International Electronic Journal of Algebra
Volume 22 (2017) 11-27
DOI: 10.24330/ieja.325916

ON SOME GENERALIZATIONS OF REVERSIBLE AND SEMICOMMUTATIVE RINGS

Arnab Bhattacharjee and Uday Shankar Chakraborty

Received: 29 June 2016; Revised: 20 June 2017
Communicated by Abdullah Harmancı

Abstract. The concept of strongly central reversible rings has been introduced in this paper. It has been shown that the class of strongly central reversible rings properly contains the class of strongly reversible rings and is properly contained in the class of central reversible rings. Various properties of the above-mentioned rings have been investigated. The concept of strongly central semicommutative rings has also been introduced and its relationships with other rings have been studied. Finally an open question raised in [D. W. Jung, N. K. Kim, Y. Lee and S. J. Ryu, Bull. Korean Math. Soc., 52(1) (2015), 247-261] has been answered.

Mathematics Subject Classification (2010): 16S99, 16U80
Keywords: Strongly central reversible ring, strongly central semicommutative ring, symmetric ring, trivial extension

1. Introduction

Throughout this article all rings are associative with identity unless otherwise stated. Let R be a ring. Denote the polynomial ring with an indeterminate x over R by $R[x]$, the power series ring with an indeterminate x over R by $R[[x]]$, center of R by $Z(R)$, set of idempotent elements of R by $E(R)$, set of nilpotent elements of R by $N(R)$, the n by n full (resp., upper triangular) matrix ring over R by $M_n(R)$ (resp., $U_n(R)$). Following the literature, we use $D_n(R) = \{(a_{ij}) \in U_n(R) : \text{all diagonal entries are equal}\}$ and $V_n(R) = \{(b_{ij}) \in D_n(R) : b_{st} = b_{(s+1)(t+1)}$ for $s = 1, \dots, n-2$ and $t = 2, \dots, n-1\}$. Use E_{ij} for the matrix with (i, j)-entry 1 and other entries 0; $\mathbb{N}$ (resp., $\mathbb{Z}_n$) denotes the ring of positive integers (resp., integers modulo n), and $\mathbb{H}$ denotes the ring of real quaternions.

A ring R is called *reduced* if it has no nonzero nilpotent elements and is called *central reduced* [1] if all the nilpotent elements of R is central. Lambek [15] called a ring R *symmetric* if $abc = 0$ implies $acb = 0$ for all $a, b, c \in R$. A ring R is said to be Armendariz [22] if for any $f(x) = \sum_{i=0}^{m} a_i x^i, g(x) = \sum_{j=0}^{n} b_j x^j \in R[x]$, $f(x)g(x) = 0$

implies $a_ib_j = 0$ for all $0 \leq i \leq m, 0 \leq j \leq n$ and central Armendariz [2] if for any $f(x) = \sum_{i=0}^{m} a_ix^i, g(x) = \sum_{j=0}^{n} b_jx^j \in R[x]$, $f(x)g(x) = 0$ implies $a_ib_j \in Z(R)$ for all i, j. A ring R is called *reversible* [7] if $ab = 0$ implies $ba = 0$ for all $a, b \in R$ and is called *central reversible* [10] if $ab = 0$ implies $ba \in Z(R)$ for all $a, b \in R$. A ring R is called *semicommutative* [17] if $ab = 0$ implies $aRb = (0)$ for all $a, b \in R$ and is called *central semicommutative* [20] if $ab = 0$ implies $aRb \subset Z(R)$ for all $a, b \in R$. The relationship among these classes of rings is given by

$$\begin{array}{ccc}
\text{Symmetric} & \Longleftarrow & \text{Reduced} \\
\Downarrow & & \\
\text{Reversible} & \Longrightarrow & \text{Semicommutative} \\
\Downarrow & & \Downarrow \\
\begin{array}{c}\text{Central}\\ \text{Reversible}\end{array} & \stackrel{?}{\Longrightarrow} & \begin{array}{c}\text{Central}\\ \text{Semicommutative}\end{array}
\end{array}$$

A ring R is called strongly reversible [26] if the polynomial ring $R[x]$ is reversible. The class of reversible rings strictly contains the class of strongly reversible rings. Similarly a ring R is called strongly semicommutative [25] if the polynomial ring $R[x]$ is semicommutative. Every strongly semicommutative ring is semicommutative but the converse is false.

In this paper we have defined strongly central reversible and strongly central semicommutative rings as generalizations of strongly reversible and strongly semicommutative rings respectively. We have also investigated various properties of these rings and their relationships with other known rings. Lastly an open question left unanswered in [9] has been answered.

2. Strongly central reversible rings

Definition 2.1. A ring R is called *strongly central reversible* if $R[x]$ is central reversible, equivalently, whenever $f(x), g(x) \in R[x]$ satisfy $f(x)g(x) = 0$, then $g(x)f(x)$ is central in $R[x]$.

Remark 2.2.
(1) *All commutative, reduced, strongly reversible rings are strongly central reversible.*
(2) *The class of strongly central reversible rings is closed under subrings and direct products.*
(3) *Every strongly central reversible ring R is central reversible, and converse holds if R is Armendariz* [9, Proposition 2.14].

