Ikonion Journal of Mathematics
https://dergipark.org.tr/tr/pub/ikjm
Research Article
Open Access
https://doi.org/10.54286/ikjm.1304566

ISSN: 2687-6531

A New Soft Set Operation: Complementary Soft Binary Piecewise Star (*) Operation

Aslıhan Sezgin * [1], Ahmet Mücahit Demirci [2]

Keywords:
Soft set,
Soft set operations,
Conditional Complements.

Abstract — Soft set theory, introduced by Molodtsov, is a crucial mathematical tool for dealing with uncertainty and it has many applications both as theoretically and in applications. Since the beginning, different types of soft set operations have been defined and used in different forms. In this article, we define a new type of soft set operation called the complementary soft binary piecewise star operation and examine its fundamental algebraic properties. Furthermore, by examining the distribution of complementary soft binary piecewise star operations over other type of soft set operations, we aim to identify the relationship between this new soft set operation and others to contribute to the soft set literature. Since proposing a new soft set operation and deriving its algebraic properties and implementations provide several new perspectives for dealing with problems related to parametric data, with the inspiration by this new soft set operation, researchers may be able to propose new cryptographic or decision methods based on soft sets and they may systematically explore soft set algebraic structures associated with new soft set operations.

Subject Classification (2020): 03E75, 20N02.

1. Introduction

Problems in many fields, such as economics, environmental sciences, health sciences, and engineering, have certain uncertainties that prevent them from being successfully solved using classical methods. There are three well-known basic theories that can be considered as mathematical tools for dealing with uncertainty as probability theory, fuzzy set theory, and interval mathematics. However, since each of these theories has its own drawbacks, Molodtsov [21] introduced soft set theory as a mathematical tool to overcome these uncertainties. Since then, this theory has ben applied to many fields including information systems, decision making [6-9,23,24], optimization theory, game theory, operations research, measurement theory, algebraic structures [25] and so on. Soft sets and fuzzy soft sets, one of the first theories in which parameterization tools are used to manage the decision process of uncertainty problems as accurately as possible were discussed in detail by Dalkılıç [10] and these sets were reevaluated and the concept of pure (fuzzy) soft sets was proposed with its properties and examples. First contributions as regards soft set operations were made by Maji et. al [20] and Pei and Miao [26]. After then, several soft set operations (restricted and extended soft set operations)

* aslihan.sezgin@amasya.edu.tr (Corresponding Author)
[1] Amasya University, Department of Mathematics and Science Education, Faculty of Education, 05100, Amasya, Turkey;
[2] Amasya University, Department of Mathematics, Graduate School of Natural and Applied Sciences, 05100, Amasya, Turkey.
Article History: Received: 28.05.2023 — Accepted: 05.09.2023— Published: 12.10.2023

were introduced and examined by Ali et. al [2]. Basic properties of soft set operations were discussed and the interconnections of soft set operations with each other were illustrated by Sezgin and Atagün [29]. They also defined the notion of restricted symmetric difference of soft sets and investigated its properties. A new soft set operation called extended difference of soft sets was defined by Sezgin et.al [37] and extended symmetric difference of soft sets was defined and its properties were investigated by Stojanovic [42]. When the studies are examined, we see that the operations in soft set theory proceed under two main headings, as restricted soft set operations and extended soft set operations. For more about the studies regarding the operation of soft sets, we refer to: [3,4,11-19,22,27-44]

Two conditional complements of sets, i.e. the inclusive complement and the exclusive complement, were proposed as new concepts in set theory, and the relationships between them were studied by Çağman [5]. Inspired by this work, some new complements and then several new additions and to soft set theory as new restricted and extended soft set operations were defined by Aybek [4]. Akbulut [1], Demirci [11], Sarıalioğlu [28] defined a new type of extended operation by changing the form of extended soft set operations using the complement at the first and second row of the piecewise function of extended soft set operations and studied the basic properties of them in detail. Moreover, a new type of soft difference operations was defined in Eren and Çalışıcı [12] and by being inspired this study Yavuz [44] defined some new soft set operations, which ise called soft binary piecewise operations and their basic properties were studied in detail. Also, Sezgin and Sarıalioğlu [36], Sezgin and Atagün [30], Sezgin and Aybek [31], Sezgin et. al [32,33], Sezgin and Çağman [34] continued their work on soft set operations by defining a new type of soft binary piecewise operation. They changed the form of soft binary piecewise operation by using the complement at the first row of the soft binary piecewise operations.

The purpose of this work is to contribute to the soft set theory literature by describing a new soft set operation called the “complementary soft binary piecewise star operation". For this aim, the definition of the operation and its examples are given, and algebraic properties such as closure, associativity, unity, inverse, and abelian properties of this new operation are examined in detail. In particular, we aim to contribute to the soft set literature by obtaining the distributions of complementary soft binary piecewise star operations over other types of soft set operations. The concept of soft set operations is a core concept similar to basic number operations in classical algebra and basic set operations in classical set theory. Proposing new soft set operations and obtaining algebraic properties and their implementations offers new perspectives for solving problems involving parametric data as regards decision-making methods and new cryptographic methods. Also, studying the algebraic structure of soft sets from the perspective of new operations provides deep insight into the algebraic structure of soft sets. This document is organized as follows: Section 2 reminds the basic definitions regarding soft sets. Section 3 provides definitions and examples of the new soft set operation. This is followed by a full analysis of the algebraic properties of this new soft set operation, including closure, associativity, unity, inverse, and abelian properties. To enhance the knowledge of soft sets, Section 4 presents the distribution of complementary soft binary piecewise star operation over extended soft set operations, complementary extended soft set operations, soft binary piecewise operations, complementary soft binary piecewise operations and restricted soft set operation. The conclusion section considers the significance of the research findings and their possible impact on the subject.

2. Preliminaries

In this section, some basic concepts related to soft set theory are compiled and given.

Definition 2.1. Let U be the universal set, E be the parameter set, $P(U)$ be the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U where F is a set-valued function such that $F: A \to P(U)$ [21].

Here, note that Çağman and Enginoğlu [6] redefined the definition of Molodstov's soft sets; however in this paper, we use the Molodtsov's soft set definition by staying faithful to the original definition. Throughout this paper, the set of all the soft sets over U (no matter what the parameter set is) is designated by $S_E(U)$. Let A be a fixed subset of E and $S_A(U)$ be the collection of all soft sets over U with the fixed parameters set A. Clearly, $S_A(U)$ is a subset of $S_E(U)$ and, in fact, all the soft sets are the elements of $S_E(U)$.

Definition 2.2. $(D, \aleph)$ is called a relative null soft set (with respect to the parameter set $\aleph$), denoted by $\emptyset_\aleph$, if $D(t) = \emptyset$ for all $t \in \aleph$ and $(D, \aleph)$ is called a relative whole soft set (with respect to the parameter set $\aleph$), denoted by $U_\aleph$ if $D(t) = U$ for all $t \in \aleph$. The relative whole soft set U_E with respect to the universe set of parameters E is called the absolute soft set over U [2].

Definition 2.3. For two soft sets $(D, \aleph)$ and (J, R), we say that $(D, \aleph)$ is a soft subset of (J, R) and it is denoted by $(D, \aleph) \tilde{\subseteq} (J, R)$, if $\aleph \subseteq R$ and $D(t) \subseteq J(t)$, $\forall t \in \aleph$. Two soft sets $(D, \aleph)$ and (J, R) are said to be soft equal if $(D, \aleph)$ is a soft subset of (J, R) and (J, R) is a soft subset of $(D, \aleph)$ [26].

Definition 2.4. The relative complement of a soft set $(D, \aleph)$, denoted by $(D, \aleph)^r$, is defined by $(D, \aleph)^r = (D^r, \aleph)$, where $D^r: \aleph \to P(U)$ is a mapping given by $(D, \aleph)^r = U \backslash D(t)$ for all $t \in \aleph$ [2]. From now on, $U \backslash D(t)$=$[D(t)]'$ will be designated by $D'(s)$ for the sake of designation.

Two conditional complements of sets as new concepts of set theory, that is, inclusive complement and exclusive complement were defined by Çağman [5]. For the ease of illustration, we show these complements as + and θ, respectively. These complements are binary operations and are defined as follows: Let Q and R be two subsets of U. R-inclusive complement of Q is defined by, Q+R= Q'∪R and R-Exlusive complement of Q is defined by $Q\theta$ R = Q'∩R'. Here, U refers to a universe, Q' is the complement of Q over U. For more information, we refer to [35].

The relations between these two complements were examined in detail by Sezgin et.al [35] and they also introduced such new three complements as binary operations of sets as follows: Let Q and R be two subsets of U. Then, Q*R=Q'∪R', $Q\gamma$R= Q'∩R, Q$\boldsymbol{\lambda}$R=Q∪R'. These set operations were also conveyed to soft sets by Aybek [4] and restricted and extended soft set operations were defined and their properties were examined.

As a summary for soft set operations, we can categorize all types of soft set operations as follows: Let "∇" be used to represent the set operations (i.e., here ∇ can be ∩, ∪,\, Δ, +,θ, *, λ,γ), then restricted soft set operations, extended soft set operations, complementary extended soft set operations, soft binary piecewise operations, complementary soft binary piecewise operations are defined in soft set theory as follows:

Definition 2.5. Let $(D, \aleph)$ and (J, R) be soft sets over U. The restricted ∇ operation of $(D, \aleph)$ and (J, R) is the soft set (Y,S), denoted by $(D, \aleph)\nabla_R(J, R) = (Y, S)$, where $S = \aleph \cap R \neq \emptyset$ and $\forall t \in S$, $Y(t)$ =$D(s)\nabla J(t)$ [2,29].

Definition 2.6. Let $(D,\aleph)$ and (J,R) be soft sets over U. The extended ∇ operation of $(D,\aleph)$ and (J,R) is the soft set (Y,S) denoted by, $(D,\aleph)\nabla_\varepsilon(J,R) = (Y,S)$, where $S = \aleph \cup R$ and $\forall t \in S$,

$$Y(s) = \begin{cases} D(t), & t \in \aleph \setminus R, \\ J(t), & t \in R \setminus \aleph, \\ D(t)\nabla J(t), & t \in \aleph \cap R. \end{cases}$$

[2,4,20,37,42]

Definition 2.7. Let $(D,\aleph)$ and (J,R) be soft sets over U. The complementary extended ∇ operation of $(D,\aleph)$ and (J,R) is the soft set (Y,S) denoted by, $(D,\aleph) \overset{*}{\nabla_\varepsilon} (J,R) = (Y,S)$, where $S = \aleph \cup R$ and $\forall t \in S$,

$$Y(s) = \begin{cases} D'(t), & t \in \aleph \setminus R \\ J'(t), & t \in R \setminus \aleph, \\ D(t)\nabla J(t), & t \in \aleph \cap R. \end{cases}$$

[1,28].

Definition 2.8. Let $(D,\aleph)$ and (J,R) be soft sets over U. The soft binary piecewise ∇ operation of $(D,\aleph)$ and (J,R) is the soft set $(Y,\aleph)$, denoted by $(D,\aleph)\underset{\nabla}{\tilde{}}(J,R) = (Y,\aleph)$, where $\forall s \in \aleph$,

$$Y(s) = \begin{cases} D(s), & s \in \aleph \setminus R \\ D(s)\nabla J(s), & s \in \aleph \cap R \end{cases}$$

[12,44].

Definition 2.9. Let $(D,\aleph)$ and (J,R) be soft sets over U. The complementary soft binary piecewise ∇ operation of $(D,\aleph)$ and (J,R) is the soft set $(Y,\aleph)$ denoted by, $(D,\aleph) \underset{\nabla}{\overset{*}{\sim}} (J,R) = (Y,\aleph)$, where $\forall s \in \aleph$;

$$Y(s) = \begin{cases} D'(s), & s \in \aleph \setminus R \\ D(s)\nabla J(s), & s \in \aleph \cap R \end{cases}$$

[30-34,36].

