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Analysis of Middle School Mathematics Applications Textbook Activities Based on Model-Eliciting Principles

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Abstract

Model-eliciting activities (MEA) represent a distinct form of problem-solving exercises that deviate from conventional problem-solving approaches. They encompass complex real-life scenarios characterized by multiple feasible solutions, demanding non-routine thinking with open-ended possibilities. Lesh and Doerr (2003) posit that MEA conform to specific principles, encompassing model construction, reality, self-evaluation, model externalization (construct certification), model generalization, and effective prototype principles. This study examines the compatibility of tasks in Turkey's middle school mathematics applications textbooks (grades 5-8) with the principles of model-eliciting activities (MEA). The analysis focuses on five principles: reality, model construction, self-evaluation, model documentation, and model generalization. The findings reveal varying degrees of compatibility across different grades. The reality and model generalization principles show more robust compatibility, while the model construction and model documentation principles have mixed levels of compatibility. The self-evaluation principle demonstrates varied compatibility. The study highlights strengths and areas for improvement in the tasks' alignment with MEA principles and emphasizes the importance of real-life relevance and model application. Suggestions are made to enhance explicit guidance in model construction and documentation. The study provides implications for curriculum design, teacher professional development, instructional strategies, student engagement, assessment practices, and future research in mathematics education. However, limitations, such as the absence of student perspectives and contextual factors, should be considered when interpreting the findings.

Keywords: Middle school, Mathematical modeling, Model-eliciting activities, Modelling principles, Textbook

Ortaokul Matematik Uygulamaları Ders Kitabı Etkinliklerinin Model Oluşturma Prensiplerine Dayalı Analizi

Öz

Model-oluşturma etkinlikleri (MOE), geleneksel problem çözme yaklaşımlarından farklı olan özel bir problem çözme yöntemini temsil eder. Bu etkinlikler çoğunlukla birden fazla doğru çözüme sahip karmaşık gerçek hayat problemlerini kapsar. Model-oluşturma etkinlikleri açık uçlu ve rutin olmayan problemlerdir. Lesh ve Doerr (2003), MOE'nin gerçeklik, model oluşturma, öz-değerlendirme, model dokümantasyon (model belgelendirme), model genelleme ve etkili prototip prensiplerine uygun olması gerektiğini belirtmektedir. Bu çalışma, Türkiye'deki ortaokul matematik uygulamaları ders kitaplarında (5-8. Sınıfla) yer alan etkinlilikleri model-oluşturma prensipleriyle uyumluluğunu incelemektedir. Etkinliklerin analizi, gerçeklik, model oluşturma, öz-değerlendirme, model dokümantasyon ve model genelleme olmak üzere beş prensibe odaklanmaktadır. Bulgular, farklı sınıf seviyelerinde model oluşturma prensipleri ile farklı derecelerde uyumluluk ortaya koymuştur. Etkinliklerde gerçeklik ve model genelleme prensipleri daha güçlü bir uyumluluk gösterirken, model oluşturma ve model dokümantasyon prensipleri zayıf uyumluluk düzeylerine sahiptir. Öz-değerlendirme prensibi çeşitli uyumluluk dereceleri göstermektedir. Bu çalışma, matematik uygulamaları ders kitaplarındaki etkinliklerin model oluşturma prensipleriyle uyumunda güçlü yönleri ve iyileştirmeye açık alanlarını vurgulamakta ve etkinliklerin gerçek hayatla ve model uygulaması ile ilişkili olmasının önemini vurgulamaktadır. Etkinliklerin model oluşturma ve dokümantasyon prensiplerine göre geliştirilmesi gerektiği ve bunun için öneriler sunulmaktadır. Bu çalışma, müfredat geliştirme, öğretmen gelişimi, öğretim stratejileri, öğrenci katılımı, değerlendirme uygulamaları ve gelecek araştırmalar için çıkarımlar ve öneriler sunmaktadır. Ancak bulguların yorumlanması sırasında, öğrenci perspektiflerinin ve bağlamsal faktörlerin eksikliği gibi sınırlamalar dikkate alınmalıdır.

Anahtar Sözcükler: Ders kitapları, Matematiksel modelleme, Model oluşturma etkinlikleri, Modelleme prensipleri, Ortaokul

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INTRODUCTION

The investigation of the teaching and learning of mathematical modeling holds considerable significance within mathematics education research (Borromeo Ferri, 2017; Niss & Blum, 2020). However, using mathematical modeling activities in educational settings and examining their compatibility still need to be expanded (Frejd, 2012).

Mathematical modeling activities aim to establish meaningful connections between mathematical concepts and real-world scenarios (Lesh, Doerr, Carmona, & Hjalmarson, 2003). Within the field of mathematics education, the overarching objective is to equip students with the capacity to effectively employ their mathematical knowledge, skills, and abilities in order to address problems that arise within authentic contexts (Blum & Leiß, 2007).

Mathematical modeling serves as a pivotal approach in forging a bridge between real-life problems and the realm of mathematics. It entails the translation of real-world phenomena into mathematical representations (Bukova Güzel, 2016). Numerous countries participating in international comparative assessments, such as the Programme for International Student Assessment and the Third International Mathematics and Science Study, endeavor to evaluate students' aptitude for applying mathematical principles to authentic situations and successfully resolving related problems. The outcomes of these evaluations have prompted participating nations to undertake substantial modifications within their educational systems and curricular frameworks (English, 2006).

Turkey is counted among those countries that have enacted significant revisions to their mathematics curricula in direct response to the findings yielded by these assessments. In response to its low ranking in education, Turkey has undertaken measures to address this issue by implementing reforms within the mathematics education system. The revised curriculum recognizes the need for more than relying solely on rote memorization to develop strong mathematical abilities, encompassing mathematical interconnections, the practical application of concepts, and effective problem-solving (Ministry of National Education [MoNE], 2018a). Consequently, thoughtfully crafted activities incorporating textbooks and multimedia resources are introduced, explicitly focusing on fostering authentic problem-solving and modeling experiences that cater to students' levels and interests (MoNE, 2018b). Notably, the curriculum for the Mathematics Applications course in middle grades adopts a student-centered approach that places significant emphasis on conceptual understanding. Within this course, the modeling approach is embraced, highlighting the practical application of mathematics in everyday life contexts.

This study examined the mathematics textbooks for the Mathematics Applications course prepared by the Ministry of National Education for middle school (grades 5-8) within the model eliciting activities principles defined by Lesh et al. (2000).

LITERATURE REVIEW

Mathematical modeling is prominent in mathematics education, giving students opportunities to articulate their thoughts, engage in decision-making during problem-solving, and construct their own mathematical structures within real-world contexts. This iterative process enhances students' motivation to learn and nurtures their logical and spatial reasoning abilities (Korkmaz, 2010).