Example 2.3. *A central reversible ring need not be strongly central reversible. We refer to the example in* [9, Example 2.12]. *Let* $A = \mathbb{Z}_2[a_0, a_1, a_2, b_0, b_1, b_2, c]$ *be the free algebra of polynomials with zero constant terms in noncommutating indeterminates* $a_0, a_1, a_2, b_0, b_1, b_2, c$ *over* $\mathbb{Z}_2$. *Note that* A *is a ring without identity and consider the ideal* I *of* $\mathbb{Z}_2 + A$ *generated by*

$$a_0b_0,\ a_0b_1 + a_1b_0,\ a_0b_2 + a_1b_1 + a_2b_0,\ a_1b_2 + a_2b_1,\ a_2b_2,\ a_0rb_0,\ a_2rb_2,$$
$$b_0a_0,\ b_0a_1 + b_1a_0,\ b_0a_2 + b_1a_1 + b_2a_0,\ b_1a_2 + b_2a_1,\ b_2a_2,\ b_0ra_0,\ b_2ra_2,$$
$$(a_0 + a_1 + a_2)r(b_0 + b_1 + b_2),\ (b_0 + b_1 + b_2)r(a_0 + a_1 + a_2),\ \text{and } r_1r_2r_3r_4r_5,$$

where $r, r_1, r_2, r_3, r_4, r_5 \in A$. *Then clearly* $A^5 \in I$. *Let* $R = (\mathbb{Z}_2 + A)/I$ *and consider* $R[x] \cong (\mathbb{Z}_2 + A)[x]/I[x]$. *Then by* [9, Example 2.12], R *is central reversible whereas* $R[x]$ *is not central reversible as* $(a_0 + a_1x + a_2x^2)(b_0c + b_1cx + b_2cx^2) \in I[x]$ *but* $(b_0c + b_1cx + b_2cx^2)(a_0 + a_1x + a_2x^2) \notin Z(R[x])$ *as* $b_1ca_0 + b_0ca_1$ *is not central. Hence* R *is not strongly central reversible.*

Example 2.4. *A strongly central reversible ring need not be strongly reversible. For a commutative reduced ring* R, $D_3(R)$ *is both Armendariz* [11, Proposition 2] *and central reversible* [14, Example 2.2], *and so by Remark 2.2 (3),* $D_3(R)$ *is strongly central reversible, which is not reversible by* [12, Example 1.5] *and hence not strongly reversible.*

Lemma 2.5. [23, Theorem 2.24] *A ring* R *is central reduced if and only if* $R[x]$ *is central reduced.*

Proposition 2.6. *Every central reduced ring is strongly central reversible.*

Proof. Let R be a central reduced ring, and let $f(x), g(x) \in R[x]$ such that $f(x)g(x) = 0$. Then $g(x)f(x) \in N(R[x]) \subset Z(R[x])$ and hence R is strongly central reversible. $\square$

Proposition 2.7. *Let* $\{R_\lambda : \lambda \in \Lambda\}$ *be rings. Then the following are equivalent:*

(1) R_λ *is strongly central reversible for each* $\lambda \in \Lambda$.
(2) *The direct product* $\prod_{\lambda \in \Lambda} R_\lambda$ *is strongly central reversible.*
(3) *The direct sum* $\bigoplus_{\lambda \in \Lambda} R_\lambda$ *is strongly central reversible.*

Corollary 2.8. *Let* R *be a ring and* $e \in E(R)$. *Then* R *is strongly central reversible if and only if both* eR *and* $(1 - e)R$ *are strongly central reversible.*

Proof. ($\Longrightarrow$) Follows from Remark 2.2 (2). ($\Longleftarrow$) Follows from Proposition 2.7. $\square$

Proposition 2.9. *For a ring R, the following are equivalent:*

(1) *R is strongly central reversible.*
(2) *$R[x_1, x_2, \cdots, x_n]$ is strongly central reversible.*
(3) *$R[[x_1, x_2, \cdots, x_n]]$ is strongly central reversible.*

Proof. The equivalence of (1), (2) and (3) can be established by showing that R is strongly central reversible $\iff$ $R[x]$ is strongly central reversible.

The argument here is essentially due to [3, Theorem 2]. ($\implies$) Let R be a strongly central reversible ring, and let $f(t), g(t) \in R[x][t]$ with $fg = 0$. We can write $f(t) = f_0 + f_1 t + \cdots + f_n t^n$ and $g(t) = g_0 + g_1 t + \cdots + g_m t^m$ where $f_i, g_i \in R[x]$. Let $k = \deg(f_0) + \cdots + \deg(f_n) + \deg(g_0) + \cdots + \deg(g_m)$ where the degree is as polynomials in x and the degree of zero polynomial is taken to be 0. Then $f(x^k) = f_0 + f_1 x^k + \cdots + f_n x^{kn}$, $g(x^k) = g_0 + g_1 x^k + \cdots + g_m x^{km} \in R[x]$ and the set of coefficients of the f_i's (resp., g_i's) equals the set of coefficients of $f(x^k)$ (resp., $g(x^k)$). Since $f(t)g(t) = 0$ and x commutes with elements of R, $f(x^k)g(x^k) = 0$. Since, R is strongly central reversible, therefore, $g(x^k)f(x^k) \in Z(R[x])$ and hence $g(t)f(t) \in Z(R[x][t])$. ($\impliedby$) Obvious as R can be considered as a subring of $R[x]$. □

Proposition 2.10. *Let R be a central Armendariz ring with the property that $ab \in Z(R)$ implies $ba \in Z(R)$ for $a, b \in R$. Then R is strongly central reversible.*

Proof. Let $f(x) = \sum_{i=0}^{m} a_i x^i, g(x) = \sum_{j=0}^{n} b_j x^j \in R[x]$ such that $f(x)g(x) = 0$. Since, R is central Armendariz, therefore, $a_i b_j \in Z(R)$ for all i, j. By hypothesis, $b_j a_i \in Z(R)$ for all i, j. Since, $Z(R)$ is a subring of R, therefore, $g(x)f(x)$ is central in $R[x]$ and hence R is strongly central reversible. □

Recall that for a ring R and an (R, R)-bimodule M, the *trivial extension* of R by M is the ring $T(R, M) = R \oplus M$ with usual addition and the following multiplication: $(r_1, m_1)(r_2, m_2) = (r_1 r_2, r_1 m_2 + m_1 r_2)$. This ring is isomorphic to the matrix ring $\{\left(\begin{smallmatrix} r & m \\ 0 & r \end{smallmatrix}\right) : r \in R \text{ and } m \in M\}$. Also for a ring R, $R[x]/(x^n) \cong V_n(R)$ for any positive integer n, where (x^n) is the ideal generated by x^n.

