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Teaching Practices Article

A new encryption task for mathematically gifted students: Encryption arising from patterns

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Abstract

The concept of encryption is noteworthy in terms of both familiarizing mathematically gifted students with technological developments and working with mathematically challenging tasks. Once the proper foundations are established, students can begin to formalize encryption and decryption with algebraic formulas. Encryption can be an important resource for developing functional thinking. Based on the given information, this study designed an encryption algorithm through linear patterns that can be presented as a teaching task in classroom environments to students who are learning at elementary school level and explained the implementation process. The task named "Encryption arising from patterns" is considered important in terms of both creating an encryption algorithm and providing content for the development of mathematical patterns and therefore functional thinking. In the task of "Encryption arising from patterns", the general term of the linear pattern was created by starting from two prime numbers. The numbers corresponding to the first 29 terms of this linear pattern have been calculated. The letters of the alphabet and the terms of the pattern were paired in order. Then, Caeser's Cipher was applied to the letters in the alphabet. Thus, the numbers corresponding to the key letters were assigned to the letters in plaintext. The letters of plaintext were sent to the receiver along with the numbers corresponding to the key letters and the first three steps of the linear pattern, and the encryption task was completed.

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Introduction

The education of mathematically gifted students is one of the current topics of discussion in mathematics education (Leikin, 2021). Mathematically gifted students exert more mental effort on mathematical and complex tasks (Leikin et al., 2017). In this context, National Council of Teachers of Mathematics (NCTM, 2016) emphasizes that mathematically promising students should be provided with differentiated instruction that will develop their mathematical abilities, curiosity and creativity in mathematical learning environments inside or outside the school. The concept of encryption is noteworthy in terms of both familiarizing mathematically gifted students with technological developments and working with mathematically challenging tasks. In addition, one of the cognitive characteristics of mathematically gifted students is generalizing mathematical structures (Leikin, 2021; Paz-Baruch et al., 2022). Generalization, which is one of the elements of mathematical thinking, is related to mathematical patterns (Assmus and Fritzlar, 2022).

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People have been trying to hide the content of their written messages since the discovery of writing and have developed different methods of doing so (Holden, 2017). Encryption that ensures privacy plays a key role in ensuring data security. In the modern world, encryption can be done with many methods such as computer-based cryptography (Miller & Bossomaier, 2021). Ciphers and encryption are mathematically interesting (Holden, 2017). Cryptography is an important topic for students from primary school to university, as it provides students with the opportunity to solve problems in different contexts (Kaur, 2008).

One of the most important goals of mathematics education is to enable students to establish the connection between mathematics and daily life (NCTM, 2000). In this context, studies have been conducted in which encryption tasks were designed and implemented in mathematics education with the aim of enabling students to establish the connection between information security, technology, daily life, and mathematics (Chua, 2006, 2008; Erol & Saygı, 2021; Ho, 2018; Katrancı & Özdemir, 2013; Kaur, 2008; Özdemir & Erdoğan, 2011; Patterson, 2021). In the studies conducted, it was noticed that the pattern context was not emphasized based on the mathematical function structure of encryption. However, once the proper foundations are established, students can begin to formalize encryption and decryption with algebraic formulas (Patterson, 2021). This is one of the reasons for designing the encryption task based on patterns. Cryptology algorithms consist of mathematical functions (Holden, 2017). Although functions are included in the secondary mathematics curriculum in most countries, studies are being conducted on how to approach functional thinking with elementary students (Schliemann et al., 2012). Functional thinking focuses on the relationship between two (or more) co-varying quantities (Smith, 2008). Patterns are one of the most important tools that deal with the relationship between co-varying quantities in elementary school and enable students to think functionally (Blanton et al., 2015).

NCTM (2000) emphasized that opportunities should be created for students at all levels from preschool to high school to understand patterns, relationships and functions. Encryption can be an important resource for developing functional thinking. Encryption can also be a fun way of teaching mathematics as it is a popular and interesting science today. Based on the given information, this study designed an encryption algorithm through linear patterns that can be presented as a teaching task in classroom environments to mathematically gifted students who are learning at elementary school level and explained the implementation process. The task named "Encryption arising from patterns" is considered important in terms of both creating an encryption algorithm and providing content for the development of mathematical patterns and therefore functional thinking. In addition, it is important to design encryption tasks that require mathematical challenge for mathematically gifted students. Providing encryption task for mathematically gifted students can offer foresight to teachers or curriculum developers in their differentiated program preparation studies.

