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A Note on The Maximum Circle Inverses of Lines in The Maximum Plane

Süheyla Ekmekçi [1*]

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Abstract — In this study, the images of lines under inversion in maximum circle are analytically examined. It is observed that the image of line not passing through the center of the inversion is not a maximum circle, but the closed curve containing at least one parabola arc. Some properties regarding the images of lines are introduced. Then, the images of line segments are examined according to the positions of their end points. In addition, it is seen that the inversion in maximum circle transforms the pencil of parallel lines (except line passing the center) to the set of the closed curves passing the center of inversion. Also, the images of the concurrent lines under inversion with respect to a maximum circle are considered and the results are presented.

Subject Classification (2020): 51B20; 51F99; 51K99.

1. Introduction and Preliminaries

In the real plane and space, the distance between two points is measured in various ways using different distance functions. The most well-known distance functions include the Euclidean distance, the maximum distance, the taxicab distance. The analytical planes equipped with these distance functions are the non-Euclidean geometries. There have been many studies that contributed to the literature in non-Euclidean planes and spaces, [1-4,6,12,21]. Maximum distance, also known as the Chebyshev distance or L_∞-distance (the limit of the L_p-distances when $p \to \infty$), is a way of measuring the distance between any two points. In analytical plane, the maximum distance between two points $A_1 = (x_1, y_1)$ and $A_2 = (x_2, y_2)$ is given by $d_M(A_1, A_2) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$. Its simple formula and intuitive interpretation make it a useful tool in computer science and engineering applications, especially when dealing with problems that involve movement in a grid or lattice.

Inversion in a circle is a geometric transformation such that a point in analytical plane invert another point. Apollonoius of Perga introduced the inversion in circle in his work with the title "Plane Loci". In the 1830s, Steiner studied the inversion in circle, systematically. Many researchers have studied the inversions in circle and contributed to them development since then, [7]. Moreover, inversion maps in an elipse, a sphere, an ellipsoid, parallel lines, central cones, star shape sets were defined and introduced, [8],[11],[18-20]. Besides, inversions with respect to a circle and a sphere in some non-Euclidean geometries such as the taxicab geometry, the Chinese Checkers geometry, the maximum

* sekmekci@ogu.edu.tr
[1] Department of Mathematics and Computer Science, Faculty of Science, Eskişehir Osmangazi University, Eskişehir, Turkey.
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geometry were defined and their basic properties were given, [5], [9], [10], [14-17], [22]. In this article, the images of lines under inversion in maximum circle have been analytically examined. It has been observed that the image of line not passing through the center of the inversion is not a maximum circle, but the closed curve. Properties regarding their images as the position of lines have been introduced. Then, the images of line segments have been examined according to the fact that the end points of them lie in the regions determined by the separator lines passing through the center of inversion. In addition, it has been seen that the inversion in maximum circle transforms the pencil of parallel lines (except line passing the center) to the set of the closed curves passing the center of inversion. Furthermore, the images of the concurrent lines under inversion with respect to a maximum circle have been examined and the results were presented.

In sequel, the some definitions, concepts and theorems required for this study are summarized as the following:

Definition 1.1. Let A_1 and A_2 be two points whose coordinates are (x_1, y_1) and (x_2, y_2) in analytical plane, respectively. The maximum distance between these points is

$$d_M(A_1, A_2) = \max\{|x_1 - y_1|, |x_2 - y_2|\}.$$

It is clear by definition 1.1 that the maximum distance between these points is equal to the greatest of the lengths of the line segments parallel to the coordinates axes in the right triangle with the hypotenuse A_1A_2.

The maximum plane is the analytical plane endowed with the maximum distance and symbolized by $\mathbb{R}^{\mathbf{2}}_{\mathbf{M}}$. The maximum plane closely resembles the Euclidean plane with the exception of its distance measurement. While the points and lines of the maximum plane are the same as the Euclidean plane, and the angles are measured the same way as the Euclidean plane, the defining characteristic lies in its distance function. Krause classified lines depending on their slope as the following definition:

Definition 1.2. Let m be the slope of the line l in maximum plane. The line l is called the steep line, the gradual line and the separator line in the cases of $| m |>1$, $| m |<1$ and $| m |=1$, respectively. In the special cases that the line l is parallel to x-axis or y-axis, l is named as the horizontal line or the vertical line, respectively,[13].