Proposition 2.11. *Let R be a reduced ring. Then $R[x]/(x^n)$ is strongly central reversible for any positive integer n.*

Proof. For any positive integer n, $R[x]/(x^n)$ is strongly reversible by [26, Proposition 3.5] and hence $R[x]/(x^n)$ is strongly central reversible by Remark 2.2 (1). □

Corollary 2.12. *Let R be a reduced ring. Then $T(R, R)$ is strongly central reversible.*

Proof. We have $T(R,R) \cong R[x]/(x^2)$. The rest follows from Proposition 2.11. $\square$

We next generalize Corollary 2.12 as follows:

Proposition 2.13. *Let R be a central reduced ring. Then $T(R,R)$ is strongly central reversible.*

Proof. Let $u = x + (x^2)$ so that $R[x]/(x^2) = R[u] = R + Ru$, where u commutes with elements of R and $u^2 = 0$ in $R[u]$. Let $f, g \in R[u][t]$ with $fg = 0$. We can write $f = f_0 + f_1 u$, $g = g_0 + g_1 u$ where $f_0, f_1, g_0, g_1 \in R[t]$. Note that

$$0 = fg = (f_0 + f_1 u)(g_0 + g_1 u) = f_0 g_0 + (f_0 g_1 + f_1 g_0)u$$

as $f_1 g_1 u^2 = 0$. This gives $f_0 g_0 = 0, f_0 g_1 + f_1 g_0 = 0$. Using Lemma 2.5, $R[t]$ is central reduced. Therefore, $f_0 g_0 = 0$ implies $g_0 f_0$ is nilpotent in $R[t]$ and so is central in $R[t]$. Now,

$$f_0 g_1 + f_1 g_0 = 0.$$

Multiplying by g_0 from left, we get

$$g_0 f_0 g_1 + g_0 f_1 g_0 = 0.$$

Using commutativity of $g_0 f_0$, we get

$$g_1 g_0 f_0 + g_0 f_1 g_0 = 0.$$

Again multiplying by f_1 from right and using commutativity of $g_0 f_0$, we get

$$g_1 f_1 g_0 f_0 + g_0 f_1 g_0 f_1 = 0.$$

Again multiplying by g_0 from right and using $f_0 g_0 = 0$, we get

$$g_0 f_1 g_0 f_1 g_0 = 0.$$

This gives $(g_0 f_1)^3 = 0$. Therefore, $g_0 f_1$ is nilpotent in $R[t]$ and so is central in $R[t]$. By similar computations, it can be shown that $f_0 g_1 + f_1 g_0 = 0$ yields $g_1 f_0$ is central in $R[t]$. Thus, $gf = g_0 f_0 + (g_1 f_0 + g_0 f_1)u$ is central in $R[u][t]$. Hence, $R[u] = R[x]/(x^2) \cong T(R,R)$ is strongly central reversible. $\square$

From Proposition 2.13, one may suspect that if R is strongly central reversible, then $T(R,R)$ is strongly central reversible. However the following example eradicates the possibility.

Example 2.14. *Consider the ring $R = T(\mathbb{H}, \mathbb{H})$. By* [26, Corollary 3.6]*, R is strongly reversible and so R is strongly central reversible. Let $S = T(R,R)$. Note that*

$$\begin{pmatrix} \begin{pmatrix} 0 & j \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & j \\ 0 & 0 \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} \end{pmatrix} = 0$$

in S, but

$$\begin{pmatrix} \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} 0 & j \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & j \\ 0 & 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -2i \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}$$

is not central in S, showing that $S = T(R, R)$ is not central reversible and hence not strongly central reversible.

Next we shall give an example to show that a homomorphic image of a strongly reversible (hence strongly central reversible) ring need not be strongly central reversible.

Example 2.15. *Consider the ring $R = D_3(\mathbb{H})$ and the ideal $I = \left(\begin{smallmatrix} 0 & \mathbb{H} & \mathbb{H} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix}\right)$ of R. Clearly the mapping $\phi : R/I \to T(\mathbb{H}, \mathbb{H})$ given by*

$$\phi \begin{pmatrix} a & 0 & 0 \\ 0 & a & b \\ 0 & 0 & a \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$$

is an isomorphism, showing that $R/I \cong T(\mathbb{H}, \mathbb{H})$. We know $\mathbb{H}$ being a division ring is reduced and therefore $T(\mathbb{H}, \mathbb{H})$ is strongly reversible by [26, Corollary 3.6] *and so $T(\mathbb{H}, \mathbb{H})$ is strongly central reversible, however R is not even central reversible. This may be verified as follows:*

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = 0$$

in $D_3(\mathbb{H})$, but

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

is not central in $D_3(\mathbb{H})$, showing that $D_3(\mathbb{H})$ is not central reversible.

Recall that an element u of a ring R is called *right* (resp. *left*) *regular* if $ur = 0$ (resp. $ru = 0$) implies $r = 0$ for $r \in R$. An element is called *regular* if it is both right and left regular. For a ring R, we denote by Δ, a multiplicatively closed subset of R consisting of central regular elements. Let $\Delta^{-1}R$ be the localization of R at Δ. Then we have the following results:

Lemma 2.16. *For a ring R and an element $x \in R$, $x \in Z(R)$ implies $(x/u) \in Z(\Delta^{-1}R)$ for all $u \in \Delta$.*

Proposition 2.17. *A ring R is strongly central reversible if and only if $\Delta^{-1}R$ is strongly central reversible.*

Proof. ($\Longrightarrow$) Let $f(x) = \sum_{i=0}^{m}(a_i/u_i)x^i$, $g(x) = \sum_{j=0}^{n}(b_j/v_j)x^j \in \Delta^{-1}R[x]$ such that $f(x)g(x) = 0$. Let $F(x) = (u_m \cdots u_0)f(x)$ and $G(x) = (v_n \cdots v_0)g(x)$. Then $F(x)$, $G(x) \in R[x]$ such that $F(x)G(x) = 0$. Since, R is strongly central reversible, therefore, $G(x)F(x) \in Z(R[x])$. Using Lemma 2.16, we get $g(x)f(x) \in Z(\Delta^{-1}R[x])$, showing that $\Delta^{-1}R$ is strongly central reversible. ($\Longleftarrow$) Obvious as R can be considered as a subring of $\Delta^{-1}R$. $\square$

The ring of Laurent polynomials in x over a ring R consisting of all formal sums $\sum_{i=k}^{n} r_i x^i$ with usual addition and multiplication, where $r_i \in R$ and $k, n \in \mathbb{Z}$, is denoted by $R[x, x^{-1}]$.