3. Complementary Soft Binary Piecewise Star (∗) Operation And Its Properties

Definition 3.1. Let $(D,\aleph)$ and (J,R) be soft sets over U. The complementary soft binary piecewise star ($*$) operation of $(D,\aleph)$ and (J,R) is the soft set $(Y,\aleph)$ denoted by, $(D,\aleph) \underset{*}{\overset{*}{\sim}} (J,R) = (Y,\aleph)$, where $\forall s \in \aleph$,

$$Y(s) = \begin{cases} D'(s), & s \in \aleph \setminus R \\ D'(s) \cup J'(s), & s \in \aleph \cap R \end{cases}$$

Example 3.2. Let $E=\{e_1,e_2,e_3,e_4,e_5\}$ be the parameter set $\aleph=\{e_1, e_3, e_5\}$ and $R=\{e_1, e_2, e_4\}$ be the subsets of E and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the initial universe set. Assume that $(D,\aleph)$ and (J,R) are the soft sets over U defined as follows: $(D,\aleph)=\{(e_1, \{h_1,h_3\}), (e_3,\{h_2,h_4\}), (e_5,\{h_2,h_4,h_5\})\}$, $(J,R)=\{(e_1,\{h_1,h_4\}), (e_2,\{h_2,h_3\}),(e_4,\{h_4,h_5\})\}$. Let $(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,R)=(Y,\aleph)$. Then,

$$Y(s)=\begin{cases} D'(s), & s\in\aleph\setminus R \\ D'(s)\cup J'(s), & s\in\aleph\cap R \end{cases}$$

Since $\aleph=\{e_1, e_3, e_5\}$ and $\aleph\setminus R=\{e_3, e_5\}$, so $Y(e_3) = D'(e_3)=\{h_1,h_3,h_5\}$, $Y(e_5) = D'(e_5)=\{h_1,h_3\}$. And since $\aleph\cap R=\{e_1\}$ so $Y(e_1)=D'(e_1)\cup J'(e_1)=\{h_2,h_4,h_5\}\cup\{h_2,h_3,h_5\}=\{h_2,h_3,h_4,h_5\}$. Thus, $(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,R)=\{(e_1,\{h_2,h_3,h_4,h_5\}), (e_3,\{h_1,h_3,h_5\}), (e_5,\{h_1,h_3\})\}$.

Algebraic Properties of the Operation

1) The set $S_E(U)$ is closed under the operation $\overset{*}{\underset{*}{\sim}}$. That is, when $(D,\aleph)$ and (J,R) are two soft sets over U, then so is $(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,R)$. Hence, the set $S_E(U)$ is closed under the operation $\overset{*}{\underset{*}{\sim}}$.

2) $[(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,\aleph)]\overset{*}{\underset{*}{\sim}}(Y,\aleph)\neq(D,\aleph)\overset{*}{\underset{*}{\sim}}[(J,\aleph)\overset{*}{\underset{*}{\sim}}(Y,\aleph)]$

Proof: Let $(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,\aleph)=(T,\aleph)$, where $\forall s\in\aleph$;

$$T(s)=\begin{cases} D'(s), & s\in\aleph\setminus\aleph=\emptyset \\ D'(s)\cup J'(s), & s\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(T,\aleph)\overset{*}{\underset{*}{\sim}}(Y,\aleph)=(M,\aleph)$, where $\forall s\in\aleph$;

$$M(s)=\begin{cases} T'(s), & s\in\aleph\setminus\aleph=\emptyset \\ T'(s)\cup Y'(s), & s\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus,

$$M(s)=\begin{cases} T'(s), & s\in\aleph\setminus\aleph=\emptyset \\ [D(s)\cap J(s)]\cup Y'(s), & s\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(J,\aleph)\overset{*}{\underset{*}{\sim}}(Y,\aleph)=(L,\aleph)$, where $\forall s\in\aleph$;

$$L(s)=\begin{cases} J'(s), & s\in\aleph\backslash\aleph=\emptyset \\ J'(s)\cup Y'(s), & s\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(D,\aleph)\overset{*}{\underset{*}{\sim}}(L,\aleph)=(N,\aleph)$, where $\forall s\in\aleph$;

$$N(s)=\begin{cases} D'(s), & s\in\aleph\backslash\aleph=\emptyset \\ D'(s)\cup L'(s), & s\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus,

$$N(s)=\begin{cases} D'(s), & s\in\aleph\backslash\aleph=\emptyset \\ D'(s)\cup[J(s)\cap Y(s)], & s\in\aleph\cap\aleph=\aleph \end{cases}$$

It is seen that $(M,\aleph)\neq(N,\aleph)$.

That is, for the soft sets whose parameter set are the same, the operation $\overset{*}{\underset{*}{\sim}}$ does not have associativity property.

Moreover, we have the following:

3) $[(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,R)]\overset{*}{\underset{*}{\sim}}(Y,S)\neq(D,\aleph)\overset{*}{\underset{*}{\sim}}[(J,R)\overset{*}{\underset{*}{\sim}}(Y,S)]$

Proof: Let $(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,R)=(T,\aleph)$, where $\forall s\in\aleph$;

$$T(s)=\begin{cases} D'(s), & s\in\aleph\backslash R\ \ 7 \\ D'(s)\cup J'(s), & s\in\aleph\cap R \end{cases}$$

Let $(T,\aleph)\overset{*}{\underset{*}{\sim}}(Y,S)=(M,\aleph)$, where $\forall s\in\aleph$;

$$M(s)=\begin{cases} T'(s), & s\in\aleph\backslash S \\ T'(s)\cup Y'(s), & s\in\aleph\cap S \end{cases}$$

Thus,

$$M(s)=\begin{cases} D(s), & s\in(\aleph\backslash R)\backslash S=\aleph\cap R'\cap S' \\ D(s)\cap J(s), & s\in(\aleph\cap R)\backslash S=\aleph\cap R\cap S' \\ D(s)\cup Y'(s), & s\in(\aleph\backslash R)\cap S=\aleph\cap R'\cap S \\ [D(s)\cap J(s)]\cup Y'(s), & s\in(\aleph\cap R)\cap S=\aleph\cap R\cap S \end{cases}$$

Let $(J,R)\underset{*}{\overset{*}{\sim}}(Y,S)=(K,R)$, where $\forall s\in R$;

$$K(s)=\begin{cases} J'(s), & s\in R\backslash S \\ J'(s)\cup Y'(s), & s\in R\cap S \end{cases}$$

Let $(D,\aleph)\underset{*}{\overset{*}{\sim}}(K,R)=(S,\aleph)$, wher $\forall s\in\aleph$;

$$S(s)=\begin{cases} D'(s), & s\in\aleph\backslash R \\ D'(s)\cup K'(s), & s\in\aleph\cap R \end{cases}$$

Thus,

$$S(s)=\begin{cases} D'(s), & s\in\aleph\backslash R \\ D'(s)\cup J(s), & s\in\aleph\cap(R\backslash S)=\aleph\cap R\cap S' \\ D'(s)\cup[J(s)\cap Y(s)], & s\in\aleph\cap(R\cap S)=\aleph\cap R\cap S \end{cases}$$

Here let's handle $s\in\aleph\backslash R$ in the second equation of the first line. Since $\aleph\backslash R=\aleph\cap R'$, if $s\in R'$, then $s\in S\backslash R$ or $s\in(R\cup S)'$. Hence, if $s\in\aleph\backslash R$, then $s\in\aleph\cap R'\cap S'$ or $s\in\aleph\cap R'\cap S$. Thus, it is seen that $(M,\aleph)\neq(S,\aleph)$, that is, for the soft sets whose parameter set are not the same, the operation $\underset{*}{\overset{*}{\sim}}$ does not have associativity property on the set $S_E(U)$.

4) $(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)\neq(J,R)\underset{*}{\overset{*}{\sim}}(D,\aleph)$.

Proof: Let $(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)=(Y,\aleph)$. Then, $\forall s\in\aleph$;

$$Y(s)=\begin{cases} D'(s), & s\in\aleph\backslash R \\ D'(s)\cup J'(s), & s\in\aleph\cap R \end{cases}$$

Let $(J,R)\underset{*}{\overset{*}{\sim}}(D,\aleph)=(T,R)$. Then, $\forall s\in R$;

$$T(s)=\begin{cases} J'(s), & s\in R\backslash\aleph \\ J'(s)\cup D'(s), & s\in R\cap\aleph \end{cases}$$

Here, while the parameter set of the soft set of the left hand side is $\aleph$; the parameter set of the soft set of the right hand side is R. Thus, by the definition of soft equality

$$(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)\neq(J,R)\underset{*}{\overset{*}{\sim}}(D,\aleph).$$

Hence, the operation $\underset{*}{\overset{*}{\sim}}$ does not have commutative property in the set $S_E(U)$, where the parameter sets of the soft sets are different. However it is easy to see that

$$(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,\aleph)=(J,\aleph)\underset{*}{\overset{*}{\sim}}(D,\aleph).$$

That is to say, the operation $\underset{*}{\overset{*}{\sim}}$ does have commutative property where the parameter sets of the soft sets are the same.

5) $(D,\aleph)\underset{*}{\overset{*}{\sim}}(D,\aleph)=(D,\aleph)^r$.

Proof: Let $(D,\aleph)\underset{*}{\overset{*}{\sim}}(D,\aleph)=(Y,\aleph)$. Then, $\forall s\in\aleph$;

$$Y(s)=\begin{cases} D'(s), & s\in\aleph\setminus\aleph=\emptyset \\ D'(s)\cup D'(s), & s\in\aleph\cap\aleph=\aleph \end{cases}$$

Here, $\forall s\in\aleph$, $Y(s)=D'(s)\cup D'(s)=D'(s)$, hence $(Y,\aleph)=(D,\aleph)^r$. That is, the operation $\underset{*}{\overset{*}{\sim}}$ does not have idempotency property on the set $S_E(U)$.

6) $(D,\aleph)\underset{*}{\overset{*}{\sim}}\emptyset_\aleph=\emptyset_\aleph\underset{*}{\overset{*}{\sim}}(D,\aleph)=U_\aleph$.

Proof: Let $\emptyset_\aleph=(S,\aleph)$. Hence, $\forall s\in\aleph$; $S(s)=\emptyset$. Let $(D,\aleph)\underset{*}{\overset{*}{\sim}}(S,\aleph)=(Y,\aleph)$. Then, $\forall s\in\aleph$,

$$Y(s)=\begin{cases} D'(s), & s\in\aleph\setminus\aleph=\emptyset \\ D'(s)\cup S'(s), & s\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, $\forall s\in\aleph$, $Y(s)=D'(s)\cup S'(s)=D'(s)\cup U=U$. Hence $(Y,\aleph)=U_\aleph$.

7) $(D,\aleph)\underset{*}{\overset{*}{\sim}}\emptyset_E=\emptyset_E\underset{*}{\overset{*}{\sim}}(D,\aleph)=U_\aleph$.

Proof: Let $\emptyset_E=(S,E)$. Hence $\forall s\in E$; $S(s)=\emptyset$. Let $(D,\aleph)\underset{*}{\overset{*}{\sim}}(S,E)=(Y,\aleph)$. Thus, $\forall s\in\aleph$,

$$Y(s)=\begin{cases} D'(s), & s\in\aleph\text{-}E=\emptyset \\ D'(s)\cup S'(s), & s\in\aleph\cap E=\aleph \end{cases}$$

Hence, $\forall s\in\aleph$, $Y(s)=D'(s)\cup S'(s)=D'(s)\cup U=U$, so $(Y,\aleph)=U_{\aleph}$.