Mathematical modeling entails translating real-life situations into mathematical solutions and using mathematical models. Its integration into the pedagogical process is indispensable, as the National Council of Teachers of Mathematics (NCTM, 2000) and the Ministry of National Education (MoNE, 2018a) underscored. Careful planning is imperative to ensure the effective implementation of mathematical modeling, involving the creation and resolution of modeling activities to enhance instructional efficacy. Consequently, mathematical modeling should be viewed as an enduring experience throughout mathematics education (Bracke & Geiger, 2011).

Since 2013, mathematical modeling has been incorporated into the mathematics education programs in Turkey, prioritizing the cultivation of logical and spatial reasoning skills and employing formulas, models, tables, plots, and graphs to articulate mathematical concepts (MoNE, 2018b). Through the medium of mathematical modeling, students acquire a profound comprehension of mathematics, apply logical and analytical thinking to real-life situations, and develop a positive disposition toward the subject (Blum & Borromeo Ferri, 2009; Galbraith, 2007; Gould, 2013; Suh et al., 2017).

Textbooks serve as indispensable instruments for bridging the gap between educational program content and classroom environments. They facilitate the dissemination of curriculum-based knowledge to students and guide educators in the instructional process (Thompson, 2014; Stylianides, 2014). Consequently, well-designed textbooks that align with the curriculum are expected to offer comprehensive and accurate lesson content (Altun, Arslan, & Yazgan, 2004). Modeling activities embedded within textbooks provide valuable insights into classroom practices associated with mathematical modeling.

Given the pivotal role of textbooks in facilitating the integration of mathematical modeling into instructional settings, it is imperative to investigate how the emphasis on mathematical modeling in the curriculum is reflected in these educational resources. While mathematical modeling encompasses diverse interpretations within the curriculum, textbooks may also present modeling in various formats (Alacacı, 2015).

Mathematical modeling presents a distinct perspective compared to conventional problem-solving approaches (Lesh & Doerr, 2003). Traditional problems typically present prepackaged data and often possess a single solution method. However, real-life situations seldom conform to such straightforward scenarios. In modeling activities, students are challenged to independently comprehend the given problem, establish relationships between mathematical concepts and operations, and generate reusable, shareable, and debatable models as problem solutions. These characteristics differentiate mathematical modeling problems from conventional problems, thereby furnishing students with diverse perspectives (Niss, Blum, & Galbraith, 2007; Lesh & Harel, 2003; Mousoulides, Christou, & Sriraman, 2008; Stillmann, 2019).

Traditional problem-solving approaches typically involve interpreting symbolically expressed questions to extract meaning. In contrast, modeling activities adopt a distinct process whereby students mathematize meaningful real-life situations by creating symbolic representations (Lesh et al., 2000).

Modeling activities can be characterized as collaborative endeavors that facilitate the logical representation of real-life situations, enabling students to generate and refine their own mathematical structures, and develop modeling competencies and creativity (Kaiser & Sriraman, 2006; Lu & Kaiser, 2022). Unlike traditional mathematics education, these activities do not rely on specific formulas for problem-solving. Instead, students construct mathematical models tailored to real-life situations, extending beyond the confines of traditional mathematical concepts (Chamberlin & Moon, 2005; Lesh & Zawojewski, 2007). The models developed by students are not solely utilized to solve the immediate problem but also serve as valuable tools for addressing similar problems (Lesh & Harel, 2003). Through the construction of models that apply to real-life contexts, are shareable among peers, and are reusable for future problem-solving, students engage in processes of explanation, evaluation, and revision of their mathematical thinking (Chamberlin & Moon, 2005; Doerr & O'Neill, 2011; Dominguez, 2010; Eric, 2008; Lesh et al., 2000; Lesh & Caylor, 2007; Yoon, Dreyfus, & Thomas, 2010).

In model-building activities focused on real-life problems, students assume the role of assisting individuals in decision-making. Throughout this process, they employ mathematical interpretation to make sense of the real-life situation. As students construct their models, their solutions reveal their thoughts and reasoning about the problem. Consequently, model-building activities are often characterized as "thought-revealing activities" and promote knowledge construction by students (Chamberlin & Moon, 2005; Meerwaldt, Borromeo Ferri, & Nevers, 2023).

The creation of model-building activities involves six principles that contribute to their effectiveness. These principles include the reality principle, the model construction principle, the self-assessment principle, the model documentation principle, the model generalization principle, and the effective prototype principle. These principles assist teachers in selecting and developing appropriate activities that cater to students with varying levels of achievement, including those with average or low performance (Lesh et al., 2000).

The reality principle emphasizes the importance of problem situations reflecting students' real lives, allowing them to interpret the situation based on their knowledge and experiences. Textbooks and tests often offer problems that align with this principle.

The model construction principle entails presenting problem situations that require students to construct a model for their solution. Students should recognize the need to construct, modify, expand, or simplify a model based on the given problem. They must engage in activities that involve constructing, defining, explaining, manipulating, predicting, and controlling a structurally significant system.

The self-assessment principle emphasizes students' ability to make decisions and find solutions to problem situations independently, without relying on teacher support. Students should be able to evaluate the sufficiency of their solutions. Throughout each modeling cycle, students should assess whether existing solutions require revision, determine the appropriate direction, and select the most practical alternative solution for a specific purpose.

The model documentation principle necessitates that the problem situation contains expressions that prompt students to explain their solution process to others (such as an advisor or customer) comprehensibly. The problem should require students to articulate their thoughts about the problem situation, including goals, potential solution approaches, and more. This principle is crucial as it supports learning and documenting while facilitating self-assessment and metacognitive reflection.

The model generalization principle stipulates that the problem situation should guide students in creating a generalizable model. The conceptual model developed should apply to other situations with specific modifications. However, textbooks and test problems often provide models or tools for traditional problems, with students being directed to produce and apply single solution paths. Even applied problems that incorporate real-life situations frequently demand only specific answers, limiting opportunities for model generation.

The effective prototype principle states that the model employed to solve the problem situation should be memorable to students, allowing them to recall it even after a significant period of time.

In conjunction with using models, model-eliciting activities enable students to apply their acquired knowledge and imbue the topic with personal meaning by transforming real-life situations into mathematical ones (Yoon, Dreyfus, & Thomas, 2010). Consequently, model-eliciting activities enhance students' learning and play a crucial role in teachers' effective instructional planning and communication with students (Lesh et al., 2000; Mousoulides, Christou, & Sriraman, 2008). Given their significance, model-eliciting activities are essential for effective mathematics instruction (Borromeo Ferri, 2017).