Corollary 2.18. *For a ring R, the following are equivalent:*

(1) *R is strongly central reversible.*
(2) *$R[x]$ is strongly central reversible.*
(3) *$R[x, x^{-1}]$ is strongly central reversible.*

Proof. (1) $\Longleftrightarrow$ (2) Follows directly from Proposition 2.9.

(2) $\Longleftrightarrow$ (3) Let $S = \{1, x, x^2, \ldots\}$. Then clearly S is a multiplicatively closed subset of $R[x]$ consisting of central regular elements such that $R[x, x^{-1}] = S^{-1}R[x]$. The rest follows from Proposition 2.17. $\square$

3. Strongly central semicommutative rings

Definition 3.1. A ring R is called *strongly central semicommutative* if $R[x]$ is central semicommutative, equivalently, whenever $f(x), g(x) \in R[x]$ satisfy $f(x)g(x) = 0$, then $f(x)h(x)g(x)$ is central in $R[x]$ for any $h(x) \in R[x]$.

Remark 3.2. (1) *All commutative, reduced, strongly semicommutative rings are strongly central semicommutative.*
(2) *The class of strongly central semicommutative rings is closed under subrings and direct products.*

(3) *Every strongly central semicommutative ring R is central semicommutative, and converse holds if R is Armendariz* [24, Theorem 2.21].

Example 3.3. *A central semicommutative ring need not be strongly central semicommutative. Let $A = \mathbb{Z}_2[a_0, a_1, a_2, a_3, b_0, b_1]$ be the free algebra (with identity) over $\mathbb{Z}_2$ generated by six indeterminates $a_0, a_1, a_2, a_3, b_0, b_1$. Let I be the ideal of R generated by*

$$a_0b_0,\ a_0b_1 + a_1b_0,\ a_1b_1 + a_2b_0,\ a_2b_1 + a_3b_0,\ a_3b_1,$$
$$a_0a_j(0 \leq j \leq 3),\ a_3a_j(0 \leq j \leq 3),\ a_1a_j + a_2a_j(0 \leq j \leq 3),$$
$$b_ib_j(0 \leq i, j \leq 1),\ b_ia_j(0 \leq i \leq 1, 0 \leq j \leq 3).$$

Let $R = A/I$. Then R is semicommutative by [18, Claim 6] *and so R is central semicommutative. But $R[x]$ is not central semicommutative by* [24, Example 2.20] *and hence R is not strongly central semicommutative.*

A ring which is both Armendariz and central semicommutative (hence strongly central semicommutative) but not semicommutative, is difficult to construct. So we leave the following as an open question.

Question 3.4. *Are strongly central semicommutative rings strongly semicommutative?*

Proposition 3.5. *Every central reduced ring is strongly central semicommutative.*

Proof. Let R be a central reduced ring, and let $f(x), g(x) \in R[x]$ such that $f(x)g(x) = 0$. Then $[g(x)f(x)]^2 = 0$. Therefore, $g(x)f(x) \in N(R[x]) \subset Z(R[x])$ as $R[x]$ is also central reduced by Lemma 2.5. Let $h(x) \in R[x]$. Now, $[f(x)h(x)g(x)]^3 = f(x)h(x)[g(x)f(x)]h(x)[g(x)f(x)]h(x)g(x) = f(x)[h(x)]^3g(x)[g(x)f(x)]^2 = 0$. Thus, $f(x)h(x)g(x) \in N(R[x]) \subset Z(R[x])$. Hence R is strongly central semicommutative. □

As noted in [9, pp. 255], a central semicommutative ring need not be central reversible [9, Example 2.6], however, whether a central reversible ring is central semicommutative is still an open question. We shall next give an example to show that a strongly central semicommutative ring need not be strongly central reversible. However whether a strongly central reversible ring is strongly central semicommutative has not been answered here.

Example 3.6. *Let F be a field. By* [4, Example 4.10], *$R = F\langle a, b : ab = 0\rangle$ is both Armendariz and semicommutative, and so R is strongly semicommutative by* [22, Proposition 4.6] *and hence R is strongly central semicommutative by Remark*

3.2 (1), but R is not central reversible by [9, Example 2.6] *and so it is not strongly central reversible.*

Question 3.7. *Are strongly central reversible rings strongly central semicommutative?*

Proposition 3.8. *Let $\{R_\lambda : \lambda \in \Lambda\}$ be rings. The following are equivalent:*

(1) *R_λ is strongly central semicommutative for each $\lambda \in \Lambda$.*
(2) *The direct product $\prod_{\lambda\in\Lambda} R_\lambda$ is strongly central semicommutative.*
(3) *The direct sum $\bigoplus_{\lambda\in\Lambda} R_\lambda$ is strongly central semicommutative.*

Corollary 3.9. *Let R be a ring and $e \in E(R)$. Then R is strongly central semicommutative if and only if both eR and $(1-e)R$ are strongly central semicommutative.*

Proof. ($\Longrightarrow$) Follows from Remark 3.2 (2). ($\Longleftarrow$) Follows from Proposition 3.8. □

Proposition 3.10. *For a ring R, the following are equivalent:*

(1) *R is strongly central semicommutative.*
(2) *$R[x_1, x_2, \cdots, x_n]$ is strongly central semicommutative.*
(3) *$R[[x_1, x_2, \cdots, x_n]]$ is strongly central semicommutative.*

Proof. The equivalence of $(1), (2)$ and (3) can be established by showing that R is strongly central semicommutative $\Longleftrightarrow$ $R[x]$ is strongly central semicommutative.

