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Research Article

The process of solving skill-based problems about multipliers and multiples by advanced learners in math and gifted students[1]

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Abstract

Globalization has also been reflected in mathematics education, and as a result, international exams, in which Turkey also participate, have become widespread since the 20th century. After participating in international exams in Turkey, it started to make changes in the questions in the exams it held throughout the country. The most striking of these changes is the preparation of skill-based questions that measure metacognitive skills in the High School Entrance System exam. In order to solve such questions effectively, as emphasized in the mathematics curriculum, the problem solving process must be managed well and correctly. In this research, it is aimed to examine the processes of solving skill-based questions about multipliers and multiples. The difficulties experienced by the students in the process of solving skill-based questions and the points they missed were determined. In this study, case study, one of the qualitative research methods, was used. The study group consists of 8 students, 4 of whom are highly successful in mathematics and 4 of whom are gifted, studying in the 8th grade. The Factors and Multiples Achievement Test consisting of 10 open-ended questions was developed in order to measure the students' skills on the subject of multipliers and multiples. While applying the Multipliers and Multiples Achievement Test, we conducted a clinical interview with each student and enabled the students to solve the questions by thinking aloud. We made the descriptive analysis of the obtained data according to the Polya' problem solving steps. While presenting the findings, we made use of the tables and interpreted them according to each question and step. As a result of the research, we determined that the students who successfully realized the step of understanding the problem were able to perform the other steps and solve the questions correctly. We also found that students who are successful in mathematics try to apply Polya's problem solving steps more and are more successful than gifted students. According to these results, we recommend that students use Polya's problem-solving steps effectively while solving problems and that the research results should be investigated in more detail.

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Introduction

Science has existed since the beginning of human history and is in constant development. The basis of all fields of science has modeled the mathematical mindset (Alpaslan, 2011). Mathematics is a science that people have used and

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developed throughout history to explain facts they do not know and to solve the difficulties in their daily lives. Today, there are a large number of people who have difficulty learning mathematical subjects (Altıntas, Ilgun, & Karadag, 2022). It is thought that this situation can be improved by a good mathematics education.

Evaluation of Mathematical Talent at International Level and Turkiye

Mathematics education in Turkey is carried out in a planned manner, based on the NCTM (National Council of Mathematics Teachers) standards, according to the general objectives determined by the Ministry of National Education (Umay, Akkus, & Duatepe-Paksu, 2006). The most striking purpose among the general objectives stated in the mathematics curriculum is the one that focuses on the problem solving process of the student (MoNET, 2018). It is not possible to evaluate the problem solving process, which is one of the high-level skills, with traditional assessment and evaluation approaches (Bal & Doganay, 2010). In order to evaluate the problem solving process, alternative assessment and evaluation approaches including different question types should be applied (Ozturk & Sahin, 2013). In this context, studies such as the International Study of Reading Skills Development (PIRLS), the International Study of Mathematics and Science Trends (TIMSS) and the Program for International Student Assessment (PISA) have emerged to measure high-level skills in the world (Demir, 2010).

Turkiye regularly participates in international measurement and evaluation research. The data obtained from these studies affect the applied central exams (Erden, 2020). This effect has been seen the most in the High School Entrance System (LGS), which has been used to select students for secondary education institutions since 2018 (Kaya, 2019). In LGS, skill-based questions, which are called "new generation questions" among the public, have begun to be asked (Uzun, 2021).

Skill-based questions are questions to measure many high-level skills, especially problem solving (Kablan & Bozkus, 2021). It is observed that students have difficulties because these questions are not compatible with the mathematics curriculum applied in Turkey, they are long and measure high-level skills (Cepni, 2020; Erden, 2020). In addition, although teachers are sensitive to these questions, they experience problems due to their lack of knowledge and mistakes in teaching methods (Kablan & Bozkus, 2021). In order to eliminate these problems, sample skill-based questions about the courses in LGS are published by the General Directorate of Assessment, Evaluation and Examination Services every month. However, it is thought that all of these questions consist of multiple-choice questions and are lacking in developing students' problem-solving skills (Kertil, Gulbagci Dede, & Ulusoy, 2021).

Success in mathematics is directly related to good problem solving. In order to solve the problem, the problem solving process must be managed very well (Altintas, 2009). In the world and in Turkey, the problem solving process of Polya (1957) is generally accepted. According to Polya, the problem solving process should consist of four stages. These stages are understanding the problem, devise a plan (translate), carry out the plan (solve), look back (check and interpret). It is thought that students should manage the problem solving process well in order to be successful in the skill-based questions that they generally have difficulty with. From this point of view, examining the process of solving the skill-based questions of the students emerges as a problem situation.

Mathematical Ability and Problem Solving Skills and Processes

When the studies on the problem solving process are examined, in general, how the participants solve the problems according to different problem solving approaches and the strategies they use are examined. In addition, there are studies in the literature examining the difficulties encountered in the problem solving process. Karatas and Guven (2004) determined that students use variables to define the problem in the stage of understanding the problem, and that students who cannot do this have difficulties in establishing equations and finding solutions. Firat and Kocak (2019) observed that successful students use metacognitive strategies such as underlining and re-reading the problem while reading the problem. Karatas and Guven (2004), Demir (2019), Ahmetoglu (2021), Altintas, Ilgun, and Angay (2022) emphasized in their studies that the most difficult problem solving is in the "understanding step" and mistakes are made. Umurbek (2020) concluded that the students who successfully completed the "understanding the problem phase" were successful in the steps of "making a plan" and "implementing the plan", they used similar solutions while implementing the plan, but did not complete the "control step". In their study, Aydemir and Kubanc (2014) determined that students who answered the problems correctly performed metacognitive processes such as understanding the problem, solving it in different ways, expressing it in their own sentences and controlling. Yilmaz (2021), in his study with prospective teachers from different branches, observed that most of the pre-service teachers in all branches had difficulties during the evaluation step of the solution. In his study, Sipahi (2021) stated that the group that uses the problem solving stages the most and sequentially is the gifted students. Koç-Koca (2022) found that gifted students are successful in the problem solving process, but they do not control the solutions, they apply the strategies correctly, and they use different solutions by developing more strategies for the problems they have difficulty with.

Considering the studies, there are quite a few studies examining the problem-solving processes of students. However, it has been observed that there are not enough studies examining the solution processes of students' skill-based questions. In addition, it is thought that examining the problem solving processes of students who are successful in mathematics and gifted students among themselves will contribute to the literature and fill an important gap. In this study, skill-based question solving processes of 8th grade students who will be placed in secondary education institutions according to their success status by taking an exam at the end of the academic year, in which skill-based questions are asked, were examined. It is thought that examining the process of solving the skill-based questions of the students is important in terms of determining the points that they have difficulties and lack in this process. Examining the process of solving skill-based questions by successful and gifted students will be very helpful to students and their parents whose aim is to settle in qualified secondary education institutions. In addition, it is thought that it will support the mathematics teachers who prepare students for this exam in terms of seeing and intervening the problems that their students experience while solving skill-based questions. Examining the students' process of solving skill-based questions will increase success in national and international exams by identifying the places where there are problems in problem solving. In this context, it is thought that the research will help institutions and authorities in organizing a mathematics curriculum.

