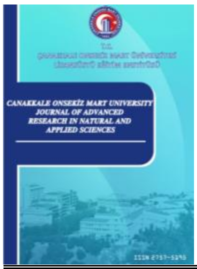
e-ISSN 2757-5195



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Journal of Advanced Research in Natural and Applied Sciences

Open Access

doi.org/10.28979/jarnas.1318975 2023, Vol. 9, Issue 4, Pages: 1005-1019 dergipark.org.tr/tr/pub/jarnas

Position Control Using Trajectory Tracking and State Estimation of a Quad-rotor

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Article History
Received: 23.06.2023
Accepted: 04.09.2023
Published: 20.12.2023

Research Article

Abstract − Quad-rotor aircrafts are unmanned aerial vehicles that have gained significant popularity in recent years and have been developed for use in many areas. Such vehicles are capable of vertical take-off and landing and are used in various applications. To operate a quad-rotor aircraft efficiently and safely, fundamental issues such as mathematical modeling, control, and state estimation need to be studied. Mathematical modeling involves creating a holistic model of the various subsystems of the aircraft including aerodynamic, kinematic, dynamic and control systems. The control system is a mechanism used for the aircraft to perform the desired movements. State estimation techniques are used to obtain and predict information about the state of the aircraft. This study includes position control using a trajectory generation algorithm. Attitude estimation of the quad-rotor is improved with the Explicit Complementary Filter (ECF) and the state estimations is improved with the Extended Kalman Filter (EKF). Different from other studies, the results are obtained by feeding the model with a state estimation filter. The performances of the filters used for state estimation are compared.

Keywords − *Quad-rotor, position control, trajectory tracking, state estimation*

1. Introduction

Recently, unmanned aerial vehicles (UAVs) have been ever more attracting the attention of researchers and different kinds of industries due to their versatility, flexibility, low research and development costs (Pines & Bohorquez, 2012; Yoon & Doh, 2022).

Navigating a mobile platform from its initial point to a desired destination is a complex challenge in the field of robotics. A technique to produce time-optimal trajectories with the help of parametric functions was presented in (Bouktir, Haddad & Chettibi, 2008). The vehicle's movement along a path was governed by a continuously increasing function. Their numerical approach was designed to work in cases involving minimum time transfers problems. To move a quad-rotor as quickly as feasible from an initial to a final position, (Chamseddine, Li, Zhang, Rabbath & Theilliol, 2012) suggested a flatness-based trajectory planner. They used a Sliding Mode Controller (SMC) and a Linear Quadratic Regulator (LQR). Implementing the controller using the linearized model of the vehicle resulted in effectively constraining the roll and pitch angles, demonstrating the success of this approach.

The researchers in (Hoffmann, Waslander & Tomlin, 2008) developed a tracking controller that linked waypoints using line segments at a designated speed. A trajectory tracking system built on model predictive control was presented in (Castillo, Moreno & Valavanis, 2007). Their controller was tested on a miniature helicopter to track the waypoints. To decrease the computational cost the control was tested on a simplified vehicle model.

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The proportional-integral-derivative (PID) position and velocity tracking controllers and the predictive controller were compared in their research. An optimization-based framework of excavation trajectory generation was developed in (Yang, Long, Song, Pan & Zhang, 2021), various optimizing criteria for different terrains shapes were studied.

One of the traditional problems in quad-rotor is dealing with state estimation. There are literature surveys on filtering techniques to estimate the parameters involved in state estimation. The Kalman filter and the Complementary filter are frequently employed methods in linear systems due to their frequency filtering characteristics (Mahony, Hamel & Ptlimlin, 2008). The Kalman filter is the subject of numerous studies (Hetenyi, Gatzy & Blazovics, 2016; Beck et al., 2016; Sebesta & Boizot, 2014; Hall, Knoebel & McLain, 2008) in the field of flight control. The standard Kalman filter is adapted to calculate the attitude of the air vehicle in (Hall et al., 2008). The estimated values are obtained through the utilization of an enhanced Kalman multiplicative filter. The estimation process incorporates data from accelerometers, gyroscopes, and GPS to gather information. An Unscented Kalman Filter (UKF) was suggested in (De Marina, Pereda, Giron-Sierra & Espinosa, 2012) three-axis attitude determination method as the observer was utilized. It was reported that the method functioned well, but the computational cost was high.

In this study, position control using the state estimation and trajectory tracking of a quad-rotor is implemented. Different from other studies, the results are obtained by feeding the model with a state estimation filter. The performances of the filters used for state estimation are compared.

In Section 2, the mathematical model of the quad-rotor, along with its kinematics, dynamics, and control, is introduced. Section 3 discusses the filter methods developed for state estimation. The experimental results and a comparison of the designed filters are presented in Section 4. Finally, in Section 5, the study concludes by highlighting the accomplishments and discussing potential future work.

2. Quad-rotor Mathematical Model

The mathematical model characterizes the motion and behavior of the system based on the input parameters of the model and external factors affecting the system. It relates inputs to outputs. By utilizing the mathematical model, it becomes feasible to anticipate the position as well as the state of the quad-rotor by knowing the propellers' four angular velocities.

The quad-rotor aircraft is capable of maneuvering in three-dimensional space by harnessing the forces generated by its four engines. To maintain the angular momentum, rotors 2 and 4 rotate counterclockwise, while rotors 1 and 3 rotate clockwise. Figure 1 depicts the impact of these rotor forces.