The argument here is essentially due to [3, Theorem 2]. ($\Longrightarrow$) Let R be a strongly central semicommutative ring, and let $f(t), g(t) \in R[x][t]$ with $fg = 0$. We can write $f(t) = f_0 + f_1 t + \cdots + f_n t^n$ and $g(t) = g_0 + g_1 t + \cdots + g_m t^m$ where $f_i, g_i \in R[x]$. Let $h(t) = h_0 + h_1 t + \cdots + h_p t^p \in R[x][t]$ where $h_i \in R[x]$. Let $k = \deg(f_0) + \cdots + \deg(f_n) + \deg(g_0) + \cdots + \deg(g_m) + \deg(h_0) + \cdots + \deg(h_p)$ where the degree is as polynomials in x and the degree of zero polynomial is taken to be 0. Then $f(x^k) = f_0 + f_1 x^k + \cdots + f_n x^{kn}$, $g(x^k) = g_0 + g_1 x^k + \cdots + g_m x^{km}$, $h(x^k) = h_0 + h_1 x^k + \cdots + h_p x^{kp} \in R[x]$ and the set of coefficients of the f_i's, g_i's and h_i's equal to the set of coefficients of $f(x^k)$, $g(x^k)$ and $h(x^k)$ respectively. Since $f(t)g(t) = 0$ and x commutes with elements of R, $f(x^k)g(x^k) = 0$. Since, R is strongly central semicommutative, therefore, $f(x^k)h(x^k)g(x^k) \in Z(R[x])$ and hence $f(t)h(t)g(t) \in Z(R[x][t])$. Since, $h(t) \in R[x][t]$ is arbitrary, therefore, $R[x]$ is strongly central semicommutative. ($\Longleftarrow$) Obvious as R can be considered as a subring of $R[x]$. □

Proposition 3.11. *Let R be a central Armendariz ring with the property that $ab \in Z(R)$ implies $aRb \subset Z(R)$ for $a, b \in R$. Then R is strongly central semicommutative.*

Proof. Let $f(x) = \sum_{i=0}^{m} a_i x^i, g(x) = \sum_{j=0}^{n} b_j x^j \in R[x]$ such that $f(x)g(x) = 0$. Since, R is central Armendariz, therefore, $a_i b_j \in Z(R)$ for all i, j. By hypothesis, $a_i R b_j \subset Z(R)$ for all i, j. Since $Z(R)$ is a subring of R, therefore, $f(x)R[x]g(x) \subset Z(R[x])$ and hence R is strongly central semicommutative. □

Proposition 3.12. *Let R be a reduced ring. Then $R[x]/(x^n)$ is strongly central semicommutative for any positive integer n.*

Proof. For any positive integer n, $R[x]/(x^n)$ is strongly semicommutative by [25, Example 3.9] and hence $R[x]/(x^n)$ is strongly central semicommutative by Remark 3.2 (1). □

Corollary 3.13. *Let R be a reduced ring. Then $T(R, R)$ is strongly central semicommutative.*

We next generalize Corollary 3.13 as follows:

Proposition 3.14. *Let R be a central reduced ring. Then $T(R, R)$ is strongly central semicommutative.*

Proof. Let $u = x + (x^2)$ so that $R[x]/(x^2) = R[u] = R + Ru$, where u commutes with elements of R and $u^2 = 0$ in $R[u]$. Let $f, g \in R[u][t]$ with $fg = 0$. We can write $f = f_0 + f_1 u$, $g = g_0 + g_1 u$ where $f_0, f_1, g_0, g_1 \in R[t]$. Note that

$$0 = fg = (f_0 + f_1 u)(g_0 + g_1 u) = f_0 g_0 + (f_0 g_1 + f_1 g_0)u$$

as $f_1 g_1 u^2 = 0$. This gives $f_0 g_0 = 0, f_0 g_1 + f_1 g_0 = 0$. Using Lemma 2.5, $R[t]$ is central reduced. Therefore, $f_0 g_0 = 0$ implies $(g_0 f_0)^2 = 0$ and so $g_0 f_0$ is central in $R[t]$. Now,

$$f_0 g_1 + f_1 g_0 = 0.$$

Multiplying by g_0 from left and using commutativity of $g_0 f_0$, we get

$$g_1 g_0 f_0 + g_0 f_1 g_0 = 0.$$

Again multiplying by f_1 from right and using commutativity of $g_0 f_0$, we get

$$g_1 f_1 g_0 f_0 + g_0 f_1 g_0 f_1 = 0.$$

Again multiplying by g_0 from right and using $f_0 g_0 = 0$, we get

$$g_0 f_1 g_0 f_1 g_0 = 0.$$

This gives $(g_0 f_1)^3 = 0$. Therefore, $g_0 f_1$ is nilpotent in $R[t]$ and so is central in $R[t]$. By similar computations, it can be shown that $f_0 g_1 + f_1 g_0 = 0$ yields $g_1 f_0$ is central in $R[t]$. Let $h = h_0 + h_1 u \in R[u][t]$, where $h_0, h_1 \in R[t]$. For $i = 0, 1$,

$$(f_0 h_i g_0)^3 = f_0 h_i (g_0 f_0) h_i (g_0 f_0) h_i g_0 = f_0 h_i^3 g_0 (g_0 f_0)^2 = 0$$

as $g_0 f_0$ is central in $R[t]$. Similarly, $(f_0 h_i g_1)^4 = 0 = (f_1 h_i g_0)^4$ for $i = 0, 1$. Since, $R[t]$ is central reduced, therefore, $f_0 h_i g_0,\ f_0 h_i g_1,\ f_1 h_i g_0 \in Z(R[t])$ for $i = 0, 1$. Thus, $fhg \in Z(R[u][t])$ and hence $R[u] = R[x]/(x^2) \cong T(R, R)$ is strongly central semicommutative. □

From Proposition 3.14, one may suspect that if R is strongly central semicommutative then $T(R, R)$ is strongly central semicommutative. However the following example eradicates the possibility.

