Research Article

Shannonian Maximum Entropy Balking Threshold Mechanism (BTM) for a Stable $M/G/1$ Queue with Significant Applications of $M/G/1$ Queue Theory to Augmented Reality (AR)

Ismail A. Mageed

Abstract— **An exposition is undertaken to analytically derive validate the Shannonian Maximum Entropy BTM for the underlying stable queue. Most importantly, the analytic derivation of the upper and lower bounds 0f the absolute difference between Shannonian Cumulative service time distribution functions (CDFs) with and without balking. Typical numerical experiments are provided. Additionally, some applications of $M/G/1$ queue theory to AR are given. Some challenging open problems are addressed combined with closing remarks and future research directions.**

Index Terms— **BTM, Maximum Entropy (ME), CDF, energy, $M/G/1$ queue theory, AR.**

I. INTRODUCTION

IN principle, this current work supplies a complementary part of the research conducted (c.f., [1]). As the authors now feel the task completion and demonstration with both analytic expressions as well as illustrative data to interpret the newly devised research results.

It is commonly thought that obtaining precise performance distributions is often impossible, which encourages the use of alternative solution analysis methods such as ME and the diffusion approximation [2-7]. Using queueing models with balking, multiple real-world systems, such as commercial service systems and call centres where clients may learn about expected delays, and traffic-flow control in transportation networks, have all been effectively appraised [8-17].

2. PANORAMIC OVERVIEW ON [1]

I A MAGEED is with Department of Computer Science, University of Bradford, United Kingdom,(e-mail: drismail664@gmail.com).

https://orcid.org/0000-0002-3691-0773

Manuscript received May 24, 2023; accepted Jan 29, 2024.
DOI: 10.17694/bajece.1302056

In [15], a ME solution to QLD for a stable $M/G/1$ queue, includes a generalized geometric distribution (GGeo), as well as average Queue length (MQL), normalization status, and prior knowledge about the queue's stability (or flow conservation), via optimisation constraint p(0) (or equivalent use).

$$F_t = 1 - \frac{2}{C^2+1} e^{-\frac{2}{C^2+1}\mu t}, t \geq 0, \mu > 0 \tag{1}$$

where the mean is $(1/\mu)$, and the square coefficient of variation is C^2 (SCV).

The discrete Half Normal distribution (dHN) [15] reads

$$p(n) = \frac{\theta^n q^{\frac{n(n-1)}{2}}}{\sum_{n=0}^{\infty} \theta^n q^{\frac{n(n-1)}{2}}} = p(0)\theta^n q^{\frac{n(n-1)}{2}}, n = 0,1,2 \ldots, \theta > 0, 0 < q < 1 \tag{3}$$

θ and q are Lagrangian coefficients-dependent parameters.

Theorem 1 [17] The GdHN is the ME distribution given by

$$p(n) = \begin{cases} p(0), & n = 0 \\ p(0)\tau_s \phi^n r^{\frac{n(n-1)}{2}}, & n = 1,2,3 \ldots \end{cases}, \tau_s, \phi, r > 0, \tag{4}$$

where τ_s, ϕ and r are distribution parameters in terms of the Lagrangian coefficients, such that $(x_3)^{-1} = \tau_s = 2/(1 + C_{s,1,S}^2)$, $\left(x_1 x_2^{-(2E[N]-1)}\right) = \phi$ and $x_2 = r^{\frac{1}{2}}$, x_1, x_2 and x_3 are the Lagrangian coefficients corresponding to the first moment, variance and $p(0)$ prior moment information constraints respectively, with $p(0)$ written as

$$p(0) = \left(1 + \sum_{n=1}^{\infty} \tau_s \phi^n r^{\frac{n(n-1)}{2}}\right)^{-1} \tag{5}$$

Theorem 2[1] The exactness of $p(n)(\text{c.f.}, (4))$ holds for $f_{s,B}(\mu, \tau_s, x_2, \beta, t)$ is GE_V- type, which reads as

$$f_{s,B}(\mu, \tau_s, x_2, \beta, t) = (1 - \tau_s^V) u_0(t) + \mu (\tau_s^V)^2 e^{-\mu \tau_s^V t} \tag{6}$$

where $\tau_s^V = \tau_s r^{\frac{\beta}{2}}$, $u_0(t)$ reads as

$u_0(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$
such that
$\int_{-\infty}^{\infty} u_0(t)dt = 1, (\tau_s, r)$ (c.f., (4)) and β is the inequality parameter, satisfying

$$n^2 = \alpha n + \beta \quad (7)$$

We call α the factorization parameter.

