Journal of Thermal Engineering, Vol. 3, No. 4, Special Issue 5, pp. 1338-1345, August, 2017
Yildiz Technical University Press, Istanbul, Turkey

# VASCULAR STRUCTURES FOR SMART FEATURES: SELF-COOLING AND SELF-HEALING

E. Cetkin[1,*]


## ABSTRACT

Here we show how smart features of self-cooling and self-healing can be gained to mechanical systems with embedded vascular structures. Vascular structures mimic the circulatory system of animals. Similar to blood distribution from heart to the animal body, vascular channels provide the distribution of coolant and/or healing agent from a point to the entire body of a mechanic system. Thus the mechanic system becomes capable of cooling itself under unpredictable heat attacks and capable of healing itself as cracks occur due to applied mechanical loads. These smart features are necessary for advanced devices, equipment and vehicles. The essential design parameter is vascularization in order to provide smart features. There are distinct configurations for vascularization such as radial, tree-shaped, grid and hybrids of these designs. In addition, several theories are available for the shape optimization of vascular structures such as fractal theory and constructal theory. Unlike fractal theory, constructal theory does not include constraints based on generic algorithms and dictated assumptions. Therefore, constructal theory approach is discussed in this paper. This paper shows how smart features can be gained to a mechanical system while its weight decreases and its mechanical strength increases simultaneously.


***Keywords:*** *Vascular, Constructal, Smart-features, Self-cooling, Self-healing*

## INTRODUCTION

Uncovering the design with the smallest resistance to the flow of heat, fluid and stresses is the fundamental of technological improvement. The reason of this is that the flow of heat, fluid and stresses are functions of the design. Using higher-conductivity material increases the heat transfer rate from a heat exchanger but the challenge is finding the best shape for minimizing thermal resistances by using a given material with fixed volume. The same challenge is also valid for minimization of resistances to the flow of fluids and stresses. Uncovering the best performing design ensures the usage of scarce materials and energy wisely, i.e. where they are necessary.

There are two well-known theories in the literature for design optimization: constructal theory [1-4] and fractal theory [5-6]. Fractal theory discusses that the design should repeat a pattern that displays at every scale, i.e. the shape of a tree should be the same for every regions of the tree: from trunk to the branches [5-6]. However, the designs in nature do not confirm this theory. In addition, Bejan and Lorente [2] showed that the fractal designs do not provide the smallest resistance to the fluid flow in tree-shaped architectures but constructal designs do.

Constructal theory was stated by Adrian Bejan in 1996 as “For a finite-size system to persist in time (to live), it must evolve in such a way that it provides easier access to the imposed currents that flow through it” [1]. This theory actually uncovers that there is no optimum design but best performing designs for a known time, constraints and conditions. As these time, constraints and conditions change, the design should also be morphed in order to survive. For example, some bacteria colonies such as living in lassen volcanic national park (California, USA) are capable of living in highly acidic regions. Even though these bacteria live in very harsh conditions (in terms of pH), they cannot be called the optimum or best bacteria kinds because they have evolved to live in these conditions. In technology, this trend of shape change in time (evolution) is also necessary and valid. For example, Walkman was revolutionary because of gaining mobility to music players. However, mobile music players have evolved from Walkman to mp3 players and then to the applications in smart phones. The brand of Walkman has forgotten because it have not evolved as fast as technology does. There are many similar examples in biology, geophysics, physics, chemistry and engineering. This shows that constructal theory is a unifying theory of animate and inanimate as discussed in the literature [7-8]. Constructal theory has been used in distinct fields such as biology, chemistry, physics, geophysics and engineering in order to show how design affects the performance in lungs, bacteria colonies, river-deltas, Eiffel tower, lightning bolts, snow-flakes and so on [1-4, 7-8].

*This paper was recommended for publication in revised form by Regional Editor Tolga Taner*
*[1]Department of Mechanical Engineering, Izmir Institute of Technology, Izmir, TURKEY*
**E-mail address: erdalcetkin@iyte.edu.tr*
*Manuscript Received 16 June 2016, Accepted 4 September 2016*

In summary, constructal theory unifies and connects distinct fields, and it is valid for animate and inanimate. The best performing design for given conditions and constraints is the constructal design, and this constructal design changes as conditions and constraints change, i.e. the design is dynamic. Furthermore, the design is not restricted by generic algorithms and dictated design assumptions in constructal theory, i.e. the design is morphed freely in order to minimize resistances.

