JOURNAL OF AMASYA UNIVERSITY THE INSTITUTE OF SCIENCES AND TECHNOLOGY (JAUIST)

ISSN: 2717-8900 https://doi.org/10.54559/jauist.1317676

Contact pseudo-slant submanifolds of a para-Sasakian manifold according to type 1, type 2, type 3 cases

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Abstract — This paper aims to present work on contact pseudo-slant submanifolds of para-Sasakian manifolds. The study includes the definitions and some results on type 1, type 2, and type 3 contact pseudo-slant submanifolds. The results were interpreted by taking into account the parallelism and geodesicity of the tensors reduced to the submanifold. Additionally, minimal anti-invariant and invariant submanifolds were evaluated for the type 1, type 2, and type 3 cases of the tensors reduced to the submanifold.

Keywords: *Contact pseudo-slant submanifold, para-Sasakian manifold, geodesic submanifold*

Subject Classification (2020): 53C15, 53C25

1. Introduction

The differential geometry of slant submanifolds has shown an increasing development since Chen [1] defined slant submanifolds in complex manifolds as a natural generalization of both invariant and anti-invariant submanifolds. Since then, many research articles have appeared on the existence of these submanifolds in different known spaces. The slant submanifolds of an almost contact metric manifolds were defined and studied by Lotta [2]. After, these submanifolds were studied by Cabrerizo et al. [3] in the setting of Sasakian manifolds.

The notion of semi-slant submanifolds of an almost Hermitian manifold was introduced by Papagiuc [4]. Hemi-slant submanifolds were first introduced by Carrizo [5], and he called them pseudo-slant submanifolds. Recently, there have been many studies conducted on this subject [6-9]. Finally, Chanyal [10] has studied slant submanifolds on an almost paracontact metric manifold.

In this paper, we study pseudo-slant submanifolds of a para-Sasakian (p-Sasakian) manifold. In Section 2, we review basic formulas and definitions for a p-Sasakian manifold and its submanifolds, which will be used later. In Section 3, we recall the definition and some basic results of a contact pseudo-slant submanifold of almost paracontact metric manifold. We obtain some results for these submanifolds in the setting of a p-Sasakian manifolds. We also research the geodetic states of the distributions.

2. Preliminaries

Let $\widetilde{M}$ be an n-dimensional contact manifold with contact form η, i.e., $\eta \wedge d\eta \neq 0$. It is well known that a contact manifold admits a vector field ξ called the characteristic vector field, shuch that $\eta(\xi) = 1$ and

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Article History: Received: 20 Jun 2023 — Accepted: 02 Jul 2023 — Published: 02 Jul 2023

$d\eta(X,\xi) = 0$, for every $X \in \Gamma(T\widetilde{M})$. Furthermore, $\widetilde{M}$ admits a Rieman metric g and a vector field ϕ of type (1,1) shuch that

$$\phi^2 X = X - \eta(X)\xi, \eta(X) = g(X,\xi), g(X,\phi Y) = d\eta(X,Y) \tag{2.1}$$

We then say that (ϕ,ξ,η,g) is a contact metric structure. A contact metric is said to be a Sasakian if

$$(\widetilde{\nabla}_X \phi)Y = g(X,Y)\xi - \eta(Y)X \tag{2.2}$$

in which case

$$\widetilde{\nabla}_X \xi = \phi X, \hat{\mathrm{R}}(\mathrm{X},\mathrm{Y})\xi = \eta(Y)X - \eta(X)Y \tag{2.3}$$

We provide a structure similar to Sasakian but not having contact.

An n-dimensional diferentiable manifold is said to admit an almost paracontact Rieman structure (ϕ,ξ,η,g), where ϕ of type (1,1) tensor field ξ is a vector field, η ia a 1-form and g is a Rieman metric on $\widetilde{M}$ such that

$$\phi\xi = 0, \eta(\phi X) = 0, \eta(\xi) = 1, \eta(X) = g(X,\xi) \tag{2.4}$$

$$\phi^2 X = X - \eta(X)\xi, g(\phi X,\phi Y) = g(X,Y) - \eta(X)\eta(Y) \tag{2.5}$$

for any vector fields X, Y on $\widetilde{M}$. The equation $\eta(\xi) = 1$ is equivalent to $|\eta| \equiv 1$, and then ξ is just the metric dual of η. If (ϕ,ξ,η,g) satisfy the equations