8) $(D,\aleph)\underset{*}{\overset{*}{\sim}}U_{\aleph}=U_{\aleph}\underset{*}{\overset{*}{\sim}}(D,\aleph)=(D,\aleph)^r$.

Proof: Let $U_{\aleph}=(T,\aleph)$. Hence, $\forall s\in\aleph$, $T(s)=U$. Let $(D,\aleph)\underset{*}{\overset{*}{\sim}}(T,\aleph)=(Y,\aleph)$. Hence, $\forall s\in\aleph$;

$$Y(s)=\begin{cases} D'(s), & s\in\aleph\setminus\aleph=\emptyset \\ D'(s)\cup T'(s), & s\in\aleph\cap\aleph=\aleph \end{cases}$$

Hence, $\forall s\in\aleph$, $Y(s)=D'(s)\cup T'(s)=D'(s)\cup\emptyset=D'(s)$, so $(Y,\aleph)=(D,\aleph)^r$.

9) $(D,P)\underset{*}{\overset{*}{\sim}}U_E=U_E\underset{*}{\overset{*}{\sim}}(D,\aleph)=(D,\aleph)^r$.

Proof: Let $U_E=(T,E)$. Hence, $\forall s\in E$, $T(s)=U$. Let $(D,\aleph)\underset{*}{\overset{*}{\sim}}(T,E)=(Y,\aleph)$, then $\forall s\in\aleph$;

$$Y(s)=\begin{cases} D'(s), & s\in\aleph\text{-}E=\emptyset \\ D'(s)\cup T'(s), & s\in\aleph\cap E=\aleph \end{cases}$$

Hence, $\forall s\in\aleph$, $Y(s)=D'(s)\cup T'(s)=D'(s)\cup\emptyset=D'(s)$, so $(Y,\aleph)=(D,\aleph)^r$

10) $(D,\aleph)\underset{*}{\overset{*}{\sim}}(D,\aleph)^r=(D,\aleph)^r\underset{*}{\overset{*}{\sim}}(D,\aleph)=U_{\aleph}$.

Proof: Let $(D,\aleph)^r=(Y,\aleph)$, so $\forall s\in\aleph$, $Y(s)=D'(s)$. Let $(D,P)\underset{*}{\overset{*}{\sim}}(Y,\aleph)=(T,\aleph)$, so $\forall s\in\aleph$,

$$T(s)=\begin{cases} D'(s), & s\in\aleph\setminus\aleph=\emptyset \\ D'(s)\cup Y'(s), & s\in\aleph\cap\aleph=\aleph \end{cases}$$

Hence, $\forall s\in\aleph$, $T(s)=D'(s)\cup Y'(s)=D'(s)\cup D(s)=U$, so $(T,\aleph)=U_{\aleph}$.

11) $[(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)]^r=(D,\aleph)\,\widetilde{\cap}\,(J,R)$.

Proof: Let $(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,R)=(Y,\aleph)$. Then, $\forall s\in\aleph$,

$$Y(s)=\begin{cases} D'(s), & s\in\aleph\setminus R \\ D'(s)\cup J'(s), & s\in\aleph\cap R \end{cases}$$

Let $(H,\aleph)^r=(T,\aleph)$, so $\forall s\in\aleph$,

$$T(s)=\begin{cases} D(s), & s\in\aleph\setminus R \\ D(s)\cap J(s), & s\in\aleph\cap R \end{cases}$$

Thus, $(T,\aleph)=(D,\aleph)\,\tilde{\cap}\,(J,R)$.

In classical theory, $\aleph\cup R=\emptyset \Leftrightarrow \aleph=\emptyset$ and $R=\emptyset$. Now, we have the following:

12) $(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,R)=\emptyset_{\aleph} \Leftrightarrow (D,\aleph)=U_{\aleph}$ and $(J,R)=U_{\aleph\cap R}$.

Proof: Let $(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,R)=(T,\aleph)$. Hence, $\forall s\in\aleph$,

$$T(s)=\begin{cases} D'(s), & s\in\aleph\setminus R \\ D'(s)\cup J'(s), & s\in\aleph\cap R \end{cases}$$

Since $(T,\aleph)=\emptyset_{\aleph}$, $\forall s\in\aleph$, $T(s)=\emptyset$. Hence, $\forall s\in\aleph\setminus R$, $D'(s)=\emptyset$, thus $D(s)=U$ and $\forall s\in\aleph\cap R$, $T(s)=D'(s)\cup J'(s)=\emptyset \Leftrightarrow \forall s\in\aleph\cap R$, $D'(s)=\emptyset$ and $J'(s)=\emptyset \Leftrightarrow \forall s\in\aleph$, $D(s)=U$ and for $\forall s\in\aleph\cap R$, $J(s)=U \Leftrightarrow (D,\aleph)=U_{\aleph}$ and $(J,R)=U_{\aleph\cap R}$.

13) $(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,\aleph)=\emptyset_{\aleph} \Leftrightarrow (D,\aleph)=(J,\aleph)=U_{\aleph}$.

Proof: Let $(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,\aleph)=(T,\aleph)$. Hence, $\forall s\in\aleph$,

$$T(s)=\begin{cases} D'(s), & s\in\aleph\setminus\aleph=\emptyset \\ D'(s)\cup J'(s), & s\in\aleph\cap\aleph=\aleph \end{cases}$$

Since $(T,\aleph)=\emptyset_{\aleph}$, $\forall s\in\aleph$, $T(s)=\emptyset$. Hence, $\forall s\in\aleph$, $T(s)=D'(s)\cup J'(s)=\emptyset \Leftrightarrow \forall s\in\aleph$, $D'(s)=\emptyset$ and $J'(s)=\emptyset \Leftrightarrow \forall s\in\aleph$, $D(s)=U$ and $J(s)=U \Leftrightarrow (D,\aleph)=(J,\aleph)=U_{\aleph}$.

In classical theory, for all $\aleph$, $\emptyset\subseteq\aleph$. Now, we have the following:

14) $\emptyset_{\aleph}\,\tilde{\subseteq}\,(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,R)$ and $\emptyset_{R}\,\tilde{\subseteq}\,(J,R)\overset{*}{\underset{*}{\sim}}(D,\aleph)$.

In classical theory, for all $\aleph, \aleph \subseteq U$. Now, we have the following:

15) $(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R) \mathrel{\tilde{\subseteq}} U_{\aleph}$ and $(J,R)\underset{*}{\overset{*}{\sim}}(D,\aleph) \mathrel{\tilde{\subseteq}} U_R$

In classical theory, $\aleph \subseteq \aleph \cup R$. Now, we have the following:

16) $(D,\aleph)^r \mathrel{\tilde{\subseteq}} (D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)$, however $(J,R)^r$ needs not to be a soft subset of $(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)$.

Proof: Let $(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)=(Y,\aleph)$. First of all, $\aleph \subseteq \aleph$. Moreover, $\forall s\in\aleph$,

$$Y(s)=\begin{cases} D'(s), & s\in\aleph\setminus R \\ D'(s)\cup J'(s), & s\in\aleph\cap R \end{cases}$$

Since $\forall s\in\aleph\setminus R$, $D'(s)\subseteq D'(s)$ and $\forall s\in\aleph\cap R$, $D'(s)\subseteq D'(s)\cup J'(s)$, hence $\forall s\in\aleph$, $D'(s)\subseteq Y(s)$. Therefore, $(D,\aleph)^r \mathrel{\tilde{\subseteq}} (Y,\aleph)=(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)$,

19) $(D,\aleph)^r \mathrel{\tilde{\subseteq}} (D,\aleph)\underset{*}{\overset{*}{\sim}}(J,\aleph)$, moreover $(J,\aleph)^r \mathrel{\tilde{\subseteq}} (D,\aleph)\underset{*}{\overset{*}{\sim}}(J,\aleph)$.

Proof: Let $(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,\aleph)=(Y,\aleph)$. First of all, $\aleph \subseteq \aleph$. Moreover, $\forall s\in\aleph$,

$$Y(s)=\begin{cases} D'(s), & s\in\aleph\setminus\aleph=\emptyset \\ D'(s)\cup J'(s), & s\in\aleph\cap\aleph=\aleph \end{cases}$$

Since $\forall s\in\aleph, Y(s)=D'(s)\subseteq D'(s)\cup J'(s)$, hence $(D,\aleph)^r \mathrel{\tilde{\subseteq}} (Y,\aleph) = (D,\aleph)\underset{*}{\overset{*}{\sim}}(J,\aleph)$.

4. Distribution Rules

In this section, distributions of complementary soft binary piecewise star $(*)$ operation over other soft set operations such as extended soft set operations, complementary extended soft set operations, restricted soft set operations, soft binary piecewise operations and complementary soft binary piecewise operation are examined in detail and many interesting results are obtained.

Proposition 4.1. Let $(D,\aleph)$, (J,R) and (Y,S) be soft sets over U. Then, for the distributions of complementary soft binary piecewise star $(*)$ operation over extended soft set operations, we have the followings:

i) $(D,\aleph)\underset{*}{\overset{*}{\sim}}[(J,R)\cap_{\varepsilon}(Y,S)] = [(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)] \cup_{\varepsilon} [(D,\aleph)\underset{*}{\overset{*}{\sim}}(Y,S)]$.

Proof: Let's first handle the left hand side of the equality and let $(J,R)\cap_{\varepsilon}(Y,S)=(M,R\cup S)$, so $\forall s\in R\cup S$,

$$M(s)=\begin{cases} J(s), & s\in R\setminus S \\ Y(s), & s\in S\setminus R \\ J(s)\cap Y(s), & s\in R\cap S \end{cases}$$

Let $(D,\aleph)\overset{*}{\underset{*}{\sim}}(M,R\cup S)=(N,\aleph)$, $\forall s\in\aleph$,

$$N(s)=\begin{cases} D'(s), & s\in\aleph\setminus(R\cup S) \\ D'(s)\cup M'(s), & s\in\aleph\cap(R\cup S) \end{cases}$$

Hence

$$N(s)=\begin{cases} D'(s), & s\in\aleph\setminus(R\cup S)=\aleph\cap R'\cap S' \\ D'(s)\cup J'(s), & s\in\aleph\cap(R\setminus S)=\aleph\cap R\cap S' \\ D'(s)\cup Y'(s), & s\in\aleph\cap(S\setminus R)=\aleph\cap R'\cap S \\ D'(s)\cup[(J'(s)\cup Y'(s)], & s\in\aleph\cap R\cap S=\aleph\cap R\cap S \end{cases}$$

Now let's handle the right hand side of the equality, that is, $[(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,R)]\cup_{\varepsilon}[(D,\aleph)\overset{*}{\underset{*}{\sim}}(Y,S)]$. Assume that $(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,R)=(V,\aleph)$, then for $\forall s\in\aleph$,

$$V(s)=\begin{cases} D'(s), & s\in\aleph\setminus R \\ D'(s)\cup J'(s), & s\in\aleph\cap R \end{cases}$$

Now let $(D,\aleph)\overset{*}{\underset{*}{\sim}}(Y,S)=(W,\aleph)$. Then, $\forall s\in\aleph$,

$$W(s)=\begin{cases} D'(s), & s\in\aleph\setminus S \\ D'(s)\cup Y'(s), & s\in\aleph\cap S \end{cases}$$

Assume that $(V,\aleph)\cup_{\varepsilon}(W,\aleph)=(T,\aleph)$, then $\forall s\in\aleph$,

$$T(s)=\begin{cases} V(s), & s\in\aleph\setminus\aleph=\emptyset \\ W(s), & s\in\aleph\setminus\aleph=\emptyset \\ V(s)\cup W(s), & s\in\aleph\cap\aleph=\aleph \end{cases}$$

$$T(s)=\begin{cases} D'(s)\cup D'(s), & s\in(\aleph\setminus R)\cap(\aleph\setminus S) \\ D'(s)\cup[D'(s)\cup Y'(s)], & s\in(\aleph\setminus R)\cap(\aleph\cap S) \\ [D'(s)\cup J'(s)]\cup D'(s), & s\in(\aleph\cap R)\cap(\aleph\setminus S) \\ [D'(s)\cup J'(s)]\cup[D'(s)\cup Y'(s)], & s\in(\aleph\cap R)\cap(\aleph\cap S) \end{cases}$$