## RESULTS AND DISCUSSION

Self-healing and self-cooling applications in engineered systems are biologically inspired. Similar to distribution of blood in the human body for keeping its temperature uniform and providing healing when a cut occurs on the body, distributing coolant and/or healing agent to the entire volume of a mechanical system provides self-cooling and/or self-healing capability to it. There are two main methods for self-healing: vascularization and microcapsules. White et al. showed that a damaged structure can be healed with embedded microcapsules which are filled with healing agents [9]. As cracks damages the microcapsules, microcapsules crack and the healing agent inside fills the gaps and polymerizes. Therefore, this healing method can be used on time for the lifetime of the mechanical system. In addition, literature shows that the mechanical strength of a structure drops after healing occurs [10-11]. Moreover, literature also shows that after the structure is damaged the applied mechanical force is unloaded for structure to be healed during experiments [12-13]. In addition, literature shows that the conductivity of a self-healing structure can be restored [14-15].

Unlike embedding microcapsules filled with healing agents, embedding a vascular structure in which healing agent flows in engineered structure enables the structure of healing countless time [16-18]. This embedded structure is similar to the blood veins in circulatory system of warm blooded animals. There are different kinds of vascular architectures in the literature such as: radial, grid, hybrid and tree-shaped designs. Each architecture has its positives and negatives. Radial and tree-shaped designs provide the smallest pressure drop in comparison with grid and hybrid structures [19-22]. However, grid and hybrid structures bathe the entire volume more uniformly in comparison with radial and tree-shaped designs [19-22]. In self-healing and self-cooling, it is essential to bathe the entire volume with coolant and/or healing agent due to random and unpredictable characteristics of heating and mechanical loads. The pressure drop of hybrid of grid and tree-shaped designs is smaller than grid designs and greater than tree-shaped architectures. In addition, this hybrid architecture bathes the entire volume almost as good as grid designs [22]. Therefore, hybrid of grid and tree-shaped designs became the best option for smart features.

Literature also shows that the vascularization gains self-cooling capability to a structure [16-25]. Similar to healing agent flow inside the vascular channels, a coolant flows through the embedded vascular channels. The cooling performance of the vascularized structure is affected by the volume fraction, the complexity of the design, the pressure difference which governs the flow and the flow direction [19-22]. For a given set of conditions (such as boundary conditions) and constraints (such as volume fraction), there is an optimal design which provides the smallest peak temperature. This optimal design should be morphed to the new optimal design as conditions and constraints change. Therefore, the biomimetic designs not necessarily the best performing desing, i.e. if the objectives of the biological design is different than the engineered design. Cetkin [26] shows that a better performing cooling channel configuration exists than the sinusoidal channels which are inspired from the vascular channels in epidermis. This dynamic behavior of design is in accord with constructal theory. Therefore, the optimal design for a given set of conditions and constraints can be called the constructal design. As time passes, the structure should be morphed into the next constructal design if not it cannot survive. This trend is valid for animate and inanimate (natural or engineered).

Furthermore, the cooling requirements can be deterministic and random. The deterministic cooling requirements are due to heat sources which are known and steady, and random ones are unsteady and diverse. A structure is protected from random and deterministic heat sources via embedded vascular cooling channels [20]. Random cooling requirements are responsible of damaging the structure which is designed to work in steady state due to their unpredictable nature.

In addition of gaining smart features to a structure, vascularization provides mechanical strength with light weight which are essential for advanced vehicles [19-22, 27-28]. The strength of a structure decreases due to removal of the material in order to create vascular channels. However, if the material volume is fixed, the removed material is placed outside of the structure [19, 29]. It is also known from the strength of materials that the centerline of a structure is not stressed under bending. Therefore, removing the material from center and placing it around the vascular channels increase mechanical strength of the structure, i.e. the material is put where it is loaded the most. Therefore, the strength of the structure increases with vascularization. Furthermore, if the structure is heated then the effect of thermal stresses cannot be neglected. Cetkin et al. shows when the effect of thermal stresses can and cannot be neglected [29]. Coolant flow in the vascular channels decreases the peak temperature and creates a more uniform temperature distribution in the solid domain; so, thermal stresses decrease greatly. Therefore, self-cooling structures have greater mechanical strength than non-vascularized structures under great heat fluxes.