Problem of Study

The aim of this research is to examine in detail the process of solving skill-based questions prepared on the subject of "multipliers and multiples" of students who are advanced learners in math and gifted students according to the Polya' problem solving steps . For this purpose, answers are sought for the following problems and sub-problems.

The problem statement of this research is, "How is the process of solving skill-based questions about multipliers and multiples of students who are advanced learners in math and gifted students?" has been determined. Sub-problems;

- How are students' problem-solving processes according to Polya's problem-solving steps?
- What are the problem solving processes of students who are advanced learners in math and gifted students?

Method

Research Model

The qualitative method, which facilitates comprehension and inference, provides the researcher with the opportunity to research and collect data in detail due to its flexible structure (Karatas, 2015). Qualitative research method was used in order to reveal how the 8th grade students who participated in the research thought, interpreted and solved the questions by making associations while solving the skill-based questions on the subject of "multipliers and multiples". In this research, technically, a case study was used. Case study is a research method that allows the researcher to examine in detail a situation or event that he cannot control. In this research model, answers are sought for "why" and "how" questions. It is generally used when there are multiple data sources and when seeking answers to scientific questions (Yıldırım & Simsek, 2016; Buyukozturk, Cakmak, Akgun, Karadeniz & Demirel, 2018).

Study Group

The study group of the research consists of 8 students, 6 girls and 2 boys, studying in the 8th grade in secondary schools in the central district of Kars in the 2022-2023 academic year. 4 of these students were educated in the field of general mental ability at the Science and Art Center (SAC), and 4 of them were students with high mathematics achievement who did not go to SAC but studied at the school where the researcher works. While determining the study group, criterion sampling method was used. The main purpose of the criterion sampling method is to work on the criteria that the researcher has previously determined. In this sampling method, the researcher can use a criterion determined by himself or someone else (Yildirim & Simsek, 2016). While choosing the study group, whether the students were selected in the SAC general mental ability field and the success of the mathematics course were determined as criteria. The reason why these students were chosen is because they thought that they would be able to solve skill-based questions in a more organized and understandable way and to convey their thoughts more easily. Students receiving support education at SAC are students who have been identified as gifted students by the Guidance Research Center. On the other hand, the 5th, 6th and 7th grade mathematics grades of the students who did not go to SAC are above 90 points. While selecting the students in SAC, support was received from the director of the institution and the guidance teacher working in SAC. Grade averages of students who did not attend SAC were obtained through the e-school system. In addition, the students from whom data will be collected voluntarily participated in this research and the necessary permissions were obtained from their families. Students were given a code to be used throughout the study, and these codes were used instead of students' names.

Table 1. Structures of participants

No	Talent Status	Grade	Gender	Code
1	Advanced learner in math	8th	F	ALM1
2	Advanced learner in math	8th	M	ALM2
3	Advanced learner in math	8th	F	ALM3
4	Advanced learner in math	8th	F	ALM4
5	Gifted student	8th	M	GS1
6	Gifted student	8th	F	GS2
7	Gifted student	8th	F	GS3
8	Gifted student	8th	F	GS4

Multipliers and Multiples Achievement Testing and Development Process

The data were collected by applying the achievement test developed by the researchers. While the achievement test was being developed, the purpose of the test was determined first. Since the aim of the research was to solve the skill-based questions of the 8th grade students, the first topic of the 8th grade curriculum, "Multipliers and Multiples", was chosen. After determining the subject on which the achievement test will be prepared, the course and test books related to the subject were examined by the researcher. After the examination was completed, 3 outcomes and 16 questions were prepared on the subject of "Multiples and Multiples". Thus, the draft test was created. A table of specifications has been prepared in order to distribute the questions in a balanced way on the basis of outcomes and to show which outcome they measure. It is very important to ensure the validity of the achievement test while developing it, and it is a frequently preferred method to seek expert opinion (Treagust, 1988; Calik & Ayas, 2003; Buyukozturk, 2020). For this purpose, the draft test prepared by the researcher was evaluated by 2 mathematics teachers, 2 academicians and 1 Turkish teacher who are experts in their fields and have experience in writing questions, in terms of language, scope, appearance, content, etc. analyzed for features. The figures and pictures of the questions were drawn by an expert art teacher in the digital environment in accordance with the level of the students. The draft form was applied to 108 students in the 9th grade of Science High School who volunteered to participate in the study, and item analysis was performed. Reliability analysis was conducted to determine how sensitively the prepared test measures the feature it measures and to what extent the results are free from errors. The KR-20 reliability value of the 16-question draft test was 0.614. After 6 questions were eliminated after item analysis, the KR-20 reliability value was 0.585 for 10 questions in the final test. Since the calculated KR-20 reliability value is greater than 0.5, the test can be said to be a moderately reliable test (Salvucci, Walter, Conley, Fink, & Saba, 1997; Tan, 2009).

Clinical Interview and Think Aloud Technique

The clinical interview is one of the non-traditional methods used to investigate the problem solving process in mathematics education. The problem solving process, which is considered to be a very complex process, and the detailed examination of student behaviors in this process were made with the help of clinical interviews. In the clinical interview, the students were asked “What do you think about the problem?”, “How do you intend to solve the problem?”, “Are you sure about the result you found?”, “Can you explain the procedures you have done?” Information about the problem solving process can be obtained by asking questions such as (Karatas & Guven, 2003).

In the clinical interview, the thinking aloud technique was used to learn all the details of the problem solving process. Thinking aloud technique is an evaluation technique in which the individual conveys everything he thinks and does during the problem solving process. The data obtained in the thinking aloud technique is recorded by a voice recorder or camera. Then this data is dumped. The purpose of applying this technique is to examine the cognitive process of the student in detail during the problem solving process and to minimize data loss (Ozkubat & Ozmen, 2018).

Pilot Scheme

For the pilot scheme, the final test was applied to 5 students, 3 girls and 2 boys, who were successful in the mathematics course, and a clinical interview was conducted. As a result of the pilot scheme, it was seen that 40 minutes was sufficient to conduct clinical interviews with the students. This pilot scheme helped to identify the critical behaviors in the table to be created to examine the data. In addition, in the intervention, it was determined which questions to be asked in order to encourage students to speak and convey their thoughts.

Intervention

While collecting the data, an achievement test was applied to the students participating in the study. While applying the achievement test, a clinical interview was conducted using a voice recorder in order to make in-depth analysis with the students and to minimize data loss. The achievement test application and clinical interview were conducted by determining the appropriate times for the students and the researcher. Gifted students' applications were made at SAC, and advanced learner in math applications were made in their own schools. Thus, the students participating in the research were not adversely affected by the data collection process.

