{\pi}\left(\frac{sin(\omega t)}{1} + \frac{sin(3\omega t)}{3} + \frac{sin(5\omega t)}{5}\right) - \frac{C_2}{\tau_2 C_1} + \left\{\frac{I_m}{2\alpha}(1 - e^{-\alpha t}) + \frac{I_m T^2 \alpha}{\pi}\sum_{k=1}^{\infty}\frac{[1-(-1)^k]}{k[(2k\pi)^2 + (\alpha T)^2]}\left[\frac{2\pi k}{T}\sin\left(\frac{2k\pi t}{T}\right) - \frac{2k\pi}{\alpha T}\cos\left(\frac{2k\pi t}{T}\right) + \frac{2k\pi}{\alpha T}(e^{-\alpha t})\right]\right\} \tag{24d}$$

The solutions obtained determine the state of the circuit in transient mode. In order to determine the steady state, the values of the sought functions at $t \to \infty$ must be determined.

For example, for the current $i_2(t)$:

$$\lim_{t\to\infty} i_2(t) = \frac{C_2}{C_1\tau_2}\left\{\frac{I_m}{2\alpha}(1 - exp(-\alpha t) + \frac{I_m T^2 \alpha}{\pi}\sum_{k=1}^{\infty}\frac{[1-(-1)^k]}{k[(2\pi k)^2 + (\alpha t)^2]}\left[\left(sin\left(\frac{2\pi kt}{T}\right) - \frac{2\pi k}{T\alpha}cos\left(\frac{2\pi kt}{T}\right)\right)\right]\right\} = \frac{C_2}{C_1\tau_2}\left\{\frac{I_m}{2\alpha} + \frac{\alpha I_m T^2}{\pi}\sum_{k=1}^{\infty}\frac{[1-(-1)^k]}{k[(2\pi k)^2 + (\alpha t)^2]}\sqrt{1 + \left(\frac{2\pi k}{\alpha T}\right)^2}\, sin\left(\frac{2\pi kt}{T} - \varphi_k\right)\right\} \tag{25}$$

Here, $\varphi_k = arctg\left(\frac{2\pi k}{\alpha T}\right)$. In the settled regime, it will be $\mathrm{i}_2(\mathrm{t}) = \mathrm{i}_2^0(\mathrm{t})$.

$$i_2^0(t) = \frac{C_2}{C_1\tau_2}\left[\frac{A}{2\alpha} + \frac{AT}{\pi}\sum_{k=1}^{\infty}\frac{[1-(-1)^k]\, sin\left(\frac{2\pi kt}{T} - \varphi_k\right)}{k\left[\sqrt{(2\pi k)^2 + (\alpha t)^2}\right]}\right] \tag{26}$$

$$i_1^0(t) = \frac{A}{2}\left(1 - \frac{C_2}{C_1\tau_2}\right) + \frac{A}{\pi}\left[\sum_{k=1}^{\infty}\left(2sin\left(\frac{2\pi kt}{T}\right) - \frac{sin\left(\frac{2\pi kt}{T} - \varphi_k\right)}{\sqrt{(2\pi k)^2 + (\alpha t)^2}}\right)\right] \tag{27}$$

Conclusion

For active RC-based linear electric circuits, two capacitor circuits were examined and mathematical models obtained from the initial equations were found. It has been determined that these mathematical models meet all the requirements of Cauchy problems using Ohm and Kirchhoff's law.

After the circuit equations were defined, solutions of differential equations were realized with certain notations. Since these equations are used in a circuit with two capacitors, they are rearranged as a second order differential equation and an intrinsic solution is obtained. It has been found that if this self-solution gives us a function $i(t)$ it allows us to construct the current and voltage equations on the capacitors depending on the current.

In order to better analyse the complex electronic circuits, the Fourier series expansions are calculated with the Cauchy problems together with the Dirichlet conditions depending on the $i(t)$ function. These calculations allowed to find the voltage equations on the capacitors. Along with the voltage equations, the current equations on the capacitor depending on t time are also found.

The special cases that are frequently encountered in practice, namely the special case where the current is periodic, and the special case where the current is a unit step, were analysed and determined. Special cases made it possible to solve for various parameters of circuits. In addition, numerical analysis of accepted transients on a circuit is also done.

Author Contribution

Numan Derda Özkutlu, Hafız Alisoy and *Gülizar Alisoy,* performed the experimental process. *Numan Derda Özkutlu and Hafız Alisoy,* performed literature review, data collection, and theoretical analysis. *Hafız Alisoy,* performed supervision. The authors read and approved the article.

Ethic

There are no ethical issues with the publication of this article.

Conflict of Interest

The authors state that there is no conflict of interest.

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