Similar to how advanced capabilities of self-healing and self-cooling require vascularization in order to bathe the entire volume with coolant fluid and/or healing agent, vascularization is also essential in order to decrease the resistances of the distribution of energy, goods and water [1-4, 30]. A factory distributes all its products to the cities located around the world and collects raw materials around the world similar to distribution and collection of coolant and/or healing agent in advanced materials. These distribution and collection kinds of flows are examples of flows from a point to an area (or volume).

### Vascularized Structures with Uniform and Non-uniform Heating

A structure can be bathed by coolant and/or healing agent with embedded vascular channels while its mechanical strength increases and its weight decreases. Discussion of some vascular channel configurations with uniform and non-uniform heating is required in order to uncover how the coolant and/or healing agent is distributed throughout the structure with random (non-uniform heating) and prescribed (uniform heating) boundary conditions. The objective is to find the design with the best cooling performance (i.e. uncovering the design which bathes the entire volume with coolant and/or healing agent).

Here we show that the mechanical strength and thermal performance of a heated and mechanically loaded circular plate can be increased with embedded radial and tree-shaped vascular structures in it. The diameter and thickness of the circular plate are D and H, and their ratio is D/H = 10 which is fixed, Fig. 1a [19]. The solid volume is fixed, so is the volume of the vascular channel network. The plate is subjected to uniformly distributed force and uniform heat flux, both acting from below, Fig. 1a.

The dimensionless governing equations (the conservation of the mass, the conservation of the momentum for the fluid domain, the energy equations for fluid and solid domains, the generalized Hooke's law and the conservation of momentum equations for solid domain) were solved in a finite element software [31]. Mesh test was also performed to confirm mesh independency of the results.

The heat flux and the mechanical load are applied on the bottom surface of the plate as shown in Fig. 1a. The pressure difference between inlet and outlet is non-dimensionalized as Bejan number [32, 33].

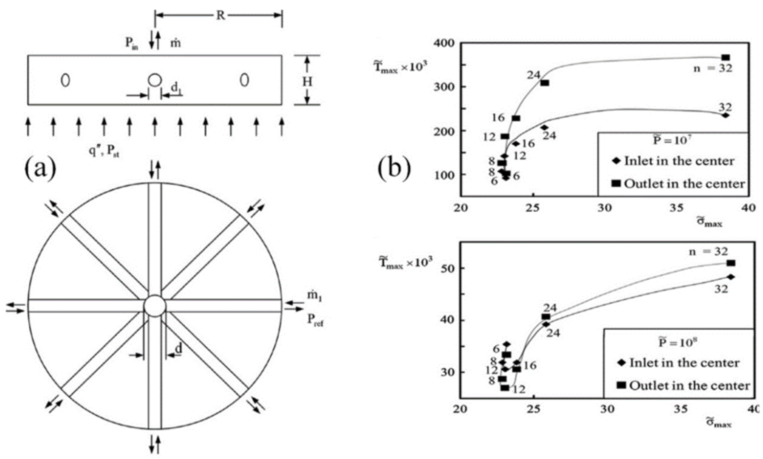


**Figure 1.** (a) Radial channel configuration embedded in the circular plate. (b) The effect of the number of cooling ducts on the dimensionless peak temperature and the dimensionless peak stress [19]

$$\tilde{P}_{max} = \frac{(P_{in} - P_{ref})D^2}{\mu\alpha} \tag{1}$$

where μ and α are dynamic viscosity and thermal diffusivity of the fluid. The value of $\tilde{P}_{max}$ represents the dimensionless overall pressure difference between the coolant inlet and outlet. The flow is laminar in all the channel configurations, Re < 2000.