Before starting to collect data, the purpose of the research was explained so that the students could feel comfortable, and an environment was tried to be created that would enable the application to take place in a conversational atmosphere. It was observed that the students were excited during the clinical interview. Therefore, the researcher tried to comfort the participants by making the necessary speeches. The students participating in the research were informed that their names and information would be kept strictly confidential. It is stated that the code will be used instead of their names. In addition, it was emphasized that the audio recording would not be listened to by anyone other than the researcher. It has been specifically stated that the study will not affect the student's school grades and the results will not be shared with other teachers. Thus, the students participating in the study were provided to participate in the clinical interview comfortably.

During the clinical interview, “What does it say about the problem?”, “What do you need to do?”, “How did you do it?”, “Are you sure?” such questions were asked. Thus, the problem solving process was tried to be examined in detail. During the clinical interview, students were asked to think aloud while answering in order to reveal what they were thinking. It has been specifically stated that they can freely say what they think without hesitation. Enough time was given for the students to think and respond where they had difficulty. Clinical interviews lasted an average of 40 minutes. The data collection process was completed in 1 month.

Data Analysis

Polya's (1957) problem solving theory was used to analyze the data obtained in this study in terms of problem solving process. Each problem-solving step in this theory is considered as a step, and the critical behaviors that participants are expected to show in each step are created with the help of Polya (1957) and the data obtained from the pilot application. In addition, while the questions in the achievement test were analyzed descriptively, the study of Umurbek (2020) was used. The data collected as a result of the clinical interviews with the students were transcribed and coded in accordance with the relevant steps and behaviors in the table.

Findings

The process of solving skill-based questions of the students participating in the research was examined for each question with the help of the data obtained from the interviews. The processes experienced by each student for each problem are shown with the help of tables. While interpreting the skill-based problem solving processes, Polya's (1957) level of realizing the problem solving steps (understanding the problem, devise a plan (translate), carry out the plan (solve), look back (check and interpret) was taken into account.

Findings of the Process of Solving the First Skill-Based Problem
Problem 1.

The divisors of a positive integer are also the factors of that integer. An integer is divided by its factors without a remainder.

After school, Fahrettin goes to help his father's toy shop. In the toy shop there are two boxes with small toys. Each of these boxes contains 50 toys. After selling for a few days, Fahrettin counted the toys in the boxes and reached the following information:

- The number of toys left in box 1 has a prime factor of 2.
- The number of toys remaining in the 2nd box is not a prime number, but has 1 prime factor.
- The numbers of toys remaining in boxes 1 and 2 are co-prime.

What is the maximum number of toys sold in two boxes?

Table 2. Analysis of the first skill-based problem by Polya's problem solving steps

Steps	Behaviours	ALM1	ALM2	ALM3	ALM4	GS1	GS2	GS3	GS4
Understanding the problem	HKP		√	√	√	√		√	√
	DWG	√	√	√	√	√	√		√
	DR			√	√				√
	EPOW		√	√	√	√			√
Devise a plan	CDSP			√					√
	IDSP	√	√		√			√	
	USP					√	√		
	ESP	√	√	√	√			√	
Carry out the plan	CISP	√	√		√			√	√
	IISP			√					
	RPS	√	√	√		√	√		√
Look back	CO	√		√	√				
	RDM								
RFS		F	F	F	F	E	E	F	T

HKP: Highlighting Key Points **DWG**: Determining what's given **DR**: Determining of Requested **EPOW**: Expressing the Problem in Own Words **CDSP**: Correctly Determining the Solution Path **IDSP**: Inorrectly Determining the Solution Path **USP**: Undetermining of Solution Path **ESP**: Expressing the Solution Path **CISP**: Correctly Implementing the Solution Path **IISP**: Incorrectly Implementing the Solution Path **RPS**: Returning the Previous Steps **CO**: Checking the Operations **RDM**: Resolving with a different method **RFS**: Result Found by Student **E**: Empty **T**: True **F**: False **ALM**: Advanced Learner in Math **GS**: Gifted Student

As seen in Table 2, it is understood that all students, except ALM1 and GS2, emphasized the important points in the *understanding the problem ste*p, and all students, except GS3, determined what was given. However, it was observed that students other than ALM3, ALM4 and GS4 could not determine what was desired, and ALM1, GS2 and GS3 could not express the problem in their own words. It was observed that the students had difficulties in the *devise a plan step* and only ALM3 and GS4 could find a correct solution. In *the carry out the plan step*, it was determined that only GS4 implemented its plan without errors and reached the correct result. In addition, it was observed that all students except ALM4 and GS3 returned to the previous steps. It is also among the findings that ALM1, ALM3 and ALM4 control their operations, although the answers given in *the look back step* are incorrect.

Findings of the Process of Solving the Second Skill-Based Problem
Problem 2.

The divisors of a positive integer are also the factors of that integer. An integer is divided by its factors without a remainder.

Oyku works in a 36-floor workplace. When he went to work in the morning, he took the elevator with a certain number of people on the ground floor (0. floor). The elevator stops at each floor where the positive factors of 36 are, and does not stop at the other floors. If the floor number of the floors where the elevator stops is odd, 2 people got off, and if it is double, 1 person got off. When he came to the last floor and stopped, there was no one left in the elevator after the last descent.
Since there is no elevator except for the ground floor, how many people got on the elevator except Oyku at the beginning?

Table 3. Analysis of the second skill-based problem by Polya's problem solving steps

Steps	**Behaviours**	**ALM1**	**ALM2**	**ALM3**	**ALM4**	**GS1**	**GS2**	**GS3**	**GS4**
Understanding the problem	HKP		√	√	√	√		√	√
	DWG	√	√	√	√	√	√	√	√
	DR	√	√	√	√			√	√
	EPOW		√						√
Devise a plan	CDSP	√	√	√	√	√	√	√	√
	IDSP								
	USP								
	ESP	√	√		√			√	
Carry out the plan	CISP		√	√	√				√
	IISP	√				√	√	√	
	RPS		√		√		√		√
Look back	CO	√			√		√		
	RDM								
RFS		F	T	T	T	F	F	F	T

HKP: Highlighting Key Points **DWG**: Determining what's given **DR**: Determining of Requested **EPOW**: Expressing the Problem in Own Words **CDSP**: Correctly Determining the Solution Path **IDSP**: Inorrectly Determining the Solution Path **USP**: Undetermining of Solution Path **ESP**: Expressing the Solution Path **CISP**: Correctly Implementing the Solution Path **IISP**: Incorrectly Implementing the Solution Path **RPS**: Returning the Previous Steps **CO**: Checking the Operations **RDM**: Resolving with a different method **RFS**: Result Found by Student **E**: Empty **T**: True **F**: False **ALM**: Advanced Learner in Math **GS**: Gifted Student

As seen in Table 3, it was understood that in the *Understanding the problem* step, all of the students except ALM1 and GS2 emphasized the important points, all of them determined what was given, and all of the students except GS1 and GS2 determined what was desired. In addition, it was determined that none of the students except ALM2 and GS4 could express the problem in their own words. In the *Devise a plan* step, it was observed that all of the students determined the solution path correctly and ALM1, ALM2, ALM4 and GS3 expressed the solution path. In the *Carry out the plan* step, it was understood that successful students, except for ALM1, reached the correct result by solving the question carefully and without making any mistakes. It was determined that gifted students made mistakes in their solutions and found the answer wrong, considering it was due to lack of attention other than GS4. It is also among the findings that the students performed the *Look back* step by checking the solutions made by only ALM1, ALM4 and GS2.