Example 3.15. *We consider the ring in Example 2.14, i.e., $S = T(R, R)$, where $R = T(\mathbb{H}, \mathbb{H})$. The trivial extension $R = T(\mathbb{H}, \mathbb{H})$ strongly semicommutative by* [25, Example 3.10] *and therefore by Remark 3.2 (1), R is strongly central semicommutative. Clearly,*

$$\begin{pmatrix} \begin{pmatrix} 0 & j \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & j \\ 0 & 0 \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} \end{pmatrix} = 0$$

in $S = T(R, R)$, but

$$\begin{pmatrix} \begin{pmatrix} 0 & j \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & j \\ 0 & 0 \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} j & 0 \\ 0 & j \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} j & 0 \\ 0 & j \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -2k \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}$$

is not central in S, showing that $S = T(R, R)$ is not central semicommutative and hence not strongly central semicommutative.

We shall next show that a homomorphic image of a strongly semicommutative (hence strongly central semicommutative) ring is not strongly central semicommutative.

Example 3.16. *Consider the ring $R = U_2(\mathbb{H})$ and the ideal $I = \left(\begin{smallmatrix} 0 & \mathbb{H} \\ 0 & 0 \end{smallmatrix}\right)$ of R. Clearly the mapping $\phi : R/I \to T(\mathbb{H}, \mathbb{H})$ given by*

$$\phi \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$$

is an isomorphism and so $R/I \cong T(\mathbb{H}, \mathbb{H})$. The trivial extension $T(\mathbb{H}, \mathbb{H})$ is strongly semicommutative by [25, Example 3.10] *and so it is strongly central semicommutative. However $R = U_2(\mathbb{H})$ is not even central semicommutative. This can be verified as follows:*

$$\begin{pmatrix} k & j \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & j \\ 0 & k \end{pmatrix} = 0$$

in $R = U_2(\mathbb{H})$, but

$$\begin{pmatrix} k & j \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & i \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & j \\ 0 & k \end{pmatrix} = \begin{pmatrix} 0 & i \\ 0 & 0 \end{pmatrix}$$

is not central in $R = U_2(\mathbb{H})$, showing that $R = U_2(\mathbb{H})$ is not central semicommutative.

Proposition 3.17. *A ring R is strongly central semicommutative if and only if $\Delta^{-1}R$ is strongly central semicommutative.*

Proof. ($\Longrightarrow$) Let $f(x) = \sum_{i=0}^{m}(a_i/u_i)x^i$, $g(x) = \sum_{j=0}^{n}(b_j/v_j)x^j \in \Delta^{-1}R[x]$ such that $f(x)g(x) = 0$. Let $h(x) = \sum_{k=0}^{l}(c_k/w_k)x^k \in \Delta^{-1}R[x]$. Let $F(x) = (u_m \cdots u_0)f(x)$, $G(x) = (v_n \cdots v_0)g(x)$, $H(x) = (w_l \cdots w_0)h(x)$. Then $F(x)$, $G(x)$, $H(x) \in R[x]$ such that $F(x)G(x) = 0$. Since R is strongly central semicommutative, therefore, $F(x)H(x)G(x) \in Z(R[x])$. Using Lemma 2.16, we get $f(x)h(x)g(x) \in Z(\Delta^{-1}R[x])$, showing that $\Delta^{-1}R$ is strongly central semicommutative. ($\Longleftarrow$) Obvious as R can be considered as a subring of $\Delta^{-1}R$. □

Corollary 3.18. *For a ring R, the following are equivalent:*

(1) *R is strongly central semicommutative.*
(2) *$R[x]$ is strongly central semicommutative.*
(3) *$R[x, x^{-1}]$ is strongly central semicommutative.*

4. Concepts related to Armendariz and semicommutative rings

Proposition 4.1. *Let R be a central reduced ring. Then the trivial extension $T(R, R)$ is*

(1) *central Armendariz,*
(2) *central reversible,*
(3) *central semicommutative.*

Proof. (1) Let $u = x+(x^2)$ so that $R[x]/(x^2) = R[u] = R+Ru$, where u commutes with elements of R and $u^2 = 0$ in $R[u]$. Let $f, g \in R[u][t]$ with $fg = 0$. We can write $f = f_0 + f_1u$, $g = g_0 + g_1u$ where $f_0, f_1, g_0, g_1 \in R[t]$. Note that

$$0 = fg = (f_0 + f_1u)(g_0 + g_1u) = f_0g_0 + (f_0g_1 + f_1g_0)u$$

as $f_1g_1u^2 = 0$. This gives $f_0g_0 = 0, f_0g_1 + f_1g_0 = 0$. Using Lemma 2.5, $R[t]$ is central reduced. Therefore, $f_0g_0 = 0$ implies g_0f_0 is nilpotent in $R[t]$ and so is central in $R[t]$. Now,

$$f_0g_1 + f_1g_0 = 0.$$

Multiplying by g_0 from left, we get

$$g_0f_0g_1 + g_0f_1g_0 = 0.$$

Using commutativity of g_0f_0, we get

$$g_1g_0f_0 + g_0f_1g_0 = 0.$$

Again multiplying by f_1 from right and using commutativity of g_0f_0, we get

$$g_1f_1g_0f_0 + g_0f_1g_0f_1 = 0.$$

Again multiplying by g_0 from right and using $f_0g_0 = 0$, we get

$$g_0f_1g_0f_1g_0 = 0.$$

This gives $(f_1g_0)^3 = 0$. Therefore, f_1g_0 is nilpotent in $R[t]$ and so is central in $R[t]$. By similar computations, it can be shown that $f_0g_1+f_1g_0 = 0$ yields f_0g_1 is central in $R[t]$. Thus, $f_0g_1,\ f_1g_0 \in Z(R[t])$. Hence, $R[u] = R[x]/(x^2) \cong T(R,R)$ is central Armendariz.

