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Research Article/Araştırma Makalesi

Fractal Analysis of S&P 500 Sector Indexes

S&P 500 Sektör Endekslerinin Fraktal Analizi

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Abstract

In this study multifractal properties of S&P 500 sector indexes are investigated with Multifractal Detrended Fluctuation Analysis (MF-DFA). The MF-DFA is a signal processing technique that is used to describe the multifractal properties of a time series data. It is an extension of Detrended Fluctuation Analysis (DFA), which is a widely utilized method for estimating the scaling behavior of a time series. Main idea behind MF-DFA is to decompose a time series into multiple scales using a coarse-graining procedure, and then to estimate the scaling behavior of each scale using DFA. This gives a set of scaling exponents that describe the multifractal features of the time series. Our MF-DFA results indicates the presence of multifractality in all S&P 500 sector indexes. Since these indexes are multifractal, we can conclude that they possess properties such as scaling variability, nonlinear dynamics, self-similarity, long-range dependence, multiscale correlations and nonstationary.

Jel Codes: *G10, G15, G19*
Keywords: *Multifractality, MF-DFA, Multifractal Detrended Fluctuation Analysis, Fractal Theory, S&P 500*

Öz

Bu çalışmada S&P 500 sektör endekslerinin çoklu fraktal özellikleri Çoklu Fraktal Eğilimden Arındırılmış Dalgalanma Analizi (ÇF-EADA) ile incelenmiştir. ÇF-EADA zaman serisi verilerinin çoklu fraktal özelliklerini tarif etmek için kullanılan bir sinyal işleme tekniğidir. Bu yöntem zaman serilerinin ölçekleme davranışını tahmin etmek için kullanılan Eğilimden Arındırılmış Dalgalanma Analizi (EADA) yönteminin bir uzantısıdır. ÇF-EADA yönteminin arkasında yatan temel fikir bir zaman serisini kaba ölçekli bir işlem kullanarak birden fazla ölçeğe ayırmak ve ardından EADA yöntemiyle her ölçeğin ölçeklenme davranışını tahmin etmektir. Bu, zaman serilerinin çok fraktal özelliklerini tanımlayan bir dizi ölçeklendirme üssü verir. ÇF-EADA sonuçlarımız, tüm S&P 500 sektör endekslerinde çoklu fraktalitenin varlığını göstermektedir. Bu indeksler çoklu fraktal olduğundan, ölçekleme değişkenliği, doğrusal olmayan dinamikler, kendine benzerlik, uzun menzilli bağımlılık, çok ölçekli korelasyonlar ve durağan olmama gibi özelliklere sahip oldukları sonucuna varabiliriz.

Jel Kodları: *G10, G15, G19*
Anahtar Kelimeler: *Çoklu Fraktalite, ÇF-EADA, Çoklu Fraktal Eğilimden Arındırılmış Dalgalanma Analizi, Fraktal Teori, S&P 500*

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1. Introduction

The efficient market hypothesis (EMH), which is suggested by Fama (1970), is a theory that asserts that financial markets are efficient in that they fully and immediately incorporate all publicly available information into asset prices. In other words, the market price of an asset reflects all information that is currently known about that asset, and it is impossible to consistently achieve returns that are higher than the market average by using information that is already publicly available. However, EMH is criticized in the literature (Malkiel, 2003; Degutis & Novickytė, 2014; Ying et al., 2019) and it is shown that some financial markets cannot be explained by EMH. Fractal theory, which is firstly proposed by Mandelbrot (1982), can be used to explain financial markets. Fractal theory is a field of mathematics work on complex and irregular shapes, which can be found in many natural and man-made systems. Fractals are geometric structures that display self-similarity and exhibit same structure at different scales. Fractal geometry was initially used to describe irregular patterns and shapes in nature, such as the branching of trees, the distribution of galaxies, and the contours of coastlines. However, fractal theory has since been applied to many other fields, including economics, physics, and computer graphics. Fractal theory has many practical applications, including in the design of computer graphics, the study of financial markets, and the analysis of natural phenomena such as earthquakes and weather patterns (Makletsov et al., 2019; Duan et al., 2021; Wang et al., 2021).

Originally Hurst (1951, 1957) suggested rescaled range (R/S) analysis to analyze fractal systems in the hydrology. Rescaled range (R/S) analysis utilized to study the fractal properties of time series data. R/S analysis involves calculating the range of a time series over distinct time scales, and then rescaling this range to account for the size of the time scale. The R/S statistic is calculated specifically by splitting the time series data's range by its standard deviation. The R/S statistic is then plotted against the time scale on a log-log scale, and the slope of the resulting line is used to compute the Hurst exponent. Long term persistency of a time series can be evaluated by the Hurst exponent. A Hurst exponent value of 0.5 implies that the time series is uncorrelated and random, while values greater than 0.5 imply positive autocorrelation, and values less than 0.5 imply negative autocorrelation. R/S analysis has been successfully applied in the fields such as finance, hydrology, geology, and climate science, to study the properties of time series data (Outcalt et al., 1997; Gilmore et al., 2002; Resta, 2012; Raimundo & Okamoto Jr, 2018). It is particularly useful for studying phenomena that exhibit self-similarity or long-term memory, such as stock prices, river flows, and weather patterns.

However, Lo (1991) subsequently showed the shortcomings of R/S analysis such as sensitivity to short-term autocorrelation. To avoid these shortcomings Peng et al. (1994) suggested a method called Detrended Fluctuation Analysis (DFA). DFA is like R/S analysis in that it is used to evaluate the long-term persistence in a time series, but it differs in its approach to detrending the data. In DFA, the time series is first split into equal-sized segments, and each section is then fitted with a polynomial trend line. The trend line is then subtracted from the data, leaving only the fluctuations around the trend. The root mean square deviation of these fluctuations is then calculated over different time scales, and the results are plotted on a logarithm-logarithm scale. The slope of the resulting line is used to calculate the scaling exponent, which reflects the long-term persistence in the time series. Like R/S analysis, DFA applied in various of fields like finance, neuroscience, and geophysics, to study the properties of time series data (Ivanova & Ausloos, 1999; Talkner & Weber, 2000; Kurnaz, 2004; De Moura et al., 2009; Kuznetsov & Rhea, 2017). It is particularly useful for analyzing non-stationary time series, where the statistical characteristics of the data change over time. DFA can help identify long-range correlations in the data that are not easily captured by traditional statistical methods.

It was later revealed that the data encountered in many fields did not show monofractal scaling behavior. Since such systems cannot be expressed with a single scaling coefficient, these systems are called multifractal. To analyze such multifractal systems Multifractal Detrended Fluctuation Analysis (MF-DFA) method was suggested by Kantelhardt et al. (2002). MF-DFA is an extension of DFA, which is a widely utilized technique for studying the scaling characteristics of a time series. The basic idea of MF-DFA is to divide a time series into non-overlapping segments with same length and then calculate the local fluctuation of the data within each segment after removing the local trend by detrending. The detrending procedure involves fitting a polynomial function of a certain order to each segment and subtracting it from the data. Once the local fluctuation of the data has been calculated for each segment, the scaling exponent is estimated for each scale by using a weighted least-squares regression method. The scaling exponent at each scale is then used to estimate the singularity spectrum. The singularity spectrum provides a way to describe the multifractal features of the time series, including the degree of heterogeneity and the degree of self-similarity at different scales. A time series with a broad singularity spectrum is said to be highly multifractal and a time series with a narrow singularity spectrum is said to be less multifractal. MF-DFA has been used in many applications, including in the analysis of financial markets, where it has been used to study the multifractal properties of stock returns, volatility, and trading volume (Cao et al., 2013; Rizvi et al., 2014; Stošić et al., 2015; Mensi et al., 2017; Shahzad et al., 2017; Ali et al., 2018; Mensi et al., 2018; Ruan et al., 2018; Zhu & Zhang, 2018; Tiwari et al., 2019; Milos et al., 2020).

Multifractal time series have several key properties:

Scaling Variability: Multifractal time series exhibit scaling variability, meaning that the statistical characteristics of the data change at different scales or resolutions.

