Research Article

Thermal insulation performance curves for exterior walls in heating and cooling seasons

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ABSTRACT

Determination of thermal insulation performance (i.e. optimum insulation thickness, energy saving and payback period) is a tedious and time-consuming task that requires a thorough knowledge in thermal insulation engineering and economics. The main goal of this paper is to make the determination of insulation performance simple and timesaving by introducing thermal insulation performance curves (TIPCs) from which the insulation performance can easily be found for any climate condition and all economic factors related to energy and insulation. These curves were generated based on a life-cycle cost analysis (LCCA) method. The curves can be easily read based on a single factor, called the f-factor, which comprises the number of degree-day, coefficient of performance, present worth factor, energy cost, and insulation cost. With the gain of heating and cooling degree days (i.e. HDD and CDD), TIPCs can be used for both heating and cooling loads. TIPCs cover commonly used insulation materials for building walls with thermal conductivities range from 0.020 to 0.055 W/m K. TIPCs were validated against published data.

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INTRODUCTION

Despite the availability of many mega projects globally to produce electricity from renewable energy, the use of petroleum products, natural gas and coal still accounts for approximately 70% of the total electricity generated worldwide. Space heating and cooling of buildings are considered as one of the most important sectors of electricity consumption, which in 2016 accounted for approximately 40% of the total electricity generated [1, 2]. To reduce the energy requirement for space heating and cooling, an appropriate engineering design should be made for the building envelope; by selecting proper type of construction and insulation materials that would minimize heat loss/gain

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from buildings envelope and reduce loads for space heating-cooling and thus achieving energy conservation [3]. Hence, extensive scientific studies have been conducted on the development and testing of new materials of insulation. In the literature, there is over 40 types of thermal insulation materials have been developed, many of them are now available in the market and used in buildings. They have a wide range of thermal conductivities, ranging from 0.004 W/m K for vacuum insulation panel [4] to 0.485 W/m K for cement-based lightweight composites [5]. Al-Homoud [6] classified the available insulation materials into six categories, namely; organic, inorganic, metallic, aerogel, insulations made from waste materials and composites. These materials have been used for floor, wall, ceiling and roof insulation. For the external walls of building, the most widely used insulation materials are polyurethane (0.020-0.030 W/m K), extruded polystyrene (0.025-0.035 W/m K), expanded polystyrene foam (0.030-0.040 W/m K), glass wool (0.030-0.046 W/m K), rock wool (0.033-0.046 W/m K), fiberglass (0.033-0.040 W/m K), and cellulose (0.046-0.054 W/m K) [4, 6–8]. The thermal conductivities of these materials range from 0.020 to 0.054 W/m K.

Insulation thickness is directly proportional to the investment cost and is inversely proportional to the heating and cooling transmission loads (i.e. operating costs). The economic thermal insulation thickness at which these costs are optimized (i.e. minimum thickness in relation with the total cost) is called as the "optimum insulation thickness" (OIT) [9]. For external walls, the OIT has been estimated for different climate conditions based on cooling [9–14] and heating loads [15–21]. Most of the available studies on OIT were performed under static conditions in which heating and cooling energy requirements were estimated using a common method called the degree-day method (heating degree-day (HDD) or cooling degree-day (CDD)). On the other hand, few studies have been analytically analyzed the dynamic behavior of multilayer envelopes [22–24]. The most commonly used financial method to optimize the thermal insulation thickness of external walls is the life cycle cost analysis (LCCA) method [9, 25, 26]. LCCA considers energy uses during the lifetime of the building, which can be taken as 10 years [26], 20 years [27], 25 years [28], 30 years [29], or 50 years [30]. Using the present worth factor (PWF), the net energy savings due to the use of insulation material during the lifetime of the building is estimated in its present value.

For decision making purposes, design engineers used to rely on performance curves rather than conducting a comprehensive study for each specific case. For example, engineers would not conduct LCCA study for selecting the optimum insulation type and thickness. Instead, they will select the cheapest available insulation material in the market with rough thickness, although mostly this could not be the optimum and economic choice. Up to the best of our knowledge, however, there were no thermal insulation performance curves (TIPCs) have been reported in the literature. Therefore, the goal of this work is to develop TIPCs from which the insulation performance characteristics can easily be found for any climate condition and all economic factors related to energy and insulation. These curves are generated based on LCCA method coupled with DD method for heating and cooling processes. TIPCs are constructed to cover commonly used insulation materials for building walls with thermal conductivities reported above, i.e. 0.020-0.055 W/m K. Unlike the current literature, this study is the first that focus on, f-factor curves in cooling and heating period for different insulation materials and OIT. In addition, curves for the payback period and energy saving values for different thermal resistances (between 0.4 to 0.8 m^2 K/W) are shown. Moreover, the results of this research not only point to the need to change energy policies, but also benefit from the selection of the most suitable insulation material by providing useful and practical results in the construction process of buildings.