$$n^2 = \alpha n + \beta \quad (8)$$

Corollary 3

The family of families $\{F_{s,B}(\mu, \tau_s, x_2, \beta, t)\}$ are the corresponding GEV type CDFs for $\{f_{s,B}(\mu, \tau_s, x_2, \beta, t)\}$ (c.f., (6)) are determined by

$$F_{s,B}(\mu, \tau_s, x_2, \beta, t) = 1 - \tau_s^V e^{-\mu\tau_s^V t} \quad (9)$$

with (τ_s, β, r) (c.f., Theorem 2) and $\tau_s^V = \tau_s r^{\frac{\beta}{2}}$. When $q \to 1$, $F_{s,B}(\mu, \tau_s, x_2, \beta, t)$(c.f., (9)) $\to$ $F_s(t) = 1 - \tau_s e^{-\tau_s \mu t}$ (c.f., [15]) with $\tau_s = \frac{2}{C_{S,1,S}^2+1}$.

Corollary 4(c.f., [1])

For $f_{B,s}$ of Equ. (6), mean values and SCVs read as follows:

$$E(s_{B,S}) = \frac{1}{\mu} \quad (10)$$

$$E(s_{B,S}^2) = \frac{2}{\mu^2 \tau_s^V} \quad (11)$$

$$C_{B,S}^2 = \frac{E(s_{B,S}^2)}{(E(s_{B,S}))^2} - 1 = \frac{(2-\tau_s^V)}{\tau_s^V} \quad (12)$$

Provided that (τ_s, β, r) (c.f., Theorem 2) and $\tau_s^V = \tau_s r^{\frac{\beta}{2}}$.

[1] mainly contributes to:

- First time ever reaveling of the SDF of the Shannonian Balking solution of the stable $M/G/1$ queueeing system.
- First time ever reaveling of the corresponding family 3.of families of CDFs of the derived SDFs.
- First time ever reaveling of the corresponding family of families of mean values and SCVs.
- The generalization of the results in [15].

The following theorem is essential in the undertaken proofs.

Preliminary Theorem (PT) [18]

Let f be a function that is defined and differentiable on an open interval (c,d).
(I) If $f'(x) > 0\ \forall x \in (c,d)$,then f is increasing on (c,d) (13)
(II) If $f'(x) < 0\ \forall x \in (c,d)$, then f is decreasing on (c,d) (14)

This paper's road map is. Section 2 overviews the short paper upon which this current paper is aimed to be an extension of it. In section 3, we derive the new results. Section 4 deals with experimental setup to numeriacally demonstrate the validity of our new rersults. Section 5 is devoted to the provision of some potential applicatications of $M/G/1$ theory to AR. Some influential emerging open problem from this paper combined with closing remarks and the next phase of research are given in section 6.

3. NEW RESULTS

In what follows, two new theorems are devised, namely, the Shannonian maximum entropy BTM and the Shannonian Balking Differrence (SBD)Theorem.

In the following Theorem, we are defining for the first time, the generic threshold for up to which Shannonian Balking service time cumulative distribution function for the $M/G/1$ queue will be monotonic decreasing and after which, it will be increasing. This will open a new ground to understand the effect of the parameters of the on the behaviour of the Shannonian Balking service time cumulative distribution function.

Theorem 3 For $F_{s,B}(\mu, \tau_s, x_2, \beta, t)$ (c.f., (9)), it holds that:

(i) $F_{s,B}(\mu, \tau_s, x_2, \beta, t)$ is forever increasing in μ, t for all values of $t \geq 0$, $\mu > 1, 1 > \tau_s > 0, x_2, \beta$ are real numbers (15)

(ii) $F_{s,B}(\mu, \tau_s, x_2, \beta, t)$ is forever increasing in τ_s, $1 > \tau_s > 0$ if and only if we can find $\varrho > 0$ satisfying

$$\varrho = -\left(\frac{(\ln(\mu_{\text{inimum}} x_2^{\beta} t)}{\ln(\tau_s)_{\text{minimum}}}\right) - 1 \quad (16)$$

μ, t for all values of $t \geq 0$,
$\mu > 1, 1 > \tau_s > 0, x_2, \beta$ are real numbers.

(iii) $F_{s,B}(\mu, \tau_s, x_2, \beta, t)$ is forever decreasing in τ_s, $1 > \tau_s > 0$ if and only if we can find $\varrho > 0$ satisfying

$$t \leq \frac{1}{\mu x_2^{\beta} \tau_s} \quad (17)$$

μ, t for all values of $t \geq 0$,
$\mu > 1, 1 > \tau_s > 0, x_2, \beta$ are real numbers.