Findings of the Process of Solving the Third Skill-Based Problem
Problem 3.

The divisors of a positive integer are also the factors of that integer. An integer is divided by its factors without a remainder.

Ali and Ahmet, two friends, topped up different amounts of internet packages to their mobile phones. After topping up the internet packages, they learned that the mobile phone company was running a campaign. According to this campaign, internet is given as a gift equal to the sum of the different prime factors of the internet package topped up the mobile phone. After the gifts were added, they saw that the total amount of internet on their line was equal. Since it is known that Ali has topped up 12 GB of internet package, how many GB of internet package could Ahmet have topped up?

A) 6 B) 8 C) 10 D) 15

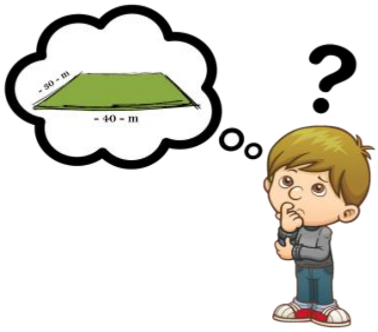
Table 4. Analysis of the third skill-based problem by Polya's problem solving steps

Steps	Behaviours	ALM1	ALM2	ALM3	ALM4	GS1	GS2	GS3	GS4
Understanding the problem	HKP		√	√	√	√	√	√	√
	DWG	√	√	√	√	√	√	√	√
	DR		√	√	√		√	√	√
	EPOW								
Devise a plan	CDSP			√	√		√	√	
	IDSP		√			√			√
	USP	√							
	ESP		√	√				√	√
Carry out the plan	CISP		√	√	√	√	√		√
	IISP							√	
	RPS							√	
Look back	CO				√				
	RDM								
RFS		E	F	T	T	F	T	E	F

HKP: Highlighting Key Points **DWG**: Determining what's given **DR**: Determining of Requested **EPOW**: Expressing the Problem in Own Words **CDSP**: Correctly Determining the Solution Path **IDSP**: Incorrectly Determining the Solution Path **USP**: Undetermining of Solution Path **ESP**: Expressing the Solution Path **CISP**: Correctly Implementing the Solution Path **IISP**: Incorrectly Implementing the Solution Path **RPS**: Returning the Previous Steps **CO**: Checking the Operations **RDM**: Resolving with a different method **RFS**: Result Found by Student **E**: Empty **T**: True **F**: False **ALM**: Advanced Learner in Math **GS**: Gifted Student

As seen in Table 4, it was understood that in the *understanding the problem* step, all of the students except ALM1 emphasized the important points, all of them determined the given ones, and all of them determined the asked one except ALM1 and GS1. However, it was determined that none of the students could express the problem in their own words. In the *devise a plan* step, it was observed that ALM3, ALM4, GS2 and GS3 of the students determined the right solution, B1 could not determine the solution, and ALM2, GS1 and GS4 chose the wrong solution because of the difficulties they experienced during the understanding of the problem. In the *carry out the plan* step, it was seen that among the students who could determine the solution, only they made a mistake while applying the GS3's plan, and could not give any answer despite returning to the previous steps. It is among the findings that only GS4 of the students tried the other options after answering and performed the *look back* step.

Findings of the Process of Solving the Fourth Skill-Based Question
Problem 4.



Faruk wants to divide his rectangular garden of 40 m in length and 30 m in width into equal squares and plant saplings in the corner of each square. Since each of the saplings Faruk wants to plant is 15 TL, how much should Faruk spend at least for the saplings he will plant?

Table 5. Analysis of the fourth skill-based problem by Polya's problem solving steps

Steps	Behaviours	ALM1	ALM2	ALM3	ALM4	GS1	GS2	GS3	GS4
Understanding the problem	HKP		√	√	√	√			√
	DWG	√	√	√	√	√		√	√
	DR	√	√	√	√	√			√
	EPOW								
Devise a plan	CDSP	√	√	√	√	√			√
	IDSP							√	
	USP						√		
	ESP	√	√	√	√	√		√	√
Carry out the plan	CISP		√	√	√	√			√
	IISP	√						√	
	RPS	√		√					
Look back	CO				√				
	RDM	√							
RFS		F	T	T	T	T	E	F	T

HKP: Highlighting Key Points **DWG**: Determining what's given **DR**: Determining of Requested **EPOW**: Expressing the Problem in Own Words **CDSP**: Correctly Determining the Solution Path **IDSP**: Incorrectly Determining the Solution Path **USP**: Undetermining of Solution Path **ESP**: Expressing the Solution Path **CISP**: Correctly Implementing the Solution Path **IISP**: Incorrectly Implementing the Solution Path **RPS**: Returning the Previous Steps **CO**: Checking the Operations **RDM**: Resolving with a different method **RFS**: Result Found by Student **E**: Empty **T**: True **F**: False **ALM**: Advanced Learner in Math **GS**: Gifted Student

As seen in Table 5, it is understood that in the *understanding the problem* step, all students except ALM1, GS2 and GS3 emphasized important points, all students except GS2 determined what was given, all students except GS2 and GS3 determined what was given, but none of the students could express the problem in their own words. . In the *devise a plan* step, it was observed that the students determined the solution path correctly, except for GS2, which could not determine any solution path, and GS3, which did not determine the solution path incorrectly, and the students who determined the solution path were able to express the solution paths. In the *carry out the plan* step, it was determined that the students who determined the solution path correctly, except for ALM1 and GS3, implemented their plans correctly and reached the right result. It was seen that ALM1 and ALM3 returned to the previous steps to solve the question. Among the findings obtained from the students, ALM1 solved the question again with a different method and ALM4 performed the *look back* step by checking its operations.

Findings of the Process of Solving the Fifth Skill-Based Question
Problem 5.

Florist Mrs. Serpil has 120 roses and 96 carnations in her shop. Mrs. Serpil wants to create bouquets for Mother's Day, each with a single flower type. How many bouquets can Serpil create, with at least 10 flowers in each bouquet?