The Significance of the Study

Existing research has acknowledged the incorporation of mathematical modeling within mathematics education programs (Güç, 2015; Bukova-Güzel, Tekin-Dede, Hıdıroğlu, Kula-Ünver & Özaltun-Çelik, 2016). However, there is a notable gap in the literature concerning comprehensive examinations of mathematics textbooks from the perspective of mathematical modeling. Consequently, conducting a study in this area presents an opportunity to contribute significantly to the existing body of knowledge. The primary objective of this study is to evaluate the extent to which middle school mathematics textbooks, aligned with the mathematics applications course curriculum currently implemented, integrate mathematical modeling. Additionally, the study aims to assess the degree of alignment between the modeling activities included in the textbooks and the principles of model creation. The anticipated outcomes of this investigation hold substantial value for curriculum developers, program designers, researchers, and teachers.

By considering the outcomes and insights related to mathematical modeling highlighted in the curriculum, curriculum designers and program developers can enhance the inclusion of detailed explanations pertaining to mathematical modeling. Similarly, curriculum developers have the opportunity to make appropriate modifications to the course content and textbooks, specifically with regard to mathematical modeling. This process may entail reevaluating the understanding of modeling and incorporating necessary revisions. Ultimately, the findings obtained from this study possess the potential to provide valuable guidance to these educational stakeholders, enabling them to enhance the integration of mathematical modeling within the domain of mathematics education.

The Aim of the Study

The primary objective of this study was to investigate the model-eliciting activities incorporated within the mathematics applications textbooks designed for middle school students (grades 5-8) by the Ministry of National Education, with the specific intention of enhancing mathematical modeling skills. These activities were examined according to the model-eliciting principles defined by Lesh et al. (2000).

The study addressed the following research questions:

1. What is the level of compatibility between the problems presented in the middle school mathematics applications textbook and the principles underlying model-eliciting activities?

2. How does the degree of compatibility between the problems and the principles of model-eliciting activities vary across the grade levels of 5th to 8th?

By investigating these research questions, the study aimed to provide valuable insights into the extent to which the model-eliciting activities in mathematics applications textbooks align with the model-eliciting principles. Additionally, the study aimed to shed light on any variations in the level of compatibility across different grade levels within the middle school context

METHOD

Research Design

This study employed the qualitative research method of document analysis to systematically scrutinize and assess the content of written documents (Wach, 2013). Document analysis is a systematic and rigorous approach that facilitates the comprehensive evaluation and examination of various types of documents, irrespective of their format as printed or electronic materials. The primary objective of document analysis is to develop a profound understanding, derive meaningful interpretations, and generate valuable knowledge pertaining to the specific research topic under investigation (Corbin & Strauss, 2008).

Documents encompass a wide array of recorded texts and images without the researcher's intervention. They serve as invaluable data sources for researchers and encompass diverse materials such as advertisements, agendas, notes, books, brochures, maps, tables, newspapers, artworks, survey data, radio-TV program scenarios, and other relevant sources (Labuschagne, 2003). By meticulously examining and interpreting these documents, researchers can glean insightful perspectives and obtain pertinent information that aligns with their research objectives.

Data Collection

The data for this research were collected from the textbooks used in the elective course "Mathematical Applications" taught in middle schools (MoNE, 2018b). This course aims to provide students with opportunities to practice math applications suitable for their grade level, enhance their mathematical knowledge and skills, foster a love for mathematics, and develop a positive attitude toward the subject. Five textbooks were examined as part of this study, namely: "5th Grade 1st Semester Mathematical Applications" and "5th Grade Mathematical Applications" for 5th graders, "6th Grade Mathematical Applications" for 6th graders, "7th Grade Mathematical Applications" for 7th graders, and "8th Grade Mathematical Applications" for 8th graders. As the 5th-grade curriculum comprises two semesters, both textbooks were considered. However, only one textbook per grade level was analyzed for the 6th, 7th, and 8th grades.

Each problem in the textbooks was assigned a unique code indicating the grade level and its order of appearance in the book. For example, the code "6.P.3" represents the third problem in the "6th Grade Mathematical Applications" textbook, while "5.1.P.2" corresponds to the second problem in the "5th Grade 1st Semester Mathematical Applications" textbook. Problems that appeared in both the "5th Grade 1st Semester Mathematical Applications" and "5th Grade Mathematical Applications" textbooks were excluded from the study. The codes for these repeated problems were as follows: 5.P.3, 5.P.4, 5.P.5, 5.P.6, 5.P.7, 5.P.8, 5.P.10, 5.P.12, 5.P.13, 5.P.14, 5.P.15, and 5.P.16, totaling 12 problems that were not examined. Additionally, the problem coded as "5.P.27" in the "5th Grade Mathematical Applications" textbook was excluded as it did not meet the study's criteria, describing only a game and its rules. Hence, 13 problems from the "5th Grade Mathematical Applications" textbook were not included. In total, there were 21 problems examined in the "5th Grade 1st Semester Mathematical Applications" textbook, 23 remaining problems after excluding the 13 problems from the "5th Grade Mathematical Applications" textbook, 32 problems in the "6th Grade Mathematical Applications" textbook, 41 problems in the "7th Grade Mathematical Applications" textbook, and 40 problems in the "8th Grade Mathematical Applications" textbook. Consequently, a total of 157 problems were analyzed in this study.

Data Analysis

This research analyzed problems within the framework of Lesh et al.'s (2000) model-eliciting principles. The focus of the investigation was to assess the alignment of the problems with five specific principles: the reality principle, the model construction principle, the self-assessment principle, the model documentation principle, and the model generalization principle. The effective prototype principle was excluded from evaluation due to the requirement for a sufficient time interval for assessment (Dost, 2019; Urhan & Dost, 2018).

The evaluation process employed three distinct categories to determine the compatibility of the problems: "compatible," "partially compatible," and "incompatible." However, for the principles of reality and model generalization, no "partially compatible" category was used, and the evaluation was based solely on the criteria of "compatible" or "incompatible."

Concerning the reality principle, a problem was deemed compatible if it involved a situation that students were likely to encounter daily. Conversely, it was considered incompatible if the problem did not reflect real-life scenarios (Dost, 2019).

Regarding the model construction principle, a problem was considered compatible if students were explicitly instructed to create a coherent model related to the given problem situation, such as by formulating an equation or constructing a mathematical expression. Problems that indirectly required students to develop a model or provided a given model for problem-solving were categorized as partially compatible. Problems falling outside of these criteria were regarded as incompatible with the principle of model building (Dost, 2019).

For the self-assessment principle, a problem was considered compliant if it included instructions for students to engage in group discussions, explain their solution approaches, evaluate their own reasoning, and review the accuracy of their solutions without relying on teacher support. If no instructions or similar support were provided, a problem was categorized as partially compliant if the data provided was clear and adequate for finding a solution and the problem situation was appropriate for the student's level. Problems failing to meet any of these criteria were deemed non-compliant with the principle of self-assessment (Dost, 2019).