Proof.

(i)We have $F_{s,Balking}(t) = 1 - \tau_s x_2^{\beta} e^{-\mu\tau_s x_2^{\beta} t}$. By the preliminary theorem, it holds that $F_{s,Balking}(t)$ is (increasing)(decreasing) if and only if

$$\frac{\partial F_{s,Balking}(t)(\mu, \tau_s, x_2, \beta, t)}{\partial \mu} < 0 (\geq 0) respectively \quad \text{(c.f.,(13))}$$

We have

$$\frac{\partial F_{s,Balking}(t)(\mu, \tau_s, x_2, \beta, t)}{\partial \mu} = \mu(\tau_s x_2^{\beta})^2 e^{-\mu\tau_s x_2^{\beta} t} \quad (18)$$

Hence, it is notable by (13) that $\frac{\partial F_s(\mu, \tau_s, t)}{\partial \mu} > 0$ for all values, $\mu, t, \tau_s > 0, 1 > \tau_s > 0$. It follows that $F_s(t)$ is increasing for all values,$\mu, t, \tau_s > 0, 1 > \tau_s > 0$, $\beta = n(n-\alpha)$ for all positive real numbers α, β. On the other hand, let $\frac{\partial F_{s,Balking}(t)(\mu, \tau_s, x_2, \beta, t)}{\partial \mu} \leq 0$, which implies $\mu(\tau_s x_2^{\beta})^2 e^{-\mu\tau_s x_2^{\beta} t} \leq 0$. This is impossible since $\mu, t, \tau_s > 0, 1 > \tau_s > 0$ and $e^{-\mu\tau_s x_2^{\beta} t} \geq 0$. Hence, the contradiction follows.

Following the same approach, $F_{s,B}(\mu, \tau_s, x_2, \beta, t)$ is forever increasing in t for all values of $t \geq 0$, $\mu > 1, 1 > \tau_s > 0, x_2, \beta$ are real numbers.

(ii) We have $F_{s,B}(\mu,\tau_s,x_2,\beta,t)=1-\tau_s x_2^{\beta} e^{-\mu\tau_s x_2^{\beta} t}$. By the preliminary theorem, it holds that $F_{s,B}(\mu,\tau_s,x_2,\beta,t)$ is increasing in τ_s if and only if $\frac{\partial F_{s,B}(t)(\mu,\tau_s,x_2,\beta,t)}{\partial \tau_s} > 0$, (i.e.,)

$$-x_2^{\beta} e^{-\mu\tau_s x_2^{\beta} t} + \tau_s (x_2^{\beta})^2 e^{-\mu\tau_s x_2^{\beta} t} > 0 \quad (19)$$

This implies

$$-x_2^{\beta} e^{-\mu\tau_s x_2^{\beta} t}(1 - x_2^{\beta}\mu\tau_s t) > 0 \quad (20)$$

by (20), $F_{s,B}(\mu,\tau_s,x_2,\beta,t)$ is forever increasing in τ_s if and only if $(1 - x_2^{\beta}\mu\tau_s t) < 0$, which implies $t > \frac{1}{\mu x_2^{\beta}\tau_s}$. It is implied that there exists a positive number $\varrho \in (0,\infty)$ such thatif $t = \frac{((\tau_s)_{min})^{-\varrho}}{\mu_{min} x_2^{\beta}\tau_s} = \frac{((\tau_s)_{min})^{-(\varrho+1)}}{\mu_{min} x_2^{\beta}}$. Hence, it follows that

$$\varrho = -\left(\frac{\boldsymbol{ln}(\mu_{\text{inimum}} x_2^{\beta} t)}{\ln(\tau_s)_{\text{minimum}}}\right) - 1 \quad \text{(c.f., (16))}$$

(iii) Following the same argument, assume $t \leq \frac{1}{\mu x_2^{\beta}\tau_s}$. Hence, it follows that $(1 - x_2^{\beta}\mu\tau_s t) \geq 0$. Consequently, $\frac{\partial F_{s,B}(t)(\mu,\tau_s,x_2,\beta,t)}{\partial \tau_s} \leq 0$.This provides the sufficient and necessary condition for $F_{s,B}(\mu,\tau_s,x_2,\beta,t)$ to be decreasing in τ_s.