A circle is a set of points that are equidistant from a given point. Since the maximum distance used to measure this equidistance is different from the Euclidean distance, a circle in maximum plane has a different shape than in Euclidean geometry.

Definition 1.3. The set of all points on the maximum plane which are the given r maximum distance from the given point $M = (m_1, m_2)$ is called the maximum circle centered at $M = (m_1, m_2)$ and radius r.

It is seen by definition 1.3 that the maximum circle centered at the point $M = (m_1, m_2)$ and radius r is the set

$$\mathcal{C} = \{(x, y): \max\{|x - m_1|, |y - m_2| = r\}.$$

As particular case, the maximum unit circle is

$$\mathcal{C} = \{(x, y): \max\{|x|, |y| = 1\},$$

that is the square.

Every Euclidean translation preserves the maximum distance. So, it is an isometry in the maximum plane. Reflections in the coordinate axes and the separator lines through the origin and rotations about the origin by integer multiples of $\frac{\pi}{2}$ are isometries in maximum plane, [21].

2. The Inversion in the Maximum Circle

Let $\mathcal{C}$ be the maximum circle centered at the point O and radius r in $\mathbb{R}^2_{\mathrm{M}}$. The inversion in the maximum circle $\mathcal{C}$ is

$$I_{\mathcal{C}}: \mathbb{R}^2_{\mathbf{M}} \backslash \{O\} \to \mathbb{R}^2_{\mathbf{M}} \backslash \{O\}$$
$$X \to I_{\mathcal{C}}(X) = X'$$

where the point X' is on the ray $\overrightarrow{OX}$ and $d_M(O,X).d_M(O,X') = r^2$. $\mathcal{C}$ is called the circle of the maximum circle inversion; O is called the center of the maximum circle inversion; r is called the radius of the maximum circle inversion; and the point X' is called the maximum circle inverse of the point X with respect to $I_{\mathcal{C}}$. For any point X different from O in the maximum plane, the maximum circle inversion map has the property ${I_{\mathcal{C}}}^2(X) = X$.

Theorem 2.1. The maximum circle inversion maps the point (except the inversion center) inside of the maximum inversion circle to the point outside of it, and vice versa, [9,10,22].

Teorem 2.2. Let $\mathcal{C}$ be the maximum circle with centered at the point $O = (0,0)$ and radius r. If the points $P = (x, y)$ and $P' = (x', y')$ are a pair of the maximum circle inverse points by $I_{\mathcal{C}}$, then the following equality between the coordinates of P and P' are obtained

$$(x', y') = \frac{r^2}{(max\{|x|, |y|\})^2}(x, y),$$

[9,10,22].

Corollary 2.3. Let $\mathcal{C}$ be the maximum circle with centered at the point $O = (a, b)$ and radius r. For every point P different from O, if the points $P = (x, y)$ and $P' = (x', y')$ are a pair of the maximum circle inverse points with respect to $I_{\mathcal{C}}$, then the following equalities between the coordinates of P and P' are obtained

$$x' = a + \frac{r^2(x-a)}{(max\{|x-a|, |y-b|\})^2}$$
$$y' = b + \frac{r^2(y-b)}{(max\{|x-a|, |y-b|\})^2},$$

[9,10,22].

Since translations are isometries in maximum plane, no generality is lost to take the center of maximum circle inversion at origin. Therefore, throughout this study, the center O is the origin unless otherwise stated.

2.1. Images of the Lines Under the Inversion in the Maximum Circle

In this section, the images of the lines under the inversion in the maximum circle are discussed analytically and the results are presented. The inversion in a Euclidean circle leaves fixed lines passing through the inversion center, while it tranforms lines not passing through the inversion center to circles passing through the inversion center. While similar results are obtained for the image of the line passing through the center of the maximum circle inversion, it has been observed that the images of the lines not passing through the center have different shapes and properties depending on their positions. These properties are given in the following theorems.