$$d\eta = 0, \widetilde{\nabla}_X \xi = \phi X \tag{2.6}$$

$$\left(\widetilde{\nabla}_X \phi\right)Y = -g(X,Y)\xi - \eta(Y)X + 2\eta(Y)\eta(Y) \tag{2.7}$$

then $\widetilde{M}$ is called a p-Sasakian manifold or briefly, a p-Sasakian, especially, a p-Sasakian manifold $\widetilde{M}$ is called a special p-Sasakian manifold or briefly a sp-Sasakian manifold if $\widetilde{M}$ admits a 1-form η satisfiying

$$\left(\widetilde{\nabla}_X \eta\right)Y = -g(X,Y) + \eta(Y)\eta(X)$$

where $\widetilde{\nabla}$ is the Levi-Civita connections of g.

Let M denotes an immersed submanifold of a p-Sasakian manifold $\widetilde{M}$. Considering the non degenerate metric induced on M by the same symbol g as on $\widetilde{M}$. Further, the Gauss and Weingarten formulas are respectively given as,

$$\widetilde{\nabla}_X Y = \nabla_X Y + \sigma(X,Y) \tag{2.8}$$

and

$$\widetilde{\nabla}_X V = -A_V X + \nabla_X^{\perp} V \tag{2.9}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$ where,

i. $X, Y \in \Gamma(TM)$ (tangent bundle) and $V \in \Gamma(T^{\perp}M)$ (normal bundle),

ii. Induced Levi-Civita connection ∇ on M,

iii. Normal connection $\nabla^{\perp}$ on $\Gamma(T^{\perp}M)$

iv. Second fundamental form σ on M,

v. Shape operator A_V associated with the normal section V.

Moreover, the second fundamental form σ and shape operator A_V are related by

$$g(A_V X, Y) = g(\sigma(X, Y), V) \tag{2.10}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$.

The mean curvature vector H of M is given by

$$H = \frac{1}{m}\sum_{i=1}^{n} \sigma(e_i, e_i) \tag{2.11}$$

where m is the dimension of M and $\{e_1, e_2, \ldots, e_m\}$ is a local orthonormal frame of M.

A submanifold M of an paracontact metric manifold $\widetilde{M}$ is said to be totally umbilical if

$$\sigma(X, Y) = g(X, Y)H \tag{2.12}$$

where H is the mean curvature vector. A submanifold M is said to be totally geodesic if $\sigma(X, Y) = 0$, for each $X, Y \in \Gamma(TM)$ and M is said to be minimal if $H = 0$.

Let M be a submanifold of an almost paracontact metric manifold $\widetilde{M}$. Then, for any $X \in \Gamma(TM)$, we can write

$$\phi X = TX + NX \tag{2.13}$$

where TX is the tangential component and NX is the normal component of ϕX.

Similary, for $V \in \Gamma(T^{\perp}M)$, we can write

$$\phi V = tV + nV \tag{2.14}$$

where tV is the tangential component and nV is the normal component of ϕV.

Furthermore, for any $X, Y \in \Gamma(TM)$, we have $g(TX, Y) = -g(X, TY)$, $g(NX, Y) = -g(X, NY)$, and $V, U \in \Gamma(T^{\perp}M)$, we get $g(U, nV) = -g(nU, V)$. These relations show that N and n are also skew-symmetric tensor fields. Moreover, for any $X \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$, we have $g(NX, V) = -g(X, tV)$, which gives the relation between N and t.

Thus, by using (2.1), (2.13), and (2.14), we obtain

$$T^2 = I - \eta \otimes \xi - tN, NT + nN = 0 \tag{2.15}$$

and

$$Tt + tn = 0, NT + n^2 = I \tag{2.16}$$

where the covariant derivatives of the tensor field T, N, t, and n are, respectively, defined by

$$(\nabla_X T)Y = \nabla_X TY - T\nabla_X Y \tag{2.17}$$

$$(\nabla_X N)Y = \nabla_X^{\perp} NY - N\nabla_X Y \tag{2.18}$$

$$(\nabla_X t)V = \nabla_X tV - t\nabla_X^{\perp} V \tag{2.19}$$

and

$$(\nabla_X n)V = \nabla_X^{\perp} nV - n\nabla_X^{\perp} V \tag{2.20}$$

for any $X, Y \in \Gamma(TM)$ and for any $V \in \Gamma(T^{\perp}M)$.