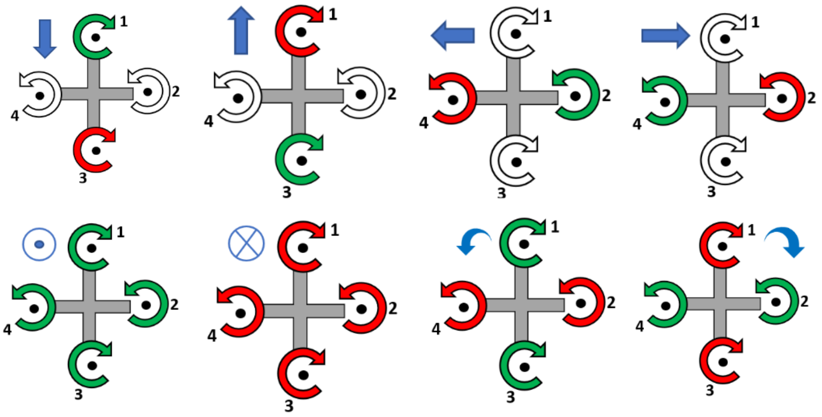

Figure 1. Four rotor movements according to lifting forces (green rings; fast rotation and red rings, slow rotation)

2.1. Quad-rotor Coordinate frames

For the mathematical model, it is crucial to establish two coordinate frames (Benic, Piljek & Kotarski, 2016);

- Earth Fixed Frame: F^E is an inertial right-hand coordinate frame, where the positive direction of the Z_E axis is from the earth. The quad-rotor attitude η and position ξ are defined within this coordinate frame (Figure 2).
- Body Fixed Frame: F^B is a coordinate frame fixed to the quad-rotor's body axis. The origin of F^B coincides with the quad-rotor's center of gravity. Linear velocities v_b, angular velocities w_b, forces f_b, and torques T_b are defined within this coordinate frame.

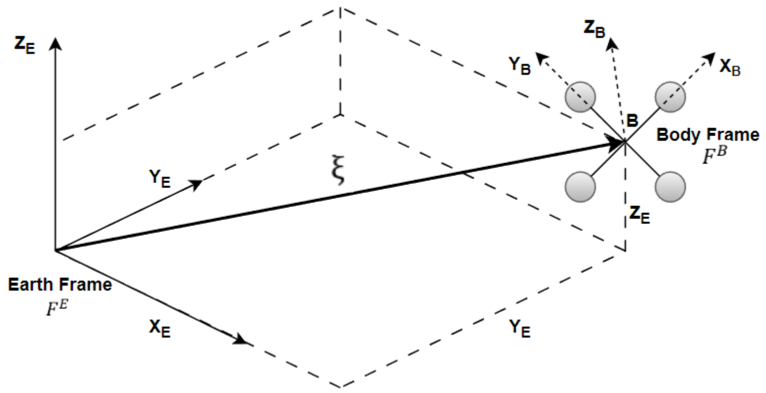

Figure 2. Earth and Body Frames

The quad-rotor's position is determined by the vector ξ, which represents the displacement between the origin of the Earth Fixed Frame and the Body Fixed Frame;

$$\xi = [x \quad y \quad z]^T. \tag{2.1}$$

The quad-rotor's attitude, denoted by η, is determined by the orientation of the Body Fixed Frame with respect to the Earth Fixed Frame. The orientation is described by three successive rotations around the coordinate axes of the Earth Fixed Frame. The quad-rotor's rotation around the y-axis is governed by the pitch angle θ. The rotation around the x-axis is determined by the roll angle φ, and the rotation around the z-axis is determined by the yaw angle ψ;

$$\eta = [\varphi \quad \theta \quad \psi]^T. \tag{2.2}$$

2.2. Quad-rotor Kinematic Model

The kinematics of a rigid body with 6 degrees of freedom are given by (Benic et al., 2016);

$$\dot{\varepsilon} = J\mathrm{v} \tag{2.3}$$

where $\dot{\varepsilon}$ is comprised of a vector linear and angular velocity relative to the F^B. The generalized velocity vector v is expressed in F^B and J represents the generalized transformation matrix. ε includes the position ξ and attitude η as follows;

$$\varepsilon = [\xi \quad \eta]^T = [x \quad y \quad z \quad \varphi \quad \theta \quad \psi]^T. \tag{2.4}$$

Generalized velocity vector v within F^B is defined in a similar manner;

$$\mathrm{v} = [v^b \quad \omega^b]^T = [u \quad v \quad w \quad p \quad q \quad r]^T. \tag{2.5}$$

The generalized rotation and transformation matrix facilitate the transfer of velocities from the Body Fixed Frame to the Earth Fixed Frame, providing a more intuitive representation of quad-rotor motion. This matrix comprises four submatrices;

$$J = \begin{bmatrix} R & 0_{3x3} \\ 0_{3x3} & T \end{bmatrix} \tag{2.6}$$

where R is the rotation matrix;

$$R = \begin{bmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\varphi - \sin\psi\cos\varphi & \cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\varphi + \cos\psi\cos\varphi & \sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi \\ -\sin\theta & \cos\theta\sin\varphi & \cos\theta\cos\varphi \end{bmatrix}. \tag{2.7}$$

To address the requirement of converting measured values between different coordinate frames, a rotation matrix is employed. This matrix enables the transfer of the linear velocity vector from one coordinate frame to another through matrix multiplication. The matrix R is used for this purpose and it is an orthonormal matrix.

Angles as well as angular velocities are acquired within the Earth Fixed Frame. To transfer angular velocities from the Body Fixed Frame to the Earth Fixed Frame, a transformation matrix T is utilized;

$$T = \begin{bmatrix} 1 & sin\varphi\ \tan\theta & cos\varphi\tan\theta \\ 0 & cos\varphi & -sin\varphi \\ 0 & \frac{sin\varphi}{\cos\theta} & \frac{cos\varphi}{\cos\theta} \end{bmatrix}. \tag{2.8}$$

To transfer angular velocities from the Earth Fixed Frame to the Body Fixed Frame, the angular velocity vector in F^E needs to be multiplied by T^{-1}.

2.3. Quad-rotor Dynamics

The dynamics of a quad-rotor are described using differential equations derived from the Newton-Euler method. These equations incorporate the mass (m) and inertia (I) of the body, accounting for the dynamics of a rigid body with six degrees of freedom.