Theorem 2.1.1. The inversion in the maximum circle maps the lines passing through the center of inversion to themselves, [9, 10, 22].

Theorem 2.1.2. The inversion in maximum circle maps a line not passing through the center of the inversion to the closed curve passing through the center of the maximum inversion.

Proof. Suppose that $I_{\mathcal{C}}$ and ℓ are the inversion in the maximum circle $\mathcal{C}$ centered at $O=(0,0)$ with the radius r and the line with the equation $ax+by+c=0$ where $a,b,c\in\mathbb{R}$ and $a^2+b^2\neq 0, c\neq 0$, respectively. $I_{\mathcal{C}}$ maps the point $X=(x,y)$ on the line ℓ to the maximum inverse point $X'=(x',y')$ satisfying the equation

$$ax'+by'+\frac{c}{\mathrm{r}^2}(\max\{|\mathrm{x}'|,|\mathrm{y}'|\})^2=0.$$

This means that the image is the closed curve consisting the union of two parabola arcs with equations $ax'+by'+\frac{c}{r^2}(x')^2=0$ for $|x'|>|y'|$ and $ax'+by'+\frac{c}{r^2}(y')^2=0$ for $|\mathrm{x}'|\leq|\mathrm{y}'|$. They are not maximum pabola arcs. It is seen that the both of arcs pass through O and the line $ax'+by'=0$ is tangent to them at O. Since the directrices of two parabolas have the equations $y=\frac{(a^2+b^2)r^2}{4bc}$ and $x=\frac{(a^2+b^2)r^2}{4ac}$, parabola arcs are on two orthogonal parabolas whose vertices are $\mathrm{T}=(\lambda,\lambda\frac{m}{2})$ and $\mathrm{S}=(\frac{\beta}{2m},\beta)$ where m is the slope of ℓ, $\lambda=-\frac{ar^2}{2c}$ and $\beta=-\frac{br^2}{2c}$, (Fig.1 (a)). In addition, in cases of $|m|<2$, $|m|>\frac{1}{2}$ and $m\in\left[-2,-\frac{1}{2}\right]\cup\left[\frac{1}{2},2\right]$, only the vertex T , only the vertex S and both vertices lie on the image, respectively. If the line ℓ does not intersect $\mathcal{C}$, $d_M(O,X)>r$ for every point X on ℓ. From Theorem 2.1, $d_M(O,X')<r$ where the point X' is the maximum circle inverse of the point X. And, the image of ℓ is in interior of $\mathcal{C}$. Thus, $I_{\mathcal{C}}$ maps the line ℓ not intersecting the inversion circle to the closed curve not intersecting the inversion circle.

In the case of $a=0$ or $b=0$, the line ℓ is parallel to the coordinate axis. Suppose that $a=0$. Then $I_{\mathcal{C}}$ maps the point $X=(x,y)$ on the line ℓ to the maximum inverse point $X'=(x',y')$ satisfying the equation

$$by'+\frac{c}{\mathrm{r}^2}(\max\{|\mathrm{x}'|,|\mathrm{y}'|\})^2=0.$$

This means that the image is the closed curve consisting the union of the line segment and the parabola arc. The the parabola arc is on the parabola whose vertex is the point O, whose directrix is the line with $y=\frac{br^2}{4c}$, and whose axis of symmetry is the y- axis. So, it is observed that $I_{\mathcal{C}}(\ell)$ pass through O and the x-axis is tangent to $I_{\mathcal{C}}(\ell)$ at O. Note that the parabola arc is symmetric about the axis of symmetry. Also, it is seen that the line segment in the image $I_{\mathcal{C}}(\ell)$, the directrix and the line ℓ are parallel, (Fig.1 (b)). If the line ℓ meet $\mathcal{C}$, then $I_{\mathcal{C}}$ leaves the intersection points fixed. In special cases for $a=\pm1, b=\pm1$, line ℓ is a separator line. $I_{\mathcal{C}}$ transforms the separator line ℓ with the equation $x+y+c=0$ to the closed curve with the equation

$$x'+y'+\frac{c}{r^2}(max\{|x'|,|y'|\})^2=0.$$

It states that the image $I_{\mathcal{C}}(\ell)$ consists of two parabola arcs such that they are on two orthogonal parabolas passing through the center of the maximum inversion. Since the reflection in the separator line $y' = x'$ through the inversion center maps each of the parabola arcs to the other, the image is symmetric about the line $y' = x'$. If the separator line ℓ does not meet the inversion circle, then the image is interior of the inversion circle. In the other case, the intersection points are invariant under I_C.