By direct calculations, we obtain the following formulas

$$(\nabla_X T)Y = A_{NY}X + t\sigma(X,Y) + g(X,Y)\xi - \eta(Y)X \tag{2.21}$$

and

$$(\nabla_X N)Y = n\sigma(X,Y) - \sigma(X,TY) \tag{2.22}$$

for any $X, Y \in \Gamma(TM)$,

Similary, we obtain

$$(\nabla_X t)V = A_{nV}X - TA_V X \tag{2.23}$$

and

$$(\nabla_X n)V = -NA_V X - \sigma(tV, X) \tag{2.24}$$

for any $X \in \Gamma(TM)$ and for any $V \in \Gamma(T^{\perp}M)$.

Lemma 2.1. If M is an immersed submanifold of a p-Sasakian manifold $\widetilde{M}$ with $\xi \in \Gamma(TM)$, then

$$\nabla_X \xi = TX \tag{2.25}$$

and

$$\sigma(X,\xi) = NX \tag{2.26}$$

$$A_V \xi = -tV \tag{2.27}$$

$X \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$.

Proof.

In (2.8), if $Y = \xi$ is written, we have

$$\widetilde{\nabla}_X \xi = \nabla_X \xi + \sigma(X,\xi)$$

Using (2.13), the tangential and normal parts of the last equation give, respectively, us

$$\nabla_X \xi = TX$$

and

$$\sigma(X,\xi) = NX$$

Besides, in (2.10), if $Y = \xi$ is written and from (2.26), we have

$$g(A_V X, \xi) = g(NX, V) = -g(X, \mathrm{t}V) \qquad \square$$

Definition 2.2. [2] Let M be a submanifold of a p-Sasakian manifold $\widetilde{M}$. For each non-zero vector X tangent to M at x, the angle $\theta(x)$, $\theta(x) \in \left[0, \frac{\pi}{2}\right]$, between ϕX and $T_x M$ is called the slant angle or the Wirtinger angle of M. If the slant angle is constant, then the submanifold is also called the slant submanifold. If $\theta = 0$ the submanifold is invariant submanifold. If $\theta = \frac{\pi}{2}$, then it is called anti-invariant submanifold. If $\theta(x) \in \left(0, \frac{\pi}{2}\right)$, then it is called proper-slant submanifold.

We prove the following characterization theorem for slant submanifold.

Theorem 2.3. [10] Let M be a slant submanifold of an almost paracontact metric manifold $(\widetilde{M}, \phi, \xi, \eta, g)$ such that $\xi \in TM$. Then,

i. M is slant of type 1 if and only if for any time like (space-like) vector field $X \in \chi(M) - \langle \xi \rangle$, TX is time like (space-like), and there exists a constant $\lambda \in (1, +\infty)$ such that

$$T^2 = \lambda(I - \eta \otimes \xi)$$

We write $\lambda = \cosh^2\theta$, with $\theta \rangle 0$.

ii. M is slant of type 2 if and only if for any time like (space-like) vector field $X \in \chi(M) - \langle \xi \rangle$, TX is time like (space-like), and there exists a constant $\lambda \in (0,1)$ such that

$$T^2 = \lambda(I - \eta \otimes \xi)$$

We write $\lambda = \cos^2\theta$, with $\theta \in \left(0, \frac{\Pi}{2}\right)$.

iii. M is slant of type 3 if and only if for any time like (space-like) vector field $X \in \chi(M) - \langle \xi \rangle$, TX is time like (space-like), and there exists a constant $\lambda \in (-\infty, 0)$ such that

$$T^2 = \lambda(I - \eta \otimes \xi)$$

We write $\lambda = -\sinh^2\theta$, with $\theta \rangle 0$. In each case θ is called the slant angle.

Corollary 2.4. [10] Let M be a slant submanifold of an almost paracontact metric manifold $(\widetilde{M}, \phi, \xi, \eta, g)$ with slant angle θ. Then, for any $X, Y \in \Gamma(TM)$, we have

If M is of type 1, then

$$g(TX, TY) = -\cosh^2\theta\{g(X, Y) - \eta(X)\eta(Y)\} \tag{2.28}$$

and

$$g(NX, NY) = \sinh^2\theta\{g(X, Y) - \eta(X)\eta(Y)\} \tag{2.29}$$

If M is of type 2, then

$$g(TX, TY) = -\cos^2\theta\{g(X, Y) - \eta(X)\eta(Y)\} \tag{2.30}$$

and

$$g(NX, NY) = -\sin^2\theta\{g(X, Y) - \eta(X)\eta(Y)\} \tag{2.31}$$

If M is of type 3, then

$$g(TX, TY) = \sinh^2\theta\{g(X, Y) - \eta(X)\eta(Y)\} \tag{2.32}$$

and

$$g(NX, NY) = -\cosh^2\theta\{g(X, Y) - \eta(X)\eta(Y)\} \tag{2.33}$$

Proof.