Thus,

$$T(s)=\begin{cases} D'(s), & s\in ℵ\cap R'\cap S' \\ D'(s)\cup Y'(s), & s\in ℵ\cap R'\cap S \\ D'(s)\cup J'(s), & s\in ℵ\cap R\cap S' \\ [D'(s)\cup J'(s)]\ \cup\ [D'(s)\cup Y'(s)], & s\in ℵ\cap R\cap S \end{cases}$$

It is seen that (N,ℵ)= (T,ℵ).

ii) $(D,ℵ)\underset{*}{\overset{*}{\sim}}[(J,R)\cup_{\varepsilon}(Y,S)]=[(D,ℵ)\underset{*}{\overset{*}{\sim}}(J,R)]\cup_{\varepsilon}[(D,ℵ)\underset{*}{\overset{*}{\sim}}(Y,S)]$, where $ℵ\cap R\cap S=\emptyset$.

iii) $(D,ℵ)\underset{*}{\overset{*}{\sim}}[(J,R)\lambda_{\varepsilon}(Y,S)]=[(D,ℵ)\underset{*}{\overset{*}{\sim}}(J,R)]\ \tilde{\cap}\ [(Y,S)\underset{\lambda}{\overset{*}{\sim}}(D,ℵ)]$, where $ℵ\cap R'\cap S=\emptyset$.

iv) $(D,ℵ)\underset{*}{\overset{*}{\sim}}[(J,R)\backslash_{\varepsilon}(Y,S)]=[(D,ℵ)\underset{*}{\overset{*}{\sim}}(J,R)]\ \tilde{\cup}\ [(Y,S)\underset{\lambda}{\overset{*}{\sim}}(D,ℵ)]$ where $ℵ\cap R'\cap S=\emptyset$.

v) $[(D,ℵ)\cup_{\varepsilon}(J,R)]\underset{*}{\overset{*}{\sim}}(Y,S)=[(D,ℵ)\underset{*}{\overset{*}{\sim}}(Y,S)]\cap_{\varepsilon}[(J,R)\underset{*}{\overset{*}{\sim}}(Y,S)]$

Proof: Let's first handle the left hand side of the equality and let $(D,ℵ)\cap_{\varepsilon}(J,R)=(M,ℵ\cup R)$, so $\forall s\in ℵ\cup R$,

$$M(s)=\begin{cases} D(s), & s\in ℵ\backslash R \\ J(s), & s\in R\backslash ℵ \\ D(s)\cup J(s), & s\in ℵ\cap R \end{cases}$$

Let $(M,ℵ\cup R)\underset{*}{\overset{*}{\sim}}(Y,S)=(N,ℵ\cup R)$, so $\forall s\in ℵ\cup R$,

$$N(s)=\begin{cases} M'(s), & s\in (ℵ\cup R)\backslash S \\ M'(s)\cup Y'(s), & s\in (ℵ\cup R)\cap S \end{cases}$$

Thus,

$$N(s)=\begin{cases} D'(s), & s\in (ℵ\backslash R)\backslash S=ℵ\cap R'\cap S' \\ J'(s), & s\in (R\backslash ℵ)\backslash S=ℵ'\cap R\cap S' \\ D'(s)\cap J'(s), & s\in (ℵ\cap R)\backslash S=ℵ\cap R\cap S' \\ D'(s)\cup Y'(s), & s\in (ℵ\backslash R)\cap S=ℵ\cap R'\cap S \\ J'(s)\cup Y'(s), & s\in (R\backslash ℵ)\cap S=ℵ'\cap R\cap S \\ [D'(s)\cap J'(s)]\cup Y'(s), & s\in (ℵ\cap R)\cap S=ℵ\cap R\cap S \end{cases}$$

Now let's handle the right hand side of the equality: $[(D,ℵ)\underset{*}{\overset{*}{\sim}}(Y,S)]\cap_{\varepsilon}[(J,R)\underset{*}{\overset{*}{\sim}}(Y,S)]$. Let $(D,ℵ)\underset{*}{\overset{*}{\sim}}(Y,S)=(V,ℵ)$, so $\forall s\in ℵ$,

$$V(s)=\begin{cases} D'(s), & s\in\aleph\setminus S \\ D'(s)\cup Y'(s), & s\in\aleph\cap S \end{cases}$$

Let $(J,R)\underset{*}{\overset{*}{\sim}}(Y,S)=(W,R)$, so $\forall s\in R$,

$$W(s)=\begin{cases} J'(s), & s\in R\setminus S \\ J'(s)\cup Y'(s), & s\in R\cap S \end{cases}$$

Assume that $(V,\aleph)\cap_{\varepsilon}(W,R)=(T,\aleph\cup R)$, so $\forall s\in\aleph\cup R$,

$$T(s)=\begin{cases} V(s), & s\in\aleph\setminus R \\ W(s), & s\in R\setminus\aleph \\ V(s)\cap W(s), & s\in\aleph\cap R \end{cases}$$

Thus,

$$T(s)=\begin{cases} D'(s), & s\in(\aleph\setminus S)\setminus R=\aleph\cap R'\cap S' \\ D'(s)\cup Y'(s), & s\in(\aleph\cap S)\setminus R=\aleph\cap R'\cap S \\ J'(s), & s\in(R\setminus S)\setminus\aleph=\aleph'\cap R\cap S' \\ J'(s)\cup Y'(s), & s\in(R\cap S)\setminus\aleph=\aleph'\cap R\cap S \\ D'(s)\cap J'(s), & s\in(\aleph\setminus S)\cap(R\setminus S)=\aleph\cap R\cap S' \\ D'(s)\cap[J'(s)\cup Y'(s)], & s\in(\aleph\setminus S)\cap(R\cap S)=\emptyset \\ [D'(s)\cup Y'(s)]\cap J'(s), & s\in(\aleph\cap S)\cap(R\setminus S)=\emptyset \\ [D'(s)\cup Y'(s)]\cap[J'(s)\cup Y'(s)], & s\in(\aleph\cap S)\cap(R\cap S)=\aleph\cap R\cap S \end{cases}$$

It is seen that $(N,\aleph\cup R)=(T,\aleph\cup R)$.

vi) $[(D,\aleph)\cap_{\varepsilon}(J,R)]\underset{*}{\overset{*}{\sim}}(Y,S)=[(D,\aleph)\underset{*}{\overset{*}{\sim}}(Y,S)]\cup_{\varepsilon}[(J,R)\underset{*}{\overset{*}{\sim}}(Y,S)]$

vii) $[(D,\aleph)\lambda_{\varepsilon}(J,R)]\underset{*}{\overset{*}{\sim}}(Y,S)=[(D,\aleph)\underset{*}{\overset{*}{\sim}}(Y,S)]\cap_{\varepsilon}[(J,R)\underset{\lambda}{\overset{*}{\sim}}(Y,S)]$, where $\aleph\cap R\cap S'=\aleph'\cap R\cap S=\emptyset$.

viii) $[(D,\aleph)\setminus_{\varepsilon}(J,R)]\underset{*}{\overset{*}{\sim}}(Y,S)=[(D,\aleph)\underset{*}{\overset{*}{\sim}}(Y,S)]\cup_{\varepsilon}[(J,R)\underset{\lambda}{\overset{*}{\sim}}(Y,S)]$, where $\aleph\cap R\cap S'=\aleph'\cap R\cap S=\emptyset$.

Proposition 4.2. Let $(D,\aleph)$, (J,R) and (Y,S) be soft sets over U. Then, for the distributions of complementary soft binary piecewise star $(*)$ operation over complementary extended soft set operations, we have the followings:

i) $(D,\aleph)\underset{*}{\overset{*}{\sim}}[(J,R)\overset{*}{\theta}_{\varepsilon}(Y,S)]=[(D,\aleph)\underset{+}{\overset{*}{\sim}}(J,R)]\cup_{\varepsilon}[(D,\aleph)\underset{+}{\overset{*}{\sim}}(Y,S)]$, where $\aleph\cap R\cap S=\emptyset$

Proof: Let's first handle the left hand side of the equality. Assume $(J,R)\overset{*}{\theta_{\varepsilon}}(Y,S)=(M,R\cup S)$, so $\forall s\in R\cup S$,

$$M(s)=\begin{cases} J'(s), & s\in R\setminus S \\ Y'(s), & s\in S\setminus R \\ J'(s)\cap Y'(s), & s\in R\cap S \end{cases}$$

Let $(D,\aleph)\overset{*}{\underset{*}{\sim}}(M,R\cup S)=(N,\aleph)$, then $\forall s\in\aleph$,

$$N(s)=\begin{cases} D'(s), & s\in\aleph\setminus(R\cup S) \\ D'(s)\cup M'(s), & s\in\aleph\cap(R\cup S) \end{cases}$$

Hence,

$$N(s)=\begin{cases} D'(s), & s\in\aleph\setminus(R\cup S)=\aleph\cap R'\cap S' \\ D'(s)\cup J(s), & s\in\aleph\cap(R\setminus S)=\aleph\cap R\cap S' \\ D'(s)\cup Y(s), & s\in\aleph\cap(S\setminus R)=\aleph\cap R'\cap S \\ D'(s)\cup[(J(s)\cup Y(s)], & s\in\aleph\cap R\cap S=\aleph\cap R\cap S \end{cases}$$

Now let's handle the right hand side of the equality: $[(D,\aleph)\overset{*}{\underset{+}{\sim}}(J,R)]\cup_{\varepsilon}[(D,\aleph)\overset{*}{\underset{+}{\sim}}(Y,S)]$. Let $(D,\aleph)\overset{*}{\underset{+}{\sim}}(J,R)=(V,\aleph)$, so $\forall s\in\aleph$,

$$V(s)=\begin{cases} D'(s), & s\in\aleph\setminus R \\ D'(s)\cup J(s), & s\in\aleph\cap R \end{cases}$$

Let $(D,\aleph)\overset{*}{\underset{+}{\sim}}(Y,S)=(W,\aleph)$, hence $\forall s\in\aleph$,

$$W(s)=\begin{cases} D'(s), & s\in\aleph\setminus S \\ D'(s)\cup Y(s),, & s\in\aleph\cap S \end{cases}$$

Assume that $(V,\aleph)\cup_{\varepsilon}(W,\aleph)=(T,\aleph)$, hence $\forall s\in\aleph$,

$$T(s)=\begin{cases} V(s), & s\in\aleph\setminus\aleph=\emptyset \\ W(s), & s\in\aleph\setminus\aleph=\emptyset \\ V(s)\cup W(s), & s\in\aleph\cap\aleph=\aleph \end{cases}$$

Hence,

$$T(s)=\begin{cases} D'(s)\cup D'(s), & s\in(\aleph\setminus R)\cap(\aleph\setminus S) \\ D'(s)\cup[D'(s)\cup Y(s)], & s\in(\aleph\setminus R)\cap(\aleph\cap R) \\ [D'(s)\cup J(s)]\cup D'(s), & s\in(\aleph\cap R)\cap(\aleph\setminus S) \\ [D'(s)\cup J(s)]\cup[D'(s)\cup Y(s)], & s\in(\aleph\cap R)\cap(\aleph\cap S) \end{cases}$$

Thus,

$$T(s)=\begin{cases} D'(s), & s\in\aleph\cap R'\cap S' \\ D'(s)\cup Y(s), & s\in\aleph\cap R'\cap S \\ D'(s)\cup J(s), & s\in\aleph\cap R\cap S' \\ [D'(s)\cup J(s)]\cup[D'(s)\cup Y(s)], & s\in\aleph\cap R\cap S \end{cases}$$