Theorem 4

The difference $\mathcal{D}$ of values between $F_{s,Balking}(\mu,\tau_s,x_2,\beta,t)$ (c.f., (9)) and,$F_s(\mu,\tau_s,t)$(c.f., [15]) satisfies

$$\left(1 - F_s(\mu,\tau_s,t)\right)\left(x_2^{\beta} e^{-\mu\tau_s(x_2^{\beta}-1)t} + 1\right)$$
$$\geq \mathcal{D} \geq \left(1 - F_s(\mu,\tau_s,t)\right)(x_2^{\beta} e^{-\mu\tau_s(x_2^{\beta}-1)t} - 1), \quad (21)$$
$$\mathcal{D} = |F_{s,Balking}(\mu,\tau_s,x_2,\beta,t) - F_s(\mu,\tau_s,t)|$$

Proof

We have

$$\mathcal{D} = |F_{s,Balking}(\mu,\tau_s,x_2,\beta,t) - F_s(\mu,\tau_s,t)|$$
$$= |\tau_s x_2^{\beta} e^{-\mu\tau_s x_2^{\beta} t} - \tau_s e^{-\mu\tau_s t}|$$
$$= \tau_s e^{-\mu\tau_s t} |x_2^{\beta} e^{-\mu\tau_s(x_2^{\beta}-1)t} - 1|$$
$$= \left(1 - F_s(\mu,\tau_s,t)\right) |x_2^{\beta} e^{-\mu\tau_s(x_2^{\beta}-1)t} - 1| \quad (22)$$

By the modulus inequality,

$$|a| + |b| \geq |a - b| \geq |a| - |b| \quad (23)$$

$$\left(1 - F_s(\mu,\tau_s,t)\right)\left(x_2^{\beta} e^{-\mu\tau_s(x_2^{\beta}-1)t} + 1\right)$$
$$\geq \mathcal{D} \geq \left(1 - F_s(\mu,\tau_s,t)\right)(x_2^{\beta} e^{-\mu\tau_s(x_2^{\beta}-1)t} - 1).$$ QED

P.S. As $\beta \to 0$, $\mathcal{D} \to 0$, as$F_{s,Balking}(\mu,\tau_s,x_2,\beta,t) \to F_s(\mu,\tau_s,t)$.

In the following section, we are showing numerically the effect of the variance parameter on the increasability and decreasility of the cumulative balking service time distribution function.

4. EXPERIMENTAL SETUP

Case study One, $n = 1$

For $n = 1, \alpha = \beta = 0.5, \tau_s = 0.5, 0.6, 0.7, \mu \in [2, 7.5]$. Let $x_2 = 0.8$. Looking at figure 1,it can be seen that the threshold value is $t = 0.1$. By the Shannonian Balking Threshold Theorem, this threshold exists by the existence of a positive real number $\varrho\epsilon(0,2)$ such that

$$\varrho = -\left(\frac{(\boldsymbol{ln}(\mu_{\text{inimum}} x_2^{\beta} t)}{\ln(\tau_s)_{\text{minimum}}}\right) - \mathbf{1}$$

By calculations, we have

$$\varrho = -\left(\frac{(\boldsymbol{ln}((0.1)(2)(0.8^{0.5})}{ln\, 0.5}\right) - \mathbf{1} = 1.482892142.$$

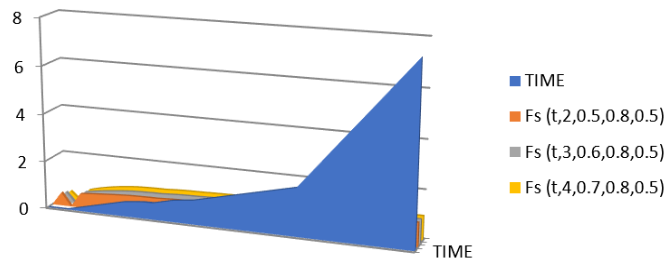

Figure 1. How time impacts $F_s(\mu,\tau_s,t), n = 1$

Case study Two, $n = 3$

Let $n = 3, \alpha = 2, \beta = 3, \tau_s = 0.5, \mu \in [2, 7.5]$. Let $x_2 = 3.5$. Observing figure 2, we can see that the threshold value is $t = 0.1$. By the Shannonian Balking Threshold Theorem, this threshold exists by the existence of a positive real number $\varrho\epsilon(0,\infty)$ *such that*

$$\varrho = -\left(\frac{(ln(\mu_{\text{inimum}} x_2^{\beta} t)}{\ln(\tau_s)_{\text{minimum}}}\right) - 1$$

By calculations, we have $\varrho = -\left(\frac{(ln((0)(2)(2^2)}{ln\, 0.5}\right) - 1$. Hence, it holds that $\varrho \to \infty$. in other words, the threshold is satisfied for a sufficiently large ϱ.