(2) Follows from Proposition 2.13.

(3) Follows from Proposition 3.14. □

Example 4.2. (1) *A central Armendariz ring need not be central semicommutative. Let F be a field and let $A = F\langle a,b\rangle$ be the free algebra with noncommuting indeterminates a, b over F. Let I be the ideal of A generated by b^2. Then $R = A/I$ is Armendariz (hence central Armendariz), but not central semicommutative by* [5, Example 2.7].

(2) *A central semicommutative ring need not be central Armendariz. Consider the ring in* [8, Example 2]. *Let $A = \mathbb{Z}_2[a_0, a_1, a_2, b_0, b_1, b_2, c]$ be the free algebra with zero constant terms in noncommutating indeterminates $a_0, a_1, a_2, b_0, b_1, b_2, c$ over $\mathbb{Z}_2$. Let I be the ideal of Z_2+A generated by a_0b_0,*

$a_1b_2+a_2b_1$, $a_0b_1+a_1b_0$, $a_0b_2+a_1b_1+a_2b_0$, a_2b_2, $(a_0+a_1+a_2)r(b_0+b_1+b_2)$ *with* $r \in A$ *and* $r_1r_2r_3r_4$ *with* r_1, r_2, r_3, $r_4 \in A$. *Then* $A^4 = I$. *Let* $R = (\mathbb{Z}_2 + A)/I$. *By* [8, Example 2], *R is semicommutative and so R is central semicommutative.*

We identify $a_0, a_1, a_2, b_0, b_1, b_2, c$ *with their images in R for simplicity. Then R is not central Armendariz by* [5, Example 1.5 (2)] *as* $(a_0 + a_1x + a_2x^2)(b_0 + b_1x + b_2x^2) = 0$ *but* $a_0b_1c \neq ca_0b_1$.

The classes of central reversible and central semicommutative rings are closed under subrings ([20, Lemma 2.5], [9, Lemma 2.11]). Therefore, for a central reversible (resp., central semicommutative) ring R, eRe is central reversible (resp., central semicommutative) where $e \in E(R)$. However the converse is not true in general.

Example 4.3. *Let F be a field and let* $R = M_2(F)$. *Since,* $E_{11}E_{21} = 0$, *but* $E_{21}E_{11} = E_{21} \notin Z(R)$ *and* $E_{11}E_{12}E_{21} = E_{11} \notin Z(R)$, *therefore, R is neither central reversible nor central semicommutative. But* $E_{11} \in E(R)$ *and* $E_{11}RE_{11} \cong F$, *which is both central reversible as well as central semicommutative.*

A ring R is called *nil-semicommutative* [6, Definition 2.1] if $a, b \in R$ satisfy $ab \in N(R)$, then $aRb \subset N(R)$.

A ring R is called *weak symmetric* [19, Definition 1] if $abc \in N(R)$ implies $acb \in N(R)$ for all $a, b, c \in R$.

Kim et al. in [13, Proposition 1.1] proved that a ring R is nil-semicommutative if and only if R is weak symmetric.

A ring R is called *weakly semicommutative* [16, Definition 2.1] if for any $a, b \in R$, $ab = 0$ implies $arb \in N(R)$ for all $r \in R$.

Nil-semicommutative rings are weakly semicommutative, but not conversely [6, Example 2.2]. By [24, Theorem 2.3], every central semicommutative ring is nil-semicommutative, but not conversely [24, Example 2.5]. By [14, Lemma 2.16], every central reversible ring is weakly semicommutative, but not conversely [14, Example 2.12]. We shall next prove that every central reversible ring is nil-semicommutative.

Lemma 4.4. [21, Proposition 2.1] *For a ring R, the following are equivalent:*

(1) *R is nil-semicommutative,*
(2) $aR \subset N(R)$ *for all* $a \in N(R)$,
(3) $Ra \subset N(R)$ *for all* $a \in N(R)$.

Proposition 4.5. *Every central reversible ring is nil-semicommutative.*

Proof. Let R be a central reversible ring, and let $a \in N(R)$. Then there exists $n \in \mathbb{N}$ such that $a^n = 0$. Let $r \in R$. Then $a^n r = 0$. This gives $a(a^{n-1}r) = 0$. Since R is central reversible, therefore, $a^{n-1}ra \in Z(R)$. This gives

$$(a^{n-1}ra)ra = ra(a^{n-1}ra) = ra^n ra = 0$$

from which we get $a(a^{n-2}(ra)^2)r = 0$. Using central reversibility of R, we get $a^{n-2}(ra)^3 \in Z(R)$. This gives

$$[a^{n-2}(ra)^3]ra = ra[a^{n-2}(ra)^3] = ra^{n-1}ra(ra)^2 = 0$$

from which we get $a^{n-3}(ra)^5 \in Z(R)$. Proceeding in this way, we get $(ra)^{2n} = 0$. Therefore, $ra \in N(R)$. Since, $r \in R$ is arbitrary, therefore, $Ra \subset N(R)$. Hence R is nil-semicommutative. □

Converse of Proposition 4.5 is however not true in general as shown in the following example:

Example 4.6. *Let F be a field. Consider the ring $R = U_3(F)$. By* [6, Proposition 2.5]*, R is nil-semicommutative. Consider the elements $a = E_{13}+E_{23}$, $b = E_{11}+E_{12}$ of R. Then $ab = 0$, but $ba = 2E_{13} \notin Z(R)$. Therefore, R is not central reversible.*

By [9, Proposition 1.4], a ring R is reversible if and only if $ab \in E(R)$ implies $ba \in E(R)$ for $a, b \in R$. By the proof of [9, Proposition 1.6], if R is a ring and $a, b, c \in R$ such that $abc \in E(R)$ implies $acb \in E(R)$, then R is symmetric. Jung et al. in [9, pp. 249] left the converse as an open question, i.e., does every symmetric ring R satisfy the condition $abc \in E(R)$ implies $acb \in E(R)$? We shall next give an example to show that even a division ring need not satisfy the above property.