The model documentation principle was assessed based on whether problems contained instructions for students to explain their models in detail to others, such as friends or customers. Problems requiring students to express their ideas and explanations without mandating formal documentation were partially compliant. Problems that did not prompt students to explain their ideas or share their models with an institution, individual, or group were classified as non-compliant with the principle of model documentation (Dost, 2019).

Regarding the model generalization principle, a problem was considered compliant if the model established to solve a specific problem could be applied to different criteria or individuals. In such cases, the solution obtained could be utilized by others facing similar situations. Model-eliciting activities inherently adhere to this principle, as the models created during the problem-solving process should apply to comparable problem situations. Problems failing to meet these criteria were deemed non-compliant with the principle of model generalization (Dost, 2019).

The second author conducted the coding procedure and subsequently shared the data and coding criteria with the first author. Following this, the researchers compared their respective codes and found a high level of agreement, reaching 92%. The researchers thoroughly reviewed instances where divergent coding was identified, with careful consideration of the coding criteria. Through a process of negotiation, consensus was achieved. Any inconsistencies or discrepancies identified in the coding were resolved.

Research Ethics

In the present study, publicly accessible mathematics applications textbooks for grades 5 to 8 were examined concerning the principles of model-eliciting activities. As such, the nature of this study did not necessitate approval from an ethics committee.

FINDINGS

This section presents the findings of the compatibility of the tasks in Turkey’s middle school mathematics applications textbook for grades 5-8 with the principles of model-eliciting activities (MoNE, 2018b).

Table 1 shows the number of tasks in the 8th-grade mathematics applications textbook and their compatibility with the model-eliciting principles.

Table 1. Principle-based Evaluation of Tasks in 8th Grade Mathematics Applications Textbook

Principles	Fully Compatible	Partially Compatible	Incompatible
Reality	8.P.1, 8.P.2, 8.P.3, 8.P.4, 8.P.6–8.P.22, 8.P.24–8.P.40		8.P.5, 8.P.23
Model Construction	8.P.5, 8.P.16, 8.P.17, 8.P.19, 8.P.23, 8.P.25, 8.P.40	8.P.1, 8.P.2, 8.P.3, 8.P.4, 8.P.6– 8.P.15, 8.P.18, 8.P.20, 8.P.21, 8.P.22, 8.P.24, 8.P.26–8.P.35, 8.P.37, 8.P.38, 8.P.39	8.P.36
Self-Evaluation	8.P.2, 8.P.3, 8.P.4, 8.P.5, 8.P.10, 8.P.15, 8.P.16, 8.P.17, 8.P.18, 8.P.20, 8.P.22–8.P.28, 8.P.30, 8.P.33, 8.P.34, 8.P.38, 8.P.39	8.P.1, 8.P.6, 8.P.7, 8.P.11, 8.P.12, 8.P.13, 8.P.14, 8.P.19, 8.P.29, 8.P.31, 8.P.32, 8.P.35, 8.P.40	8.P.9, 8.P.21, 8.P.36, 8.P.37
Model Documentation	8.P.25	8.P.1, 8.P.2, 8.P.3, 8.P.4, 8.P.5, 8.P.6, 8.P.8, 8.P.10–8.P.20, 8.P.22, 8.P.23, 8.P.24, 8.P.26, 8.P.27, 8.P.30, 8.P.33, 8.P.34, 8.P.35, 8.P.38, 8.P.39, 8.P.40	8.P.7, 8.P.9, 8.P.21, 8.P.29, 8.P.31, 8.P.32, 8.P.36, 8.P.37
Model Generalization	8.P.1–8.P.40		

Note: The symbol "–" denotes the inclusive of numbers between the specified values surrounding that symbol.

As seen in Table 1, the 8th-grade mathematics applications textbook contained 40 problems. All the problems (40 problems) were found to be fully compatible with the model generalization principles, and 38 problems were fully compatible with the reality principle. As for the model construction principle, 32 problems were found to be partially compatible. 23 problems were fully compatible with the self-evaluation principle. However, only one problem was fully compatible with the model documentation principle.

Figure 1 displays task 8.P.16, and it has been classified as fully compatible with the reality, model construction, self-evaluation, and model generalization principles and partially compatible with the model documentation principle. The scenario presented in the problem pertains to the number of words a professional printer could write per minute, which is a situation that students could encounter in their daily lives. Thus, the problem was fully compatible with the reality principle.

The students have been asked to construct a model that illustrates how the quantity of words evolves with time, with the instruction to "Write an equation that gives the number of words (W) written per minute (M). What do the numbers in the equation tell you? Explain your answer." As a result, the task has been classified as fully compatible with the model construction principle.

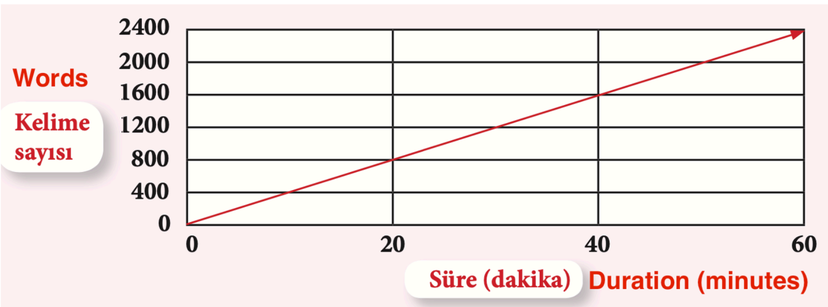
In this problem, with the statement "Determine whether the following statements are true or false. Provide evidence to support whether the statements are correct or incorrect in the context of the problem. Explain your reasoning." Students are prompted to reflect on their own concepts and techniques while solving the problem, resulting in an explicit expression of their thought processes. Since the task requires students to scrutinize their own thinking while solving it, the problem is fully compatible with the self-evaluation principle. The problem requires students to articulate their ideas while constructing the model, but it does not require them to provide documentation of their actions. As a result, the problem was categorized as partially compatible with the model documentation principle. Finally, the task was classified as fully compatible with the principle of model generalization since the model developed for the problem can be applied to different numbers of words with varying durations.

A professional printer can write 50 words per minute.

Question 1: Draw the graph of the above statement.

Question 2: Write an equation that gives the number of words (W) written per minute (M). What do the numbers in the equation tell you? Explain your answer.

Question 3: The following graph shows the number of words that Mehmet wrote for 60 minutes. Write an equation that gives the number of words written per minute according to this graph. Support your answer with mathematical reasoning.