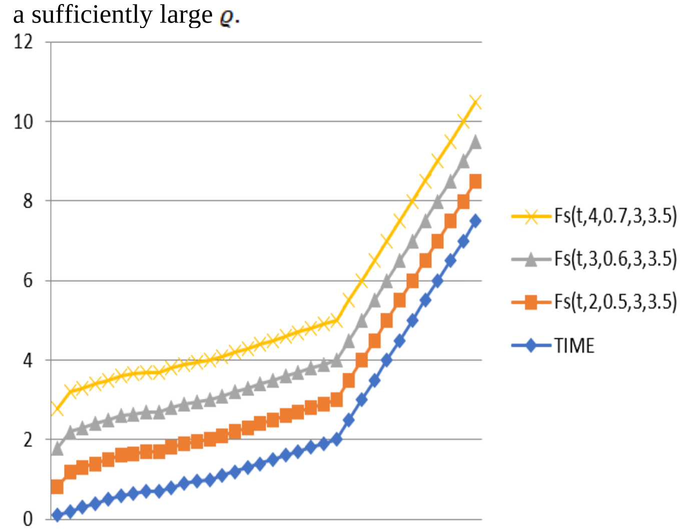

Figure 2. How time impacts $F_s(\mu,\tau_s,t), n = 3$

5. $M/G/1$ THEORY APPLICATIONS TO AR

[19] discussed the challenge of providing trusted and updated information for AR services over a terahertz (THz) cellular network and explained the relationship linking an $M/G/1$ queue dynamics and the ruin's risk, which can be used to re-define the system's reliability explicitly in PAoI.

In the context of an AR system, the authors [19] proposed a queuing model that splits the system into two as a guarantee among users-fair scheduling policy, while also allowing for urgent requests to be prioritized. This is visually demonstrated by figure 3(c, f., [19]). The proposed model aims to prevent outdated and potentially harmful information from being sent to AR users.

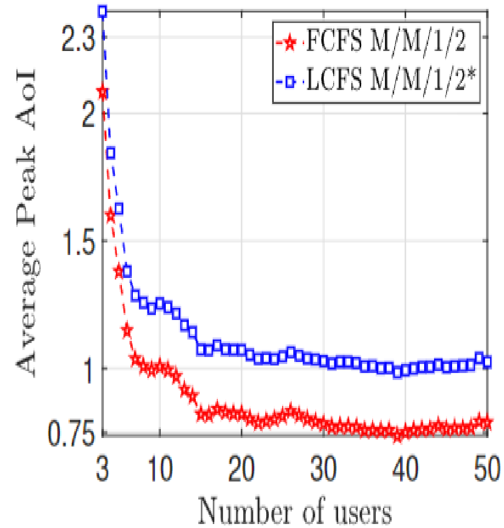

Figure 4. Average peak AoI versus user count [19].

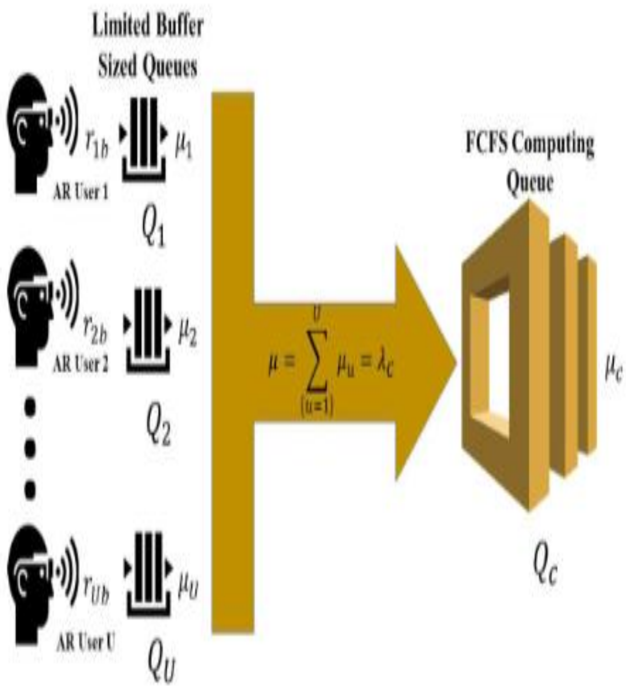

Figure 3. An illustration of the proposed queuing model for analysing the freshness of AR content updates. The model splits the queuing system into two limited-sized queues, one for prioritizing urgent requests and another for fair scheduling of requests. (c.f., [19]).