From the anti-symetry of T and Theorem 2.3, we have

$$g(TX,TY) = -g(T^2X,Y) = -\lambda\{g(X,Y) - \eta(X)\eta(Y)\} = \lambda\{g(\phi X,\phi Y)\}$$

(2.13) yields

$$g(\phi X,\phi Y) = g(TX,TY) + g(NX,NY)$$

from last two equations, we obtain

$$g(NX,NY) = (1-\lambda)g(\phi X,\phi Y)$$

Hence, the corallary follows from the values of λ in the Theorem 2.3. □

3. Contact Pseudo-Slant Submanifolds of a Para-Sasakian Manifold

Definition 3.1. [11] We say that M is a contact pseudo-slant submanifold of an almost paracontact metric manifold $\widetilde{M}$ if there exist two orthogonal distributions D_θ and $D^\perp$ on M such that

i. TM admits the orthogonal direct decomposition $TM = D^\perp \oplus D_\theta$, $\xi \in \Gamma(D_\theta)$,

ii. The distribution $D^\perp$ is anti-invariant (totally-real), i.e., $\phi D^\perp \subset (T^\perp M)$,

iii. The distribution D_θ is a slant with slant angle $\theta \neq \frac{\pi}{2}$, that is, the angle between D_θ and $\phi(D_\theta)$ is a constant.

From the definition, it is clear that if $\theta = 0$, then the contac pseudo-slant submanifold is a semi-invariant submanifold, $\theta = \frac{\pi}{2}$ submaifold becomes an anti-invariant.

We suppose that M is a contact pseudo-slant submanifold of an almost paracontact metric manifold $\widetilde{M}$.

Furthermore, let $d_1 = \dim(D^\perp)$ and $d_2 = \dim(D_\theta)$. We distinguish the following six cases.

i. If $d_2 = 0$, then M is an anti-invariant submanifold.

ii. If $d_1 = 0$ and $\theta = 0$, then M is invariant submanifold.

iii. If $d_1 = 0$ and $\theta \in \left(0,\frac{\pi}{2}\right)$, then M is a proper slant submanifold.

iv. If $\theta = \frac{\pi}{2}$ then, M is an anti-invariant submanifold.

v. If $d_2 d_1 \neq 0$ and $\theta = 0$, then M is a semi-invariant submanifold.

vi. If $d_2 d_1 \neq 0$ and $\theta \in \left(0,\frac{\pi}{2}\right)$, then M is a contact pseudo-slant submanifold.

If we denote the orthogonal complementary of φTM in $T^\perp M$ by μ, then the normal bundle $T^\perp M$ can be decomposed as follows:

$$T^\perp M = N(D^\perp) \oplus N(D_\theta) \oplus \mu \tag{3.1}$$

Theorem 3.2. The necessary ond sufficient condition for submanifold M of a p-Sasakian manifold $\widetilde{M}$ to be a contact pseudo-slant submanifold is that $\exists$ a distribution D on M and a constant $\lambda \in (-\infty,+\infty)$ satisfiying

i. $D = \{X \in \Gamma(TM): T^2X = -\lambda X\}$

ii. $TX = 0$, for tangent vectorfield X orthogonal to D

Further, λ can be $\cosh^2\theta$, $\cos^2\theta$, or $-\sinh^2\theta$ [10].

Proof.

From Theorem 2.3 (*i*-*iii*), the proof of the theorem is obvious. □

Definition 3.3. A contact pseudo-slant submanifold M of p-Sasakian manifold $\widetilde{M}$ is said to be D_θ-geodesic (resp. $D^\perp$-geodesic) if $\sigma(X,Y)=0$, for $X,Y\in\Gamma(D_\theta)$ (resp. $\sigma(Z,W)=0$, for $Z,W\in\Gamma(D^\perp)$. If for any $X\in\Gamma(D_\theta)$ and $Z\in\Gamma(D^\perp)$, $\sigma(X,Z)=0$, the M is called mixed geodesic submanifold.