Question 4: Determine whether the following statements are true or false. Provide evidence to support whether the statements are correct or incorrect in the problem context. Explain your reasoning.

a: A person can decide whether someone is faster or slower than Mehmet by comparing both of their graphs.
b: A person can decide whether someone is faster or slower than Mehmet by comparing both of their equations.

Figure 1. *Model Eliciting Activity 8.P.16 in 8th Grade Mathematics Applications Textbook.*

In the 7th-grade mathematics applications textbook, 41 problems existed, and they were categorized regarding their compatibility with the model-eliciting principles (Table 2). Although most of the problems (38 problems) were fully compatible with the reality principle, three were found to be incompatible with it. 12 problems were fully compatible, and 12 problems were partially compatible with the model construction principle. As for the self-evaluation principle, 29 problems were found to be fully compatible. However, 11 problems were partially compatible, and one problem was incompatible with the self-evaluation principle. Regarding the model documentation principle, 3 problems fully conformed to the principle, 29 problems partially conformed, and 9 did not conform. All the problems were fully compatible with the model generalization principle.

Table 2. Principle-based Evaluation of Tasks in 7th Grade Mathematics Applications Textbook

Principles	Fully Compatible	Partially Compatible	Incompatible
Reality	7.P.1- 7.P.12, 7.P.14-7.P.22, 7.P.24–7.P.31, 7.P.33–7.P.41		7.P.13, 7.P.23, 7.P.32
Model Construction	7.P.4, 7.P.5, 7.P.7, 7.P.8, 7.P.9, 7.P.10, 7.P.12, 7.P.16, 7.P.18, 7.P.20, 7.P.21, 7.P.22, 7.P.23, 7.P.27, 7.P.29, 7.P.30, 7.P.31, 7.P.33, 7.P.37, 7.P.41	7.P.1, 7.P.2, 7.P.3, 7.P.6, 7.P.11, 7.P.13, 7.P.14, 7.P.17, 7.P.19, 7.P.24, 7.P.25, 7.P.26, 7.P.28, 7.P.32, 7.P.34, 7.P.35, 7.P.36, 7.P.38, 7.P.39, 7.P.40	7.P.15
Self-Evaluation	7.P.1, 7.P.2, 7.P.4–7.P.12, 7.P.15, 7.P.17, 7.P.18, 7.P.21, 7.P.22, 7.P.23, 7.P.24, 7.P.25, 7.P.29, 7.P.30, 7.P.32–7.P.38, 7.P.41	7.P.3, 7.P.13, 7.P.14, 7.P.16, 7.P.19, 7.P.20, 7.P.26, 7.P.27, 7.P.28, 7.P.31, 7.P.39	7.P.40
Model Documentation	7.P.8, 7.P.19, 7.P.33	7.P.1, 7.P.2, 7.P.4, 7.P.5, 7.P.6, 7.P.7, 7.P.9, 7.P.10, 7.P.11, 7.P.12, 7.P.15, 7.P.16, 7.P.17, 7.P.18, 7.P.21, 7.P.22, 7.P.23, 7.P.24, 7.P.25, 7.P.29, 7.P.30, 7.P.32, 7.P.34, 7.P.35, 7.P.36, 7.P.37, 7.P.38, 7.P.39, 7.P.41	7.P.3, 7.P.13, 7.P.14, 7.P.20, 7.P.26, 7.P.27, 7.P.28, 7.P.31, 7.P.40
Model Generalization	7.P.1–7.P.41		

Task 7.P.13 from the 7th-grade mathematical applications textbook is illustrated in Figure 2. This problem was classified as incompatible with the reality principle because students may not face similar circumstances in their lives. In this problem, no direct expression leads students to construct a model. However, the data on the problem is clear and sufficient for a solution. Since this activity indirectly requires students to construct a model, it was categorized as partially compatible with the model construction principle. In terms of the self-evaluation principle, the data in the problem are clear and sufficient for a solution, and the problem situation is appropriate for the level of the students. As a result, the problem is partially compatible with the self-evaluation principle. The activity does not prompt or imply that students should provide a record or explanation of their solution process. Thus, it was classified as not compatible with the model documentation principle. Finally, the problem scenario in this activity involves a model that can be applied to various criteria and individuals. Therefore, the activity was classified as fully compatible with the model generalization principle.

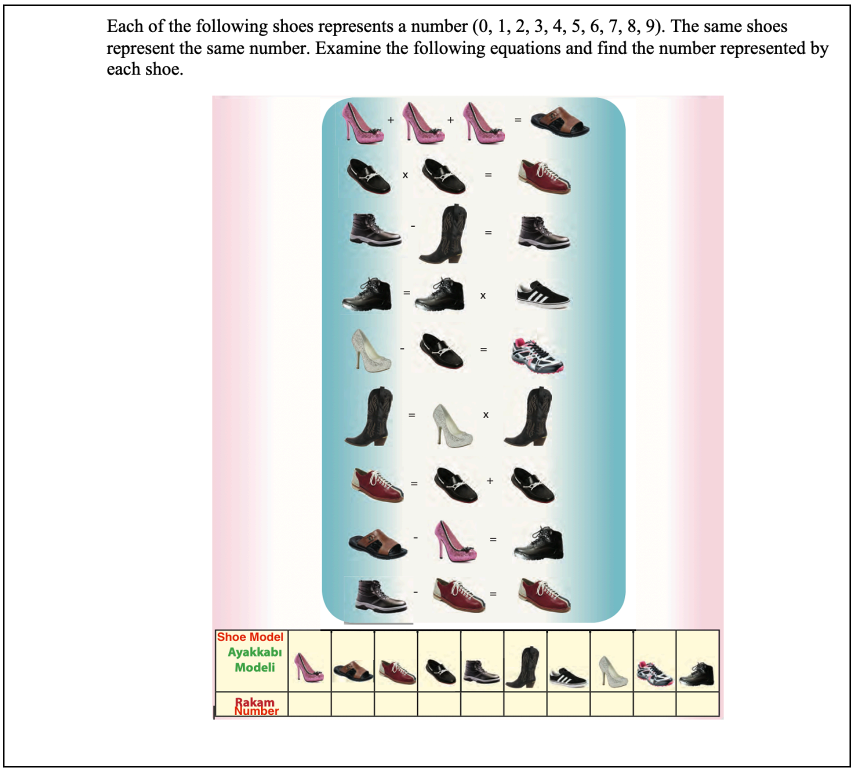


Figure 2. *Model Eliciting Activity 7.P.13 in 7th Grade Mathematics Applications Textbook.*