Theoretical Framework

Mathematical giftedness

According to Krutetskii (1976), one of the authoritative names in the field of mathematical ability, mathematical giftedness is a combination of mathematical abilities that manifests itself as a successful performance in a mathematical task or superior creativity in a subject. Leikin (2019) defines a mathematically gifted student as a student who demonstrates a high level of mathematical performance and produces creative mathematical ideas within a reference group. Mathematically gifted students can differentiate themselves from other students by exerting less mental effort on mathematical and complex tasks (Leikin et al., 2017). In addition, gifted students prefer to deal with challenging tasks and autonomy in learning environments (Wu et al., 2018). Mathematical challenge is defined as a mathematical challenge that a person is motivated to overcome (Leikin, 2014). In this context, it is emphasized that differentiated instruction should be provided in the education process of mathematically gifted students. Thus, mathematically gifted students will have opportunities to develop their mathematical abilities, curiosity and creativity in their mathematics learning processes (NCTM, 2016). In addition to all these, it is important to reveal and develop the abilities of mathematically gifted students with the developing technology in terms of the development of countries in mathematics, science and technology fields (Erdoğan & Erben, 2020; Sheffield, 2018). The concept of encryption is noteworthy in terms of both familiarizing mathematically gifted students with technological developments and working with mathematically challenging tasks.

Among the cognitive characteristics of mathematically gifted students are the competencies of generalizing, abstracting and noticing mathematical structures, relations and patterns (Assmus & Fritzlar, 2022; Leikin, 2021; Paz-Baruch et al., 2022). Generalization, which is one of the elements of mathematical thinking and thus algebraic thinking, is related to mathematical patterns (Assmus & Fritzlar, 2022). Although it is emphasized that more attention should be paid to patterns in studies conducted in the field of education of mathematically gifted students (Leikin & Sriraman, 2022), studies examining the patterning skills of gifted students are very limited (e.g., Amit & Neria 2008; Assmus & Fritzlar, 2022; Eraky et al., 2022; Erdogan & Gul, 2022). Therefore, it is expected that the design of an encryption task based on patterns will contribute to the field of education of mathematically gifted students.

Encryption

Derived from the words "kryptos" and "lo-gos" in Greek, the word kryptos in encryption means "hidden", logos means "establishing cause-effect relationship, logical analysis area" (Bauer, 2021). As the first example, people who examine secret messages usually use the terms code and cipher in two different meanings. "A code consists of thousands of words, phrases, letters and syllables that replace these plain text elements with code words or code numbers. In ciphers, the basic unit is letter. The work of sending secret messages with codes and ciphers is called cryptography. The work of reading such secret messages without permission is called cryptanalysis or code breaking. Cryptography and cryptanalysis together form the field of cryptology (Holden, 2017). Cryptology is the science of encryption. While classical cryptology is concerned with hiding messages, modern cryptology since 1975 is concerned with communication in the presence of enemies (Bauer, 2021).

The history of encryption dates back hundreds of years. Ancient Roman Emperor Julius Caesar, he used a simple substitution cipher known as Caesar's Cipher, which is a classic example of symmetric key encryption (Santos, 2023). Roman historian Suetonius describes Caesar's Cipher as follows: Caesar has letters to Cicero and if he has something secret to say in the last one, he writes it in code. Caesar changes the order of the letters in the alphabet and writes no words in an incomprehensible way. Anyone who wants to solve them and find their meanings should replace D, the fourth letter of the alphabet, with A, and others in the same way (Holden, 2017). That is, in Caesar's Cipher, each letter in the alphabet is encrypted with three next letters.

Ciphers and encryption are mathematically interesting (Holden, 2017). Since cryptology is a mathematical science based on number theory in general, cryptology algorithms consist entirely of mathematical functions. For this reason, one-to-one and covering functions are used to encrypt messages in cryptology, while inverse functions are used to decrypt them (Santos, 2023).

Patterns and Functional Thinking

Patterns have an important place in the development of functional thinking, which is one of the elements of algebraic thinking in mathematics education (Steele, 2008). When looking at the definitions of pattern, according to Souviney (1994), pattern; it is the systematic structuring of geometric shapes, sounds, symbols and actions. Guerrero and Rivera (2002) defined pattern as a rule between the elements of a series of mathematical objects (numbers, shapes, etc.) that are structured. Features such as understanding, creating, continuing patterns are important skills in seeing mathematical relationships, generalizing, grasping the essence of mathematics (Burns, 2000). The relationships between numbers or mathematical shapes in patterns can be classified as recursive and explicit. Obtaining the next step by using the previous step is a recursive relationship (Orton & Orton, 1999). In explicit relationship, it is about relating the step number with the term in that step. That is, when the explicit relationship is thought of as a functional relationship between dependent and independent variables (Blanton & Kaput, 2004). Functional thinking concerns the process of generalizing this relationship based on specific situations (Smith, 2008). Therefore, the role of patterns in the development of students' functional thinking cannot be denied.