Theorem 3.4. Let M be a proper contact pseudo-slant submanifold of a p-Sasakian manifold $\widetilde{M}$. If t is parallel, then

i. For type 2, M is anti-invariant submanifold.

ii. For type 3, M is invariant submanifold.

iii. M is a mixed-geodesic submanifold.

Proof.

Consider (2.22) and (2.23) which gives the relation between t and N. If t is parallel, then N is parallel, we obtain

$$n\sigma(X,Y)=0$$

for any $X\in\Gamma(D_\theta)$ and $Y\in\Gamma(D^\perp)$. Replacing X by Y in (2.22) and taking into account to N being parallel, we have

$$n\sigma(Y,TX)-\sigma(Y,T^2X)=\cos^2\theta\,\sigma(X,Y)=\sinh^2\theta\,\sigma(X,Y)=0$$

From type 2, we write

$$\cos^2\theta\,\sigma(X,Y)=0\ (\theta=\tfrac{\pi}{2}\ M\text{ is anti-invariant})$$

From type 3, we write

$$\sinh^2\theta\,\sigma(X,Y)=0$$

Thus,

$$2\sinh\theta=e^{\theta}-e^{-\theta}=0\ (\theta=0\ M\text{ is invariant})$$

Besides, for any $X\in\Gamma(D_\theta)$ and $Y\in\Gamma(D^\perp)$, $\sigma(X,Y)=0$, M is a mixed geodesic submanifold. This proves our assertion. □

Theorem 3.5. Let M be a proper contact pseudo-slant submanifold of a p-Sasakian manifold $\widetilde{M}$. If N is parallel, then either M is a $D^\perp$-geodesic or an anti-invariant submanifold of $\widetilde{M}$.

Proof.

Consider (2.22) and (2.23) which gives the relation between t and N. If t is parallel, then N is parallel, we obtain

$$TA_{NY}Z=0$$

for any $Y,Z\in\Gamma(D^\perp)$. This implies that M is either anti-invariant or $A_{NY}Z=0$. Therefore, we obtain

$$g(\sigma(Z,W),NY)=0$$

for any $Y,Z,W\in\Gamma(D^\perp)$. Moreover, by using (2.23), we conclude that

$$g(A_{nV}Z,Y)-g(TA_VZ,Y)=g(\sigma(Y,Z),nV)=0$$

for any $V \in \Gamma(T^{\perp}M)$. This tells us that M is either $D^{\perp}$-geodesic or it is an anti-invariant submanifold. □

Theorem 3.6. Let M be a contact pseudo-slant submanifold of a p-Sasakian manifold $\widetilde{M}$. If N is parallel on D_θ, then either M is a D_θ-geodesic submanifold or $\sigma(X,Y)$ is an eigenvector of n^2 with eigenvalues are $\cosh^2\theta$, $\cos^2\theta$, or $-\sinh^2\theta$, for type 1, type 2, and type 3, respectively.

Proof.

For all $Y, Z \in \Gamma(D_\theta)$. From (2.22), we have

$$n\sigma(Z,Y) - \sigma(Z,TY) = 0 \tag{3.2}$$

Besides, since D_θ is slant distribution, we get

$$n\sigma(Z,Y-\eta(Y)\xi) - \sigma(Z,T(Y-\eta(Y)\xi)) = 0$$

From (2.4) and (2.13), we get

$$n\sigma(Z,Y-\eta(Y)\xi) - \sigma(Z,TY) = 0 \tag{3.3}$$

Applying n to (3.3), we have

$$n^2\sigma(Z,Y-\eta(Y)\xi) - n\sigma(Z,TY) = 0$$

Moreover, by interchanging of Y and TY in (3.2), we have

$$n\sigma(Z,TY) - \sigma(Z,T^2Y) = 0$$

Hence, using Theorem 2.3, we obtain

$$\begin{aligned} n^2\sigma(Z,Y-\eta(Y)\xi) &= n\sigma(Z,TY) \\ &= \sigma(Z,T^2Y) \\ &= \cosh^2\theta\,\sigma(Z,Y-\eta(Y)\xi) \\ &= \cos^2\theta\,\sigma(Z,Y-\eta(Y)\xi) \\ &= -\sinh^2\theta\,\sigma(Z,Y-\eta(Y)\xi) \end{aligned}$$

This implies that either $\sigma = 0$ on D_θ or σ in an eigenvector of n^2 with eigenvalues $\cosh^2\theta$, $\cos^2\theta$, or $-\sinh^2\theta$.□

Theorem 3.7. Let M be a totally umbilical proper contact pseudo-slant submanifold of a p-Sasakian manifold $\widetilde{M}$. If t is parallel, then either M is a minimal or an anti-invariant and invariant submanifold of $\widetilde{M}$.