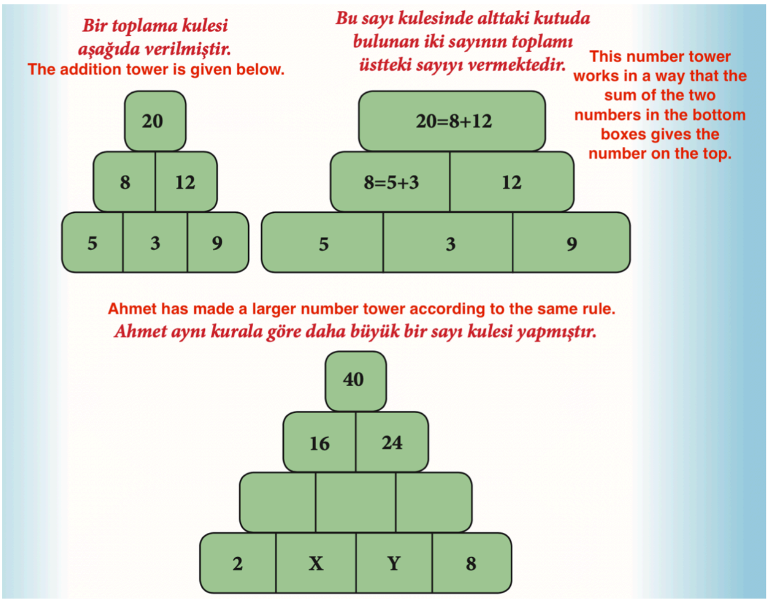
Table 3 displays the results of the compatibility analysis of the model-eliciting problems in the 6th-grade mathematics applications textbook with each principle. The table shows the problems categorized as fully, partially, or incompatible with each principle. A total of 32 problems existed in the textbook. 27 problems were fully compatible with the reality principle, even though 5 problems did not meet its requirements. 4 problems met the requirements of the model construction principle entirely, and the remaining problems only partially met the criteria. With respect to the self-evaluation principle, 22 problems met all the requirements, but 10 problems only partially complied. As for the model documentation principle, 3 problems were fully compatible, 21 problems partially conformed, and 8 failed to conform. All the problems were determined to be fully compatible with the model generalization principle.

Table 3. Principle-based Evaluation of Tasks in 6th Grade Mathematics Applications Textbook

Principles	Fully Compatible	Partially Compatible	Incompatible
Reality	6.P.1, 6.P.2, 6.P.3, 6.P.4, 6.P.5, 6.P.7–6.P.13, 6.P.15, 6.P.16, 6.P.18–6.P.24, 6.P.27–6.P.32		6.P.6, 6.P.14, 6.P.17, 6.P.25, 6.P.26
Model Construction	6.P.14, 6.P.15, 6.P.23, 6.P.32	6.P.1–6.P.13, 6.P.16–6.P.22, 6.P.24–6.P.31	
Self-Evaluation	6.P.1- 6.P.9, 6.P.12, 6.P.14, 6.P.15, 6.P.18, 6.P.20–6.P.25, 6.P.27, 6.P.30, 6.P.32	6.P.10, 6.P.11, 6.P.13, 6.P.16, 6.P.17, 6.P.19, 6.P.26, 6.P.28, 6.P.29, 6.P.31	
Model Documentation	6.P.18, 6.P.23, 6.P.32	6.P.1–6.P.9, 6.P.12, 6.P.14, 6.P.15, 6.P.17, 6.P.20, 6.P.21, 6.P.22, 6.P.24, 6.P.25, 6.P.27, 6.P.30, 6.P.31	6.P.10, 6.P.11, 6.P.13, 6.P.16, 6.P.19, 6.P.26, 6.P.28, 6.P.29
Model Generalization	6.P.1–6.P.32		

The activity depicted in Figure 3 is called 6.P.17 in the model-eliciting problems, where an addition and multiplication tower are given, and students are asked to find the empty boxes in the number towers using algebraic operations. Figure 3 represents the addition tower, how it works, and related questions. The problem was deemed incompatible with the reality principle as it does not involve a real-life situation. The problem requires students to use algebraic operations to fill in the empty boxes in the number tower, indirectly asking them to construct a model to solve the problem. Therefore, the problem was classified as partially compatible with the model construction principle. The task only mentions something about students assessing and reflecting on their problem-solving strategies with the teacher's assistance. However, the data presented in the activity are adequate and unambiguous for arriving at a solution. Thus, the problem was partially aligned with the self-evaluation principle.

The task requires students to demonstrate their understanding of whether the equations $2x+y=14$ and $x+2y=16$ are valid or not using a table. However, no specific instruction asks them to document their solution process in detail and share it with others. Therefore, the problem was partially compatible with the model documentation principle. Moreover, the number tower in the problem situation can be adjusted to represent the addition of various numbers, indicating that the solution can be utilized in comparable problem situations. Hence, the activity was categorized as fully compatible with the model generalization principle.



Question 1: Complete the empty boxes in the number tower using algebraic operations.

Question 2: After completing the number tower, use the table to show whether the equations $2x+y=14$ and $x+2y=16$ are true or not.

Figure 3. *Model Eliciting Activity 6.P.17 in 6th Grade Mathematics Applications Textbook.*

Finally, Table 4 and Table 5 present the problems in the 5th-grade mathematics applications textbook and their compatibility with the model-eliciting principles. As indicated earlier in the methodology section, there are two textbooks for fifth-grade students, one containing 21 problems (Table 4). The other contains 36 problems (Table 5), but problem 5.P.27 was not included in the analysis since it was the same problem as a problem in the other 5th-grade textbook. The findings are provided in both tables.

Both tables show that most problems in both textbooks are fully compatible with the reality principle. Only two problems were incompatible with the reality principle in each textbook. As for the model construction principle, the problems in both textbooks were found to be either fully or partially compatible. Similarly, in one textbook, most of the problems were entirely compatible, and the rest were partially compatible with the self-evaluation principle. However, in the other 5th-grade textbook, the problems' compatibility with the self-evaluation principle was vice versa. Regarding the model documentation principle, some problems were found to be fully compatible; however, most of the problems in both textbooks were either partially compatible or incompatible. Lastly, it was concluded that all the problems were fully consistent with the model generalization principle.