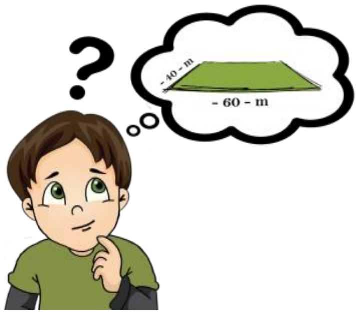
Table 6. Analysis of the fifth skill-based problem by Polya's problem solving steps

Steps	Behaviours	ALM1	ALM2	ALM3	ALM4	GS1	GS2	GS3	GS4
Understanding the problem	HKP		√	√	√				√
	DWG	√	√	√	√	√		√	√
	DR	√	√	√	√				√
	EPOW				√				
Devise a plan	CDSP	√	√		√	√			√
	IDSP							√	
	USP			√			√		
	ESP	√	√		√			√	√
Carry out the plan	CISP	√	√		√			√	√
	IISP					√			
	RPS	√	√	√				√	
Look back	CO		√		√				
	RDM								
RFS		T	T	E	T	F	E	F	T

HKP: Highlighting Key Points **DWG**: Determining what's given **DR**: Determining of Requested **EPOW**: Expressing the Problem in Own Words **CDSP**: Correctly Determining the Solution Path **IDSP**: Incorrectly Determining the Solution Path **USP**: Undetermining of Solution Path **ESP**: Expressing the Solution Path **CISP**: Correctly Implementing the Solution Path **IISP**: Incorrectly Implementing the Solution Path **RPS**: Returning the Previous Steps **CO**: Checking the Operations **RDM**: Resolving with a different method **RFS**: Result Found by Student **E**: Empty **T**: True **F**: False **ALM**: Advanced Learner in Math **GS**: Gifted Student

As seen in Table 6, ALM2, ALM3, ALM4 and GS4 emphasized the important points in the *understanding the problem* step, students other than GS2 determined what was given, students other than GS1, GS2 and GS3 determined what was desired, and only B4 expressed the problem in his own words. understandable. In the *devise a plan* step, it was observed that students other than GS3 who incorrectly determined the solution path and ALM3 and GS2, who could not produce a solution, produced a correct solution. It has been determined that students other than GS1 among the students who produce solutions express the solution path. In the *carry out the plan* step, it was observed that students other than ALM1, among the students who produced a solution, applied their plans correctly. In addition, it was observed that the students ALM1, ALM2, ALM3 and GS3 tried to solve the question by returning to the previous steps. In the *look back* step, it is among the findings that it performs this step by controlling the operations of only ALM2 and ALM4.

Findings of the Process of Solving the Sixth Skill-Based Question
Problem 6.



Mrs. Oğuzhan decided to divide his rectangular field with side lengths of 60 m and 40 m into square pieces and to plant different types of vegetables on each piece. But since Mr. Oğuzhan wants the area of each piece not to exceed 200 square meters, at least how many types of vegetables can he plant in his field?

Table 7. Analysis of the sixth skill-based problem according to Polya's problem solving steps

Steps	**Behaviours**	**ALM1**	**ALM2**	**ALM3**	**ALM4**	**GS1**	**GS2**	**GS3**	**GS4**
Understanding the problem	HKP		√	√	√				√
	DWG	√	√	√	√	√		√	√
	DR	√	√	√	√				√
	EPOW				√				
Devise a plan	CDSP	√	√		√				√
	IDSP			√				√	
	USP					√	√		
	ESP		√		√			√	
Carry out the plan	CISP	√	√		√				√
	IISP			√				√	
	RPS								
Look back	CO				√				
	RDM								
RFS		T	T	F	T	E	E	F	T

HKP: Highlighting Key Points **DWG**: Determining what's given **DR**: Determining of Requested **EPOW**: Expressing the Problem in Own Words **CDSP**: Correctly Determining the Solution Path **IDSP**: Incorrectly Determining the Solution Path **USP**: Undetermining of Solution Path **ESP**: Expressing the Solution Path **CISP**: Correctly Implementing the Solution Path **IISP**: Incorrectly Implementing the Solution Path **RPS**: Returning the Previous Steps **CO**: Checking the Operations **RDM**: Resolving with a different method **RFS**: Result Found by Student **E**: Empty **T**: True **F**: False **ALM**: Advanced Learner in Math **GS**: Gifted Student

As seen in Table 7, ALM2, ALM3, ALM4 and GS4 emphasized important points in the *understanding the problem* step, all students except GS2 determined what was given, students other than GS1, GS2 and GS3 determined what was desired, and only ALM4 explained the problem in their own words. is understood to mean. In the *devise a plan* step, it was observed that students other than ALM3 and GS3, who incorrectly determined the solution path, and GS1 and GS2, who could not determine the solution path, determined a correct solution. It was observed that ALM2, ALM4 and GS3, which determined the solution path, expressed the solution path they found. It was determined that the students who produced the right solution in *the carry out the plan* step applied their plans correctly, while the students who produced the wrong solution made mistakes while applying their plans. In this question, none of the students returned to the previous steps and only checked the operations of ALM4 and performed the *look back* step.

Findings of the Process of Solving the Seventh Skill-Based Question
Problem 7.

Construction master Mr. Deniz wants to lay the square floor bathroom with rectangular tiles with side lengths of 8 cm and 10 cm. When the side lengths of the bathroom floor are not a common multiple of the side lengths of the tile, it is necessary to break and place the necessary tiles in order to completely cover the floor. Since Mr. Deniz did not break any tiles when he finished his work, at least how many tiles did he use to cover the bathroom floor?

Table 8. Analysis of the seventh skill-based problem according to Polya's problem solving steps

Steps	Behaviours	ALM1	ALM2	ALM3	ALM4	GS1	GS2	GS3	GS4
Understanding the problem	HKP		√	√	√			√	√
	DWG	√	√	√	√	√	√	√	√
	DR	√	√	√	√				√
	EPOW		√		√				
Devise a plan	CDSP		√	√		√			√
	IDSP	√			√				
	USP						√	√	
	ESP		√	√					
Carry out the plan	CISP		√	√					√
	IISP	√			√	√			
	RPS		√		√				
Look back	CO	√							√
	RDM		√						
RFS		F	T	T	F	F	E	E	T

HKP: Highlighting Key Points **DWG**: Determining what's given **DR**: Determining of Requested **EPOW**: Expressing the Problem in Own Words **CDSP**: Correctly Determining the Solution Path **IDSP**: Incorrectly Determining the Solution Path **USP**: Undetermining of Solution Path **ESP**: Expressing the Solution Path **CISP**: Correctly Implementing the Solution Path **IISP**: Incorrectly Implementing the Solution Path **RPS**: Returning the Previous Steps **CO**: Checking the Operations **RDM**: Resolving with a different method **RFS**: Result Found by Student **E**: Empty **T**: True **F**: False **ALM**: Advanced Learner in Math **GS**: Gifted Student

As seen in Table 8, in the *understanding the problem step*, students other than ALM1, GS1 and GS2 emphasized important points, all students determined what was given, students other than GS1, GS2 and GS3 determined what was desired, and only ALM2 and ALM4 expressed the problem in their own words. understood. In the *devise a plan step*, it was observed that all students, except ALM1 and ALM4, who incorrectly determined the solution path, and GS2 and GS3, who could not determine the solution, all determined the correct solution. It has been determined that ALM2 and ALM3, which determine the solution path, express the solution path they have determined. In the *carry out the plan step*, it was observed that ALM2, ALM3 and GS4 applied the plan they determined correctly, while ALM1, ALM4 and GS1 made mistakes while applying the plan they determined. It was also observed that ALM2 and ALM4 returned to the previous steps in order to solve the problem. It is among the findings obtained that students control the operations of ALM1 and GS4, and ALM2 performs the *look back step* by solving the problem again with a different method.