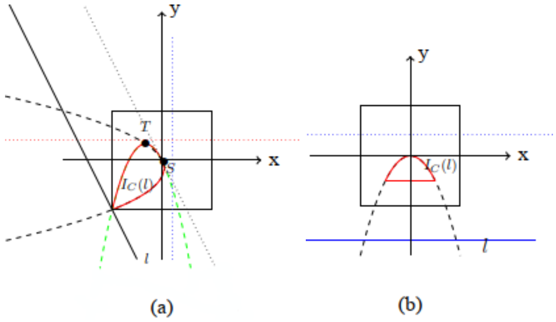

Figure 1. The maximum circle inverse of the line ℓ

Also, the following results are immediately obtained from the proof of the Theorem 2.1.2.

Corollary 2.1.3. The inversion in maximum circle maps a gradual or steep line not passing through the center of the inversion to the closed curve having the following properties:

i) It passes through the center of the maximum inversion,

ii) It consists of two parabola arcs on two orthogonal parabolas, also it pass through two vertices of them when the slope of the given line is in $\left[-2, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 2\right]$,

iii) Its tangent at the center of inversion is the line through the center parallel to the given line.

Corollary 2.1.4. The inversion in maximum circle maps a horizontal or vertical line not passing through the center of the inversion to the closed curve having the following properties:

i) It passes through the center of the maximum inversion,

ii) It consists of the line segment and the parabola arc,

iii) The coordinate axis parallel to the given line is tangent to it at the center of inversion

iv) It is symmetric about the coordinate axis perpendicular to the given line,

v) The line segment in the image, the directrix of the parabola containing the parabola arc and the given line are parallel.

Corollary 2.1.5. The inversion in maximum circle maps the seperator line not passing through the center of inversion to the closed curve having the following properties:

i) It passes through the center of the maximum inversion,

ii) It consists of two parabola arcs on two orthogonal parabolas,

iii) It is symmetric about the separator line through the center of the maximum inversion,

iv) The separator line passing through the center of inversion parallel to the given separator line is tangent to it.

2.2. The Line Segments Under the Inversion in the Maximum Circle

Let A and B be two distinct points in the maximum plane. It will be that the image of the line segment joining points A and B under the inversion in maximum circle is on the image of the line passing through points A and B. If the completion of the line segment AB passes through the center of maximum circle inversion, then the line AB inverts to itself. Therefore, the image $A'B'$ lie on the line AB. If the completion of the line segment AB does not pass through the center of maximum circle inversion, then its image is the closed curve passing through the inversion center. The inverse points A' and B' are the intersection points where the rays OA and OB meet this curve. The image of the line segment AB is the curve segment between points A' and B' such that it does not include the point O. The image $A'B'$ has different shapes depending on the position the points A and B. The classification of the images can be given by using the regions formed by the separator lines through the point O as the following:

a) Let the end points of the line segment AB be in the same region. If AB is on the line perpendicular to the coordinate axis in this region, the image $A'B'$ is a line segment. In the remain cases, the image $A'B'$ is a parabola arc, (Fig.2).

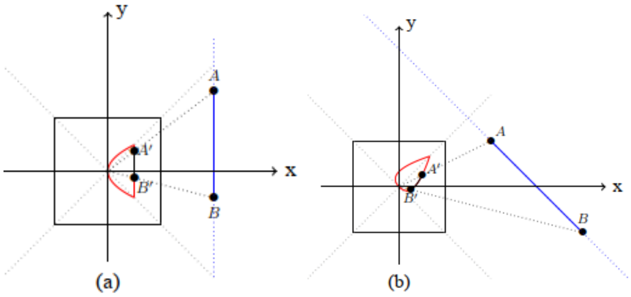

Figure 2. The maximum circle inverse of a line segment when its end points lie on same region.

b) Let the end points of the line segment AB be in two different neighboring regions. If AB is on the line parallel to the coordinate axes, then the image $A'B'$ consists of a line segment parallel to AB and a parabola arc. In the other cases, the image $A'B'$ consists of two parabola arcs, (Fig.3).