Nonlinear Dynamics: Multifractal time series are often generated by nonlinear systems with complex dynamics. The nonlinear nature of these systems can give rise to long-term correlations, non-Gaussian distributions, and other statistical properties that are not observed in linear systems.

Self-Similarity: Although multifractal time series exhibit scaling variability, they also display some degree of self-similarity. This means that the statistical characteristics of the data are similar across different scales, albeit with different scaling exponents.

Long-Range Dependence: Multifractal time series often exhibit long-range dependence, meaning that the autocorrelation function decays slowly over time. This property can have important implications for forecasting and risk management.

Multiscale Correlations: Multifractal time series exhibit correlations at multiple scales or resolutions. These correlations can be positive or negative and can be characterized by a multifractal correlation function.

Nonstationarity: Multifractal time series are typically nonstationary, meaning that the statistical characteristics of the data vary over time. This nonstationarity can be caused by changes in the underlying dynamics of the system or by external factors that affect the time series.

In this study multifractality and efficiency of the S&P 500 sector indexes are investigated by using MF-DFA. The investigation of multifractality in financial time series is a subject of interest for researchers and practitioners due to several important reasons. Our motivations for investigating the multifractality in S&P 500 sector indexes are listed below:

a) Capturing Complexity: Financial markets are highly complex systems with various agents and factors influencing their behavior. Traditional linear models often fail to fully capture the intricate dynamics present in financial data. Multifractal analysis allows researchers to explore and model this complexity better.

b) Non-Stationarity: Financial time series often exhibit non-stationary behavior, meaning their statistical properties change over time. Multifractal analysis helps to characterize this varying behavior and provides insights into the underlying mechanisms driving the changes.

c) Risk Management: Understanding the multifractal nature of financial data can have significant implications for risk management. It helps to identify periods of increased market risk or instability, which can be crucial for investors, traders, and financial institutions.

d) Market Efficiency and Anomalies: Studying multifractality can shed light on the efficiency of financial markets and the presence of anomalies. If certain time series exhibit strong multifractal properties, it may indicate inefficiencies that could be exploited for profit or that market participants should be cautious about.

e) Portfolio Diversification: Multifractal analysis can also be used to assess the diversification potential of different assets in a portfolio. Understanding how different assets' multifractal properties interact with each other can provide valuable insights into portfolio risk and performance.

f) Modeling and Prediction: Multifractal analysis can lead to the development of more accurate and robust models for financial time series. These models may provide better predictions of market movements and assist in making more informed investment decisions.

g) Market Microstructure: Studying multifractality can reveal underlying characteristics of market microstructure, such as trading patterns and liquidity dynamics, which can be essential for understanding market behavior.

h) Behavioral Finance: The multifractal approach can help researchers explore the psychological and behavioral aspects of financial markets, as it provides a more comprehensive view of market dynamics beyond traditional linear methods.

In summary, investigating the multifractality of financial time series is essential for gaining a deeper understanding of market behavior, improving risk management strategies, enhancing prediction models, and advancing the overall knowledge of complex financial systems.

In this study also market efficiencies of the S&P 500 sector indexes are investigated. The investigation of market efficiency is essential for several reasons as it provides valuable insights into the functioning and behavior of financial markets. Our motivations for investigating the market efficiencies of S&P 500 sector indexes are listed below:

a) Resource Allocation: Efficient markets are believed to allocate resources more effectively. In an efficient market, prices quickly adjust to new information, reflecting the true underlying value of assets. This facilitates better allocation of capital, promoting investments in productive and profitable ventures.

b) Investment Decisions: Understanding market efficiency is crucial for investors when making investment decisions. If a market is highly efficient, it becomes challenging to consistently outperform the market through stock picking or timing strategies, as asset prices already incorporate all available information.

c) Price Discovery: Efficient markets are better at discovering the true prices of assets. Market participants continuously incorporate new information into asset prices, leading to more accurate valuations.

d) Market Integrity: Market efficiency is linked to market integrity. In efficient markets, there is less room for price manipulation or insider trading, as prices rapidly adjust to new information.

e) Risk Management: An understanding of market efficiency is essential for risk management. Inefficient markets may be subject to greater price volatility, making risk assessment and hedging more challenging.

f) Financial Stability: Efficient markets contribute to financial stability as asset prices reflect relevant information, reducing the likelihood of price bubbles or crashes based on misinformation or speculative behavior.

g) Market Regulation: Regulatory authorities investigate market efficiency to ensure fair and transparent markets. By understanding market efficiency, regulators can identify potential areas of concern and implement appropriate measures to maintain market integrity.

h) Academic Research: Market efficiency has been a significant subject of research in finance and economics. Understanding the efficiency of different markets helps academics develop theories and models that explain market behavior and dynamics.

i) Behavioral Finance: Investigating market efficiency allows researchers to explore deviations from efficiency, leading to insights into behavioral biases and irrational investor behavior that might affect asset pricing.

j) Portfolio Management: Market efficiency affects portfolio management strategies. In efficient markets, passive investment strategies like index funds become more popular, while in inefficient markets, active management might offer opportunities for outperformance.

In summary, investigating market efficiency is crucial for various stakeholders, including investors, regulators, academics, and financial institutions. It provides valuable information about market functioning, influences investment decisions, and helps maintain market integrity and stability. Understanding market efficiency is an ongoing endeavor, and research in this area continues to evolve as markets and financial instruments change and adapt over time.

Our paper is structured as follows: In section two literature is reviewed. In section three methodology is presented. In section four data are described. In section five empirical results are given. Finally, section six concludes the study.

2. Literature Review

Rizvi et al. (2014) investigated market efficiencies of 22 stock market indexes belong to developed and Islamic countries by utilizing MF-DFA methodology. They indicated that level of market development is connected with the market efficiency and higher development corresponds to higher efficiency. Milos et al. (2020) realized a comparative investigation of the multifractal features of seven Eastern and Central European stock markets by utilizing MF-DFA. They detected multifractality in these markets and concluded that these stock markets are not efficient and mature. By utilizing asymmetric multifractal detrended fluctuation analysis Cao et al. (2013) investigated multifractal properties of Chinese stock markets. They detected multifractality in these stock markets an found multifractality degree is higher in the uptrends than downtrends. Mensi et al. (2017) investigated efficiencies of Islamic stock markets by using ten sectoral stock indexes. By utilizing MF-DFA authors demonstrated time-varying efficiency in these indexes. In the long term they detected high efficiency and in the short term they detected moderate efficiency. They also found that efficiency decreases after the beginning of the global financial crisis. Zhu and Zhang (2018) explored multifractal properties of Chinese stock market by using MF-DFA. They detected multifractal behavior in this market. They showed that multifractality is connected to the weighing order. Mensi et al. (2018) investigated efficiencies of five Gulf Council Cooperation (GCC) stock markets by utilizing MF-DFA. They demonstrated multifractality in the returns of these stock markets. They discovered persistence that varies over time and is more pronounced in the short term than the long term. Also, they showed efficiencies of GCC stock markets are less than regional and Islamic markets. Shahzad et al. (2017) analyzed power law features of eleven US stock and credit markets by using MF-DFA. They established that CDS markets exhibit lower levels of efficiency than stock markets. Also, they found that Financial and Banks credit markets possess highest efficiency and Basic Materials markets possess lowest efficiency. By using MF-DFA, Stošić et al. (2015) investigated auto-correlations in the changes of prices and volumes for thirteen global stock market indexes. They found different multifractal properties in the changes of prices and volumes. They demonstrated that price changes possess higher complexity than volume changes. Ali et al. (2018) compared the efficiencies of twelve conventional and Islamic stock markets by utilizing MF-DFA. They demonstrated that the equity markets in developed countries are more efficient than those in the BRICS. Additionally, they discovered that Islamic stock markets outperform their traditional peers in terms of efficiency. By utilizing MF-DFA, Ruan et al. (2018) demonstrated the multifractal behaviors of Hong Kong and Shanghai stock markets. They also showed that after the Connect Program efficiency of Shanghai stock market increased. Utilizing MF-DFA, Tiwari et al. (2019) looked into the multifractality and efficiency of stock markets in eight established and two emerging countries. Authors showed that considered stock markets possess multifractal and persistent properties. Also, they found that most markets possess higher efficiencies in the long-term.