Findings Regarding the Eighth Skill-Based Question Solving Process
Problem 8.

Mrs. Nilufer, who grows ivy on her balcony, wants to make two supports of equal length for the vines to wrap around. These supports will be formed by bringing equal lengths of 6 cm and 10 cm sticks end to end and gluing them together. Since Mrs. Nilufer has to pay 3 TL to the carpenter for each sticking process, how much should she pay at least for this process?

Table 9. Analysis of the eighth skill-based problem by Polya's problem solving steps

Steps	Behaviours	ALM1	ALM2	ALM3	ALM4	GS1	GS2	GS3	GS4
Understanding the problem	HKP		√	√	√	√	√	√	√
	DWG	√	√	√	√	√	√	√	√
	DR	√	√	√	√	√			√
	EPOW				√				
Devise a plan	CDSP	√	√					√	√
	IDSP			√	√	√	√		
	USP								
	ESP		√		√			√	
Carry out the plan	CISP	√	√						√
	IISP			√	√	√	√	√	
	RPS			√		√			
Look back	CO				√				
	RDM								
RFS		T	T	F	F	F	F	F	T

HKP: Highlighting Key Points **DWG**: Determining what's given **DR**: Determining of Requested **EPOW**: Expressing the Problem in Own Words **CDSP**: Correctly Determining the Solution Path **IDSP**: Incorrectly Determining the Solution Path **USP**: Undetermining of Solution Path **ESP**: Expressing the Solution Path **CISP**: Correctly Implementing the Solution Path **IISP**: Incorrectly Implementing the Solution Path **RPS**: Returning the Previous Steps **CO**: Checking the Operations **RDM**: Resolving with a different method **RFS**: Result Found by Student **E**: Empty **T**: True **F**: False **ALM**: Advanced Learner in Math **GS**: Gifted Student

As seen in Table 9, it was observed that in the understanding the problem step, students other than ALM1 emphasized important points, all students determined what was given, students other than GS2 and GS3 determined what was desired, and only ALM4 expressed the problem in their own words. In the devise a plan step, it is understood that ALM1, ALM2, GS3 and GS4 determine the correct solution path, while ALM3, ALM4, GS1 and GS2 determine the wrong solution path. It has been determined that ALM2, ALM4 and GS3, which determine the solution path, express the solution path they have determined. In the carry out the plan step, it was observed that ALM1, ALM2 and GS4 applied the plan they determined correctly, while the other students made mistakes while applying the plan they determined. In addition, it was determined that ALM3 and GS1 returned to the previous steps in order to solve the problem. It is among the findings that the students performed the look back step by only checking the operations of ALM4.

Findings of the Process of Solving the Ninth Skill-Based Question
Problem 9.

Natural numbers that have no common divisor other than 1 are called prime numbers

Yunus has an old model car. There is a screen on the dashboard of the car that shows the humidity and temperature. However, because the car is old, the ones digit of the two-digit temperature display does not work in some cases. After a certain time, Yunus noticed that the ones digit of the temperature indicator did not work when the humidity and temperature values were prime numbers. Accordingly, if 2_ is written on the temperature indicator when the humidity value is 48, how many different values can the air temperature take?

Table 10. Analysis of the ninth skill-based problem by Polya's problem solving steps

Steps	Behaviours	ALM1	ALM2	ALM3	ALM4	GS1	GS2	GS3	GS4
Understanding the problem	HKP		√	√	√	√	√	√	√
	DWG	√	√	√	√	√	√	√	√
	DR	√	√	√	√		√	√	√
	EPOW		√		√	√			√
Devise a plan	CDSP	√	√	√	√	√	√	√	√
	IDSP								
	USP								
	ESP	√	√	√	√	√		√	√
Carry out the plan	CISP		√	√			√	√	√
	IISP	√			√	√			
	RPS						√		
Look back	CO				√				
	RDM								
RFS		F	T	T	F	F	T	T	T

HKP: Highlighting Key Points **DWG**: Determining what's given **DR**: Determining of Requested **EPOW**: Expressing the Problem in Own Words **CDSP**: Correctly Determining the Solution Path **IDSP**: Incorrectly Determining the Solution Path **USP**: Undetermining of Solution Path **ESP**: Expressing the Solution Path **CISP**: Correctly Implementing the Solution Path **IISP**: Incorrectly Implementing the Solution Path **RPS**: Returning the Previous Steps **CO**: Checking the Operations **RDM**: Resolving with a different method **RFS**: Result Found by Student **E**: Empty **T**: True **F**: False **ALM**: Advanced Learner in Math **GS**: Gifted Student

As seen in Table 10, it is understood that in the understanding the problem step, all students except ALM1 emphasized important points, all students determined what was given, all students except GS1 determined what was desired, and ALM2, ALM4, GS1 and GS4 expressed the problem in their own words. In the devise a plan step, it was observed that all students determined the correct solution path and all students, except GS2, expressed the solution they determined. In the carry out the plan step, it was determined that all students except ALM1, ALM4 and GS1 applied the plan correctly and GS2 returned to the previous steps to solve the problem. It is among the findings that the students performed the look back step by only checking the operations of ALM4.

Findings of the Process of Solving the Tenth Skill-Based Question
Problem 10.

Natural numbers that have no common divisor other than 1 are called prime numbers

Meltem and Ipek are two close friends living on the 3rd and 4th floors of the same apartment building, respectively. In this apartment, there are 10 apartments on each floor and the apartment numbers are named with the floor name first. For example, the apartment numbers on the 5th floor are named as 50, 51, 52, ..., 59. Ipek and Meltem's flat numbers have the same unit digit and the flat numbers are prime between them, so how many different values can the flat numbers of these two friends take?