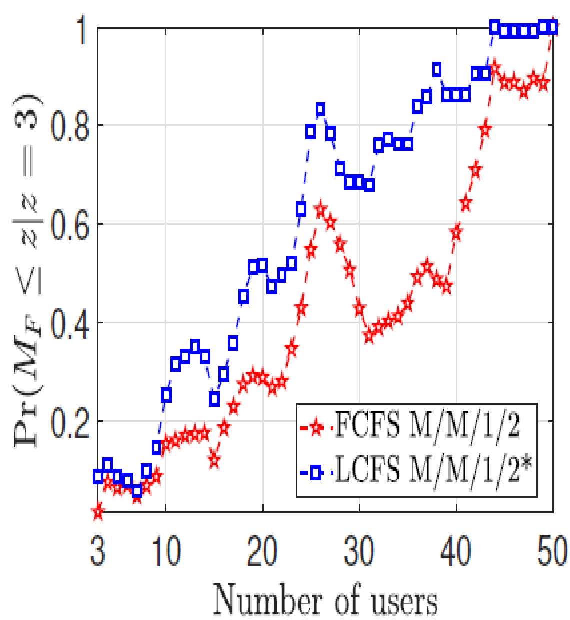

Figure 5. The probability of the worst-case peak AoI being less than z = 3 versus the number of users [19].

Figures 4 and 5(c.f., [19]) demonstrate the impact of the number of users on up-to-date information in a THz cellular network. With the increase of users, the peak average Age of Information (AoI) decreases due to the simultaneous arrival of more packets, generating information updates to the allocated server of the network.

Extended Reality (XR)[20], which includes Virtual Reality and Augmented Reality, has the potential to revolutionize virtual and telepresence experiences. Figure 6 (c,f., [20]) describes a simplified architecture of a Cloud XR system. This architecture is important for providing an immersive experience but presents challenges in terms of real-time video quality adaptation and network resource allocation.

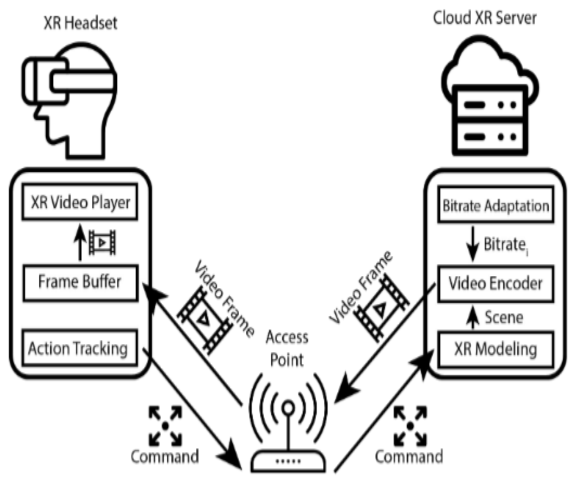

Figure 6. Cloud XR architecture's sample (c.f., [20])

The authors of [20] studied the performance of adaptive XR video streaming in the presence of high-priority interfering traffic and discussed the existing literature on the $M/G/1$ queue system and its probabilistic properties, so the authors[20] used their model to estimate the maximum bitrate of an XR video stream in a scenario where the network capacity is limited and the stalling probability should be less than 0.01.
Figure 7 shows that the actual capacity of the network for the XR video is up to 50% lower than what can be obtained with a certain equation due to the variance of the resource consumption by the commands growing with the command rate.

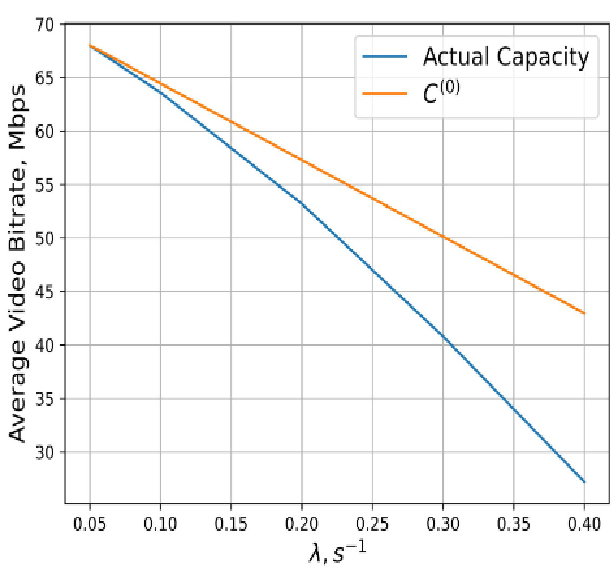

Figure 7. Network capacity estimation (c.f., [21])

The average computation delay for tasks processed by a BS can be calculated by modeling the service process as an $M/G/1$ queue and analysing its sojourn time, which includes both service time and waiting time.