3. Methodology

MF-DFA consists of several steps. Let's assume that x_t is a time series with $t = 1,2, \dots, N$.

1. Step: Compute profile with the following formula:

$$X_i = \sum_{t=1}^{i} (x_t - \bar{x}) \tag{1}$$

In the above formula $\bar{x}$ denotes the mean of the observations and computed with the following formula:

$$\bar{x} = \frac{1}{N} \sum_{t=1}^{N} x_t \tag{2}$$

2. Step: Divide the profile into non-overlapping windows of equal size of s. By this division $N_s = int(N/s)$ non-overlapping segments are obtained. There might be a little residue at the conclusion of the profile since the length of the series x_t might not be multiple of the time scale s. The identical process used at the end of the series was repeated in order to account for this residue. By this calculation $2N_s$ segments are acquired.

3. Step: Compute variances by using following formulas for the segments $v = 1,2, \dots, N_s$ and $v = N_s + 1, N_s + 2, \dots, 2N_s$.

$$F_X^2(s, v) = \frac{1}{s} \sum_{j=1}^{s} [X_{(v-1)s+j} - \hat{X}_j^v]^2 \tag{3}$$

$$F_X^2(s, v) = \frac{1}{s} \sum_{j=1}^{s} [X_{N-(v-N_s)s+j} - \hat{X}_j^v]^2 \tag{4}$$

In the formulas above $\hat{X}_j^v$ shows a polynomial of order m that fits the section v.

4. Step: Compute fluctuation function $F_X^q(s)$ with the formulas below for $q \neq 0$ and $q = 0$.

$$F_X^q(s) = \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F_X^2(s, v)]^{q/2} \right\}^{1/q} \tag{5}$$

$$F_X^q(s) = exp\left\{\frac{1}{4N_s}\sum_{v=1}^{2N_s}[F_X^2(s,v)]\right\} \tag{6}$$

5. Step: Create a log-log plot between $F_X^q(s)$ and s for each value of q. If there is a linear relationship between these variables create a power-law relationship denoted with the following formula:

$$F_X^q(s) \sim s^{h(q)} \tag{7}$$

In the equation above $h(q)$ denotes generalized Hurst exponent which measures power-law auto correlation. In monofractal time series $h(q)$ do not vary with q. However, for multifractal time series $h(q)$ varies with q. Singularity spectrum which is denoted with $f(\alpha)$ can be utilized to characterize a time series. Singularity spectrum is computed with the following formulas:

$$\alpha(q) = h(q) + qh'(q) \tag{8}$$

$$f(\alpha) = q[\alpha(q) - h(q)] + 1 \tag{9}$$

In the formula above $h'(q)$ indicates the derivative of $h(q)$ according to q. The Hölder exponent $\alpha(q)$ measures the singularity's strength, while the singularity spectrum $f(\alpha)$ measures the Hausdorff dimension of the time series which is described by $\alpha(q)$. The calculation of the mass function $\tau(q)$ is as follows:

$$\tau(q) = qh(q) - 1 \tag{10}$$

Degree of multifractality can be measured by multifractal spectrum ($\Delta\alpha$) which can be calculated as below:

$$\Delta\alpha = \alpha_{max} - \alpha_{min} \tag{11}$$

Value of $\Delta\alpha$ reflects the strength of the multifractality and a high $\Delta\alpha$ value implies high multifractality degree and a low $\Delta\alpha$ value implies low multifractality. Also Δh value defined below can be used to measure the degree of multifractality.

$$\Delta h = \text{maximum}(h(q)) - \text{minimum}(h(q)) \tag{12}$$

Similar to the $\Delta\alpha$, a high Δh value implies high multifractality degree and a low Δh value implies low multifractality. The skewness of the spectrum provides details on the main fluctuations. Right-skewed spectrum suggests that minor variations will predominate, while left-skewed spectrum suggests that huge fluctuations will.

4. Data

In this study S&P 500's sector indexes' daily closing prices of data are used. The data cover the period between 8th May 2003 and 7th February 2023 and contain 4960 observations. Data are obtained from the https://tr.investing.com/ web site. Before the analyses differences of logarithms of the data are taken with the following formula:

$$r_t = \ln(P_{t-1}) - \ln(P_t) \tag{13}$$

List and the explanations of the used sector indexes are given in the Table 1.

5. Empirical Results

To apply MF-DFA to a time series firstly three parameters must be determined. These parameters are vector of scales (s), q-order of the moments (q) and polynomial order for the detrending (m). In this study scales values are selected from 10 to 1240 in steps of 20, q-order of the moment values are selected from -10 to +10 in steps of 1 and polynomial order for the detrending is selected as 1. Plots obtained from MF-DFA of each sector index are presented in Figure 1-11. In the upper right panels of the Figures log-log plots between time scale and fluctuation function are plotted for q=-10 (in black), q=0 (in red) and q=10 (in green). In all figures it is seen that there are linear relationships between logarithms of time scales and fluctuation functions. This indicates the presence of power-law cross-correlations between time scales and fluctuation functions. In these panels slopes of the fitted lines indicates generalized Hurst exponents for q=-10, q=0 and q=10. As revealed in these panels fitted lines have different slopes and depends on q values. This implies multifractality in sector index time series. This situation is seen more clearly in the upper right panels of the figures. In these panels relationship between q-order of the moments versus generalized Hurst exponents h_q are plotted. As seen in all Figures generalized Hurst exponent values depend on q values and these are evidences of multifractal behavior in all the time series. The interval between the highest generalized Hurst exponent value (h_{-10}) and the lowest generalized Hurst exponent value (h_{10}), namely Δh used to measure the multifractality degree. Higher Δh implies higher multifractality and lower Δh implies lower multifractality. Δh values for each sector index are presented in Table 2. As seen from this table S&P 500 Energy Index (SPNY) has the highest multifractality degree, S&P 500 Materials Index (SPLRCM) has the second highest multifractality degree and S&P 500 Utilities Index (SPLRCU) has the third highest multifractality degree. Also, S&P 500 Industrials Index (SPLRCI) has the lowest multifractality degree, S&P 500 Real Estate Index (SPLRCREC) has second lowest multifractality degree and S&P 500 Financials Index (SPSY) has third lowest multifractality degree. Generalized hurst exponent for $q = 2$ (h_2) is called the Hurst exponent. The Hurst exponent can be utilized to determine how persistent a time series is over the long run. The Hurst exponent can take values between zero and one, values close to zero imply anti-persistent or mean-reverting behavior, values close to one imply persistent or trend-following behavior, and values around 0.5 imply a random walk or white noise process. In Table 1 Hurst exponent values for each sector are presented. As seen in this table Hurst exponent values for SPSY, SPLRCREC and SPLRCI are greater than 0.5 which implies persistent behavior and Hurst exponent values for SPLRCS, SPLRCU, SPXHC, SPNY, SPLRCT and SPLRCD are less than 0.5 which implies anti-persistent behavior. Additionally, Hurst values for SPX and SPLRCM are very close to 0.5 which implies random walk and efficient market. Generalized Hurst exponents are also presented in Table 3. In the lower right panels of Figure 1-11 relationship between q order moments and mass exponents $(\tau(q))$ are presented. As seen in these panels there are nonlinear relationships between q order moments and mass exponents and this is evidence of multifractality in the sector indexes. In the lower right panels of Figure 1-11 multifractal spectrums are presented. In these panels width of the multifractal spectrum (Δα) measures the multifractality degree and positive Δα values imply the existence of multifractality. The Δα values for each sector index are presented in Table 1. Looking at the Δα values in this table, it is seen that SPNY has the highest multifractality degree, SPLRCM has the second highest multifractality degree and SPLRCU has the third highest multifractality degree. Also, according to the Δα values SPLRCI has the lowest degree of multifractality, SPLRCREC has second lowest degree of multifractality and SPSY has third lowest degree of multifractality. Therefore both Δh and Δα values give the same results for the degree of the multifractality. Additionally, all multifractal spectrums in Figure 1-11 exhibits left-skewed spectrum indicate that large fluctuations are dominant in all sectors.