Table 11. Analysis of the tenth skill-based problem by Polya's problem solving steps

Steps	Behaviours	ALM1	ALM2	ALM3	ALM4	GS1	GS2	GS3	GS4
Understanding the problem	HKP		√	√	√	√	√	√	√
	DWG	√	√	√	√	√	√	√	√
	DR	√	√	√	√	√	√	√	√
	EPOW					√			
Devise a plan	CDSP	√	√	√	√	√	√	√	√
	IDSP								
	USP								
	ESP		√			√	√	√	√
Carry out the plan	CISP	√	√	√	√	√			√
	IISP						√	√	
	RPS		√					√	
Look back	CO		√	√	√	√	√		
	RDM								
RFS		T	T	T	T	T	F	F	T

HKP: Highlighting Key Points **DWG**: Determining what's given **DR**: Determining of Requested **EPOW**: Expressing the Problem in Own Words **CDSP**: Correctly Determining the Solution Path **IDSP**: Incorrectly Determining the Solution Path **USP**: Undetermining of Solution Path **ESP**: Expressing the Solution Path **CISP**: Correctly Implementing the Solution Path **IISP**: Incorrectly Implementing the Solution Path **RPS**: Returning the Previous Steps **CO**: Checking the Operations **RDM**: Resolving with a different method **RFS**: Result Found by Student **E**: Empty **T**: True **F**: False **ALM**: Advanced Learner in Math **GS**: Gifted Student

As seen in Table 11, it is understood that in the "Understanding the problem" step, all students except ALM1 emphasized important points, all students determined what was given and what was required, and only GS1 expressed the problem in his own words. In the "Devise a plan" step, it was observed that all students determined the correct solution path and all students, except ALM1, ALM3, and ALM4, expressed the solution they determined. In the "Carry out the plan" step, it was determined that all students, except GS2 and GS3, applied the plan correctly, and ALM1 and GS3 returned to the previous steps in order to solve the problem. It is among the findings that the students performed the k "Look back" step by controlling the operations of ALM2, ALM3, ALM4, GS1 and GS2.

Conclusion and Discussion

In this research, it is concluded that the "Understanding the problem" step is carried out by the students who answered the question correctly. Another result is that the "understanding the problem" step is the step that students have the most difficulty with, and students who cannot successfully perform this step also experience difficulties in the next steps. This result is in line with the results of the studies conducted by Karatas and Guven (2004), Demir (2019), Ahmetoglu (2021), Altintas, Ilgun and Angay (2022), who emphasized that the most difficulty in problem solving is at the level of understanding and mistakes are made. It is concluded that the students who could not perform the understanding the problem step read the question silently, did not specify the important parts with a pencil or underline the whole question and could not express the problem in their own words. This result is similar to the results of the study by Umurbek (2020). In addition, it was emphasized in the study conducted by Aydemir and Kubanc (2014) that students with high metacognitive skills expressed the problem in their own words and were able to determine what was given and what was desired.

It was concluded that the students who successfully completed the understanding the problem step determined the solution path correctly. It was observed that the students who misidentified the solution or could not determine the solution had difficulty in understanding the problem and expected support from the researcher. It was observed that some of the students expressed the solution way, and some of them tried to solve the question without expressing the solution. These behaviors are similar to the results stated in the study conducted by Umurbek (2020), that some students went to the solution without expressing the plan, and students who could not understand the question expected guidance from the researcher. However, it was determined that the students who went to the solution without expressing the solution way expressed the solution with the intervention of the researcher. Another result reached by the research is that the students generally try to realize the step of creating a plan by using the methods in the questions they have solved before. This result is in parallel with the result of the study by Karatas and Guven (2004) that the students set up equations based on the problems they had previously solved. In addition, it is observed that students try to produce solutions with different strategies when they have difficulties, and this result is similar to the results of the study conducted by Koc Koca (2022).

In this study, it is also concluded that the students who successfully perform the understanding the problem and the devise a plan step also successfully perform the carry out the plan step. This result is similar to the results of the study by Umurbek (2020) in which he emphasized that students who perform critical behaviors in the problem understanding step and choose the appropriate strategy in the devise a plan step are successful in the carry out the plan step.

It was observed that the students did not control the operations they did in general and did not solve the question again with a different method. These results are in line with the results of the studies conducted by Umurbek (2020) with students with different levels of success, by Koc Koca (2022) with gifted students, and by Yilmaz (2021) with pre-service teachers.

Advanced learner in math and gifted students

As a result of the research, advanced learner in math emerges as a result that they try to apply Polya's problem solving steps more than gifted students. This result is in line with the result of Yeşilova (2013)'s study, in which the problem solving behaviors of students with below average and above average mathematics achievement were compared. However, in the study conducted by Sipahi (2021), which examined the problem solving stages of successful and gifted students, Polya's statement that the group that applied the problem solving steps most and sequentially was the gifted students, which contradicts the result.

It was observed that advanced learner in math students made more effort to understand the problem than gifted students. It was determined that advanced learner in math students produced a solution for almost every question, even if they were wrong, and gifted students could not produce a solution for the questions they gave up more easily and had difficulties.

It was determined that advanced learner in math students acted more carefully and made fewer operational errors than gifted students in the carry out the plan step. It is thought that the reason for this situation is that some gifted students try to make operations from their minds.

It was concluded that advanced learner in math students applied the check step more than gifted students. It is thought that the reason for this situation is that gifted students are overconfident in themselves and therefore do not think that they will make mistakes. This result coincides with the result of Koc Koca (2022) stating that gifted students do not perform the check step in his study.

When the answers to the questions were examined, it was determined that advanced learner in math students were more successful as a group than gifted students, but the gifted GS4 student was more successful than all students. This result contradicts the result emphasized by Koc Koca (2022), stating that gifted students are successful in solving the questions.

Recommendations

According to the results of the study, the following suggestions can be presented to the researchers;

- It is recommended that advanced learner in math students do more detailed research on the reasons why they are more successful in problem solving than gifted students..
- In this research, Polya's problem solving approach was used. It is recommended that advanced learner in math students and gifted students examine problem solving processes with different problem solving process approaches.
- Skill-based questions were used in this research. It is recommended that advanced learner in math students' and gifted students' examine their problem-solving processes with different types of questions.
- This research was conducted with 4 advanced learner in math students and 4 gifted students. The status of these results obtained in qualitative research can also be examined in quantitative research.

According to the results of the study, the following suggestions can be presented to the applicants;

- It is recommended that teachers of advanced learner in math students and gifted students teach problem-solving steps to their students and solve problems using these steps.
- Authorities that shape education policies are recommended to enrich the mathematics curriculum with activities that can improve the problem-solving process of advanced learner in math students and gifted students.
- Teachers teaching at SAC are recommended to carry out studies that will enable gifted students to use the look back step more in the problem solving process.

Limitations of Study

- It is limited to the 2022-2023 academic year.
- Limited to 8 students, 4 advanced learner in math students, 4 gifted students.
- Limited to the 8th grade "Multipliers and Multiples" topic in the mathematics curriculum
- It is limited to the data obtained from the prepared achievement test and the clinical interview.

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Appendix

Appendix 1. Multipliers and Multiples Achievement Test (Turkish)

Soru 1.