The peak temperature and the peak stress is affected by the design. Therefore, the design corresponding the smallest resistance to the flow of heat, fluid and stress can be uncovered by freely morphing the design. Figure 1b shows the relation between the temperature, stress and number of ducts when is $10^7$ and $10^8$. The maximum stress decreases when the number of the cooling channels increases from 6 to 8, and it increases when the number of the cooling ducts increases from 8 to 32. The reason of this behavior is that the maximum stress increases in the vicinity of the junctions of the cooling ducts. Even though the peak stress is the minimum when the number of the channels is 8, neighboring designs (6 cooling channels when = $10^7$ and 12 cooling channels when = $10^8$) offer minimum peak temperatures. In summary, when the pressure drop is prescribed, it is possible to identify one design (or a group of designs) that provides the minimum peak stress and peak temperature, or vice versa. However, there is no optimal design for all the constraints and conditions.

Next consider a square plate with length L and thickness H = 0.1L with embedded vascular channels, Fig. 2a [22]. The plate is subjected to a uniformly distributed load and uniform heat flux both acting from below. The volume of the solid and the fluid are fixed. Lg is the length scale of the square area in which the grid cooling channels are embedded as shown in Fig. 2a.

The grid channels are connected to the periphery with radial channels. Coolant enters or exits from the center of the grid, and it is driven by the pressure difference maintained between the inlet and outlet boundaries. The results were obtained by solving the governing equations numerically.

Figure 2b shows that the minimum peak temperatures plotted against the peak stresses as $L_g/L$ varies. The effect of the flow direction on the peak temperature and stress is weak. $T_{peak}$ and decreases when $L_g/L < 0.25$, and the peak temperature increases as $L_g/L$ increases even though the peak stress decreases and increases. The peak stress is the minimum when the design is a hybrid of grid and trees. However, the peak temperature is the minimum with radial channels.

Consider that the heating is concentrated in a small region on the vascularized solid domain. The area of the heated spot is 1/16 of the square area of length scale $L_g$, Fig. 2 [22]. The total heating rate of the concentrated heat generation is fixed, i.e. volumetric heating rate increases as the heat generating region size decreases in order to conform fixed heating rate.

Figure 3a shows how the peak temperature changes as $L_g/L$ increases. When the concentrated heating is located in the center of the slab, $T_{peak}$ decreases as $L_g/L$. When the concentrated heating is located in the corner of the grid, $T_{peak}$ increases as $L_g/L$ increases. $T_{peak}$ is the lowest when $L_g/L = 0.25$ with the concentrated heating is in the corner, and when $L_g/L = 0.625$ with the concentrated heating is in the center. In addition, when $L_g/L = 0.375$ the peak temperature becomes almost as low as the lowest peak temperature obtained when the concentrated heating is located in the center or in the corner. Figure 3b shows the temperature distribution in the mid-plane of square domain when the heat generation is concentrated in the center of the slab and in the corner of the grid for $L_g/L = 0.25$ and 0.5. The effect of flow direction on the temperature distribution is also shown in Fig. 3b.

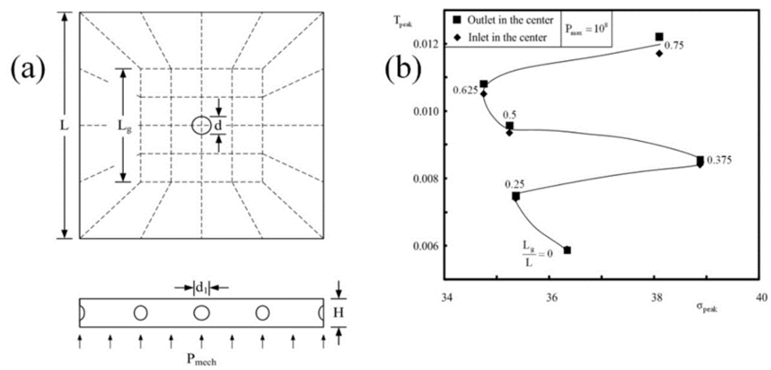


**Figure 2.** (a) Grid structure connected to the perimeter with radial channels (b) Minimum peak temperatures relative to their peak stresses as $L_g/L$ varies [22]