Table 1: Indexes of S&P 500 sectors

Symbol of the S&P 500 Sector Index	Explanation of the Index
SPLRCD	Consumer Discretionary
SPLRCI	Industrials
SPLRCM	Materials
SPLRCREC	Real Estate
SPLRCS	Consumer Staples
SPLRCT	Information Technology
SPLRCU	Utilities
SPNY	Energy
SPSY	Financials
SPX	Whole Index
SPXHC	Health Care

Figure 1: MF-DFA Results for S&P 500 Consumer Discretionary Index (SPLRCD)

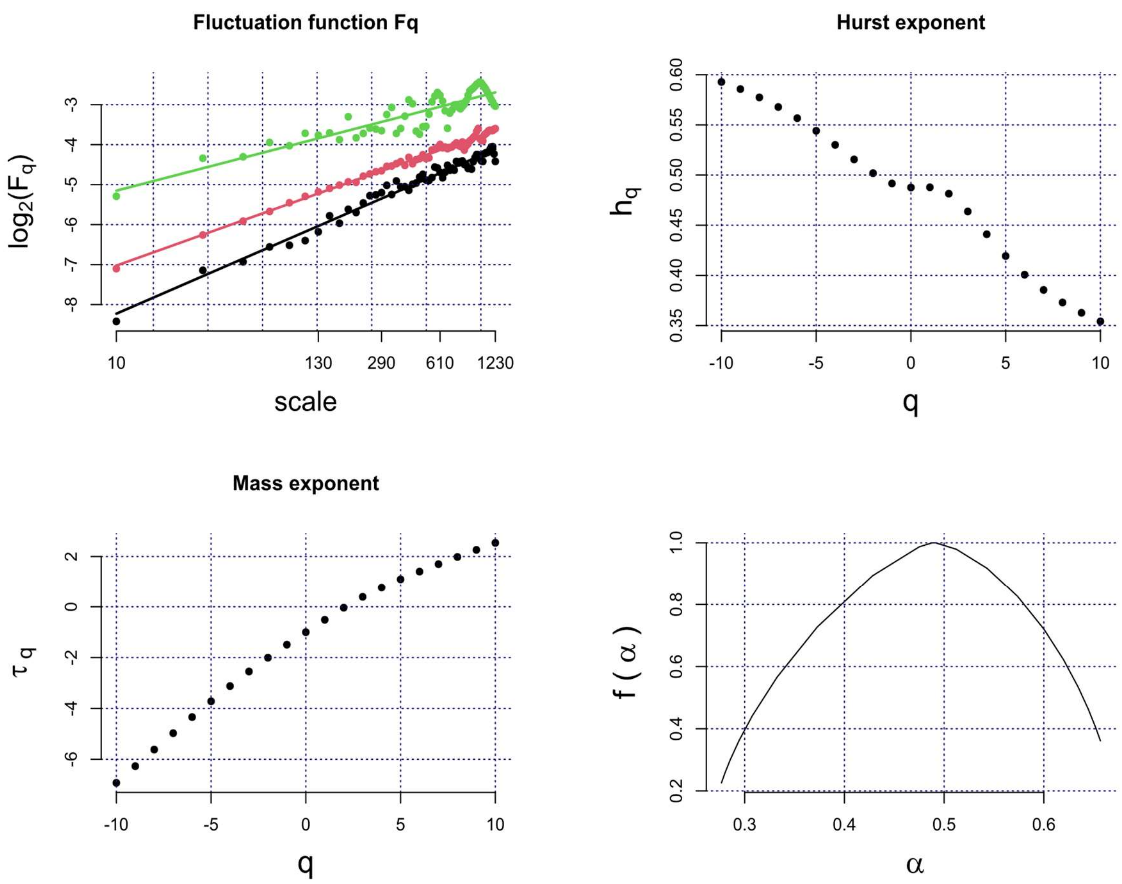

Figure 2: MF-DFA Results for S&P 500 Industrials Index (SPLRCI)

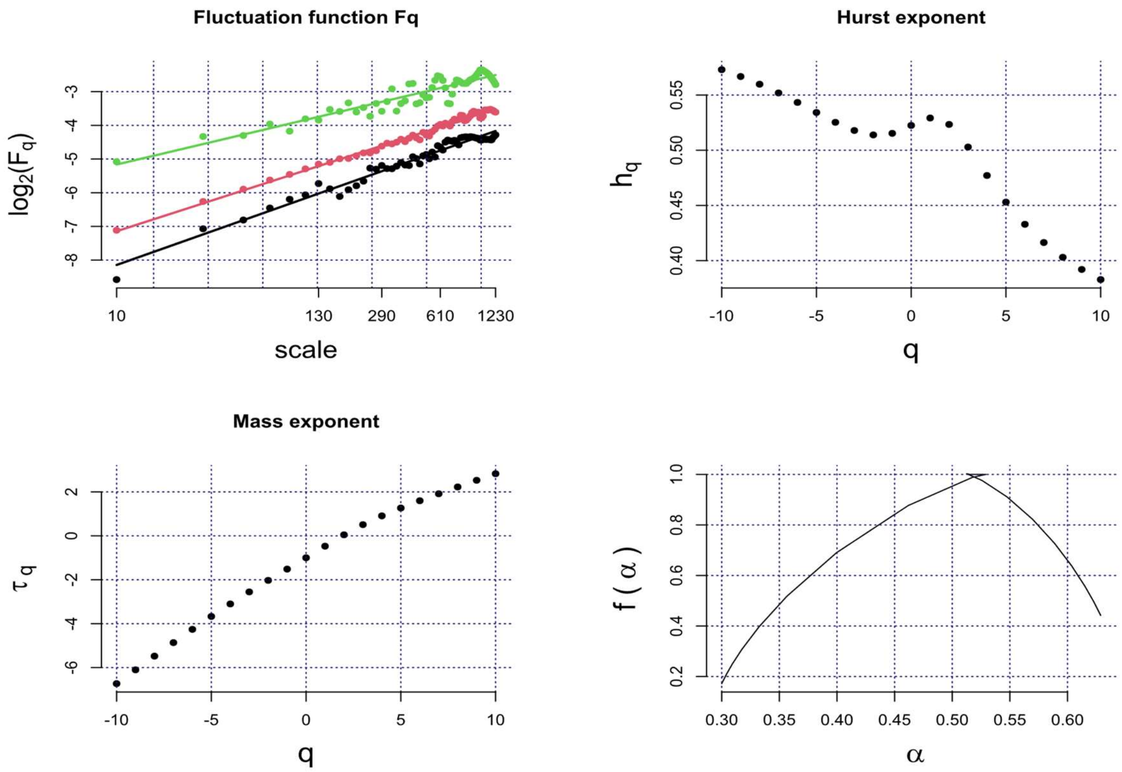

Figure 3: MF-DFA Results for S&P 500 Materials Index (SPLRCM)