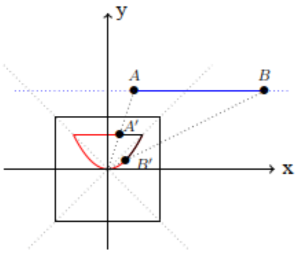

Figure 3. The maximum circle inverse of a line segment when its end points lie on neighboring regions.

c) Let the end points of the line segment AB be in two alternate regions. If AB is on the line parallel to the coordinate axis, then the image $A'B'$ is formed a line segment parallel to AB and two parabola arc. In the other cases, the image $A'B'$ consists of three parabola arcs, (Fig.4).

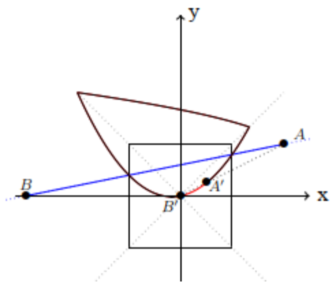

Figure 4. The maximum circle inverse of a line segment when its end points lie on alternate regions.

2.3. Images of Parallel Lines Under the Inversion in the Maximum Circle

In this section, the image of the pencil of all lines parallel to the given line is investigated. Suppose that I_C and ℓ_0 are the inversion in the maximum circle C with the center $O=(0,0)$ and radius r and the line through O with the equation $ax+by=0$ where $a^2+b^2 \neq 0$, respectively. The pencil of all lines parallel to the line ℓ_0 is the set $\{\ell : \ell \parallel \ell_0\}$, where ℓ has the line with equation $ax+by+c=0$, c $\in \mathbb{R}$. The image of this pencil under I_C is the set of inverses of all lines parallel to the line ℓ_0. It is clear from theorem 2.1.2 that the maximum circle inversion maps lines in pencil (except ℓ_0) to the closed curves through the inversion center O such that the line ℓ_0 is tangent to them at the inversion center. The set of these closed curves forms the image of the pencil under I_C. In the case that $a \neq 0$ and $b \neq 0$, it is known from theorem 2.1.2 and corollary 2.1.3 that every closed curve in the image of the pencil is the union of two parabola arcs through O and tangent to ℓ_0 at the point O. Additionally, each curve in the image is on two orthogonal parabolas where their vertices are on the lines $y = -\frac{a}{2b}x$ and $y = -\frac{2a}{b}x$, (Fig 5 (a)). In the case that $a = 0$ or $b = 0$, it is seen from theorem 2.1.2 and corollary 2.1.4 that every closed curve in the image of the pencil is the union of a parabola arc and a line segment parallel to ℓ_0. Also, all curves contain the vertex O and are symmetric about the coordinate axis. And, the line ℓ_0 is tangent to them at the point O, (Fig 5 (b)).

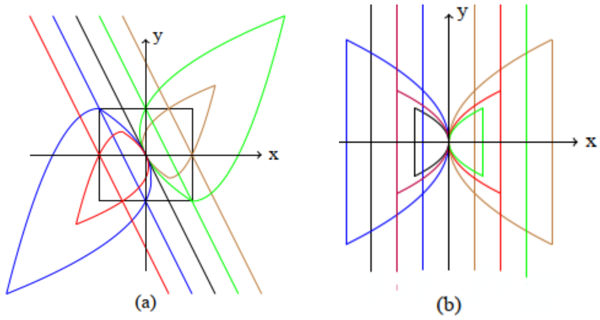

Figure 5. Images of parallel lines under the maximum circle inversion.

2.4. Images of Concurrent Lines Under the Inversion in the Maximum Circle

In this part, it is examined how their images behave when an inversion transform is applied to concurrent lines. Concurrent lines are lines that intersect at a point. It is observed that image of concurrent lines under the maximum circle inversion can vary depending on lines and the inversion circle. Also, it is well known that inversion in a Euclidean circle preserves the angles between intersecting lines. By considering inversion in maximum circle, this property is examined as the following.