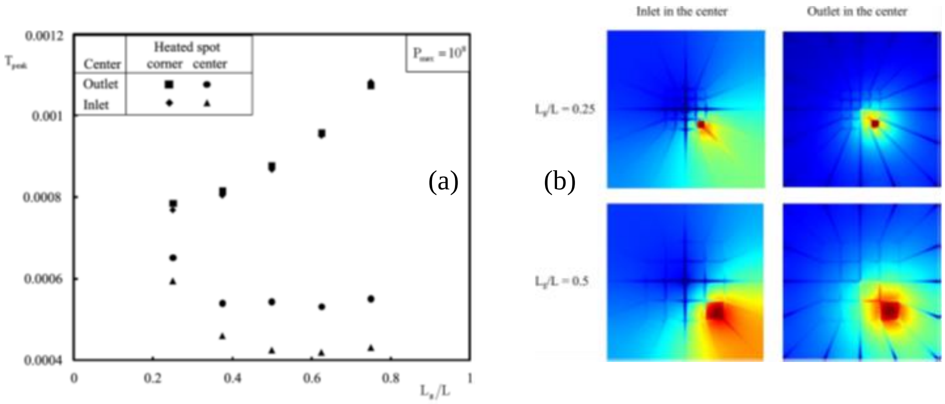


**Figure 3.** (a) Peak temperature relative to Lg/L when the flow direction and the concentrated heat generation location change. (b) The temperature distribution in the mid-plane of the slab [22]

Consider a square plate of length scale L, and thickness of H = 0.1L as shown in Fig. 4 [20]. A vascular channel network is embedded in the plate in order to keep it under an allowable temperature ceiling while the plate is heated with a concentrated and moving heat flux. The length scale of the square footprint of the heating spot is 0.1L and it moves with the constant speed of W from one edge of the plate to another. Four possible beam paths are discussed as shown in Fig. 4. The volume of the solid is fixed, so is the volume of the fluid. Coolant enters or exits from the center of the slab while the pressure difference between the inlet and exit boundaries is constant. The flow is incompressible with constant properties, and the dimensionless governing equations are time dependent, the dimensionless equations can be found in Ref. [20].

Figure 4 shows the average peak temperature of a solid structure (without embedded vascular channels) and the average peak temperature in four competing designs with embedded vascular channel configurations. The error bars indicate the maximum and minimum peak temperatures when the dimensionless time is greater than 0.1, i.e. after the entire concentrated heat flux enters the plate surface. Figure 4 also shows that a plate heated by a moving beam with an unpredictable path can be cooled by embedding vascular cooling channels in the plate. The effect of changing from no cooling to vascular cooling is dramatic with random cooling requirements similar to with prescribed cooling requirements.

Next, consider the plate with uniform heating load applied on its surface has embedded channels configured as radial, tree-shaped and their hybrid, Ref [34]. Figure 5 shows how the temperature distribution changes as the design and pressure drop (the difference between the inlet and outlet pressures) are altered. The resistance to the fluid flow is smaller in tree-shaped designs in comparison with the radial designs (for instance mass flow rate is 9 to 19% greater for the same pressure drop and volume fraction with tree-shaped design in comparison to the radial design Ref [34]). However, Figure 5 also shows that the thermal resistances in tree-shaped designs are greater than the radial designs, therefore, the peak temperature value is greater in tree-shaped designs when the pressure drop increases, for example the comparison of radial and tree-shaped designs with 290 Pa

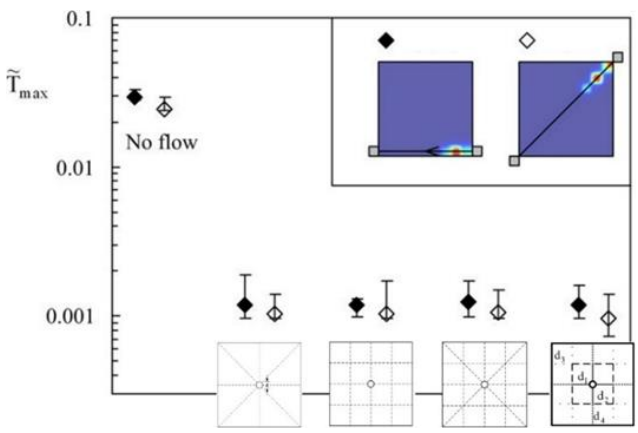


**Figure 4.** The average peak temperatures of four competing designs with vascular channels and the peak temperature of solid plate (without vascular channels) [20]