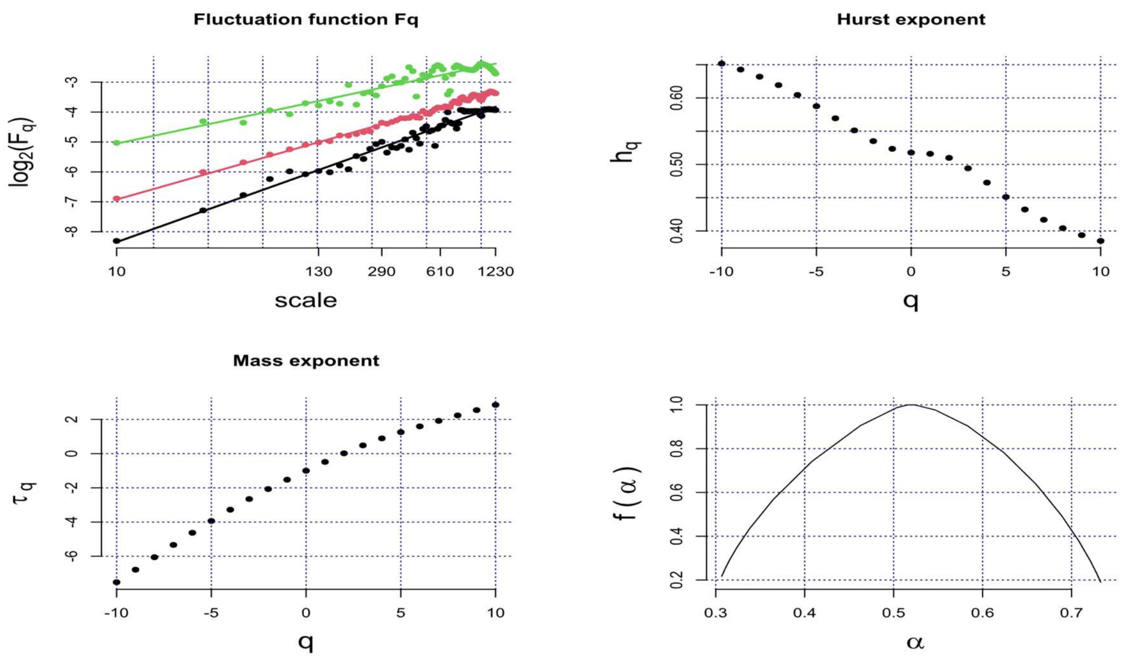

Figure 4: MF-DFA results for S&P 500 Real Estate Index (SPLRCREC)

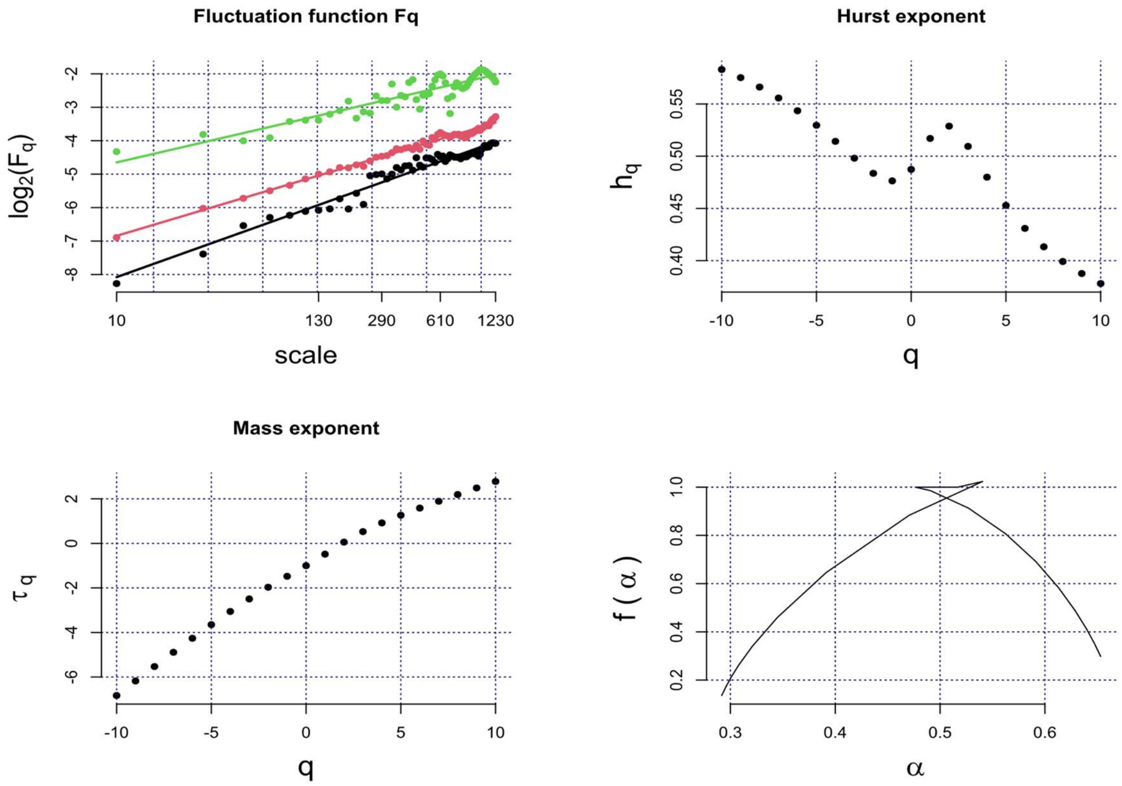

Figure 5: MF-DFA results for S&P 500 Consumer Staples Index (SPLRCS)

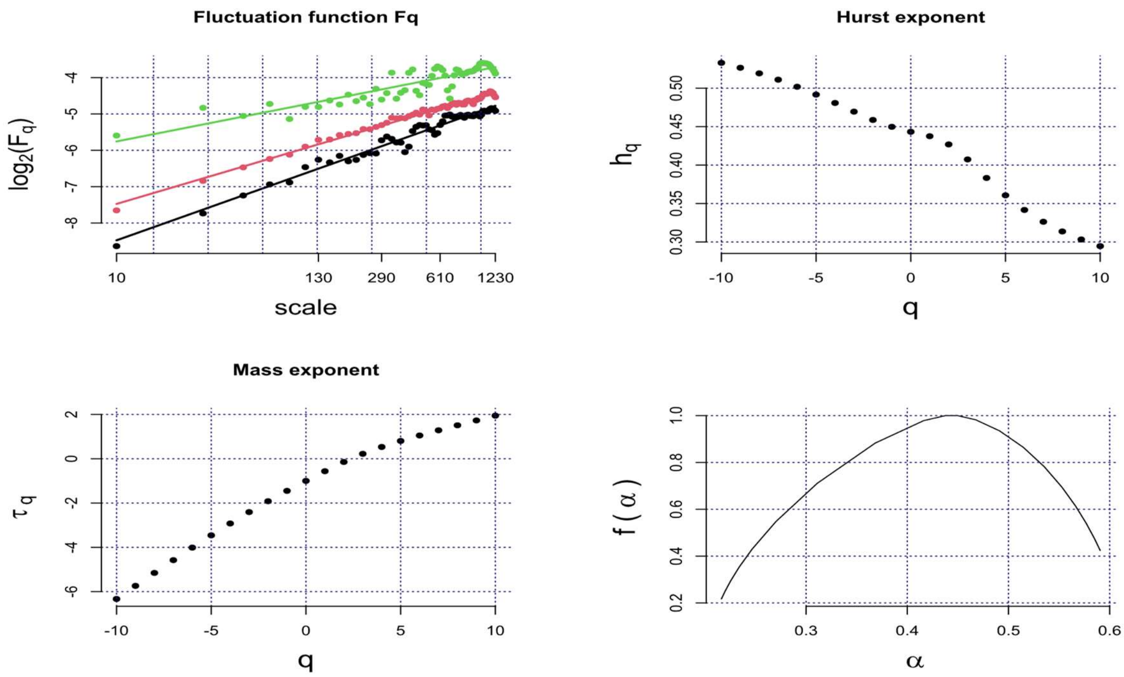

Figure 6: MF-DFA Results for S&P 500 Information Technology Index (SPLRCT)