It is seen that $(N,\aleph)=(T,\aleph)$.

ii) $(D,\aleph)\underset{*}{\overset{*}{\sim}}[(J,R)\underset{*}{\overset{*}{}}{}_{\varepsilon}(Y,S)]=[(D,\aleph)\underset{+}{\overset{*}{\sim}}(J,R)]\cup_{\varepsilon}[(D,\aleph)\underset{+}{\overset{*}{\sim}}(Y,S)]$ where $\aleph\cap R\cap S=\emptyset$.

iii) $(D,\aleph)\underset{\theta}{\overset{*}{\sim}}[(J,R)\overset{*}{+}_{\varepsilon}(Y,S)]=[(D,\aleph)\underset{+}{\overset{*}{\sim}}(J,R)]\,\tilde{\cap}\,[(Y,S)\underset{*}{\overset{*}{\sim}}(D,\aleph)]$ where $\aleph\cap R'\cap S=\emptyset$.

iv) $(D,\aleph)\underset{*}{\overset{*}{\sim}}[(J,R)\overset{*}{\gamma}_{\varepsilon}(Y,S)]=[(D,\aleph)\underset{+}{\overset{*}{\sim}}(J,R)]\,\tilde{\cup}[(Y,S)\underset{*}{\overset{*}{\sim}}(D,\aleph)]$ where $\aleph\cap R'\cap S=\emptyset$.

v) $[(D,\aleph)\underset{*}{\overset{*}{}}{}_{\varepsilon}(J,R)]\underset{*}{\overset{*}{\sim}}(Y,S)=[(D,\aleph)\tilde{\lambda}(Y,S)]\cap_{\varepsilon}[(J,R)\tilde{\lambda}(Y,S)]$.

Proof: Let's first handle the left hand side of the equality. Assume $(D,\aleph)\underset{*}{\overset{*}{}}{}_{\varepsilon}(J,R)=(M,\aleph\cup R)$ and $\forall s\in\aleph\cup R$,

$$M(s)=\begin{cases} D'(s), & s\in\aleph\setminus R \\ J'(s), & s\in R\setminus\aleph \\ D'(s)\cup J'(s), & s\in\aleph\cap R \end{cases}$$

Let $(M,\aleph\cup R)\underset{*}{\overset{*}{\sim}}(Y,S)=(N,\aleph\cup R)$ and $\forall s\in\aleph\cup R$,

$$N(s)=\begin{cases} M'(s), & s\in(\aleph\cup R)\setminus S \\ M'(s)\cup Y'(s), & s\in(\aleph\cup R)\cap S \end{cases}$$

Thus,

$$N(s)=\begin{cases} D(s), & s\in(\aleph\setminus R)\setminus S=\aleph\cap R'\cap S' \\ J(s), & s\in(R\setminus\aleph)\setminus S=\aleph'\cap R\cap S' \\ D(s)\cap J(s), & s\in(\aleph\cap R)\setminus S=\aleph\cap R\cap S' \\ D(s)\cup Y'(s), & s\in(\aleph\setminus R)\cap S=\aleph\cap R'\cap S \\ J(s)\cup Y'(s), & s\in(R\setminus\aleph)\cap S=\aleph'\cap R\cap S \\ [D(s)\cap J(s)]\cup Y'(s), & s\in(\aleph\cap R)\cap S=\aleph\cap R\cap S \end{cases}$$

Now let's handle the right hand side of the equality: $[(D,\aleph)\tilde{\lambda}(Y,S)] \cap_{\varepsilon} [(J,R)\tilde{\lambda}(Y,S)]$. Let $(D,\aleph)\tilde{\lambda}(Y,S)=(V,\aleph)$ and $\forall s\in\aleph$

$$V(s)=\begin{cases} D(s), & s\in\aleph\setminus S \\ D(s)\cup Y'(s), & s\in\aleph\cap S \end{cases}$$

Let $(J,R)\tilde{\lambda}(Y,S)=(W,R)$ and $\forall s\in R$,

$$W(s)=\begin{cases} J(s), & s\in R\setminus S \\ J(s)\cup Y'(s), & s\in R\cap S \end{cases}$$

Assume that $(V,\aleph)\cap_{\varepsilon}(W,R)=(T,\aleph\cup R)$ and $\forall s\in\aleph\cup R$,

$$T(s)=\begin{cases} V(s), & s\in\aleph\setminus R \\ W(s), & s\in R\setminus\aleph \\ V(s)\cap W(s), & s\in\aleph\cap R \end{cases}$$

Hence,

$$T(s)=\begin{cases} D(s), & s\in(\aleph\setminus S)\setminus R=\aleph\cap R'\cap S' \\ D(s)\cup Y'(s) & s\in(\aleph\cap S)\setminus R=\aleph\cap R'\cap S \\ J(s), & s\in(R\setminus S)\setminus\aleph=\aleph'\cap R\cap S' \\ J(s)\cup Y'(s), & s\in(R\cap S)\setminus\aleph=\aleph'\cap R\cap S \\ D(s)\cap J(s), & s\in(\aleph\setminus S)\cap(R\setminus S)=\aleph\cap R\cap S' \\ D(s)\cap[J(s)\cup Y'(s)], & s\in(\aleph\setminus S)\cap(R\cap S)=\emptyset \\ [D(s)\cup Y'(s)]\cap J(s), & s\in(\aleph\cap S)\cap(R\setminus S)=\emptyset \\ [D(s)\cup Y'(s)]\cap[J(s)\cup Y'(s)], & s\in(\aleph\cap S)\cap(R\cap S)=\aleph\cap R\cap S \end{cases}$$

It is seen that $(N,\aleph\cup R)=(T,\aleph\cup R)$.

vi) $[(D,\aleph)\overset{*}{\theta}_{\varepsilon}(J,R)]\overset{*}{\underset{*}{\sim}}(Y,S)=[(D,\aleph)\tilde{\lambda}(Y,S)]\cup_{\varepsilon}[(J,R)\tilde{\lambda}(Y,S)]$

vii) $[(D,\aleph)\overset{*}{\gamma}_{\varepsilon}(J,R)]\overset{*}{\underset{*}{\sim}}(Y,S)=[(D,\aleph)\underset{\lambda}{\sim}(Y,S)]\cup_{\varepsilon}[(J,R)\underset{*}{\sim}(Y,S)]$ where $\aleph\cap R\cap S'=\aleph'\cap R\cap S=\emptyset$.

viii) $[(D,\aleph)\overset{*}{+}_{\varepsilon}(J,R)]\overset{*}{\underset{*}{\sim}}(Y,S)=[(D,\aleph)\underset{\lambda}{\sim}(Y,S)]\cap_{\varepsilon}[(J,R)\underset{\lambda}{\sim}(Y,S)]$, where $\aleph\cap R\cap S'=\aleph\cap R\cap S=\emptyset$.

Proposition 4.3. Let $(D,\aleph)$, (J,R) and (Y,S) be soft sets over U. Then, for the distributions of complementary soft binary piecewise star ($*$) operation over soft binary piecewise operations, we have the followings:

i) $(D,\aleph)\overset{*}{\underset{*}{\sim}}[(J,R)\,\tilde{\cup}\,(Y,S)]=[(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,R)]\,\tilde{\cap}\,[(Y,S)\overset{*}{\underset{*}{\sim}}(D,\aleph)]$.

Proof: Let's first handle the left hand side of the equality and let $(J,R)\,\tilde{\cup}\,(Y,S)=(M,R)$, so $\forall s\in R\cup S$,

$$M(s)=\begin{cases} J(s), & s\in R\setminus S \\ J(s)\cup Y(s), & s\in R\cap S \end{cases}$$

$(D,\aleph)\underset{*}{\overset{*}{\sim}}(M,R)=(N,\aleph)$, where $\forall s\in\aleph$;

$$N(s)=\begin{cases} D'(s), & s\in\aleph\setminus R \\ D'(s)\cup M'(s), & s\in\aleph\cap R \end{cases}$$

Thus,

$$N(s)=\begin{cases} D'(s), & s\in\aleph\setminus R \\ D'(s)\cup J'(s), & s\in\aleph\cap(R\setminus S)=\aleph\cap R\cap S' \\ D'(s)\cup[J'(s)\cap Y'(s)], & s\in\aleph\cap R\cap S=\aleph\cap R\cap S \end{cases}$$

Now let's handle the right hand side of the equality: $[(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)]\ \tilde{\cap}\ [(Y,S)\underset{*}{\overset{*}{\sim}}(D,\aleph)]$. Assume that $(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)=(V,\aleph)$, then for $\forall s\in\aleph$,

$$V(s)=\begin{cases} D'(s), & s\in\aleph\setminus R \\ D'(s)\cup J'(s), & s\in\aleph\cap R \end{cases}$$

Now let $(Y,S)\underset{*}{\overset{*}{\sim}}(D,\aleph)=(W,S)$. Then, $\forall s\in S$,

$$W(s)=\begin{cases} Y'(s), & s\in S\setminus\aleph \\ Y'(s)\cup D'(s), & s\in S\cap\aleph \end{cases}$$

Assume that $(V,\aleph)\,\tilde{\cap}(W,S)=(T,\aleph)$, then $\forall s\in\aleph$,

$$T(s)=\begin{cases} V(s), & s\in\aleph\setminus S \\ V(s)\cap W(s), & s\in\aleph\cap S \end{cases}$$

Thus,

$$T(s)=\begin{cases} D'(s), & s\in(\aleph\setminus R)\setminus S=\aleph\cap R'\cap S' \\ D'(s)\cup J'(s) & s\in(\aleph\cap R)\setminus S=\aleph\cap R'\cap S \\ D'(s)\cap Y'(s), & s\in(\aleph\setminus R)\cap(S\setminus\aleph)=\emptyset \\ D'(s)\cap[Y'(s)\cup D'(s)], & s\in(\aleph\setminus R)\cap(S\cap\aleph)=\aleph\cap R'\cap S \\ [D'(s)\cup J'(s)]\cap Y'(s) & s\in(\aleph\cap R)\cap(S\setminus\aleph)=\emptyset \\ [D'(s)\cup J'(s)]\cap[Y'(s)\cup D'(s)], & s\in(\aleph\cap R)\cap(S\cap\aleph)=\aleph\cap R\cap S \end{cases}$$

Thus,

$$T(s)=\begin{cases} D'(s), & s\in(\aleph\setminus R)\setminus S=\aleph\cap R'\cap S' \\ D'(s)\cup J'(s) & s\in(\aleph\cap R)\setminus S=\aleph\cap R\cap S' \\ D'(s)\cap Y'(s), & s\in(\aleph\setminus R)\cap(S\setminus\aleph)=\emptyset \\ D'(s) & s\in(P\setminus R)\cap(S\cap\aleph)=\aleph\cap R'\cap S \\ [D'(s)\cup J'(s)]\cap Y'(s) & s\in(\aleph\cap R)\cap(S\setminus\aleph)=\emptyset \\ [D'(s)\cup J'(s)]\cap[Y'(s)\cup D'(s)], & s\in(\aleph\cap R)\cap(S\cap\aleph)=\aleph\cap R\cap S \end{cases}$$

Since $\aleph\setminus R=\aleph\cap R'$, if $s\in R'$, then $s\in S\setminus R$ or $s\in(R\cup S)'$. Hence, if $s\in\aleph\setminus R$, $s\in\aleph\cap R'\cap S'$ or $s\in\aleph\cap R'\cap S$. Thus, it is seen that $(N,\aleph)=(T,\aleph)$.