Quad-rotor has a symmetrical structure in which the four rotors in the quad-rotor are aligned with the X_B and Y_B axes. By assuming that the principal inertial axes align with the coordinate axes of F^B, the inertial matrix is simplified to a diagonal matrix with $I_{xx} = I_{yy}$.

$$\mathrm{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{2.9}$$

The dynamics of the quad-rotor are defined as follows;

$$\begin{bmatrix} m\, I_{3x3} & 0_{3x3} \\ 0_{3x3} & I \end{bmatrix} \begin{bmatrix} \dot{v}^b \\ \dot{\omega}^b \end{bmatrix} + \begin{bmatrix} \omega^b \times \left(m\, \omega^b\right) \\ \omega^b \times \left(I\, \omega^b\right) \end{bmatrix} = \begin{bmatrix} f^B \\ \tau^B \end{bmatrix} \tag{2.10}$$

where I_{3x3} is the identity matrix. The term $\dot{v}^b$ refers to the linear acceleration vector, $\dot{\omega}^b$ represents the angular acceleration vector, f^B denotes the force vector acting on quad-rotor and τ^B represents the torque vector that is exerted on the quad-rotor.

If the generalized force vector is defined as ϒ;

$$\Upsilon = [f^B \quad \tau^B]^T = [F_x \quad F_y \quad F_z \quad \tau_x \quad \tau_y \quad \tau_z]^T. \tag{2.11}$$

Equation (2.11) can be written in the following form;

$$I_B\dot{v} + C_B(v)v = \Upsilon \tag{2.12}$$

where $\dot{v}$ represents the generalized acceleration vector, I_B represents inertia matrix of the system, $C_B(v)$ represents the Coriolis-centripetal matrix. Parameters of the quad-rotor are given in Table 1.

Table 1
The parameters of quad-rotor model

Parameter	Value	Unit
m	0.95	kg
m_w	0.01	kg
l	0.23	m
g	9.81	$kg\, m/s^2$
I_w	0.000065	$kg\, m^2$
J_x	0.0075	$kg\, m^2$
J_y	0.0075	$kg\, m^2$
J_z	0.0013	$kg\, m^2$

2.4. Controller Structure of the Quad-rotor

PID is a common control scheme used in industrial applications. It is widely used for the control of quad-rotors as well. The PID controller receives the feedback signal and compares it with a setpoint or reference signal, resulting in an error signal. Its objective is to minimize the difference between the plant/process variable and the setpoint or reference signal. The controller's behavior is determined by the combination of three control actions: proportional, integral, and derivative. The outputs of these actions are combined and provided as input to the plant/process as follows;

$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt} \tag{2.13}$$

where, K_p is the proportion coefficient, the output of the error multiplied by a certain gain value and calculates the current error, K_i is the integral coefficient, the integral effect means the sum of errors the system has made in the past and K_d is the derivative coefficient, it has a proportional effect on the output of the system according to the variation of the error. That is, it calculates the estimation of the future error. A conventional PID structure can be depicted using blocks, as shown in Figure 3a. In this study, the position controller, linear velocity controller, attitude controller, and angular velocity controller were designed using PID (Figure 3b) and the values of the PID coefficients were optimized using the MATLAB optimization toolbox. The PID coefficients of controllers are given in Table 2.

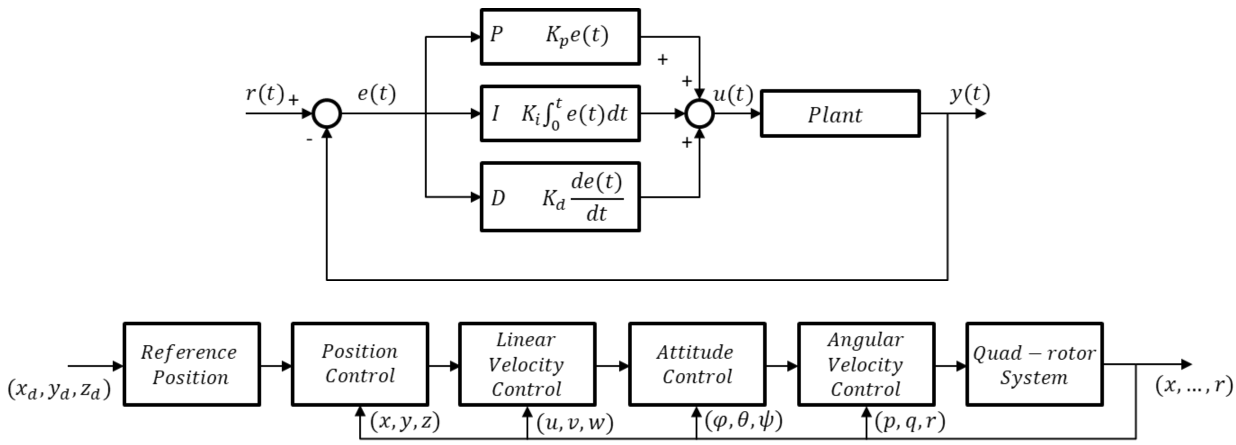

Figure 3. a) General PID controller schematic, b) The overall control blocks for the quad-rotor in this study

Table 2
PID coefficients of controllers.

Control Terms	Position	Linear Velocity	Attitude	Angular Velocity
K_p	6.05	3.1	1.2	0.8
K_i	8.15	1.5	0.8	0.69
K_d	3.2	0.05	0.03	0.017

The designed controller outputs are shown in Figure 4 and Figure 5. In Figure 4 the reference signal the one that the system should follow when the position information from the trajectory generation algorithm is presented to the quad-rotor system. The signal labeled as 'sensor measurements' contains data obtained from sensor models, where white noise is added to the original sensor data values. Additionally, another signal displays the data as the designed controller outputs. P, Q, R signals shown in Figure 4 correspond to the angular velocity parameters. The North, East Down values indicate the quad-rotor's position along the NED axes The U, V signals presented in Figure 5 are the velocity of the quad-rotor on the NED axes. One can notice that ψ and W signals are not included in the graphs, since ψ and W are not controlled indeed in the system designed for the quad-rotor. ϕ and θ are the angle between the body axis of the quad-rotor and the NED axes respectively, and they provide essential information about the quad-rotor's attitude. Instead of employing lowercase notation like (p, q, r), it becomes evident that the use of uppercase letters like P, Q, R serves to signify these signals within the simulations.