Figure 8(c.f., [21]) highlights the importance of effective service caching decisions in accommodating different types of services and improving overall system performance.
"Cached service" refers to the storage of frequently accessed data or services at base stations (BSs) in a wireless network. Figure 8 highlights the importance of cooperation in accommodating more types of services and improving overall system performance. Notably, the number of services is denoted by k.

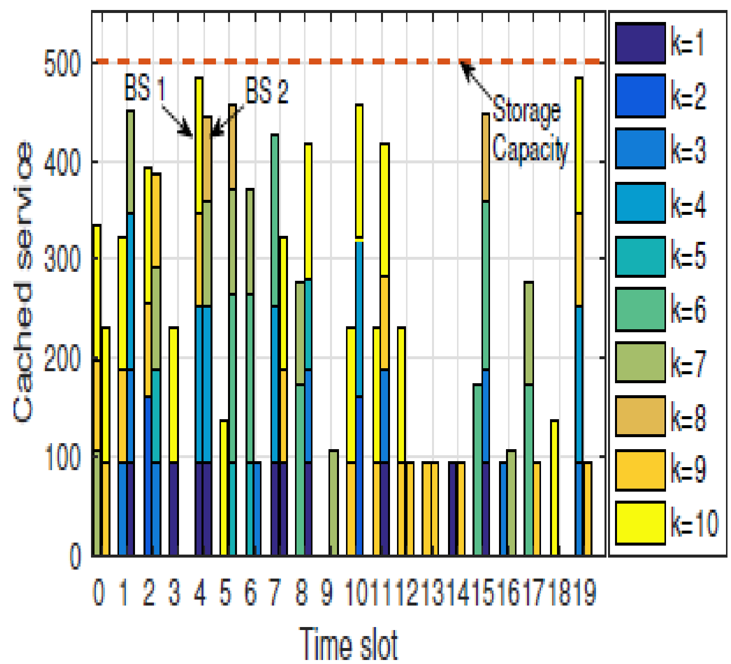

Figure 8. Cached service

6. CONCLUSION, OPEN PROBLEMS AND NEXT RESEARCH PHASE

An explanation is given to confirm the Shannonian Maximum Entropy BTM for the underlying stable queue analytically. Most crucially, the upper and lower bounds of the absolute difference between with and without Shannonian Cumulative service time distribution functions are derived analytically. There are examples of typical numerical experiments offered. In addition, several queue theory applications to AR are shown. Some difficult open problems are addressed, as well as closing remarks and future research objectives.
The current paper has several emerging open problems.

Open Problem One

Having arrived at the stage where Shannonian Balking mechanism has been fully captured in both analytic and numerical domains. The current resesarch generates some open problems as follows:

- What will the overall derivations for the undertaken balking mechanism if we add more higher order moments to the constraints?
- Coming to a situation, where other higher order entropies that generalize Shannon's entropy with the same set of constraints, which will the derivations look like? More interestingly, will the derived formalism be a generalisation to the results of [1] and this current paper?

Open Problem Two

Numerous potential open problems are generated by the established model in [19].

- Following [19], is possible to extend their established model to a more complex and realistic high-priority traffic pattern. They are asking if their model can be adapted to better account for video encoding idiosyncrasies, such as different types of frames in a video stream, their sizes, and their impact on Quality of Service (QoE). This open problem highlights the potential for further research and development in the field of network optimization.
- Can we broaden the model to include interference from other videos? Finally, a more sophisticated optimisation technique can be employed to lower the computational cost of calculating the optimal bitrate adaption function for a particular network state. Using this technique, we may perform optimisation with finer granularity and provide improved QoE to end users.

- **Open Problem Three**

In the undertaken model(c,f., [21]) gives rise to open problems, such as incorporating user-cell association decisions and allowing workload transfer among peer base stations to balance task workload.

The next phase of research includes attempting to solve the highlighted open problems and expanding on the exploration of more applications of $M/G/1$ theory to other fields of science.

REFERENCES

[1] I.A.Mageed, et al, "M/G/1 queue with Balking Shannonian Maximum Entropy Closed Form Expression with Some Potential Queueing Applications to Energy", 2022 Global Energy Conference (GEC). IEEE, 2022.

[2] E.T.Jaynes, "Information Theory and Statistical Mechanics", Physical Review, 106, 1957, 620 - 630.

[3] E.T.Jaynes, E.T., "Where do we Stand on Maximum Entropy?", in Proc. The Maximum Entropy Formalism Conference, M.I.T., USA, 1978.