ii) $(D,\aleph)\underset{*}{\overset{*}{\sim}}[(J,R)\ \tilde{\cap}\ (Y,S)] = [(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)]\ \tilde{\cap}\ [(Y,S)\underset{*}{\overset{*}{\sim}}(D,\aleph)]$ where $\aleph\cap R\cap S=\emptyset$

iii) $(D,\aleph)\underset{*}{\overset{*}{\sim}}[(J,R)\ \tilde{\lambda}\ (Y,S)] = [(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)]\ \tilde{\cap}\ [(Y,S)\underset{\lambda}{\overset{*}{\sim}}(D,\aleph)]$.

iv) $(D,\aleph)\underset{*}{\overset{*}{\sim}}[(J,R)\ \tilde{\setminus}\ (Y,S)] = [(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)]\ \tilde{\cap}\ [(Y,S)\underset{\lambda}{\overset{*}{\sim}}(D,\aleph)]$.

v) $[(D,\aleph)\ \tilde{\cap}\ (J,R)]\underset{*}{\overset{*}{\sim}}(Y,S)= [(D,\aleph)\underset{*}{\overset{*}{\sim}}(Y,S)]\ \tilde{\cup}\ [(J,R)\underset{*}{\overset{*}{\sim}}(Y,S)]$

Proof: Let's first handle the left hand side of the equality. Suppose $(D,\aleph)\ \tilde{\cap}\ (J,R)=(M,\aleph)$, so $\forall s\in\aleph$ için,

$$M(s)=\begin{cases} D(s), & s\in\aleph\setminus R \\ D(s)\cap J(s), & s\in\aleph\cap R \end{cases}$$

Let $(M,\aleph)\underset{*}{\overset{*}{\sim}}(Y,S)=(N,\aleph)$, so $\forall s\in\aleph$,

$$N(s)=\begin{cases} M'(s), & s\in\aleph\setminus S \\ M'(s)\cup Y'(s), & s\in\aleph\cap S \end{cases}$$

Thus,

$$N(s)=\begin{cases} D'(s), & s\in(\aleph\setminus R)\setminus S=\aleph\cap R'\cap S' \\ D'(s)\cup J'(s), & s\in(\aleph\cap R)\setminus S=\aleph\cap R\cap S' \\ D'(s)\cup Y'(s), & s\in(\aleph\setminus R)\cap S=\aleph\cap R'\cap S \\ [D'(s)\cup J'(s)]\cup Y'(s), & s\in(\aleph\cap R)\cap S=\aleph\cap R\cap S \end{cases}$$

Now let's handle the right hand side of the equality: $[(D,\aleph)\overset{*}{\underset{*}{\sim}}(Y,S)]\ \tilde{\cup}\ [(J,R)\overset{*}{\underset{*}{\sim}}(Y,S)]$. Let $(D,\aleph)\overset{*}{\underset{*}{\sim}}(Y,S)=(V,\aleph)$, so $\forall s\in\aleph$,

$$V(s)=\begin{cases} D'(s), & s\in\aleph\setminus S \\ D'(s)\cup Y'(s), & s\in\aleph\cap S \end{cases}$$

Let $(J,R)\overset{*}{\underset{*}{\sim}}(Y,S)=(W,R)$, so $\forall s\in R$,

$$W(s)=\begin{cases} J'(s), & s\in R\setminus S \\ J'(s)\cup Y'(s), & s\in R\cap S \end{cases}$$

Assume that $(V,\aleph)\tilde{\cup}(W,R)=(T,\aleph)$, so $\forall s\in\aleph$,

$$T(s)=\begin{cases} V(s), & s\in\aleph\setminus R \\ V(s)\cup W(s), & s\in\aleph\cap R \end{cases}$$

Hence,

$$T(s)=\begin{cases} D'(s), & s\in(\aleph\setminus S)\setminus R=\aleph\cap R'\cap S' \\ D'(s)\cup Y'(s), & s\in(\aleph\cap S)\setminus R=\aleph\cap R'\cap S \\ D'(s)\cup J'(s), & s\in(\aleph\setminus S)\cap(R\setminus S)=\aleph\cap R\cap S' \\ D'(s)\cup\ [J'(s)\cup Y'(s)], & s\in(\aleph\setminus S)\cap(R\cap S)=\emptyset \\ [D'(s)\cup Y'(s)]\ \cup J'(s), & s\in(\aleph\cap S)\cap(R\setminus S)=\emptyset \\ [D'(s)\cup Y'(s)]\ \cup\ [J'(s)\cup Y'(s)], & s\in(\aleph\cap S)\cap(R\cap S)=\aleph\cap R\cap S \end{cases}$$

It is seen that $(N,\aleph)=(T,\aleph)$.

vi) $[(D,\aleph)\ \tilde{\cup}\ (J,R)]\overset{*}{\underset{*}{\sim}}(Y,S)=[(D,\aleph)\overset{*}{\underset{*}{\sim}}(Y,S)]\ \tilde{\cap}\ [(J,R)\overset{*}{\underset{*}{\sim}}(Y,S)]$

vii) $[(D,\aleph)\ \tilde{\setminus}(J,R)]\overset{*}{\underset{*}{\sim}}(Y,S)=[(D,\aleph)\overset{*}{\underset{*}{\sim}}(Y,S)]\ \tilde{\cup}\ [(J,R)\ \tilde{\lambda}\ (Y,S)]$.

viii) $[(D,\aleph)\ \tilde{\lambda}(J,R)]\overset{*}{\underset{*}{\sim}}(Y,S)=[(D,\aleph)\overset{*}{\underset{*}{\sim}}(Y,S)]\ \tilde{\cap}\ [(J,R)\ \tilde{\lambda}\ (Y,S)]$.

Proposition 4.4. Let $(D,\aleph)$, (J,R) and (Y,S) be soft sets over U. Then, for the distributions of complementary soft binary piecewise star ($*$) operation over complementary soft binary piecewise operations we have the followings:

i) $(D,\aleph)\overset{*}{\underset{*}{\sim}}[(J,R)\overset{*}{\underset{*}{\sim}}(Y,S)]=[(D,\aleph)\overset{*}{\underset{+}{\sim}}(J,R)]\ \tilde{\cap}\ [(D,\aleph)\overset{*}{\underset{+}{\sim}}(Y,S)]$ where $\aleph\cap R\cap S'=\emptyset$

Proof: Let's first handle the left hand side of the equality, suppose $(J,R) \underset{*}{\overset{*}{\sim}} (Y,S)=(M,R)$, so $\forall s\in R$,

$$M(s)=\begin{cases} J'(s), & s\in R\backslash S \\ J'(s)\cup Y'(s), & s\in R\cap S \end{cases}$$

Let $(D,\aleph) \underset{*}{\overset{*}{\sim}} (M,R)=(N,\aleph)$, so $\forall s\in\aleph$,

$$N(s)=\begin{cases} D'(s), & s\in\aleph\backslash R \\ D'(s)\cup M'(s), & s\in(\aleph\cap R \end{cases}$$

Thus,

$$N(s)=\begin{cases} D'(s), & s\in\aleph\backslash R \\ D'(s)\cup J(s), & s\in\aleph\cap(R\backslash S)=\aleph\cap R\cap S' \\ D'(s)\cup\ [(J(s)\cap H(s)], & s\in\aleph\cap R\cap S=\aleph\cap R\cap S \end{cases}$$

Now let's handle the right hand side of the equality: $(D,\aleph) \underset{+}{\overset{*}{\sim}} (J,R)]\ \tilde{\cap}\ [(D,\aleph) \underset{+}{\overset{*}{\sim}} (Y,S)]$. Let $(D,\aleph) \underset{+}{\overset{*}{\sim}}$ $(J,R)=(V,\aleph)$, so $\forall s\in\aleph$,

$$V(s)=\begin{cases} D'(s), & s\in\aleph\backslash R \\ D'(s)\cup J(s), & s\in\aleph\cap R \end{cases}$$

Let $(D,\aleph) \underset{+}{\overset{*}{\sim}} (Y,S)=(W,\aleph)$, so $\forall s\in\aleph$,

$$W(s)=\begin{cases} D'(s), & s\in\aleph\backslash S \\ D'(s)\cup Y(s), & s\in\aleph\cap S \end{cases}$$

Assume that $(V,\aleph)\tilde{\cap}(W,\aleph)=(T,\aleph)$, so $\forall s\in\aleph$,

$$T(s)=\begin{cases} V(s), & s\in\aleph\backslash\aleph=\emptyset \\ V(s)\cap W(s), & s\in(\aleph\cap\aleph=\aleph \end{cases}$$

Thus,

$$T(s)=\begin{cases} D'(s) & s\in(\aleph\backslash R)\cap(\aleph\backslash S)=\aleph\cap R'\cap S' \\ D'(s)\cap\ [D'(s)\cup Y(s)], & s\in(\aleph\backslash R)\cap(\aleph\cap S)=\aleph\cap R'\cap S \\ [D'(s)\cup J(s)]\cap D'(s), & s\in(\aleph\cap R)\cap(\aleph\backslash S)=\aleph\cap R\cap S' \\ [D'(s)\cup J(s)]\cap[D'(t)\cup Y(t)], & s\in(\aleph\cap R)\cap(\aleph\cap S)=\aleph\cap R\cap S \end{cases}$$

Thus,

$$T(s)=\begin{cases} D'(s) & s\in(\aleph\backslash R)\cap(\aleph\backslash S)=\aleph\cap R'\cap S' \\ D'(s), & s\in(\aleph\backslash R)\cap(\aleph\cap S)=\aleph\cap R'\cap S \\ D'(s), & s\in(\aleph\cap R)\cap(\aleph\backslash S)=\aleph\cap R\cap S' \\ [D'(s)\cup J(s)]\cap[D'(t)\cup Y(t)], & s\in(\aleph\cap R)\cap(\aleph\cap S)=\aleph\cap R\cap S \end{cases}$$

Since $\aleph\backslash R=\aleph\cap R'$, if $s\in R'$, then $s\in S\backslash R$ or $s\in(R\cup S)'$. Hence, if $s\in\aleph\backslash R$, $s\in\aleph\cap R'\cap S'$ or $s\in\aleph\cap R'\cap S$. Thus, it is seen that $(N,\aleph)=(T,\aleph)$.

ii) $(D,\aleph)\underset{*}{\overset{*}{\sim}}[(J,R)\underset{\theta}{\overset{*}{\sim}}(Y,S)]=[(D,\aleph)\underset{+}{\overset{*}{\sim}}(J,R)]\ \tilde{\cup}\ [(D,\aleph)\underset{+}{\overset{*}{\sim}}(Y,S)]$, where $\aleph\cap R'\cap S=\emptyset$

iii) $(D,\aleph)\underset{*}{\overset{*}{\sim}}[(J,R)\underset{\gamma}{\overset{*}{\sim}}(Y,S)]=[(D,\aleph)\underset{+}{\overset{*}{\sim}}(J,R)]\ \tilde{\cup}\ [(Y,S)\underset{*}{\overset{*}{\sim}}(D,\aleph)]$, where $\aleph\cap R'\cap S=\emptyset$

iv) $(D,\aleph)\underset{*}{\overset{*}{\sim}}[(J,R)\underset{+}{\overset{*}{\sim}}(Y,S)]=[(D,\aleph)\underset{+}{\overset{*}{\sim}}(J,R)]\ \tilde{\cap}\ [(Y,S)\underset{*}{\overset{*}{\sim}}(D,\aleph)]$

v) $(D,\aleph)\underset{*}{\overset{*}{\sim}}[(J,R)\underset{\cap}{\overset{*}{\sim}}(Y,S)]=[(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)]\tilde{\cup}[(Y,S)\underset{*}{\overset{*}{\sim}}(D,\aleph)]$, where $\aleph\cap R\cap S'=\emptyset$ and $\aleph\cap R'\cap S=\emptyset$.