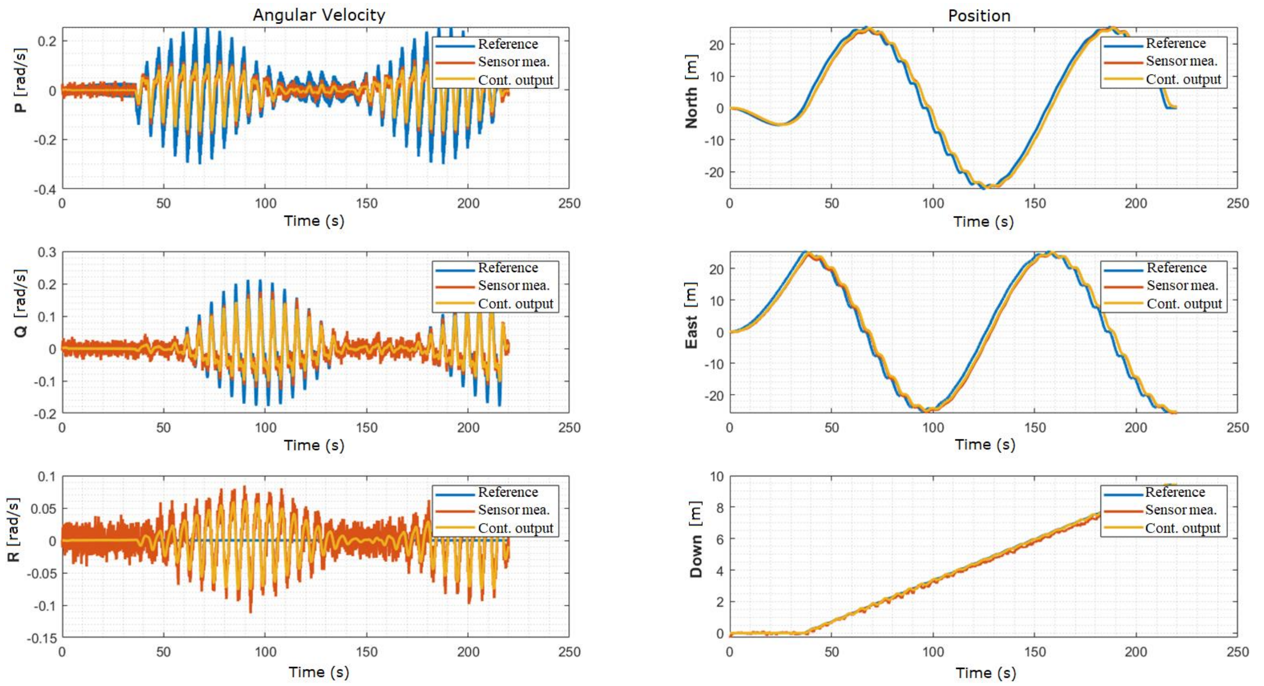

Figure 4. Angular velocity control and position control

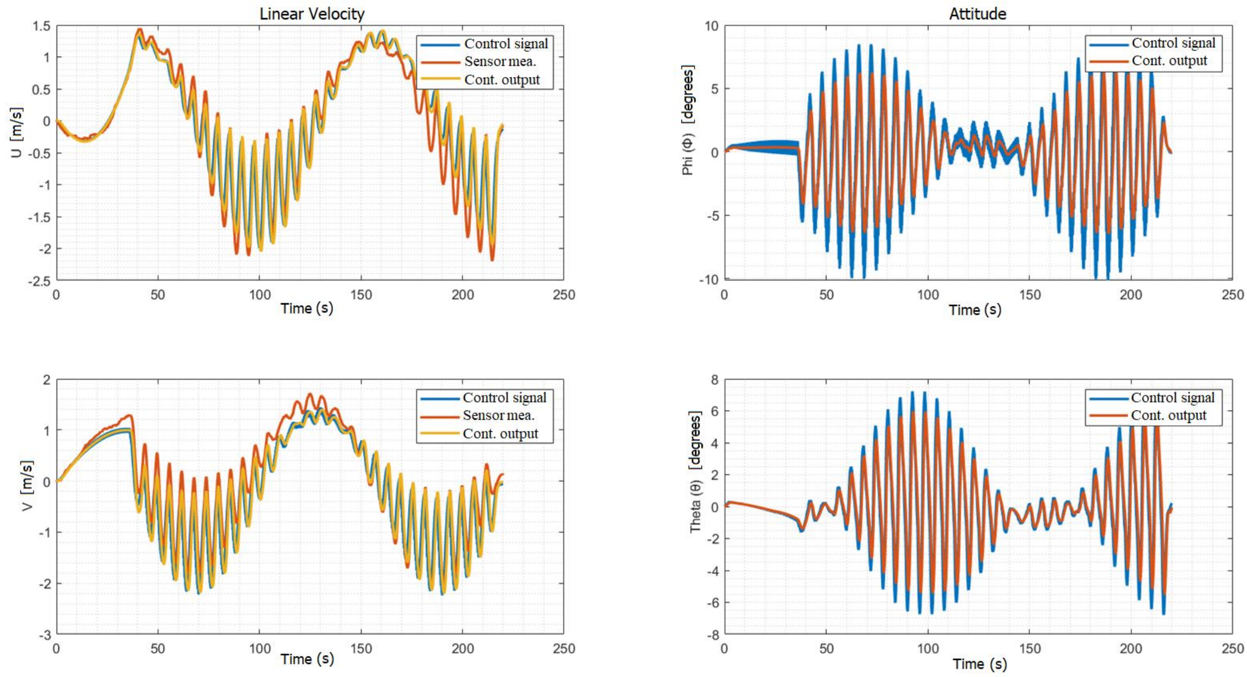

Figure 5. Linear velocity control and attitude control

2.5. Trajectory Tracking Algorithm

To initiate the trajectory tracking algorithm, the initial step involves generating a trajectory for subsequent tracking. The waypoints represent the planned path that the dynamic system should follow to efficiently achieve the tracking. The trajectory generation algorithm populates waypoints at a specified sampling rate and trajectory model. This trajectory then serves as the source for providing reference values for yaw angle, altitude, and linear velocity. The control system operates on these trajectories.