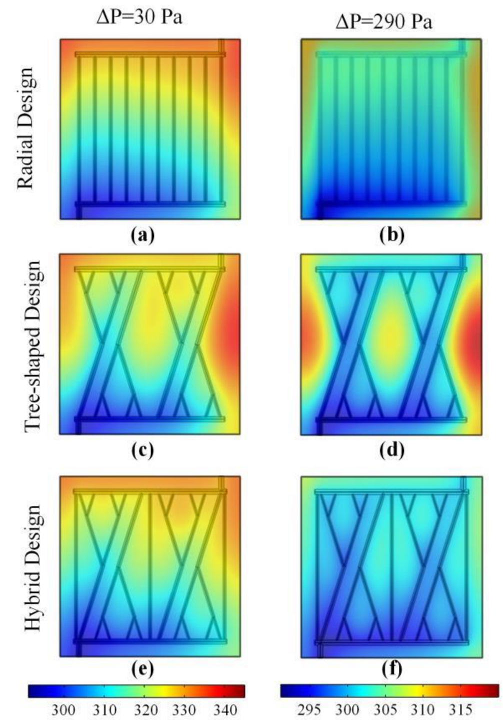


**Figure 5.** Temperature distribution of radial design for (a) 30 Pa, (b) 290 Pa, tree-shaped design for (c) 30 Pa, (d) 290 Pa, and hybrid design for (e) 30 Pa, (f) 290 Pa [34]

pressure drop value. Overall, the tree-shaped and radial designs promise to minimum resistances to the fluid flow and heat flow, respectively. The novel idea is to combining these two in one design, i.e. the hybrid design as shown in Fig. 5. Hybrid design performs almost as good as radial design and tree-shaped designs in terms of resistances to the heat and fluid flow. Both resistances are slightly greater (several per cent) than the corresponding minimum value of the best performing design.

The result of Figure 5 uncovers that in some cases the conductive resistances are in great importance. In order to increase the overall thermal conductance of a solid material high-conductivity inserts can be placed. Figure 6 shows how the locations of this high-conductivity inserts affect the thermal conductance by minimizing the peak temperature for fixed boundary conditions, Ref. [35].

Figure 6 shows that high-conductivity inserts should be embedded non-equidistantly in order to minimize thermal resistances. However, Figure 6 shows that there are family of best options, i.e. similar performing designs.

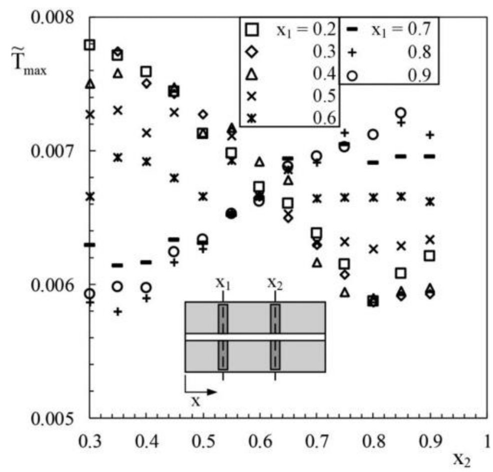


**Figure 6.** The effect of the second level high-conductivity insert locations on maximum temperature [35]

For instance, if the first insert is fixed at 0.2 position, then second insert should be located at 0.8. However, the same performance can be achieved with placing inserts at locations 0.4 and 0.75.

## CONCLUSION

This paper shows how the smart features of self-cooling and self-healing can be gained to an engineered structure. Vascularization is essential in order to bathe the entire volume of the system with coolant and/or healing agent. The best performing vascular channel networks for circular and rectangular plates with uniform and non-uniform loads are documented. Novel hybrid designs of radial and tree-shaped designs are documented. In addition, the increase in the overall thermal performance for self-cooling with embedded high-conductivity materials is uncovered.