Patterns can be categorized according to their structure and presentation forms. It is seen that pattern types are classified in different ways in the literature. Stacey (1989) classified patterns according to n. term as linear (an+b) and quadratic (an^2+bn+c). There are achievements in the elementary school mathematics curriculum for students to work with linear patterns and generalize linear patterns (Ministry of National Education, 2018). Therefore, the encryption algorithm task was structured in the context of linear patterns.

Purpose

The purpose of this study is to design an encryption algorithm task that can be used in mathematics classes at elementary level, can be enhanced with variable character table, and is based on linear patterns and prime numbers.

Structures of Math Teaching Practice

The task named "Encryption arising from patterns" can be applied to students who have the ability to generalize linear patterns. In the context of Türkiye, students who are studying at the seventh and eighth grade level can use the task of "Encryption arising from patterns". Indeed, in the elementary school mathematics curriculum, acquiring the achievement of "Expresses the rule of number patterns with a letter, finds the desired term of the pattern whose rule is expressed with a letter" at the seventh grade level is a prerequisite for performing the task. In addition, students need to have knowledge about the concept of prime numbers at previous grade levels. Therefore, the achievement of "Determines prime numbers by their properties." at the sixth grade level can also be seen as a prerequisite. The task can be applied to students individually or in groups. The duration of the task application may vary depending on student skills, but it may take one lesson hour (40 minutes). There is no need for special material for the implementation of the task. The task can be presented to students in the form of activity sheets.

Implementation of Math Teaching Practice

The steps of the encryption algorithm based on linear patterns and prime numbers, called "Encryption arising from patterns", are explained in the following section.

Assigning Numbers in Pattern Steps to Letters and Applying Caesar's Cipher

In the task of "Encryption arising from patterns", firstly, any two of the prime numbers 2, 3, 5, 7, 11, 13, 17, 19 are taken. A pattern rule is created from this prime number pair. The pattern rule is "Small prime number. Term order + Large prime number". The first three terms of the pattern are created as a numberical or figural pattern to send to the receiver. Since the Turkish alphabet consists of 29 letters, the terms in the first 29 steps of the pattern are determined. For terms greater than 29, the value (mod 29) is taken based on modular arithmetic. Since elementary school students do not see the concept of modular arithmetic, the instruction "Take the remainder of dividing the result by 29" is used. The terms of the pattern are assigned to each letter in the alphabet in order. Then, according to Caeser's Cipher, each letter in the alphabet is encrypted with three next letters and key letters are found. Thus, the numbers corresponding to the key letters are determined. As a result, the final number corresponding to each letter is reached.

Sending the Message to the Recipient

First, the first three terms of the linear pattern are designed as a numberical or figural pattern. Then, the first three terms of the pattern and the numbers of the encrypted text are sent to the receiver. To prevent the numbers from mixing with each other, a dot is placed between each number indicating a letter.

Decryption of the Message

The receiver first determines the rule of the pattern according to the first three terms of the pattern. Then, he multiplies the difference between the terms in the pattern by three (because the letters are shifted three steps according to Caeser's Cipher). The multiplication result is added to the terms of the pattern. Then, the letters in the alphabet are written in order for each term of the pattern. Thus, by finding the letters corresponding to the numbers, plaintext is reached.

Example Encryption Application

Text to be encrypted: MATEMATİK HAYATTIR ("math is life" in English)

Let the prime numbers chosen be 2 and 3. The rule of the pattern is "2.term order +3". The first three terms of the pattern are created as a numberical or figural pattern (Figure 1). Then, the first 29 steps are determined according to the rule. These terms are as follows:

5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61.

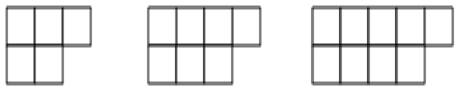


Figure 1. Presentation of the First Three Terms of the Pattern with Figural Representation

The value (mod 29) is determined for terms greater than 29. Accordingly, the new terms are as follows:
5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 1, 3.

The terms of the pattern are assigned to the letters of the alphabet sequentially (Table 1). Then, according to Caeser's Cipher, each letter in the alphabet is encrypted with three next letters and key letters are found. Thus, the numbers corresponding to the key letters are determined. Thus, the final number corresponding to each letter is reached (Table 2).