Theorem 2.4.1. The angle between the two intersecting lines is the same as the angle between their maximum circle inverses at the center of maximum circle inversion.

Proof. Let θ be the angle between the intersecting lines ℓ_1 and ℓ_2 at the point N. Suppose that I_C is the inversion in the maximum circle C with the center $O = (0,0)$, the radius r. If the lines ℓ_1 and ℓ_2 pass through the inversion center, then the intersection point of ℓ_1 and ℓ_2 must be $\mathrm{N} = O$. Since $I_C(\ell_1) = \ell_1$ and $I_C(\ell_2) = \ell_2$, angle between the images is θ. In the case that only the line ℓ_1 passes through the inversion center, then $I_C(\ell_1) = \ell_1$ and $I_C(\ell_2)$ is the closed curve whose tangent at the inversion center is parallel to ℓ_2. So, angle between $I_C(\ell_1)$ and $I_C(\ell_2)$ is θ. If ℓ_1 and ℓ_2 do not pass through the inversion center, then $I_C(\ell_1)$ and $I_C(\ell_2)$ are two closed curves through the inversion center such that tangents of $I_C(\ell_1)$ and $I_C(\ell_2)$ at the inversion center are parallel to ℓ_1 and ℓ_2, respectively. Thus, angle between $I_C(\ell_1)$ and $I_C(\ell_2)$ at the inversion center is θ.

Corollary 2.4.2. The maximum circle inverses of two orthogonal lines are orthogonal at the inversion center.

Now, considering the set of concurrent lines. This set is also called the pencil of concurrent lines. The properties regarding the image of this pencil under the maximum circle inversion are introduced in the following corollary.

Corollary 2.4.3. Let N be a point in the maximum plane. The maximum circle inverse of the pencil of concurrent lines at the point N has in the following properties:

i) If the point N is the inversion center, then I_C leaves the pencil fixed.

ii) If the point N is on the maximum inversion circle, then I_C maps all lines in the pencil, except the line ON, to closed curves passing through point N and the inversion center O, (Fig 6 (a)).

iii) If the point N is not on the maximum inversion circle, then I_C maps all lines in the pencil, except the line ON, to closed curves passing through the inversion center O and the inverse N' of N, (Fig 6 (b)).

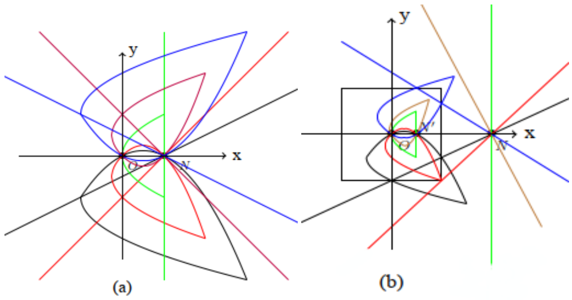

Figure 6. Images of concurrent lines under the maximum circle inversion

3. Conclusion

In this study, the focus was on the examination of the images of lines under inversion in a maximum circle. Through a detailed analytical analysis, several observations were made. It was found that when a line does not pass through the center of inversion, its image does not form a maximum circle but instead becomes a closed curve containing at least one parabolic arc. The study also introduced various properties related to the images of lines under maximum circle inversion. Furthermore, the investigation extended to the examination of line segments and their images, considering the positions of their endpoints. This analysis provided valuable insights into how the inversion in a maximum circle affects the geometric configuration consisting line segments. Another finding was that the inversion in a maximum circle transforms a pencil of parallel lines (excluding the line passing through the center of inversion) into a set of closed curves that all pass through the center of inversion. Additionally, it was examined the images of concurrent lines under inversion with respect to a maximum circle and results were obtained. These findings provide to our understanding of how parallel and concurrent lines are affected by inversion. Consequently, it is thought that the results obtained in this study contribute to the literature including the subject of inversion in non-Euclidean geometry.

Author Contributions

The author read and approved the final version of the manuscript.

Conflicts of Interest

The author declares no conflict of interest.

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