Table 4. Principle-based Evaluation of Tasks in 5th Grade* Mathematics Applications Textbook

Principles	Fully Compatible	Partially Compatible	Incompatible
Reality	5.1.P.1–5.1.P.8, 5.1.P.10–5.1.P.15, 5.1.P.17–5.1.P.21		5.1.P.9, 5.1.P.16
Model Construction	5.1.P.1, 5.1.P.2, 5.1.P.3, 5.1.P.8, 5.1.P.11, 5.1.P.12, 5.1.P.13, 5.1.P.14, 5.1.P.15, 5.1.P.17	5.1.P.4, 5.1.P.5, 5.1.P.6, 5.1.P.7, 5.1.P.9, 5.1.P.10, 5.1.P.16, 5.1.P.18, 5.1.P.19, 5.1.P.20, 5.1.P.21	
Self-Evaluation	5.1.P.1, 5.1.P.2, 5.1.P.3, 5.1.P.4, 5.1.P.5, 5.1.P.8, 5.1.P.11, 5.1.P.12, 5.1.P.13, 5.1.P.15, 5.1.P.16, 5.1.P.17, 5.1.P.18, 5.1.P.20	5.1.P.6, 5.1.P.7, 5.1.P.9, 5.1.P.10, 5.1.P.14, 5.1.P.19, 5.1.P.21	
Model Documentation	5.1.P.1, 5.1.P.2, 5.1.P.3, 5.1.P.8, 5.1.P.12, 5.1.P.18, 5.1.P.20	5.1.P.4, 5.1.P.5, 5.1.P.11, 5.1.P.13, 5.1.P.15, 5.1.P.16, 5.1.P.17	5.1.P.6, 5.1.P.7, 5.1.P.9, 5.1.P.10, 5.1.P.14, 5.1.P.19, 5.1.P.21
Model Generalization	5.1.P.1–5.1.P.21		

Table 5. Principle-based Evaluation of Tasks in 5th Grade Mathematics Applications Textbook

Principles	Fully Compatible	Partially Compatible	Incompatible
Reality	5.P.1, 5.P.9, 5.P.11, 5.P.17, 5.P.18, 5.P.19, 5.P.21–5.P.36*		5.P.2, 5.P.20
Model Construction	5.P.9, 5.P.17, 5.P.18, 5.P.21, 5.P.29, 5.P.33, 5.P.34	5.P.1, 5.P.2, 5.P.11, 5.P.19, 5.P.20, 5.P.22–5.P.28*, 5.P.30, 5.P.31, 5.P.32, 5.P.35, 5.P.36	
Self-Evaluation	5.P.9, 5.P.17, 5.P.18, 5.P.28, 5.P.36	5.P.1, 5.P.2, 5.P.11, 5.P.19–5.P.26, 5.P.29–5.P.35	
Model Documentation	5.P.9, 5.P.18	5.P.17, 5.P.28, 5.P.36	5.P.1, 5.P.2, 5.P.11, 5.P.19–5.P.26, 5.P.29–5.P.35
Model Generalization	5.P.1, 5.P.2, 5.P.9, 5.P.11, 5.P.17–5.P.36*		

*5.P.27 was not included in the analysis

Figure 4 shows 5.P.9 in the 5th Grade Mathematics Applications Textbook. When Figure 4 is examined, it is understood that this problem aims to help students discover that shapes with the same perimeter can have different areas. The problem was classified as fully compatible with the reality principle because the situation examined in the activity is likely to happen in real life. The problem requires students to determine the maximum area possible for a specified perimeter and explicitly instructs them to develop a model demonstrating the area's dimensions. Because this requirement is clearly stated, the activity was classified as entirely consistent with the model construction principle.

The activity has a statement that directly asks students to evaluate their own thoughts during the solution process by discussing them with their friends in their group. Due to this explicit expectation, the activity was classified as fully aligned with the self-evaluation principle. As for the model documentation principle, the statement "Write your solution in a way that Kamil and anyone reading it can understand" mandates that students elucidate the principle. Thus, the problem was classified as partially consistent with the model documentation principle. The activity was categorized as fully compatible with the model generalization principle because the model can be adapted and applied to similar situations.

Kamil's Sheep:

Kamil and his father are sheep farmers. To prevent the sheep from getting lost while grazing, they will enclose a flat pasture area at the edge of the village with a fence. They have 316 meters of fence. They want to have the largest possible area for the sheep to graze. Kamil's father asks him how the enclosed area should be. What should be the dimensions of the area? Find a solution by discussing it with your friends in your group. Write your solution in a way that Kamil and anyone reading it can understand. Draw the different shapes you have in mind. Calculate the dimensions and the area of the shapes. Summarize your findings in a table. Specify which shape is the most suitable.

Figure 4. *Model Eliciting Activity 5.P.9 in 5th Grade Mathematics Applications Textbook.*

Table 6 summarizes the findings regarding how much the model-eliciting activities in the 5-8 grades mathematics applications textbook align with the principles. The table provides the total number of activities classified as fully, partially, or incompatible with each principle.

Table 6. Compatibility of Problems in 5-8 Grades Mathematics Applications Textbook with Model-eliciting Principles

Principles		Total number of activities in 5-8 grade mathematics applications textbook				
		5.1.	5.	6.	7.	8.
Reality	Fully compatible	19(%90)	21(%91)	27(%84)	38(%93)	38(%95)
	Partially compatible	–	–	–	–	–
	Incompatible	2(%10)	2(%9)	5(%16)	3(%7)	2(%5)
Model Construction	Fully compatible	10(%47)	7(%30)	4(%12.5)	21(%51)	7(%17.5)
	Partially compatible	11(%53)	16(%70)	28(%87.5)	19(%47)	32(%80)
	Incompatible	–	–	–	1(%2)	1(%2.5)
Self-Evaluation	Fully compatible	14(%67)	5(%22)	22(%69)	29(%71)	23(%57.5)
	Partially compatible	7(%33)	18(%78)	10(%31)	11(%27)	13(%32.5)
	Incompatible	–	–	–	1(%2)	4(%10)
Model Documentation	Fully compatible	7(%33)	2(%9)	3(%9)	3(%7)	1(%2.5)
	Partially compatible	7(%33)	3(%13)	21(%66)	29(%71)	31(%77.5)
	Incompatible	7(%33)	18(%78)	8(%25)	9(%22)	8(%20)
Model Generalization	Fully compatible	21(%100)	23(%100)	32(%100)	41(%100)	40(%100)
	Partially compatible	–	–	–	–	–
	Incompatible	–	–	–	–	–

According to the findings presented in Table 6, although a few problems do not conform to the reality principle at each grade level, most of the problems in the textbook are fully aligned with the reality principle. Likewise, all problems in the textbook at each grade level are found to be fully compatible with the model generalization principle. Regarding the model construction principle, most problems are partially compatible at each grade level. However, the 7th-grade textbook has the highest percentage rate regarding fully compatible problems with the model construction principle. Moreover, the model construction principle is completely compatible; however, in the 7th and 8th-grade textbooks, one problem is found to be incompatible with the principle.