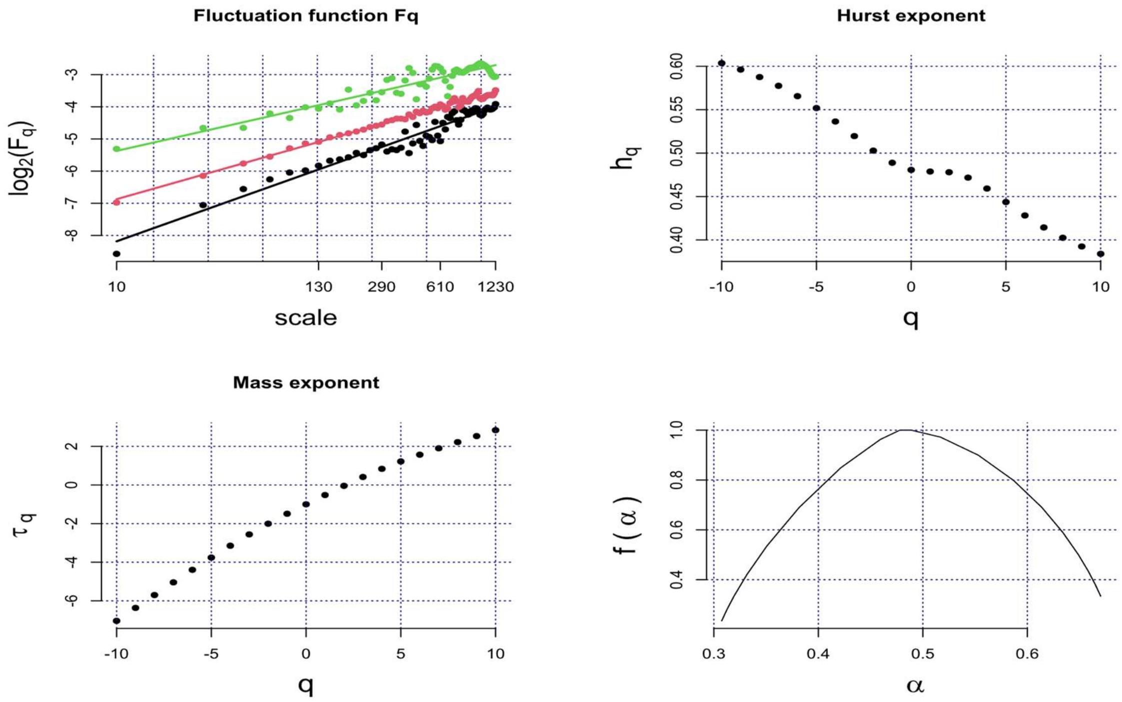

Figure 7: MF-DFA results for S&P 500 Utilities Index (SPLRCU)

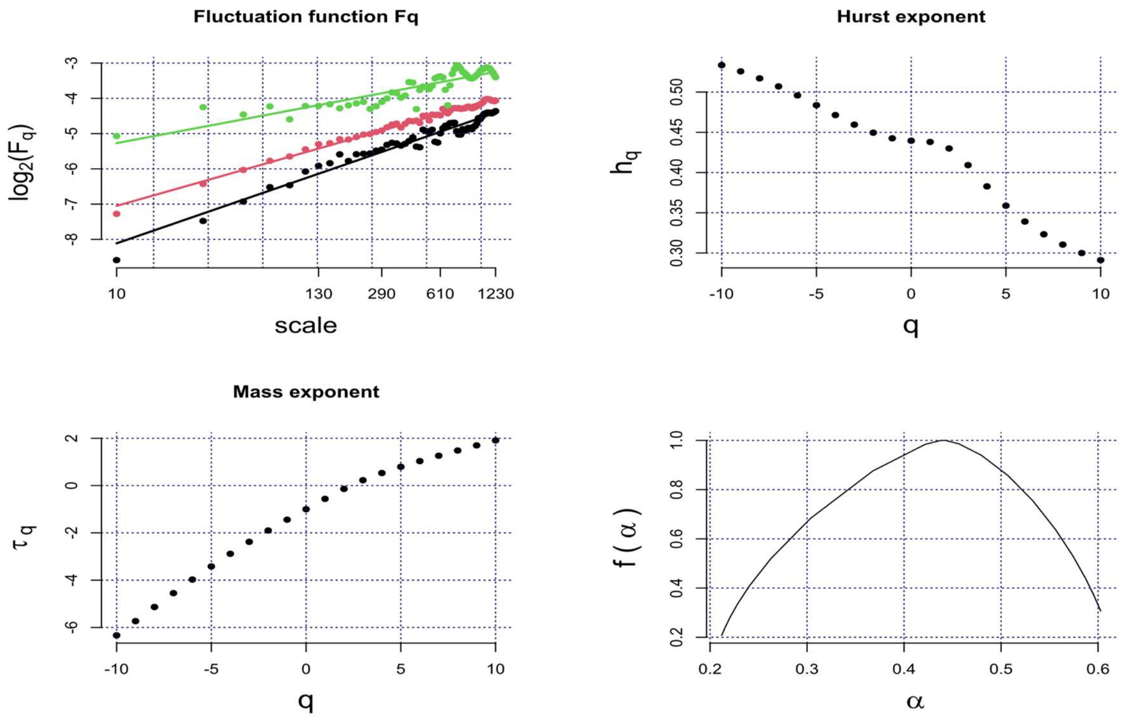

Figure 8: MF-DFA Results for S&P 500 Energy Index (SPNY)

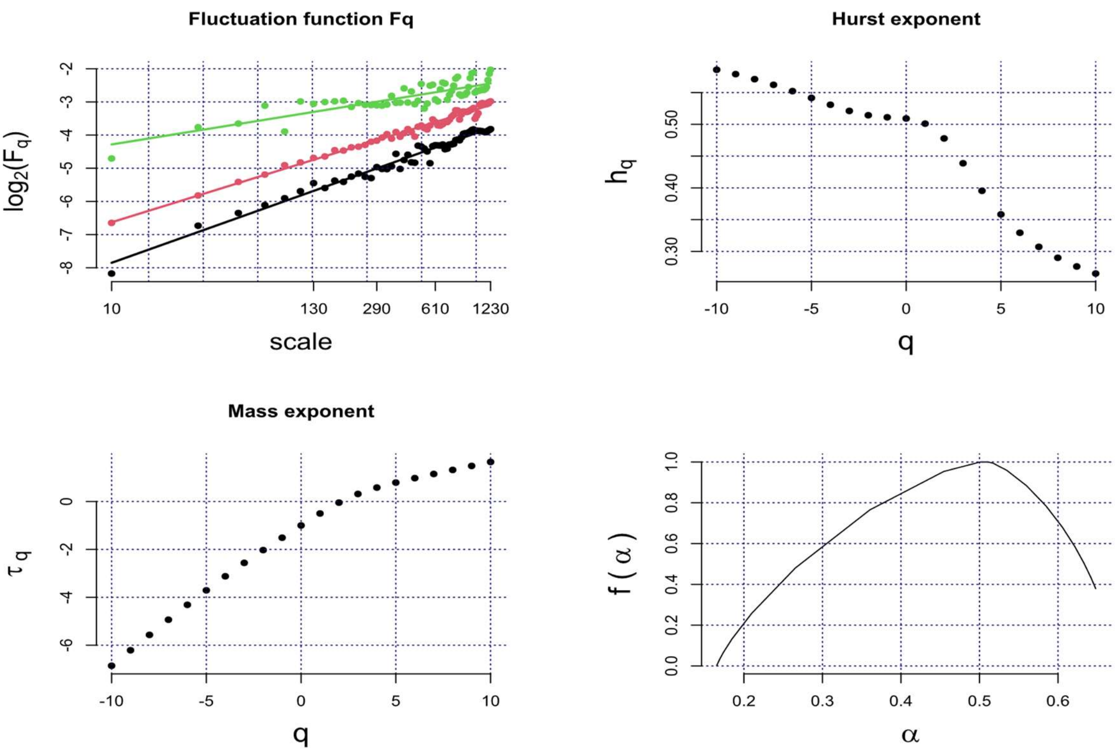

Figure 9: MF-DFA Results for S&P 500 Financials Index (SPSY)