In the trajectory generation algorithm, the desired positions, i.e. waypoints, are determined on the (x, y, z) axes and the desired speed is determined at these waypoints. A system of linear equations will be created and generated with the help of the matrix.

The position of the quad-rotor equation can be expressed as follows;

$$\begin{aligned} x(t) &= a_0 + a_1 t + a_2 t^2 + a_3 t^3 \\ y(t) &= b_0 + b_1 t + b_2 t^2 + b_3 t^3 \\ z(t) &= c_0 + c_1 t + c_2 t^2 + c_3 t^3 \end{aligned} \tag{2.14}$$

where t is time, and each one is a cubic function with real coefficients. The velocity of the quad-rotor equation can be expressed as follows;

$$\begin{aligned} v_x(t) &= \dot{x}(t) = a_1 + 2a_2 t + 3a_3 t^2 \\ v_y(t) &= \dot{y}(t) = b_1 + 2b_2 t + 3b_3 t^2 \\ v_z(t) &= \dot{z}(t) = c_1 + 2c_2 t + 3c_3 t^2 \end{aligned} \tag{2.15}$$

2.16 is obtained by solving 2.14 and 2.15 by specifying the first condition with an index value of 0 and the last condition with an index value f;

$$G = \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_0 & 3t_0^2 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix}, \quad \begin{bmatrix} x_0 \\ x_f \\ v_{x0} \\ v_{xf} \\ y_0 \\ y_f \\ v_{y0} \\ v_{yf} \\ z_0 \\ z_f \\ v_{z0} \\ v_{zf} \end{bmatrix} = \begin{bmatrix} G & 0_{4x4} & 0_{4x4} \\ 0_{4x4} & G & 0_{4x4} \\ 0_{4x4} & 0_{4x4} & G \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ b_0 \\ b_1 \\ b_2 \\ b_3 \\ c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} \tag{2.16}$$

The following expressions represent the linear matrix equations;

$$A\varkappa = \mathscr{b} \tag{2.17}$$

$$\varkappa = A^{-1}\mathscr{b} \tag{2.18}$$

and there is a very concise way of writing a system of linear equations, where A is a matrix, $\varkappa$ and $\mathscr{b}$ are vectors (usually of different sizes). As a result of the solution of the expression given in 2.16, the above linear equation results in a cubic polynomial that generates a trajectory according to the desired velocity dynamics between the start and end waypoints. Controlling the velocity dynamics as a result of generating the third-degree polynomial is important in terms of control. To control the applied force, a higher-order polynomial proposition can be made that can be reduced to the derivative of the acceleration.

Each range of waypoints is expressed by a different equation as a result of applying the trajectory generation algorithm to each one separately. The trajectory generation algorithm was given to the quad-rotor as a position reference. Thus, the quadcopter successfully followed the specified trajectory. The simulation results of trajectory tracking created according to the equations 2.14-2.18 are shown in Figure 6.

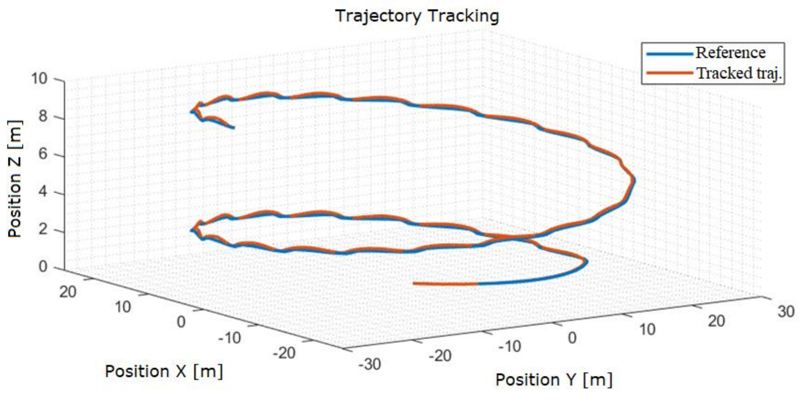

Figure 6. Trajectory tracking

3. Quad-rotor Attitude Estimation

Accurate and reliable attitude estimation is required for quad-rotor aircraft to fly successfully and perform their tasks. Attitude estimation is the process of estimating the current state of the aircraft, i.e., its position, speed, attitude, and angular velocity. A quad-rotor aircraft is usually equipped with various sensors such as accelerometers, gyroscopes, magnetometers, and barometers. These sensors measure the speed, acceleration, altitude, and orientation of the aircraft. However, the accuracy and precision of the sensors can be limited and may give inaccurate measurements due to external factors. Therefore, filtering and estimation algorithms are used in state estimation. Widely utilized algorithms, such as the Complementary Filter and Kalman Filter, analyze sensor data and rectify errors to estimate the aircraft's state. By leveraging sensor data and taking into account the physical model, these filtering algorithms make use of estimation techniques to determine the current state.