vi) $[(D,\aleph)\underset{\theta}{\overset{*}{\sim}}(J,R)]\underset{*}{\overset{*}{\sim}}(Y,S)=[(D,\aleph)\tilde{\lambda}(Y,S)]\ \tilde{\cup}\ [(J,R)\tilde{\lambda}(Y,S)]$

Proof: Let's first handle the left hand side of the equality. Let $(D,\aleph)\underset{\theta}{\overset{*}{\sim}}(J,R)=(M,\aleph)$, so $\forall s\in\aleph$,

$$M(s)=\begin{cases} D'(s), & s\in\aleph\backslash R \\ D'(s)\cap J'(s), & s\in\aleph\cap R \end{cases}$$

Let $(M,\aleph)\underset{*}{\overset{*}{\sim}}(Y,S)=(N,\aleph)$, so $\forall s\in\aleph$,

$$N(s)=\begin{cases} M'(s), & s\in\aleph\backslash S \\ M'(s)\cup Y'(s), & s\in\aleph\cap S \end{cases}$$

Thus,

$$N(s)=\begin{cases} D(s), & s\in(\aleph\backslash R)\backslash S=\aleph\cap R'\cap S' \\ D(s)\cup J(s), & s\in(\aleph\cap R)\backslash S=\aleph\cap R\cap S' \\ D(s)\cup Y'(s), & s\in(\aleph\backslash R)\cap S=\aleph\cap R'\cap S \\ [D(s)\cup J(s)]\cup Y'(s), & s\in\aleph\cap R\cap S=\aleph\cap R\cap S \end{cases}$$

Now let's handle the right hand side of the equality: $[(D,\aleph)\tilde{\lambda}(Y,S)]\ \tilde{\cup}\ [(J,R)\tilde{\lambda}(Y,S)]$. Let $(D,\aleph)\tilde{\lambda}(Y,S)=(V,\aleph)$, so $\forall s\in\aleph$,

$$V(s)=\begin{cases} D(s), & s\in\aleph\setminus S \\ D(s)\cup Y'(s), & s\in\aleph\cap S \end{cases}$$

Let $(J,R)\ \tilde{\lambda}\ (Y,S)=(W,R)$, so $\forall s\in R$,

$$W(s)=\begin{cases} J(s), & s\in R\setminus S \\ J(s)\cup Y'(s), & s\in R\cap S \end{cases}$$

Let $(V,\aleph)\ \tilde{\cup}\ (W,R)=(T,\aleph)$, so $\forall s\in\aleph$,

$$T(s)=\begin{cases} V(s), & s\in\aleph\setminus R \\ V(s)\cup W(s), & s\in\aleph\cap R \end{cases}$$

Thus,

$$T(s)=\begin{cases} D(s), & s\in((P\setminus S)\setminus R=\aleph\cap R'\cap S' \\ D(s)\cup Y'(s), & s\in(\aleph\cap S)\setminus R=\aleph\cap R'\cap S \\ D(s)\cup J(s), & s\in(\aleph\setminus S)\cap(R\setminus S)=\aleph\cap R\cap S' \\ D(s)\cup[J(s)\cup Y'(s)], & s\in(\aleph\setminus S)\cap(R\cap S)=\emptyset \\ [D(s)\cup Y'(s)]\cup J(s), & s\in(\aleph\cap S)\cap(R\setminus S)=\emptyset \\ [D(s)\cup Y'(s)]\cup J([s)\cup Y'(s)], & s\in(\aleph\cap S)\cap(R\cap S)=\aleph\cap R\cap S \end{cases}$$

It is seen that $(N,\aleph)=(T,\aleph)$.

vii) $(D,P)\underset{*}{\overset{*}{\sim}}(J,R)]\ \underset{*}{\overset{*}{\sim}}(Y,S)=\ [(D,\aleph)\tilde{\lambda}(Y,S)]\ \tilde{\cap}\ [(J,R)\ \tilde{\lambda}(Y,S)]$

viii)$[(D,P)\underset{\gamma}{\overset{*}{\sim}}(J,R)]\ \underset{*}{\overset{*}{\sim}}(Y,S)=\ [(D,\aleph)\tilde{_\lambda}(Y,S)]\ \tilde{\cup}\ [(J,R)\ \tilde{_*}(Y,S)]$, where $\aleph\cap R\cap S'=\emptyset$

ix) $[(D,P)\underset{+}{\overset{*}{\sim}}(J,R)]\ \underset{*}{\overset{*}{\sim}}(Y,S)=\ [(D,\aleph)\tilde{_\lambda}(Y,S)]\ \tilde{\cap}\ [(J,R)\underset{\lambda}{\overset{*}{\sim}}(Y,S)]$, where $\aleph\cap R\cap S=\emptyset$

Proposition 4.5. Let $(D,\aleph)$, (J,R) and (Y,S) be soft sets over U. Then, for the distributions of complementary soft binary piecewise star ($*$) operation over restricted soft set operations, we have the followings:

i)$(D,\aleph)\underset{*}{\overset{*}{\sim}}[(J,R)\ \cap_R(Y,S)]=[(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)]\cap_R\ [(D,\aleph)\underset{*}{\overset{*}{\sim}}(Y,S)]$, where $\aleph\cap R\cap S=\emptyset$.

Proof: Let's first handle the left hand side of the equality, suppose $(J,R)\ \cap_R(Y,S)=(M,R\cap S)$ and so $\forall s\in R\cap S$, $M(s)=J(s)\cap Y(s)$. Let $(D,\aleph)\underset{*}{\overset{*}{\sim}}(M,R\cap S)=(N,\aleph)$, so $\forall s\in\aleph$,

$$N(s)=\begin{cases} D'(s), & s\in\aleph\setminus(R\cap S) \\ D'(s)\cup M'(s), & s\in\aleph\cap(R\cap S) \end{cases}$$

Thus,

$$N(s)=\begin{cases} D'(s), & s\in\aleph\setminus(R\cap S) \\ D'(s)\cup\ [J'(s)\cup Y'(s)], & s\in\aleph\cap(R\cap S) \end{cases}$$

Now let's handle the right hand side of the equality: $[(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)]\ \cap_R\ [(D,\aleph)\underset{*}{\overset{*}{\sim}}(Y,S)]$. Let $(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)=(V,\aleph)$, so $\forall s\in\aleph$,

$$V(s)=\begin{cases} D'(s), & s\in\aleph\setminus R \\ D'(s)\cup J'(s), & s\in\aleph\cap R \end{cases}$$

Let $(D,\aleph)\underset{*}{\overset{*}{\sim}}(Y,S)=(W,\aleph)$, so $\forall s\in\aleph$,

$$W(s)=\begin{cases} D'(s), & s\in\aleph\setminus S \\ D'(s)\cup Y'(s), & s\in\aleph\cap S \end{cases}$$

Assume that $(V,\aleph)\cap_R(W,\aleph)=(T,\aleph)$, and so $\forall s\in\aleph$, $T(s)=V(s)\cap W(s)$,

$$T(s)=\begin{cases} D'(s)\cap D'(s), & s\in(P\setminus R)\cap(\aleph\setminus S) \\ D'(s)\cap[D'(s)\cup Y'(s)], & s\in(\aleph\setminus R)\cap(\aleph\cap S) \\ [D'(s)\cup J'(s)]\cap D'(s), & s\in(\aleph\cap R)\cap(\aleph\setminus S) \\ [D'(s)\cup J'(s)]\cap[D'(s)\cup Y'(s)], & s\in(\aleph\cap R)\cap(\aleph\cap S) \end{cases}$$

Hence,

$$T(s)=\begin{cases} D'(s), & s\in\aleph\cap R'\cap S' \\ D'(s), & s\in\aleph\cap R'\cap S \\ D'(s), & s\in\aleph\cap R\cap S' \\ [D'(s)\cap J'(s)]\cup[D'(s)\cap Y'(s)], & s\in\aleph\cap R\cap S \end{cases}$$

Considering the parameter set of the first equation of the first row, that is, $\aleph\setminus(R\cap S)$; since $\aleph\setminus(R\cap S)=\aleph\cap(R\cap S)'$, an element in $(R\cap S)'$ may be in $R\setminus S$, in $S\setminus R$ or $(R\cup S)$. Then, $\aleph\setminus(R\cap S)$ is equivalent to the following 3 states: $\aleph\cap(R\cap S')$, $\aleph\cap(R'\cap S)$ and $\aleph\cap(R'\cap S')$. Hence, $(N,\aleph)=(T,\aleph)$.

ii) $(D,\aleph)\underset{*}{\overset{*}{\sim}}[(J,R)\ \cup_R(Y,S)]\ =\ [(D,\aleph)\underset{*}{\overset{*}{\sim}}(J,R)]\ \cap_R\ [(D,\aleph)\underset{*}{\overset{*}{\sim}}(Y,S)]$.

iii) $(D,\aleph)\overset{*}{\underset{*}{\sim}}[(J,R) *_R (Y,S)] = [(D,\aleph)\overset{*}{\underset{+}{\sim}}(J,R)] \cap_R [(D,\aleph)\overset{*}{\underset{+}{\sim}}(Y,S)]$.

iv) $(D,\aleph)\overset{*}{\underset{*}{\sim}}[(J,R)\,\theta_R(Y,S)]=[(D,\aleph)\overset{*}{\underset{+}{\sim}}(J,R)]\cap_R[(D,\aleph)\overset{*}{\underset{+}{\sim}}(Y,S)]$, where $\aleph\cap R\cap S=\emptyset$.

v) $(D,\aleph)\overset{*}{\underset{*}{\sim}}[(J,R)\setminus_R(Y,S)]=[(D,\aleph)\overset{*}{\underset{*}{\sim}}(J,R)] \cap_R [(D,\aleph)\overset{*}{\underset{+}{\sim}}(Y,S)]$.

vi) $(D,\aleph)\overset{*}{\underset{*}{\sim}}[(J,R) +_R(Y,S)] = [(D,\aleph)\overset{*}{\underset{+}{\sim}}(J,R)] \cap_R [(D,\aleph)\overset{*}{\underset{*}{\sim}}(Y,S)]$.

vi) $(D,\aleph)\overset{*}{\underset{*}{\sim}}[(J,R)\,\gamma_R(Y,S)] =[(D,\aleph)\overset{*}{\underset{\gamma}{\sim}}(J,R)]\cup_R [(D,\aleph)\overset{*}{\underset{\theta}{\sim}}(Y,S)]$, where $\aleph\cap R\cap S =\emptyset$.

viii) $[(D,\aleph)\cup_R (J,R)]\overset{*}{\underset{*}{\sim}}(Y,S)= [(D,\aleph)\overset{*}{\underset{*}{\sim}}(Y,S)] \cap_R [(J,R)\overset{*}{\underset{*}{\sim}}(Y,S)]$.