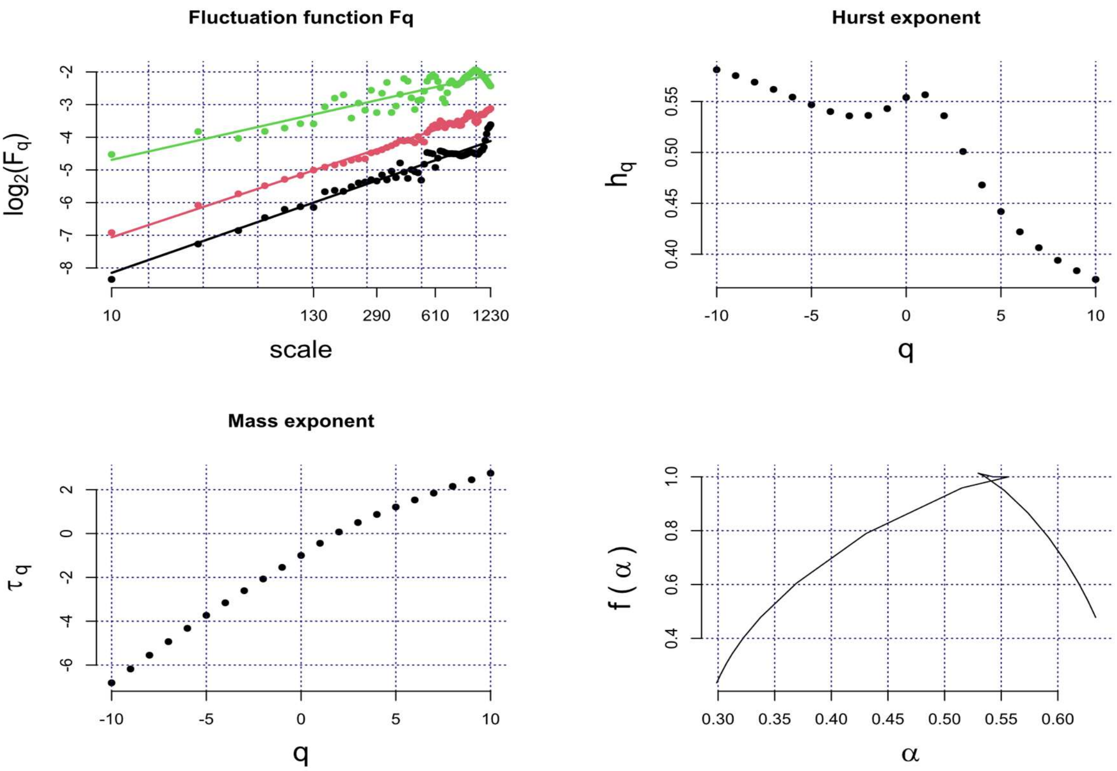

Figure 10: MF-DFA Results for S&P 500 Index (SPX)

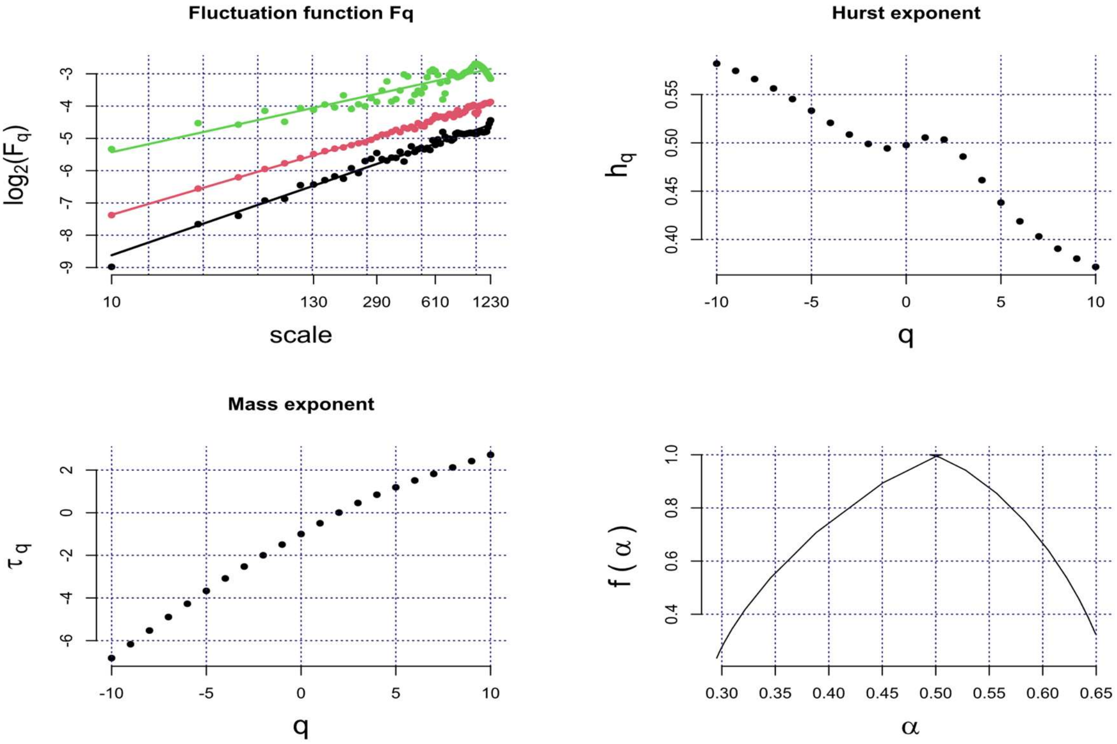

Figure 11: MF-DFA Results for S&P 500 Health Care Index (SPXHC)

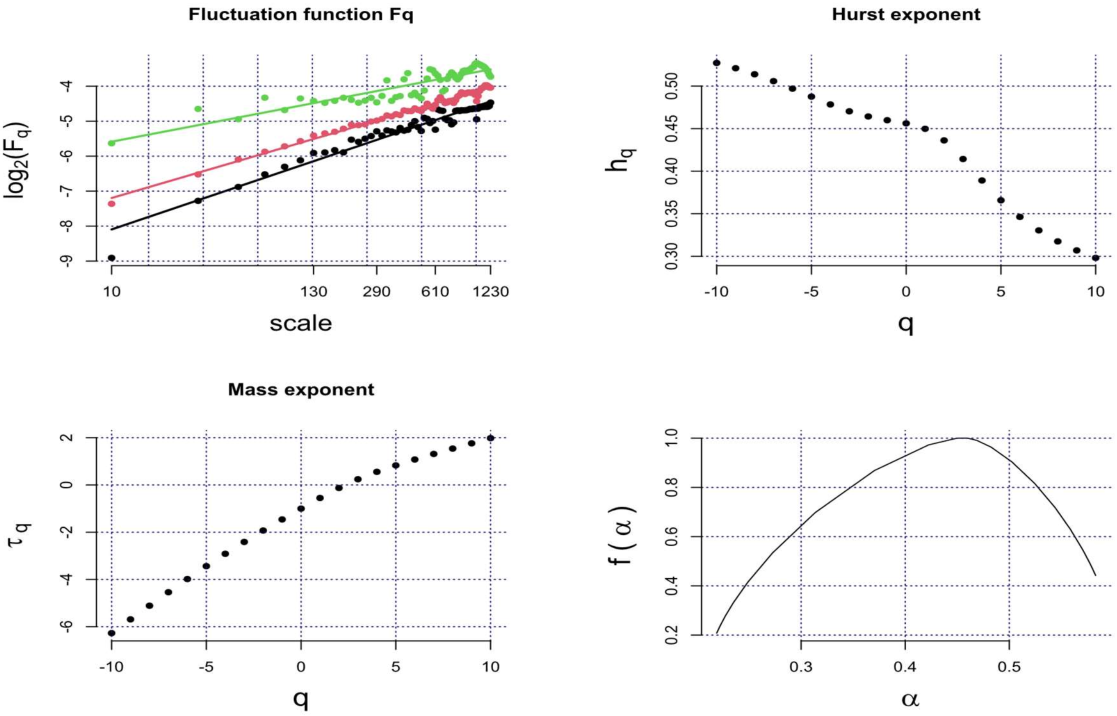

Table 2: Multifractal Spectrum (Δα), Interval Between Minimum and Maximum Generalized Hurst Exponent (Δh) and Hurst Exponent

Sector Index	Δα	Δh	Hurst Exponent
SPLRCD	0.38	0.2387	0.4814
SPLRCI	0.3288	0.1902	0.5234
SPLRCM	0.426	0.2667	0.5099
SPLRCREC	0.3615	0.2049	0.5288
SPLRCS	0.3743	0.2384	0.4269
SPLRCT	0.3627	0.2196	0.4780
SPLRCU	0.3907	0.2422	0.4299
SPNY	0.4827	0.3207	0.4778
SPSY	0.3346	0.2059	0.5360
SPX	0.3543	0.2103	0.5034
SPXHC	0.3641	0.2291	0.4362