This paper also shows that the hybrid designs combine the best features of each design that they were constructed from. Furthermore, this paper uncovers that there is no best design but family of best designs. This idea is in accord with the constructal law and the tendency in the nature. For instance, even the tree-shaped designs minimizes the resistances to the fluid flow for point to area (or volume) flows which explains why the tree roots and branches are similar to the animal lungs, none of the trees and animal lungs are identical and they vary from plant to plant and animal to animal (even for the same species).

**ACKNOWLEDGMENTS**

This work was supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under Grant No. 114M592.

## NOMENCLATURE

| | |
|---|---|
| d | diameter of cooling channels, m |
| H | plate thickness, m |
| k | thermal conductivity, W $m^{-1}$ $K^{-1}$ |
| L | rectangular plate length scale, m |
| $L_g$ | length scale of the grid region, m |
| P | pressure, N $m^{-2}$ |
| $P_{st}$ | mechanical load, N $m^{-2}$ |
| $P_{in}$ | inlet pressure, N $m^{-2}$ |
| R | circular plate radius, m |
| q'' | imposed heat flux, W $m^{-2}$ |
| T | temperature, K |
| x | horizontal direction, m |

**Greek symbols**

| | |
|---|---|
| $\alpha$ | thermal expansion coefficient, $K^{-1}$ |
| $\mu$ | dynamic viscosity, kg $m^{-1}$ $s^{-1}$ |
| $\sigma$ | normal stress, N $m^{-2}$ |

**Subscripts**

| | |
|---|---|
| in | inlet |
| st | mechanical |
| max | maximum |
| ref | reference |

**Superscript**

| | |
|---|---|
| ~ | dimensionless |

## REFERENCES

[1] Bejan, A., Advanced engineering thermodynamics, 1997, 2nd ed., Wiley, New York.
[2] Bejan, A. and Lorente, S., Design with constructal theory, 2008, Wiley, Hoboken.
[3] Bejan, A. and Zane, J.P., Design in nature: how constructal law governs evolution in biology, physics, technology, and social organization, 2012, Doubleday, New York.
[4] Bejan, A., Shape and structure, from engineering to nature, 2000, Cambridge University Press, Cambridge.
[5] Pfeifer, P. and Avnir, D., Chemistry in noninteger dimensions between 2 and 3.1. fractal theory of heterogeneous surfaces, J. Chem. Phys., 1983, 79(7), pp. 3558–3565.
[6] Mandelbrot, B.B., The fractal geometry of nature, 1983, Henry Holt and Company.
[7] Bejan, A. and Maden, J.H., The constructal unification of biological and geophysical design, Phys. Life Rev., 2009, 6(2), pp. 85–102.