As for the self-evaluation principle, most problems are partially compatible with the principle in the 5th-grade textbook. However, in 6th, 7th, and 8th-grade textbooks, the majority of the problems are fully compatible with the self-evaluation principle. In the 8th-grade textbook, four problems were found to be incompatible with the self-evaluation principle. While some problems were fully compatible with the model documentation principle at each grade level, most of the problems in each textbook were either partially or not compatible with the principle. Especially in the 5th-grade textbook, a considerable number of problems (78%) were incompatible with the model documentation principle. It can be seen from Table 7 that at each grade level, the textbook includes problems that fail to fully align with each model-eliciting principle

DISCUSSION & CONCLUSION

In this study, the tasks in Turkey's middle school mathematics applications textbooks (grades 5-8) were analyzed in terms of their compatibility with the principles of model-eliciting activities (MEA). The analysis focused on five principles: reality, model construction, self-evaluation, model documentation, and model generalization.

In the 8th-grade mathematics applications textbook, it was found that all 40 problems were fully compatible with the model generalization principle, indicating that the models developed for these problems could be applied to various scenarios. Most of the problems were also fully compatible with the reality principle, suggesting they were relevant to students' daily lives. However, only one problem was fully compatible with the model documentation principle, indicating a need for more emphasis on documenting the problem-solving process. Most activities were partially compatible with the model construction principle, indicating room for improvement in guiding students to construct explicit models. While more than half of the problems were fully compatible with the self-evaluation principle, some problems were only partially aligned, suggesting the need for more explicit prompts for reflection and reasoning.

The 7th-grade mathematics applications textbook contained 41 problems, with most problems fully compatible with reality and model generalization principles. However, three problems were found to be incompatible with the reality principle, indicating a potential lack of relevance to students' lives. While twelve problems were fully compatible with the model construction principle, several problems were only partially aligned. The self-evaluation principle was mostly fulfilled, but some problems were only partially aligned or were incompatible. Similarly, most problems partially conformed to the model documentation principle, with nine problems not conforming at all.

In the 6th-grade mathematics applications textbook, most of the 32 problems were fully compatible with the reality principle, indicating relevance to students' lives. However, five problems did not meet the requirements. Four problems fully met the requirements of the model construction principle, while the remaining problems were only partially aligned. Most problems fully met the requirements of the self-evaluation principle, but some problems were only partially compatible. The model documentation principle had limited fulfillment, with three problems fully conforming and the majority partially conforming or not conforming at all. All problems were fully compatible with the model generalization principle.

Based on the findings, it is evident that there is variability in the compatibility of the tasks in the mathematics applications textbooks with the MEA principles across different grades. Reality and model generalization principles generally showed better compatibility, indicating a stronger focus on the relevance and application of models. However, the principles of model construction and model documentation had mixed compatibility, suggesting the need for more explicit guidance and an emphasis on constructing models and documenting problem-solving processes (Lesh & Doerr, 2003; Stillman et al., 2007). The self-evaluation principle had varied compatibility, with some problems fully aligned but others only partially aligned.

The compatibility analysis revealed strengths and areas for improvement in the tasks of Turkey's middle school mathematics applications textbooks in relation to the MEA principles. The findings provide insights into the design and alignment of the tasks with these principles.

The textbooks generally demonstrated a strong emphasis on reality and model generalization principles, indicating a recognition of the importance of real-life relevance and the application of models to different scenarios (Bracke & Geiger, 2011; Lesh, Doerr, Carmona, & Hjalmarson, 2003). This highlights the positive aspects of the textbooks in providing students with opportunities to connect mathematical concepts to their everyday lives and generalize their learning (Niss, Blum, & Galbraith, 2007; Lesh & Harel, 2003; Mousoulides, Christou, & Sriraman, 2008).

However, there is room for improvement in certain areas. The model construction principle requires further attention, as most activities are only partially compatible. Providing explicit guidance and instructions for students to construct models can enhance their problem-solving abilities and promote a deeper understanding of mathematical concepts (Bukova Güzel, 2016).

The model documentation principle was the least fulfilled across all grade levels, indicating a need to enhance students' ability to articulate and document their problem-solving processes (Kaiser & Sriraman, 2006; Lesh et al., 2000). Encouraging students to explain their reasoning, provide evidence, and justify their solutions since they foster a deeper understanding of concepts, promote higher-order thinking skills and creativity, and allow individuals to apply their knowledge and skills meaningfully (Chamberlin & Moon, 2005; Korkmaz, 2010; Lu & Kaiser, 2022).

Implications

The study on the compatibility analysis of tasks in Turkey's middle school mathematics applications textbooks with the principles of model-eliciting activities (MEA) has several implications:

1. Curriculum design: The study provides insights for curriculum designers to inform the design of future mathematics textbooks and materials. Emphasizing model construction and documentation principles while focusing on reality and model generalization can enhance mathematics education.

2. Teacher professional development: Teachers can benefit from professional development opportunities to guide students in constructing models and documenting problem-solving processes. Developing teachers' knowledge and skills in these areas can promote deeper mathematical understanding.

3. Instructional strategies: Teachers can incorporate explicit guidance to help students construct models, document processes, and engage in self-evaluation, reflection, and reasoning. This enhances metacognitive skills and promotes a deeper understanding of mathematical concepts.

4. Student engagement: Incorporating real-life contexts and examples into mathematics tasks improves students' engagement by highlighting the relevance of mathematics in their everyday lives.

5. Assessment and feedback: Teachers can design assessments targeting MEA principles, assessing students' model construction, documentation, and self-evaluation skills. Providing feedback in these areas further develops problem-solving and metacognitive abilities.

6. Research opportunities: The study encourages further research in mathematics education, exploring the impact of task compatibility with MEA principles on learning outcomes, problem-solving abilities, and long-term retention of mathematical concepts.

Overall, the study has implications for curriculum design, teacher development, instructional strategies, student engagement, assessment practices, and future research in mathematics education. Addressing areas for improvement and leveraging strengths identified in the analysis can enhance mathematics instruction, promoting a more profound understanding and application of mathematical concepts.

Limitations

Limitations of the study on the compatibility analysis of tasks in Turkey's middle school mathematics applications textbooks with the principles of model-eliciting activities include:

1. Lack of student perspectives: The study primarily focuses on analyzing the tasks in the textbooks without considering students' perspectives or experiences. Student feedback and input could provide valuable insights into their engagement, understanding, and preferences regarding the tasks and their compatibility with MEA principles.

2. Lack of contextual factors: The analysis solely focuses on the tasks within the textbooks and does not consider contextual factors such as instructional practices, classroom environment, or teacher characteristics. These factors can influence the implementation and effectiveness of the tasks in promoting MEA principles.

These limitations should be considered when interpreting the study's findings and their implications for curriculum design and instructional practices.

Statements of Publication Ethics

We declare that we obey the principles of publication ethics.

Researchers’ Contribution Rate

First author 60%, the second author 40%.

Conflict of Interest

This study has no conflict of interest.

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