[4] S.-C.Fang,et al, "Entropy optimization and mathematical programming,1997, Kluwer Academic Publishers, Boston.

[5] J.Shore, and R. Johnson, "Axiomatic derivation of the principle of maximum entropy and the principle of minimum cross-entropy", IEEE Transactions onInformation Theory,1980, 26, 26-37.

[6] J.Shore, "Information theoretic approximations for M/G/1 and G/G/1 queuing systems",1982, Acta Informatica, 17, 43.

[7] J.Cantor,et al, "Information theoretic analysis for a general queueing system at equilibrium with application to queues in tandem", Acta Informatica,1986, 23 ,657-678.

[8] F.A.Haight, "Queueing with Balking", Biometrika, 1957,44 (1957) 360-369.

[9] L.Liu, " Service Systems with Balking Based on Queueing Time ,PhD Thesis", Department of Statistics and Operations Research, University of North Carolina at Chapel Hill, USA, 2007.

[10] O.J.Boxma, and B.J. Prabhu, "Analysis of an M/G/1 Queue with Customer Impatience and an Adaptive Arrival Process", 2009,Technical Report Eurandom; Vol. 2009028, Eindhoven University of Technology, Eindhoven, The Netherlands.

[11] F.Baccelli,et al, "Single-Server Queues with Impatient Customers", 1984, Advances in Applied Probability, 16, 887-905.

[12] P.M.Morse, " Queues, Inventories and Maintenance: The Analysis of Operational Systems with Variable Demand and Supply", 1958,First ed., John Wiley & Sons, Inc.

[13] A.W.Kemp, "Steady-state Markov chain models for certain q-confluent hypergeometric distributions", 2005,Journal of Statistical Planning and Inference, 135, 107.

[14] A.W.Kemp, "The Discrete Half-Normal Distribution", in: Arnold, B.C., Balakrishnan,N., J.M. Sarabia,J.M. & Minguez, R. (Eds.) Advances in Mathematical and Statistical Modeling, Birkhäuser Boston,2008, pp. 353-360.

[15] M.A.El-Affendi,and D.D.Kouvatsos," A maximum entropy analysis of the M/G/1 and G/M/1 queueing systems at equilibrium", 1983,Acta Informatica, 19,339.

[16] D.D.Kouvatsos, "A Maximum Entropy Analysis of the G/G/1 Queue at Equilibrium", 1988, The Journal of the Operational Research Society, 39, 183-200.

[17] N.Shah, "Entropy Maximisation and Queues With or Without Balking", 2014,PhD Thesis, Department of Computing, University of Bradford.

[18] P.V.Gapeev , et al, " Optimal double stopping problems for maxima and minima of geometric Brownian motions", Methodology and Computing in Applied Probability. 2022 Jun;24(2):789-813.

[19] C.Chaccour and W.Saad, "On the ruin of age of information in augmented reality over wireless terahertz (THz) networks", InGLOBECOM 2020-2020 IEEE Global Communications Conference 2020 Dec 7 (pp. 1-6). IEEE.

[20] M.Liubogoshchev, et al, "Adaptive cloud-based extended reality: Modeling and optimization", IEEE Access. 2021 Feb 26;9:35287-99.

[21] J.Xu , et al, " Joint service caching and task offloading for mobile edge computing in dense networks",In IEEE INFOCOM 2018-IEEE Conference on Computer Communications 2018 Apr 16 (pp. 207-215). IEEE.

BIOGRAPHIES

ISMAIL A MAGEED Dr. Ismail A Mageed completed his doctorate in Applied Probability at The University of Bradford, United Kingdom. His current research interests include the unification of queueing theory with information theory and information geometry. His leading research on the relativization of queuing theory and discovering the geodesic equation of motion for transient queues was greatly received by the world research community. Mageed's research on finding the analytic solutions of the longstanding simulative approach of The Pointwise Stationary Fluid Flow Approximation theory (PSFFA) was an exceptional discovery to advance PSFFA theory. Mageed has published numerous papers in many highly reputable journals and conferences. He is also a reviewer of several prestigious journals. Mageed's research has been internationally recognized as being revolutionary by providing several breakthroughs. Dr Mageed has published a chapter in a book of the best eight queueing theorists in the world by ISTE WILLEY. He is currently leading several research teams in UK, India, Africa, and Saudi Arabia. He is also a fellow of the Royal Statistical Society (RSS), a member of INTISCC (Austria), IEANG (world council of engineering) and a life member of the Islamic Society of Statistical Sciences.