[8] Bejan, A. and Lorente, S., Constructal law of design and evolution: physics, biology, technology and society, J. Appl. Phys., 2013, 113, 151301.
[9] White, S.R., Sottos, N.R., Moore, J., Geubelle, P., Kessler, M., Brown, E., Suresh, S. and Viswanathan, S., Autonomic healing of polymer composites, Nature, 2001, 409, pp. 794–794.
[10] Brown, E.N., Sottos, N.R. and White, S.R., Fracture testing of a self-healing polymer composite, Experiment. Mech., 2002, 42(4), pp. 372–379.
[11] Toohey, K.S., Sottos, N.R., Lewis, J.A., Moore, J.S. and White, S.R., Self-healing materials with microvascular networks, Nature Mater., 2007, 6, pp. 581–585.
[12] Coope, T.S., Wass, D.F., Trask, R.S. and Bond, I.P., Repeated self-healing of microvascular carbonfibre reinforced polymer composites, Smart Mater. Struct., 2014, 23(11), 115002.
[13] White, S.R., Moore, J.S., Sottos, N.R., Krull, B.P., Santa Cruz, W.A., Gergely, R.C.R., Restoration of large damage volumes in polymers, Science, 2014, 344, pp. 620–623.
[14] Kang, S., Jones, A.R., Moore, J.S., White, S.R. and Sottos, N.R., Microencapsulated carbon black suspensions for restoration of electrical conductivity, Adv. Funct. Mater., 2014, 24, pp. 2947–2956.
[15] Odom, S.A., Tyler, T.P., Caruso, M.M., Ritchey, J.A., Schulmerich, M.V., Robinson, S.J., Bhargava, R., Sottos, N.R., White, S.R., Hersam, M.C. and Moore, J.S., Autonomic restoration of electrical conductivity using polymer-stabilized carbon nanotube and grapheme microcapsules, Appl. Phys. Lett., 2012, 101, 043106.
[16] Lee, J., Kim, Y., Lorente, S. and Bejan A., Constructal design of a comb-like channel network for self-healing and self-cooling, Int. J. Heat Mass Transfer, 2013, 66, pp. 898–905.
[17] Lorente, S. and Bejan, A., Vascularized smart materials: designed porous media for self-healing and self-cooling, J. Porous Media, 2009, 12(1), pp. 1–18.
[18] Therriault, D., White, S.R. and Lewis, J.A., Chaotic mixing in three-dimensional microvascular networks fabricated by direct-write assembly, Nature Mater., 2003, 2(4), pp. 265–271.
[19] Cetkin, E., Lorente, S. and Bejan, A., Vascularization for cooling and mechanical strength, Int. J. Heat Mass Transfer, 2011, 54, pp. 2774–2781.
[20] Cetkin, E., Lorente, S. and Bejan, A., Vascularization for cooling a plate heated by a randomly moving source, J. Appl. Phys., 2012, 112, 084906.
[21] Wang, K.-M., Lorente, S. and Bejan, A., Vascular materials cooled with grids and radial channels, Int. J. Heat Mass Transfer, 2009, 52, pp. 1230–1239.
[22] Cetkin, E., Lorente, S. and Bejan, A., Hybrid grid and tree structures for cooling and mechanical strength. J. Appl. Phys., 2011, 110, 064910.
[23] Rocha, L.A.O., Lorente, S. and Bejan, A., Tree-shaped vacular wall designs for localized intense cooling, Int. J. Heat Mass Transfer, 2009, 52, pp. 4535–4544.
[24] Kim, S., Lorente, S. and Bejan, A., Vascularized materials with heating from one side and coolant forced from the other side, Int. J. Heat Mass Transfer, 2007, 50, pp. 3498–3506.
[25] Soghrati, S., Thakre, P.R., White, S.R., Sottos, N.R. and Geubelle, P.H., Computational modelling and design of actively-cooled microvascular materials, Int. J. Heat Mass Transfer, 2012, 55, pp. 5309–5321.
[26] Cetkin, E., Constructal structures for self-cooling: microvascular wavy and straight channels, J. Thermal Engineering, 2015, 1, pp. 166–174.
[27] Wang, K.-M., Lorente, S. and Bejan, A., Vascular structures for volumetric cooling and strength, J. Appl. Phys., 2010, 107, 044901.
[28] Rocha, L.A.O., Lorente, S. and Bejan, A., Vascular design for reducing hot spots and stresses, J. Appl. Phys., 2014, 115, 174904.
[29] Cetkin, E., Lorente, S. and Bejan, A., Vascularization for cooling and reduced thermal stresses, Int. J. Heat Mass Transfer, 2015, 80, pp. 858–864.
[30] Bejan, A. and Lorente, S., The constructal law and evolution of the design in nature, Phys. Life Rev., 2011, 8, pp. 209–240.
[31] See www.comsol.com for information about Comsol Multiphysics.
[32] Bhattacharje, S. and Grosshandler, W.L., The formation of a wall jet near a high temperature wall under microgravity environment, ASME HTD, 1988, 96, pp. 711–716.
[33] Petrescu, S., Comments on the optimal spacing of parallel plates cooled by forced convection, Int. J. Heat Mass Transfer, 1994, 34, p. 1283.
[34] Yenigun, O. and Cetkin, E., Constructal tree-shaped designs for self-cooling, Int. J. Heat Technology, 2016, 34, pp. 173–178.
[35] Cetkin, E., Constructal vascular structures with high-conductivity inserts for self-cooling, J. Heat Transfer, 2015, 137, 111901.