Table 1. The terms of the pattern corresponding to the letters of the alphabet

Letter	A	B	C	Ç	D	E	F	G	Ğ	H	I	İ	J	K	L	M	N	O	Ö	P	R	S	Ş	T	U	Ü	V	Y	Z
Term	5	7	9	11	13	15	17	19	21	23	25	27	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	1	3

Table 2. Numbers corresponding to the letters in the alphabet according to Caeser's Cipher

Letter	A	B	C	Ç	D	E	F	G	Ğ	H	I	İ	J	K	L	M	N	O	Ö	P	R	S	Ş	T	U	Ü	V	Y	Z
Key	Ç	D	E	F	G	Ğ	H	I	İ	J	K	L	M	N	O	Ö	P	R	S	Ş	T	U	Ü	V	Y	Z	A	B	C
Term	1	1	1	1	1	2	2	2	2	0	2	4	6	8	1	1	1	1	1	2	2	2	2	2	1	3	5	7	9

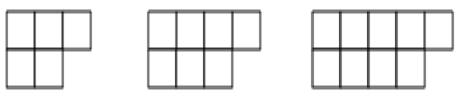
The letters of the plaintext are sent to the receiver along with the numbers of the key letters and the first three terms of the linear pattern (Figure 1 and Table 3). The transmission of the ciphertext is as follows: 12.11.28.21.12.11.28.4.8.0.11.7.11.28.28.2.22.

Table 3. Numbers corresponding to the Letters of the Ciphertext

M	A	T	E	M	A	T	İ	K	H	A	Y	A	T	T	I	R
12	11	28	21	12	11	28	4	8	0	11	7	11	28	28	2	22

Decryption of the message

The figural representation of the first three terms of the linear pattern and the cipher that came to the receiver are as follows:



12.11.28.21.12.11.28.4.8.0.11.7.11.28.28.2.22

The receiver first creates the rule of the pattern using the figural pattern: "2. Term order +3". Since the difference between the terms in the pattern is 2 and the letters are shifted 3 steps according to Caeser's Cipher, 6 is added to the terms of the pattern. Then, the letters in the alphabet are written in order for each term of the pattern. Thus, the message corresponding to the numbers is determined (Table 4). 12.11.28.21.12.11.28.4.8.0.11.7.11.28.28.2.22=MATEMATİK HAYATTIR.

Table 4. Letters of the Alphabet corresponding to the Terms of the Pattern

Terms	5	7	9	11	13	15	17	19	21	23	25	27	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	1	3
Arranged terms	11	13	15	17	19	21	23	25	27	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	1	3	5	7	9
Letter	A	B	C	Ç	D	E	F	G	Ğ	H	I	İ	J	K	L	M	N	O	Ö	P	R	S	Ş	T	U	Ü	V	Y	Z

Conclusion

In the task of "Encryption arising from patterns", the general term of the linear pattern was created by starting from two prime numbers. The numbers corresponding to the first 29 terms of this linear pattern have been calculated. The letters of the alphabet and the terms of the pattern were paired in order. Then, Caeser's Cipher was applied to the letters in the alphabet. That is, according to Caeser's Cipher, each letter in the alphabet was encrypted with three next letters. Thus, the numbers corresponding to the key letters were assigned to the letters in plaintext. The letters of plaintext were sent to the receiver along with the numbers corresponding to the key letters and the first three steps of the linear pattern, and the encryption task was completed.

When decrypting, first of all, the general rule of the pattern was found by using the first three terms of the linear pattern. The amount of increase between terms and the amount of shift in Caeser's Cipher (3 letter shift) were multiplied and added to the terms of the pattern. The letters of the alphabet were assigned to the terms of the most recently reached pattern and deciphered. When reaching the terms in pattern during encryption and decryption processes, (mod 29) was taken into account.

Limitations and Implications

OECD (2021) emphasizes the need to develop mathematical creativity and flexibility of students at different class and ability levels. Encryption algorithms can be used to develop the creativity of mathematically gifted students, which is one of their distinctive characteristics (Leikin & Sriraman, 2022). Based on the "Encryption arising from patterns" task designed in this study, mathematically gifted students could be asked to develop different encryption algorithms.

The task of "Encryption arising from patterns" was designed using linear patterns. In future studies, encryption tasks could be designed for students who are studying at more advanced levels by considering quadratic patterns. In addition, designing encryption algorithms with patterns of different structures may be one of the factors that will increase the difficulty levels of the tasks. Thus, the challenging tasks that mathematically gifted students need can be diversified.

The limitation of prime numbers to 2, 3, 5, 7, 11, 13, 17, 19 in the process of creating linear patterns is to ensure that students focus on the pattern and Caeser's Cipher rather than operational fluency. The set of prime numbers could be expanded. In the task of "Encryption arising from patterns", the letters were shifted according to Caeser's Cipher. Different substitution ciphers can be used to design encryption tasks from patterns. Applying the pattern task presented in this study in classroom environments and presenting its impact on instructional and educational outcomes will contribute to the field of mathematics education.

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