Example 4.7. *We have $\mathbb{H}$ being a division ring is symmetric. Now $i, j, k \in \mathbb{H}$ such that $ikj = 1 \in E(\mathbb{H})$, but $ijk = -1 \notin E(\mathbb{H})$.*

By [9, Proposition 1.8], if R is a ring and $a, b \in R$ satisfy $ab \in E(R)$ implies $arb \in E(R)$ for all $r \in R$, then R is semicommutative. The converse is however not true even if R is a division ring.

Example 4.8. *We have $\mathbb{H}$ being a division ring is semicommutative. Let $a = -j$, $b = j$. Then $ab = (-j)j = 1 \in E(\mathbb{H})$. But for $r = -i$, $arb = (-j)(-i)j = i \notin E(\mathbb{H})$.*

Acknowledgment. The authors would like to thank the referee for the valuable suggestions and comments.

References

[1] N. Agayev, A. Harmanci and S. Halicioglu, *Extended Armendariz rings*, Algebras Groups Geom., 26(4) (2009), 343-354.

[2] N. Agayev, G. Gungoroglu, A. Harmanci and S. Halicioglu, *Central Armendariz rings*, Bull. Malays. Math. Sci. Soc., 34(1) (2011), 137-145.

[3] D. D. Anderson and V. Camillo, *Armendariz rings and Gaussian rings*, Comm. Algebra, 26(7) (1998), 2265-2272.

[4] R. Antoine, *Nilpotent elements and Armendariz rings*, J. Algebra, 319(8) (2008), 3128-3140.

[5] H. J. Cha, D. W. Jung, H. K. Kim, J. A. Kim, C. I. Lee, Y. Lee, S. B. Nam, S. J. Ryu, Y. Seo, H. J. Sung and S. J. Yun, *On a ring property generalizing power-Armendariz and central Armendariz rings*, Korean J. Math., 23(3) (2015), 337-355.

[6] W. Chen, *On nil-semicommutative rings*, Thai J. Math., 9(1) (2011), 39-47.

[7] P. M. Cohn, *Reversible rings*, Bull. London Math. Soc., 31(6) (1999), 641-648.

[8] C. Huh, Y. Lee and A. Smoktunowicz, *Armendariz rings and semicommutative rings*, Comm. Algebra, 30(2) (2002), 751-761.

[9] D. W. Jung, N. K. Kim, Y. Lee and S. J. Ryu, *On properties related to reversible rings*, Bull. Korean Math. Soc., 52(1) (2015), 247-261.

[10] G. Kafkas, B. Ungor, S. Halicioglu and A. Harmanci, *Generalized symmetric rings*, Algebra Discrete Math., 12(2) (2011), 72-84.

[11] N. K. Kim and Y. Lee, *Armendariz rings and reduced rings*, J. Algebra, 223(2) (2000), 477-488.

[12] N. K. Kim and Y. Lee, *Extensions of reversible rings*, J. Pure Appl. Algebra, 185(1-3) (2003), 207-223.

[13] N. K. Kim, T. K. Kwak and Y. Lee, *Semicommutative property on nilpotent products*, J. Korean Math. Soc., 51(6) (2014), 1251-1267.

[14] H. Kose, B. Ungor, S. Halicioglu and A. Harmanci, *A generalization of reversible rings*, Iran. J. Sci. Technol. Trans. A Sci., 38(1) (2014), 43-48.

[15] J. Lambek, *On the representation of modules by sheaves of factor modules*, Canad. Math. Bull., 14 (1971), 359-368.

[16] L. Liang, L. Wang and Z. Liu, *On a generalization of semicommutative rings*, Taiwanese J. Math., 11(5) (2007), 1359-1368.

[17] L. Motais de Narbonne, *Anneaux semi-commutatifs et unis riels anneaux dont les id aux principaux sont idempotents*, Proceedings of the 106th National

Congress of Learned Societies (Perpignan, 1981), Bib. Nat., Paris (1982), 71-73.

[18] P. P. Nielsen, *Semi-commutativity and the McCoy condition*, J. Algebra, 298(1) (2006), 134-141.

[19] L. Ouyang and H. Chen, *On weak symmetric rings*, Comm. Algebra, 38(2) (2010), 697-713.

[20] T. Ozen, N. Agayev and A. Harmanci, *On a class of semicommutative rings*, Kyungpook Math. J., 51(3) (2011), 283-291.

[21] Y. Qu and J. Wei, *Some notes on nil-semicommutative rings*, Turkish J. Math., 38(2) (2014), 212-224.

[22] M. B. Rege and S. Chhawchharia, *Armendariz rings*, Proc. Japan Acad. Ser. A Math. Sci., 73(1) (1997), 14-17.

[23] B. Ungor, S. Halicioglu, H. Kose and A. Harmanci, *Rings in which every nilpotent is central*, Algebras Groups Geom., 30(1) (2013), 1-18.

[24] L. Wang and J. C. Wei, *Central semicommutative rings*, Indian J. Pure Appl. Math., 45(1) (2014), 13-25.

[25] G. Yang and R. Du, *Rings over which polynomial rings are semi-commutative*, Vietnam J. Math., 37(4) (2009), 527-535.

[26] G. Yang and Z. K. Liu, *On strongly reversible rings*, Taiwanese J. Math., 12(1) (2008), 129-136.

Arnab Bhattacharjee and **Uday Shankar Chakraborty** (Corresponding Author)
Department of Mathematics
Assam University, Silchar
788011 Assam, India
e-mails: arnab.math.au@gmail.com (A. Bhattacharjee)
udayhkd@gmail.com (U. S. Chakraborty)