Table 3: Generalized Hurst exponents

q	SPLRCD	SPLRCI	SPLRCM	SPLRCREC	SPLRCS	SPLRCT	SPLRCU	SPNY	SPSY	SPX	SPXHC
-10	0.5928	0.573	0.6517	0.583	0.533	0.6036	0.5334	0.5859	0.5812	0.582	0.5272
-9	0.5857	0.5668	0.6427	0.5752	0.5266	0.5962	0.5257	0.579	0.5754	0.5745	0.521
-8	0.5774	0.5598	0.632	0.5662	0.5193	0.5876	0.517	0.5711	0.569	0.566	0.5139
-7	0.5678	0.5519	0.6193	0.5557	0.5111	0.5775	0.507	0.5622	0.5619	0.5563	0.5058
-6	0.5567	0.5433	0.6045	0.5435	0.5019	0.5656	0.4958	0.5523	0.5543	0.5453	0.497
-5	0.5441	0.5342	0.5876	0.5296	0.4917	0.5519	0.4837	0.5416	0.5468	0.5333	0.4876
-4	0.5301	0.5253	0.5694	0.5142	0.4808	0.5364	0.4714	0.5308	0.5401	0.5207	0.4784
-3	0.5156	0.5179	0.5512	0.4981	0.4695	0.5196	0.4595	0.5211	0.5359	0.5086	0.4704
-2	0.5019	0.5139	0.5351	0.4836	0.4587	0.5029	0.4495	0.5143	0.5363	0.4989	0.4643
-1	0.4916	0.5153	0.5235	0.4764	0.4498	0.4889	0.4425	0.5111	0.543	0.4943	0.4599
0	0.4876	0.5225	0.5178	0.4874	0.4433	0.4807	0.4395	0.5092	0.554	0.4977	0.4562
1	0.4878	0.5292	0.516	0.517	0.4376	0.4788	0.438	0.5011	0.5566	0.5055	0.4498
2	0.4814	0.5234	0.5099	0.5288	0.4269	0.478	0.4299	0.4778	0.536	0.5034	0.4362
3	0.4637	0.5029	0.4942	0.5094	0.4074	0.4718	0.4092	0.4387	0.5009	0.4858	0.4143
4	0.441	0.4771	0.4727	0.4799	0.3833	0.4592	0.3828	0.3954	0.468	0.4614	0.3891
5	0.4193	0.453	0.4511	0.4529	0.3607	0.4437	0.3588	0.3583	0.4419	0.4382	0.3658
6	0.4007	0.4329	0.4323	0.4309	0.3417	0.4282	0.3391	0.3294	0.422	0.4188	0.3463
7	0.3855	0.4164	0.4168	0.4133	0.3263	0.4144	0.3233	0.3072	0.4064	0.4032	0.3304
8	0.373	0.403	0.4041	0.3992	0.3137	0.4025	0.3105	0.29	0.394	0.3905	0.3175
9	0.3627	0.392	0.3937	0.3877	0.3033	0.3925	0.3	0.2763	0.3838	0.3802	0.3069
10	0.3541	0.3828	0.385	0.3781	0.2946	0.384	0.2912	0.2652	0.3753	0.3717	0.2981

6. Conclusion

Multifractality refers to the property of a system or signal where different parts exhibit different degrees of scaling behavior, as characterized by their fractal dimension. One important conclusion that can be drawn about multifractality is that it is often observed in complex systems and signals that exhibit self-similarity at multiple scales. These systems and signals are typically characterized by a high degree of variability and heterogeneity, which can make it difficult to capture their properties using traditional methods such as ARIMA. In this study we investigated multifractality in the S&P 500 sector indexes by using MF-DFA methodology. Our analysis express multifractality in all the sector indexes. Since all the sector indexes possess multifractality, we can conclude that these indexes possess the key properties such as scaling variability, nonlinear dynamics, self-similarity, long-range dependence, multiscale correlations and nonstationarity. Another important conclusion that can be drawn is that multifractal properties can have important implications for the dynamics and behavior of complex systems. In finance, multifractal properties have been linked to market crashes and other extreme events. Overall, multifractality is a useful concept for analyzing and apprehending the complex scaling properties of systems and signals, and can shed light on the processes and behavior of these systems at their core.

The results obtained from our analysis indicate that the SPX and SPLRCM markets are efficient. Market efficiency implies several key characteristics and outcomes for a market. These implications are essential for investors, regulators, and other market participants to understand how the market functions and what to expect. Here are the main implications of market efficiency:

a) Quick Price Adjustments: In an efficient market, prices rapidly adjust to new information. This means that any relevant news, data, or events that affect an asset's value will be quickly incorporated into its price. As a result, it becomes challenging for investors to consistently earn abnormal returns based on publicly available information.

b) Random Price Movements: Market efficiency implies that asset prices follow a random walk or a sequence of unpredictable movements. Price changes are not systematically predictable, and any past price patterns or trends cannot be reliably used to forecast future price movements.

c) Fair Value: Efficient markets are believed to accurately reflect the fair value of assets. Prices reflect all available information, making it difficult for an asset to be significantly undervalued or overvalued in the long term.

d) Limited Arbitrage Opportunities: Market efficiency reduces the presence of arbitrage opportunities. Arbitrage involves exploiting price discrepancies between related assets, but in an efficient market, such opportunities are short-lived and quickly eliminated.

e) Active vs. Passive Investing: Market efficiency has implications for investment strategies. In highly efficient markets, passive investment strategies like index funds are more popular as they aim to replicate the overall market's returns. In less efficient markets, active management may be pursued to seek out mispriced assets.

f) Market Stability: Efficient markets tend to be more stable as prices incorporate all available information, reducing the likelihood of sudden, large price swings driven by misinformation or speculative behavior.

g) Market Integrity: Market efficiency is associated with market integrity. In efficient markets, there is less room for price manipulation or insider trading, as any attempts to distort prices are quickly corrected by new information.

h) Information Processing: An efficient market indicates that information is processed and disseminated effectively. Market participants actively analyze and react to information, leading to a more informative and well-functioning market.

Our results indicate that SPSY, SPLRCREC and SPLRCI markets are persistent. Since these markets are persistent, they exhibit positive autocorrelation, meaning that past values influence future values. In other words, if an asset's price has been increasing (or decreasing) over recent periods, it is more likely to continue that trend in the near future. This implies the existence of trends and momentum in the market. Persistence can present opportunities for trend-following strategies, where investors try to capitalize on the continuation of existing price trends. However, it also carries risks, as persistently rising prices might lead to overvaluation, while persistent declines may result in undervaluation. Persistent trends may be driven by factors such as market sentiment, investor behavior, or fundamental changes in the underlying asset. Investors should be cautious when relying solely on historical price trends, as persistent patterns can contribute to higher levels of volatility and increased risk in the market.

Also, our results imply that SPLRCS, SPLRCU, SPXHC, SPNY, SPLRCT and SPLRCD markets are anti-persistent. Since these markets are anti-persistent, they exhibit negative autocorrelation, meaning that past values have an inverse effect on future values. In other words, a price decrease (or increase) is more likely to be followed by a price increase (or decrease). Anti-persistence implies mean reversion, where the market tends to revert to its average value. Anti-persistence can lead to opportunities for mean-reversion strategies. Investors may expect that extreme price movements will be followed by a correction towards the mean or average price. Mean-reversion strategies involve buying when prices are low and selling when they are high. Anti-persistence might arise due to profit-taking behavior after price movements, or when external factors drive temporary deviations from an asset's intrinsic value.

In summary, persistence and anti-persistence in financial time series imply that markets may exhibit trends and momentum (persistence) or mean-reverting behavior (anti-persistence). Understanding these characteristics is crucial for investors, as they can inform trading strategies and risk management decisions.

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