Proof: Let's first handle the left hand side of the equality, suppose $(D,\aleph)\cup_R(J,R)=(M,\aleph\cap R)$ so, $\forall s\in\aleph\cap R$, $M(s)=D(s)\cup J(s)$. Let $(M,\aleph\cap R)\overset{*}{\underset{*}{\sim}}(Y,S)=(N,\aleph\cap R)$, so $\forall s\in\aleph\cap R$,

$$N(s)=\begin{cases} M'(s), & s\in(\aleph\cap R)\setminus S \\ M'(s)\cup Y'(s), & s\in(\aleph\cap R)\cap S \end{cases}$$

Hence,

$$N(s)=\begin{cases} D'(s)\cap J'(s), & s\in(\aleph\cap R)\setminus S= \aleph\cap R\cap S' \\ [D'(s)\cap J'(s)]\cup Y'(s), & s\in(\aleph\cap R)\cap S \end{cases}$$

Now let's handle the right hand side of the equality: $[(D,\aleph)\overset{*}{\underset{*}{\sim}}(Y,S)] \cap_R [(J,R)\overset{*}{\underset{*}{\sim}}(Y,S)]$. Let $(D,\aleph)\overset{*}{\underset{*}{\sim}}(Y,S)=(V,\aleph)$, so $\forall s\in\aleph$,

$$V(s)=\begin{cases} D'(s), & s\in\aleph\setminus S \\ D'(s)\cup Y'(s), & s\in\aleph\cap S \end{cases}$$

Let $(J,R)\overset{*}{\underset{*}{\sim}}(Y,S)=(W,R)$, so $\forall s\in R$,

$$W(s)=\begin{cases} J'(s), & s\in R\setminus S \\ J'(s)\cup Y'(s), & s\in R\cap S \end{cases}$$

Suppose that $(V,\aleph)\cap_R (W,R)=(T,\aleph\cap R)$, so $\forall s\in\aleph\cap R$, $T(s)=V(s)\cap W(s)$. Thus,

$$T(s)=\begin{cases} D'(s)\cap J'(s), & s\in(\aleph\setminus S)\cap(R\setminus S)=\aleph\cap R\cap S' \\ D'(s)\cap[J'(s)\cup Y'(s)], & s\in(\aleph\setminus S)\cap(R\cap S)=\emptyset \\ [D'(s)\cup Y'(s)]\cap J'(s), & s\in(\aleph\cap S)\cap(R\setminus S)=\emptyset \\ [D'(s)\cup Y'(s)]\cap[J'(s)\cup Y'(s)], & s\in(\aleph\cap S)\cap(R\cap S)=\aleph\cap R\cap S \end{cases}$$

It is seen that $(N,\aleph\cap R)=(T,\aleph\cap R)$,

ix) $[(D,\aleph)\cap_R(J,R)]\ \overset{*}{\underset{*}{\sim}}\ (Y,S)=[(D,\aleph)\overset{*}{\underset{*}{\sim}}(Y,S)]\cup_R[(J,R)\overset{*}{\underset{*}{\sim}}(Y,S)]$.

x) $[(D,\aleph)*_R(J,R)]\ \overset{*}{\underset{*}{\sim}}\ (Y,S)=[(D,\aleph)\tilde{\lambda}(Y,S)]\cap_R[(J,R)\tilde{\lambda}(Y,S)]$.

xi) $[(D,\aleph)\theta_R(J,R)]\ \overset{*}{\underset{*}{\sim}}\ (Y,S)=[(D,\aleph)\tilde{\setminus}(Y,S)]\cup_R[(J,R)\tilde{\setminus}(Y,S)]$.

5. Conclusion

The concept of soft set operations is an essential concept similar to fundamental operations on numbers and basic operations on sets. Proposing new soft operations and deriving their algebraic properties and implementations provide new perspectives for dealing with problems related to parametric data. The operations in soft set theory have proceed under two main headings up to now, as restricted soft set operations and extended soft set operations. In this paper, we contribute to the soft set literature by defining a new kind of soft set operation. Inspired by these ideas, in this paper we have defined a new soft set operation which we call complementary soft binary piecewise star operation. The basic algebraic properties of the operations have been investigated. Moreoverby examining the distribution rules, we have obtained the relationships between this new soft set operation and other types of soft set operations such as extended soft set operations, complementary extended soft set operations, soft binary piecewise operations, complementary soft binary piecewise operations and restricted soft set operations. This paper can be regarded as a theoretical study for soft sets and some future studies may continue by examining the distribution of other soft set operations over complementary soft binary piecewise star operation. Furthermore, since soft sets are a powerful mathematical tool for detecting insecure objects, researchers may be able to propose new cryptographic or decision methods based on soft sets with the help of this new soft set operation. Also, the study of soft algebraic structures can be handled again in terms of algebraic properties by the operations defined in this article and thus studying the algebraic structure of soft sets from the perspective of this new operation provides deep insight into the algebraic structure of soft sets.

Author Contributions

All authors contributed equally to this work. They all read and approved the final version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] Akbulut, E. (2023) New type of extended operations of soft set: Complementary extended lambda and difference operations. MSc. Thesis, Amasya University, The Graduate School of Natural and Applied Sciences Master of Science in Mathematics Department, Amasya.

[2] Ali, M.I., Feng, F., Liu, X., Min, W.K., Shabir, M. (2009) On some new operations in soft set theory. Computers and Mathematics with Applications, 57(9): 1547-1553.

[3] Ali, M.I., Shabir, M., Naz, M., (2011) Algebraic structures of soft sets associated with new operations, Computers and Mathematics with Applications 61 2647–2654.

[4] Aybek, F. (2023) New restricted and extended soft set operations. MSc. thesis, Amasya University, The Graduate School of Natural and Applied Sciences Master of Science in Mathematics Department, Amasya.

[5] Çağman N. (2021) Conditional complements of sets and their application to group theory. Journal of New Results in Science, 10 (3): 67-74.

[6] Çağman N., Enginoğlu, S. (2010) Soft set theory and uni-int decision making, Eurepean Journal of Operational Research, 207: 848-855.

[7] Dalkılıç O. (2021) A novel approach to soft set theory in decision-making under uncertainty, International Journal of Computer Mathematics, 98:10, 1935-1945.

[8] Dalkılıç O. (2021) Determining the (non-)membership degrees in the range (0,1) independently of the decision-makers for bipolar soft sets, Journal of Taibah University for Science, 15:1, 609-618.

[9] Dalkılıç O. (2022) Determining the membership degrees in the range (0, 1) for hypersoft sets independently of the decision-maker, International Journal of Systems Science, 53:8, 1733-1743.

[10] Dalkılıç O. (2022) Approaches that take into account interactions between parameters: pure (fuzzy) soft sets, International Journal of Computer Mathematics, 99:7, 1428 1437.

[11] Demirci A.M. (2023) New type of extended operations of soft set: Complementary extended plus, union and theta operations. MSc. Thesis, Amasya University, The Graduate School of Natural and Applied Sciences Master of Science in Mathematics Department, Amasya.

[12] Eren Ö.F. and Çalışıcı H. (2019) On some operations of soft sets, The Fourth International Conference on Computational Mathematics and Engineering Sciences (CMES-2019), Antalya.

[13] Feng, F., Li, Y.M., Davvaz, B., Ali, M. I., (2010) Soft Sets Combined With Fuzzy Sets and Rough Sets: A Tantative Approach, Soft Computing, 14 899–911.

[14] Fu, L., Notes on soft set operations, (2011) ARPN Journal of systems and softwares, 1, 205-208.

[15] Ge, X., Yang S., (2011) Investigations on some operations of soft sets, World academy of Science, Engineering and Technology, 75 1113-1116.

[16] Husain, S., Shamsham, Km., (2018) A study of properties of soft set and its applications, International Research Journal of Engineering and Technology, 5 (1) 363-372.

[17] Jayanta, S., (2014) On algebraic structure of soft sets, Annals of Fuzzy Mathematics and Informatics, Volume 7 (6) 1013-1020.

[18] Jiang, J., Tang, Y., Chen, Q., Wang, J., Tang, S., (2010) Extending soft sets with description logics, Computers and Mathematics with Applications, 59 2087–2096.

[19] Neog, I.J, Sut, D.K., (2011) A new Approach to the Theory of Soft Set, International Journal of Computer Applications, 32 (2) 1-6.

[20] Maji, P.K, Bismas. R., Roy, A.R. (2003) Soft set theory. Computers and Mathematics with Applications, 45 (1): 555-562.

[21] Molodtsov, D. (1999) Soft set theory-first results. Computers and Mathematics with Applications, 37 (1): 19-31

[22] Onyeozili, L.A., T.A., Gwary, (2014) A study of the fundamentals of soft set theory, Internatıonal Journal Of Scıentıfic & Technology Research, 3 (4) 132-143.

[23] Özlü, Ş., (2022a) Interval valued q- rung orthopair hesitant fuzzy choquet aggregating operators in multi-criteria decision making problems. Gazi University Journal of Science Part C: Design and Technology, 10 (4). 1006-1025.

[24] Özlü Ş., (2022b) Interval valued bipolar fuzzy prioritized weighted dombi averaging operator based on multi-criteria decision making problems. Gazi University Journal of Science Part C: Design and Technology, 10 (4). 841-857.

[25] Özlü Ş. and Sezgin A., (2021) Soft covered ideals in semigroups. Acta Universitatis Sapientiae Mathematica, 12 (2). 317-346.

[26] Pei, D. and Miao, D. (2005) From Soft Sets to Information Systems. In: Proceedings of Granular Computing. IEEE, 2: 617-621.

[27] Ping, Z., Qiaoyan, W.,(2013) Operations on Soft Sets Revisited, Journal of Applied Mathematics, Volume 2013 Article ID 105752 7 pages.

[28] Sarıalioğlu, M. (2023) New type of extended operations of soft set: Complementary extended gamma, intersection and star operations. MSc. Thesis, Amasya University, The Graduate School of Natural and Applied Sciences Master of Science in Mathematics Department, Amasya.

[29] Sezgin, A. and Atagün, A.O. (2011) On operations of soft sets. Computers and Mathematics with Applications, 61(5):1457-1467.

[30] Sezgin A. and Atagün, A.O., (2023) New soft set operation: Complementary soft binary piecewise plus operation. accepted in Matrix Science Mathematic.

[31] Sezgin A. and Aybek, F. (2023) New soft set operation: Complementary soft binary piecewise gamma operation. Matrix Science Mathematics (7) 1, 27-45.

[32] Sezgin A., Aybek F., Güngör Bilgili N. (2023a) New soft set operation: Complementary soft binary piecewise union operation. Acta Informatica Malaysia 7(1), 38-53.

[33] Sezgin A., Aybek F., Atagün A.O. (2023b) New soft set operation: Complementary soft binary piecewise intersection operation. accepted in Black Sea Journal of Engineering and Science.

[34] Sezgin A. and Çağman N. (2023) New soft set operation: Complementary soft binary piecewise difference operation. accepted in Osmaniye Korkut Ata University Journal of the Institute of Science and Technology.

[35] Sezgin A., Çağman N., Atagün A.O., Aybek F. (2023c) Complemental binary operations of sets and their application to group theory, accepted in Information Management and Computer Science.

[36] Sezgin A. and Sarıalioğlu, M. (2023) New soft set operation: Complementary soft binary piecewise theta operation. accepted in Journal of Kadirli Faculty of Applied Sciences.

[37] Sezgin, A., Shahzad A. and Mehmood, A. (2019) New Operation on Soft Sets: Extended Difference of Soft Sets. Journal of New Theory, (27): 33-42.

[38] Singh, D., Onyeozili, L.A., (2012) Some conceptual misunderstanding of the fundamentals of soft set theory, ARPN Journal of systems and softwares, 2 (9) 251-254.

[39] Singh, D., Onyeozili, L.A., (2012) Some results on Distributive and absorption properties on soft operations, IOSR Journal of mathematics, 4 (2) 18-30.

[40] Singh, D., Onyeozili, L.A., (2012) On some new properties on soft set operations, International Journal of Computer Applications, 59 (4) 39-44.

[41] Singh, D., Onyeozili, L.A., (2012) Notes on soft matrices operations, ARPN Journal of science and technology, 2(9) 861-869.

[42] Stojanovic, N.S. (2021) A new operation on soft sets: extended symmetric difference of soft sets. Military Technical Courier, 69(4): 779-791.

[43] Yang, C.F., (2008) "A note on: "Soft set theory" [Computers & Mathematics with Applications 45 (2003), no. 4-5, 555–562]," Computers & Mathematics with Applications, 56 (7) 1899–1900.

[44] Yavuz E., (2023) Soft binary piecewise operations and their properties, MSc. Thesis, Amasya University, The Graduate School of Natural and Applied Sciences Master of Science in Mathematics Department, Amasya.