3.1. Explicit Complementary Filter use for Attitude Estimation

The attitude estimation complementary filter employs low-pass filtering to refine a low-frequency attitude estimate. The attitude estimate is derived by applying high-pass filtering to biased high-frequency accelerometer data and integrating the output of the gyroscope. These estimates are then combined to obtain a comprehensive estimate of attitude. If the pitch angle and roll angle of a quad-rotor are treated as decoupled processes, it is possible to design a Single Input Single Output filter for each signal, (Buskey, Roberts, Corke, Ridley & Wyeth, 2004). Figure 7 depicts the Explicit Complementary Filter (ECF) implementation.

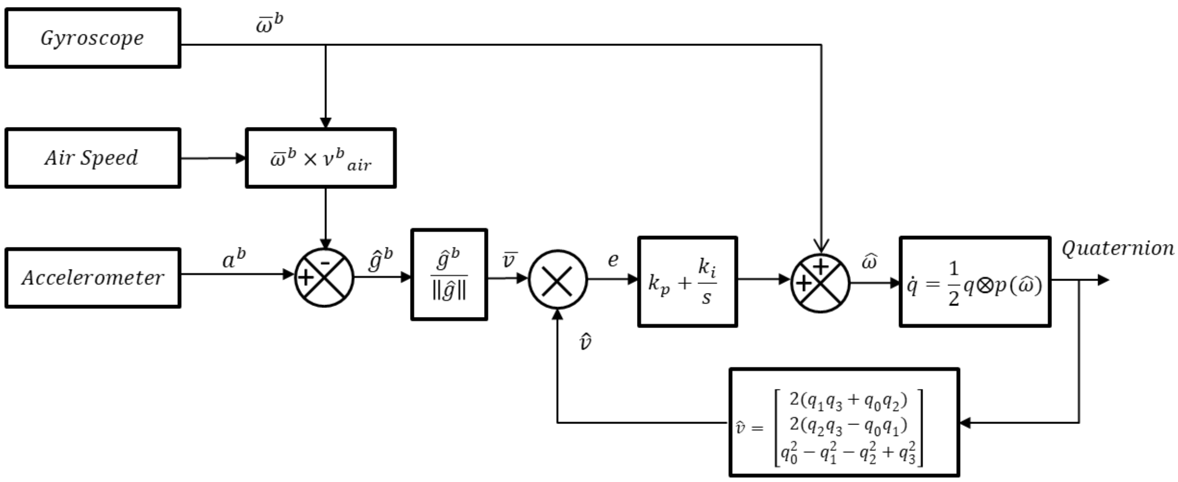

Figure 7. Acceleration compensated Explicit Complementary Filter scheme (edited from (Euston, Coote, Mahony, Kim & Hamel, 2008)

In addition to the measured angular velocities $\overline{\omega}^b$, the ECF also captures the inertial direction, indicated as $\bar{v}$;

$$\bar{v} = \frac{\hat{g}^b}{|\hat{g}^b|} \tag{3.1}$$

where the estimation of the gravitational direction $\hat{g}^b$ obtained from the system is used to derive the inertial direction $\bar{v}$.

The ECF can be represented as a quaternion as follows (Euston et al., 2008);

$$\dot{\hat{q}} = \frac{1}{2}\hat{q} \otimes p(\widehat{\omega} + \delta) \tag{3.2}$$

$$\delta = k_p e + k_i \frac{e}{s} \tag{3.3}$$

$$e = \bar{v} \times \hat{v} \tag{3.4}$$

where $\hat{q}$ represents an estimate of the system attitude in a quaternion form. The innovation term δ here is generated by a PI block and the error e represents the relative rotation between the measured inertial direction $\bar{v}$ and the predicted inertial direction $\hat{v}$. The proportional gain is denoted as k_p, while k_i represents the integral gain.

The estimated gravitational direction $\hat{v}$ aligns with the z-axis of the inertial frame;

$$\hat{v} = \begin{bmatrix} 2(q_1q_3 + q_0q_2) \\ 2(q_2q_3 - q_0q_1) \\ q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}. \quad (3.5)$$

The most common approach for compensation with the ECF involves the use of proportional or proportional-integral (PI) control. The proportional component is used to make the frequency transition between the gyro estimates obtained by the quaternion update and the attitude estimates based on accelerometers. Gyro bias is adjusted for the use of the integral term in the PI correction.

3.2. Extended Kalman Filter use for State Estimation

The Extended Kalman Filter (EKF) utilizes both the system and observation expressions to perform state estimation in nonlinear systems. The algorithm uses the final prediction state and the associated error covariance matrix to perform state estimation.

The EKF filter linearizes the system equations using the Taylor series or by taking the Jacobian. It then computes the prediction state using the linearized system equations. (Crasidis & Junkis, 2011). The observation equations are similarly linearized and provide feedback to ensure agreement between the forecast state and actual observations. This process updates the system state and covariance matrix, reducing errors and providing a more accurate estimate (Wang, Yang, Hatch & Zhang, 2004);

$$\begin{aligned} x_k &= f(x_{k-1}, u_k) + w_k \\ z_k &= h(x_k) + v_k \end{aligned} \quad (3.6)$$

where w_k represents the normal random process with zero mean and covariance matrix Q_k.v_k represents the white Gaussian noise in the measurements also with zero mean and covariance matrix R_k. u denotes the control vector, while z_k represents the output of sensors measuring the state vector component x_k of the state vector at time step k. Considering 2.4 and 2.5, the state vector can be expressed in the form as follows;

$$x_k = [x \;\; y \;\; z \;\; u \;\; v \;\; w \;\; \varphi \;\; \theta \;\; \psi]^T. \quad (3.7)$$

Expressions in 3.6 are nonlinear; therefore, we use the EKF where the model is linearized in a certain neighborhood of the considered point $(\hat{x}^k, u_k)$ via an expansion into a Taylor series (Crasidis & Junkins, 2011);

$$\begin{aligned} x^{k+1} &\approx f\left(\hat{x}^k, u_k\right) + F_k\left(x - \hat{x}^k\right) + w_k \\ z_k &\approx h\left(\hat{x}^k\right) + H\left(x_k - \hat{x}^k\right) + v_k \end{aligned} \quad (3.8)$$

where

$$F_k = \left(\frac{\partial f}{\partial x}\middle| x = \hat{x}^k\right), H_k = \left(\frac{\partial h}{\partial x}\middle| x = \hat{x}^k\right) \quad (3.9)$$