Pozitif bir tam sayının bölenleri, aynı zamanda bu tam sayının çarpanlarıdır. Bir tam sayı, çarpanlarına kalansız olarak bölünür.

Fahrettin okuldan sonra babasının oyuncak dükkânına yardıma gitmektedir. Oyuncak dükkânında içinde küçük oyuncakların bulunduğu iki tane kutu vardır. Bu kutuların her birinin içinde 50 adet oyuncak bulunmaktadır. Fahrettin birkaç gün satış yaptıktan sonra kutulardaki oyuncakları saymış ve aşağıdaki bilgilere ulaşmıştır:

- 1. kutuda kalan oyuncakların sayısının 2 tane asal çarpanı vardır.
- 2. kutuda kalan oyuncakların sayısı asal sayı olmayıp 1 tane asal çarpanı vardır.
- 1. ve 2. kutuda kalan oyuncakların sayıları aralarında asaldır.

Buna göre iki kutuda satılan toplam oyuncak sayısı **en çok** kaçtır?

Soru 2.

Pozitif bir tam sayının bölenleri, aynı zamanda bu tam sayının çarpanlarıdır. Bir tam sayı, çarpanlarına kalansız olarak bölünür.

Öykü, 36 katlı bir iş yerinde çalışmaktadır. Sabah işe gittiğinde zemin katta (0. kat) belirli bir sayıda kişiyle asansöre binmiştir. Asansör 36 sayısının pozitif çarpanlarının olduğu her bir katta durmakta bunun dışındaki katlarda durmamaktadır. Asansörün durduğu katların kat numarası tek ise 2 kişi, çift ise 1 kişi inmiştir. Son kata gelip durduğunda son inenlerin ardından asansörde kimse kalmamıştır.

Zemin kat haricinde asansöre binen olmadığına göre başlangıçta Öykü'den **hariç** kaç kişi asansöre binmiştir?

Soru 3.

Pozitif bir tam sayının bölenleri, aynı zamanda bu tam sayının çarpanlarıdır. Bir tam sayı, çarpanlarına kalansız olarak bölünür.

İki arkadaş olan Ali ve Ahmet cep telefonlarına farklı miktarlarda internet paketi yüklemişlerdir. Bu iki arkadaş internet paketlerini yükledikten sonra cep telefonu şirketinin bir kampanya yaptığını öğrenmişlerdir. Bu kampanyaya göre, cep telefonuna yüklenen internet paketi miktarının farklı asal çarpanlarının toplamı kadar internet hediye edilmektedir. Hediyeler eklendikten sonra hatlarındaki toplam internet miktarlarının eşit olduğunu görmüşlerdir. Ali'nin 12 GB internet paketi yüklediği bilindiğine göre, Ahmet kaç GB' lık internet paketi yüklemiş olabilir?

A) 6 B) 8 C) 10 D) 15

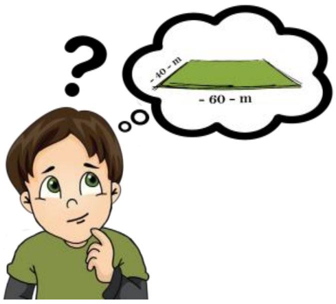
Soru 4.

Faruk, boyu 40 m, eni 30 m olan dikdörtgen şeklindeki bahçesini eş karelere ayırıp, her karenin köşesine fidan dikmek istemektedir. Faruk'un dikmek istediği fidanların tanesi 15 TL olduğuna göre Faruk, dikeceği fidanlar için **en az** kaç TL harcamalıdır?

Soru 5.

Çiçekçi Serpil Hanım'ın dükkânında 120 adet gül ve 96 adet karanfil vardır. Serpil Hanım Anneler günü için her birinde tek çiçek türü olacak şekilde buketler oluşturmak istemektedir. Serpil Hanım her bukette **en az** 10 çiçek olacak şekilde **en fazla** kaç buket oluşturabilir?

Soru 6.



Oğuzhan Bey kenar uzunlukları 60 m ve 40 m olan dikdörtgen biçimindeki tarlasını kare şeklinde parçalara ayırıp, her parçaya farklı türde sebze ekmeye karar vermiştir. Fakat Oğuzhan Bey her bir parçanın alanının 200 metrekareyi geçmemesini istediğine göre tarlasına **en az** kaç çeşit sebze ekebilir?

Soru 7.

İnşaat ustası Deniz Bey zemini kare biçimindeki banyoyu kenar uzunlukları 8 cm ve 10 cm olan dikdörtgen şeklindeki fayanslarla döşemek istemektedir. Banyo zemininin kenar uzunlukları fayansın kenar uzunluklarının ortak bir katı olmadığında, zemininin tamamen kaplanması için gerekli kadar fayans kırıp yerleştirmek gerekmektedir. Deniz Bey işini bitirdiğinde hiç fayans kırmadığına göre banyonun zemini kaplamak için en az kaç adet fayans kullanmıştır?

Soru 8

Balkonuna sarmaşık diken Nilüfer Hanım sarmaşıkların dolanması için eş uzunlukta iki tane destek yapmak istemektedir. Bu destekleri, 6 cm ve 10 cm uzunluğundaki çubuklardan boyları eşit uzunlukta olanları uç uca getirip yapıştırarak yapacaktır. Nilüfer Hanım çubukları her yapıştırma işlemi için marangoza 3 TL ödemesi gerektiğine göre, bu işlem için en az kaç TL ödeme yapması gerekmektedir?

Soru 9.

1'den başka ortak böleni olmayan doğal sayılara aralarında asal sayılar denir.

Yunus'un eski model bir arabası vardır. Arabanın gösterge panelinde nem ve sıcaklığı gösteren bir ekran bulunmaktadır. Fakat araba eski olduğu için iki basamaklı sıcaklık göstergesinin birler basamağı bazı durumlarda çalışmamaktadır. Yunus belli bir zaman sonra nem ile sıcaklık değerinin aralarında asal sayılar olduğu durumlarda sıcaklık göstergesinin birler basamağının çalışmadığını fark etmiştir. Buna göre nem değeri 48 olduğu zaman sıcaklık göstergesinde 2_ yazıyorsa, hava sıcaklığının alabileceği kaç farklı değer vardır?

Soru 10

1'den başka ortak böleni olmayan doğal sayılara aralarında asal sayılar denir.

Meltem ve İpek aynı apartmanın sırasıyla 3. ve 4. katta oturmakta olan iki yakın arkadaştır. Bu apartmanda her katta 10 daire bulunmaktadır ve daire numaraları kat ismi başta olacak şekilde isimlendirilmektedir. Örneğin 5. kattaki daire numaraları 50,51,52,...,59 şeklinde isimlendirilmektedir. İpekle Meltem'in daire numaralarının birler basamağı aynı ve daire numaraları aralarında asal olduğuna göre bu iki arkadaşın daire numaraları kaç farklı değer alabilir?