Proof.

For all $Y \in \Gamma(D^{\perp})$ and $X \in \Gamma(D_\theta)$. Consider (2.22) and (2.23) which gives the relation between t and N. If t is parallel then N is parallel, we obtain

$$n\sigma(X,Y) - \sigma(X,TY) = 0$$

Replacing X by TX in above equation, we get

$$n\sigma(TX,Y) - \sigma(TX,TY) = 0$$

For $Y \in \Gamma(D^{\perp})$, $TY = 0$. Thus,

$$n\sigma(TX,Y) = 0$$

Since M is totally umbilical, from (2.12), we have

$$ng(TX, Y)H = 0$$

Replacing X by TX in above equation and from Theorem 2.3, we obtain

$$\begin{aligned} ng(T^2X, Y)H &= -ng(TX, TY)H \\ &= -n\cosh^2\theta\, g(X, Y)H \\ &= -n\cos^2\theta\, g(X, Y)H \\ &= n\sinh^2\theta\, g(X, Y)H \\ &= 0 \end{aligned}$$

Hence, from type 2 and type 3, we have either $\theta = \frac{\pi}{2}$ (M is anti invariant), $\theta = 0$ (M is invariant), or $H = 0$ (M is minimal). □

4. Conclusion

In this article, interesting results have been obtained regarding the contact pseudo-slant submanifolds of para-Sasakian manifolds, taking into account the geodesic and parallelism situations of the tensors. These situations can be investigated on other contact metric manifolds.

Author Contributions

All the authors equally contributed to this work. They all read and approved the final version of the paper.

Conflict of Interest

All the authors declare no conflict of interest.

References

[1] B-Y. Chen, *Slant immersions*, Bulletin of the Australian Mathematical Society 41 (1990) 135–147.

[2] A. Lotta, *Slant submanifolds in contact geometry*, Bulletin Mathématique de la Société des Sciences Mathématiques de Roumanie 39 (87) (1/4) (1996)183–198.

[3] J. L. Cabrerizo, A. Carriazo, L. M, Fernandez, M. Fernandez, *Slant submanifolds in Sasakian manifolds*, Glasgow Mathematical Journal 42 (01) (2000) 125–138.

[4] N. Papaghuic, *Semi-slant submanifolds of a Kaehlarian manifold*, Analele Stiintifice ale Universitatii Al I Cuza din Iasi 40 (2009) 55–61.

[5] A. Carriazo, *New developments in slant submanifolds theory*, Applicable Mathematics in the Golden Age Narosa Publishing House, 2002, pp. 339–356.

[6] M. Atçeken, S. K. Hui, *Slant and pseudo-slant submanifolds in LCS-manifolds*, Czechoslovak Mathematical Journal 63 (138) (2013) 177–190.

[7] M. Atçeken, S. Dirik, *On the geometry of pseudo-slant submanifold of a Kenmotsu manifold*. Gulf Mathematical of Journal 2 (2) (2014) 51–66.

[8] S. Dirik, Ü. Yıldırım, *Characterizations of contact pseudo-slant submanifolds of a para-Kenmotsu manifold*, Journal of Amasya University the Institute of Sciences and Technology 3 (1) 2022 49–59.

[9] S. Dirik, M. Atçeken, Ü. Yıldırım, *Contact pseudo-slant submanifolds of an* (∈)-*para Sasakian space forms*, Journal of the International Mathematical Virtual Institute 10 (1) (2020) 59–74.

[10] S. K. Chanyal, *Slant submanifolds in an almost paracontact metric manifold*, Annals of the Alexandru Ioan Cuza University-Mathematics 67 (2) (2021) 213–229.

[11] V. A. Khan, M. A. Khan, *Pseudo-slant submanifolds of a Sasakian manifold*, Indian Journal of Pure and Applied Mathematics 38 (1) (2007) 31–42.