The expressions for extrapolation and correction of the EKF follow;

$$\hat{x}^k = f\left(\hat{x}^{k-1}, \hat{u}^{k-1}\right) \quad (3.10)$$

$$P_k = F_k P_{k-1} F_k^T + Q_k \quad (3.11)$$

$$K_k = \frac{P_k H_k^T}{H_k P_k H_k^T + R_k} \quad (3.12)$$

$$P_k = (I - K_k H_k) P_k \quad (3.13)$$

4. Experimental Results

In this section, the position control using trajectory generation and state estimation of the quad-rotor implemented in MATLAB ®2022a Simulink environment are presented. The model runs by feeding the estimation results to the controller.

The North, East, and Down (NED) position of the quad-rotor and the result of the EKF filter are shown in Figure 8. The North, East, and Down (NED) velocity of the quad-rotor and the result of the EKF filter are shown in Figure 9. Quad-rotor attitude is estimated with ECF and EKF and the results are shown in Figure 10. The data represented with the legend sensor measurements is the position/velocity/direction information obtained from the GNNS sensor model of the system.

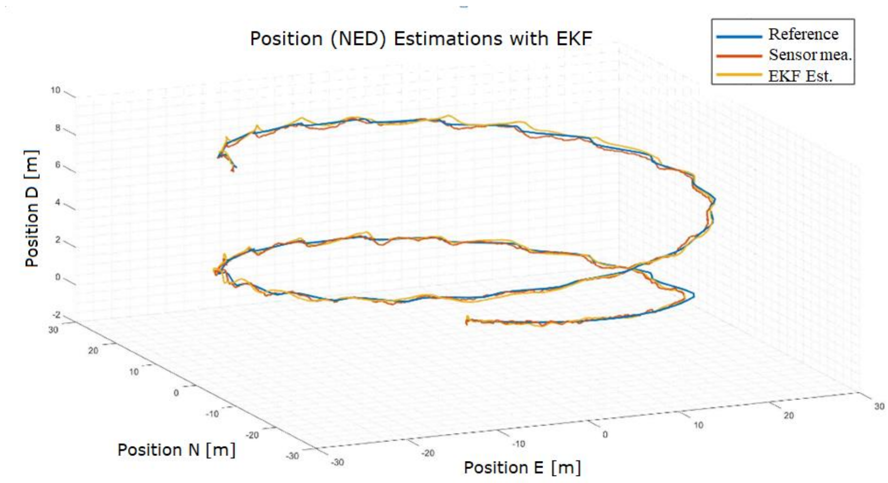

Figure 8. North, East, and Down (NED) position of the quad-rotor

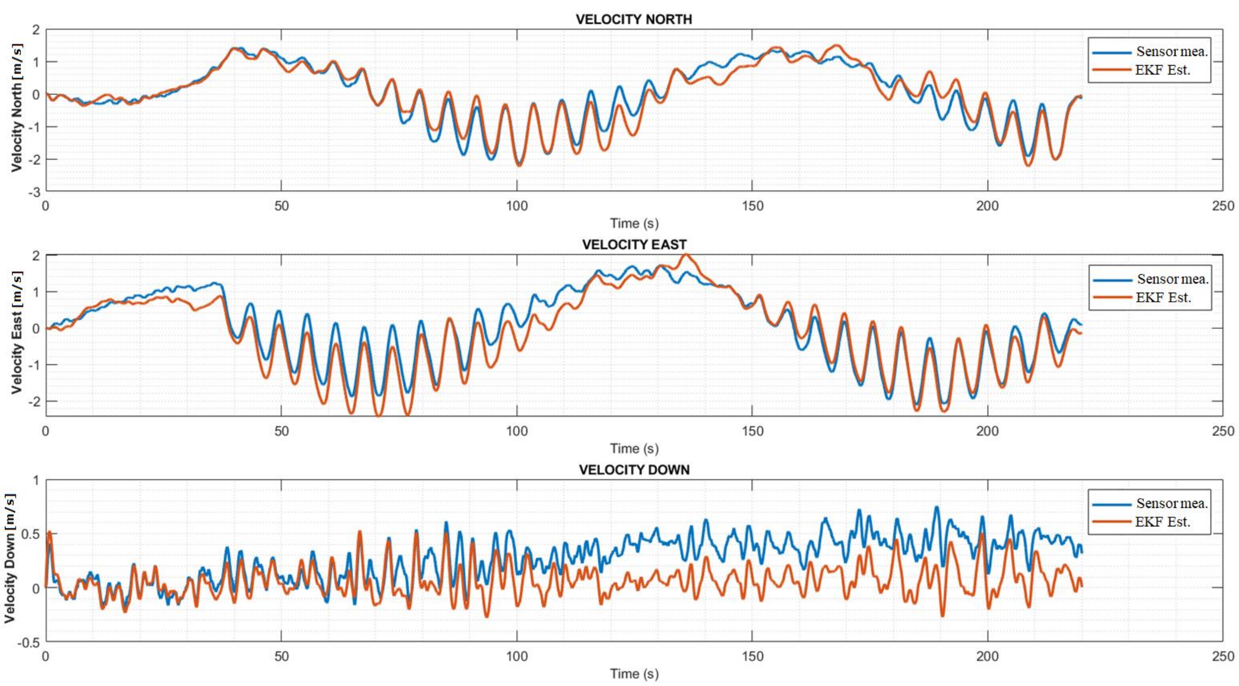

Figure 9. North, East, and Down (NED) velocity of the quad-rotor

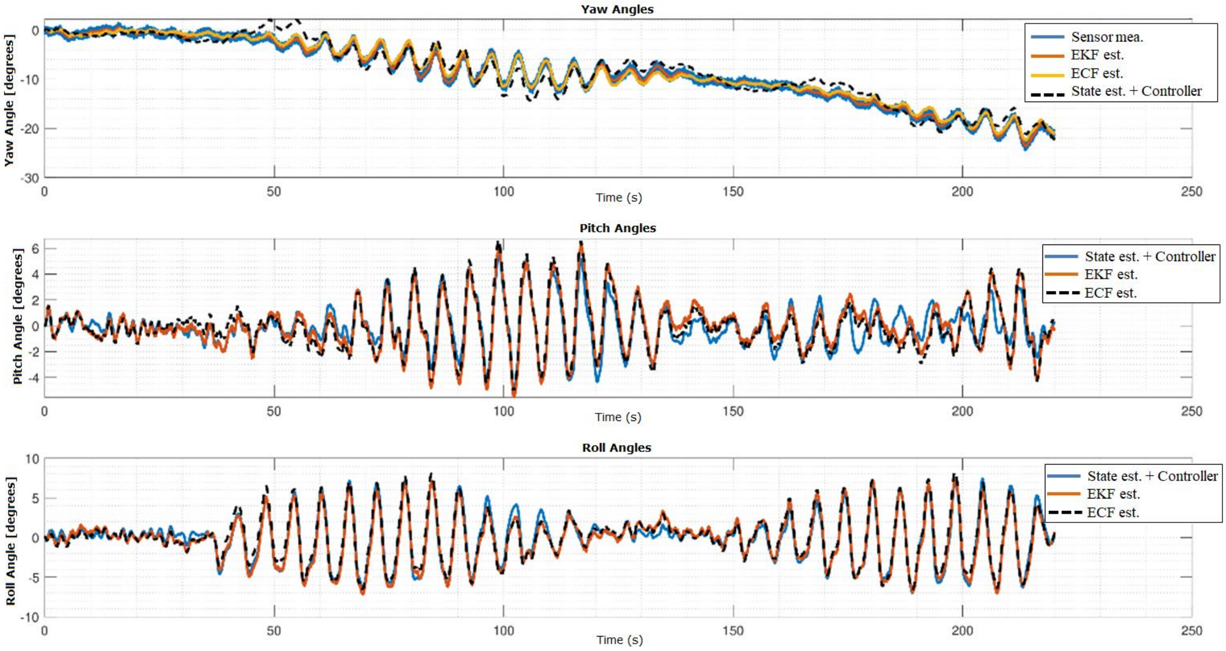

Figure 10. The attitude angles of the quad-rotor

The Root Mean Square Error (RMSE) values of the estimated states with EKF filter are given in the Table 3, since the error difference is difficult to notice on the figures. The values shown in this table vary depending on the process noise matrix and the Gaussian measurement noise matrix which are normally utilized with zero mean and covariance matrix. These matrices are formed considering parameters of the utilized sensor models and their regular working conditions. Upon examining the outcomes, it is evident that the EKF filter exhibits minimal prediction errors. Table 4 illustrates the RMSE values of the attitude estimation errors for EKF and ECF filters. The RMSE values given in this table were created by taking the attitude signal difference between filter estimations and the reference signals computed from the trajectory. The attitude estimation accuracy of the ECF filter is worse than that of the EKF filter, regarding the RMSE values of the filters' attitude estimates. This is because the ECF method is a fixed gain filter and cannot track the dynamic properties of the quad-rotor adaptively. The EKF and ECF filter estimation errors are computed for both the noisy and noise free data.

Table 3
The RMSE of the EKF state estimation

State	RMSE
Position [NED]	0.449 m
Velocity [NED]	0.25 m/s
Yaw (ψ)	0.26 °
Pitch (θ)	0.22 °
Roll (φ)	0.22 °

Table 4
Attitude estimation of filters comparison table

RMS	Yaw (ψ)	Pitch (θ)	Roll (φ)	Simulated Data Behaviour
ECF Error	1,62°	0,89°	0,76°	Noisy
EKF Error	0,62°	0,8°	0,61°	Noisy
EKF-ECF	1,55°	0,4°	0,64°	Noisy
ECF Error	0,97°	0,59°	0,51°	Noise Free
EKF Error	0,42°	0,54°	0,59°	Noise Free
EKF-ECF	1,18°	0,32°	0,58°	Noise Free

5. Conclusions

In this study, the state estimation, and position control using trajectory generation of a quad-rotor aircraft are implemented. The performances of the filters used for state estimation are compared.

Position control was executed on the quad-rotor by planning an optimal path as waypoints within the trajectory tracking algorithm. These waypoints were populated and used to create a trajectory through the trajectory generation algorithm at a specified sampling rate.

Dynamic modeling of the system was conducted, and sensor models were developed for state estimation. Explicit Complementary Filter and Extended Kalman Filter are designed for attitude estimation and state estimation respectively. Differing from other studies, we integrated the output of the designed filter into the controller while the quad-rotor tracked the trajectory. Our analysis revealed that the filters introduced minimal errors during quad-rotor trajectory tracking.

In our upcoming research, we will focus on path planning and trajectory optimization for the quad-rotor. We will also explore state estimation techniques, including the Unscented Kalman Filter (UKF), Adaptive Gain Complementary Filter (AGCF), and Particle Filters (PF), and compare their state estimates.

Author Contributions

Muharrem Mercimek: All design activities, experimental studies, measurements, and theoretical calculations were made under the supervision of Muharrem Mercimek. The manuscript was formulated, edited, and revised by the same author.

Onur Sarıpınar: The system model for the quad-rotor was created; the position controller, the trajectory tracking algorithm, and the design of the attitude estimation filters were implemented by Onur Sarıpınar. The analysis of the filters that perform state estimation was also carried out by the same author.

Conflicts of Interest

The authors declare no conflict of interest.

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