METHODOLOGY

Heat gain or loss from external walls

Heat losses in buildings may occur due to poorly insulated roofs, walls, windows, doors, unused parts, garages, plumbing pipes or ventilation sections of buildings. The heat gain or loss from external walls is [31]:

$$q = U A |T_b - T_{sa}| = U A \Delta T \qquad (1)$$

$$U = \frac{1}{\sum R} \qquad (2)$$

$$\sum R = \frac{1}{h_b A} + \frac{x_1}{k_1 A} + \cdots + \frac{x_n}{k_n A} + \frac{x_{ins}}{k_{ins} A} + \frac{1}{h_0 A} \qquad (3)$$

where subscripts b, sa, 1, n and ins are denoted to indoor base, solar-air, first wall layer, pre-insulation wall layer and insulation layer, respectively, A is the wall surface area (m^2), U is the overall heat transfer coefficient (W/m^2 K), T_i is the temperature (K), R is the total resistance (K/W), which is the sum of the total internal resistance of wall layers and the surface resistances of convective heat transfer over the inside and outside wall surfaces, h_i is the convection heat transfer coefficient (W/m^2 K), k_i is the thermal conductivity of the wall material (W/m K) and x_i is the wall layers thickness (m).

The annual heat loss/gain per unit area of external walls in terms of degree-days (DD) are calculated as:

$$q_H = 86400 \text{ HDD U} \qquad (4)$$

$$q_C = 86400 \text{ CDD U} \qquad (5)$$

where HDD and CDD are the heating and cooling degree-day numbers, respectively, which are used to estimate the buildings envelope heat transfer. For heating [20],

$$HDD = \sum_{1}^{365} (T_b - T_{sa}) \quad \text{values are counted only if } T_b \geq T_{sa} \quad (6)$$

and for cooling,

$$CDD = \sum_{1}^{365} (T_{sa} - T_b) \quad \text{values are counted only if } T_{sa} \geq T_b \quad (7)$$

Energy requirement and costs

The cost of insulation material per unit area (C_{Tins}, $/m^2) used for external walls is [9]:

$$C_{Tins} = x_{ins} \, C_{ins} \quad (8)$$

where C_{ins} is the insulation material cost per unit volume ($/m^3).

The annual energy requirement per unit area for heating (E_{AH}) and cooling (E_{AC}), in unit of J/m^2 year, are estimated as [32]:

$$E_{AH} = \frac{86400 \, HDD}{(R_{wt} + R_{ins}) \, \eta} = \frac{86400 \, HDD}{\left(R_{wt} + \frac{x_{ins}}{k_{ins}}\right) \eta} \quad (9)$$

$$E_{AC} = \frac{86400 \, CDD}{(R_{wt} + R_{ins}) \, COP} = \frac{86400 \, CDD}{\left(R_{wt} + \frac{x_{ins}}{k_{ins}}\right) COP} \quad (10)$$

where R_{wt} is the overall wall thermal resistance excluding the insulation layer (K/W), η is the efficiency of the heating system (%) and COP is the coefficient of performance.

The annual fuel consumption is calculated as [33]:

$$m_F = \frac{86400 \, HDD}{\left(R_{wt} + \frac{x_{ins}}{k_{ins}}\right) \eta \, Hv} \quad (11)$$

where Hv is the fuel lower heating value (J/kWh, J/kg or J/m^3).

The annual energy cost per unit area for heating (C_{AH}) and cooling (C_{AC}), in units of $/m^2 year, are calculated as [9, 32]:

$$C_{AH} = \frac{86400 \, HDD \, C_F}{\left(R_{wt} + \frac{x_{ins}}{k_{ins}}\right) \eta \, Hv} \quad (12)$$

$$C_{AC} = \frac{86400 \, CDD \, C_e}{\left(R_{wt} + \frac{x_{ins}}{k_{ins}}\right) COP} \quad (13)$$

where C_F is the fuel cost ($/kWh, $/kg or $/m^3) and C_e is the cost of electricity ($/kWh).

To calculate the fuel cost over a building lifetime, the present worth factor (PWF) is used. It depends on interest rate, i, (%) and the inflation rate, g, (%) [34]. For inflation, PWF is adjusted as:

$$PWF = \begin{cases} \dfrac{1 - (1 + i^*)^{-N}}{i^*} & ; \quad i \neq g \\ (1 + i)^{-1} & ; \quad i = g \end{cases} \quad (14)$$

where N is insulation material lifetime, and i* is interest rate adjusted for inflation rate:

$$i^* = \begin{cases} \dfrac{i - g}{1 + g} & ; \quad i > g \\ \dfrac{g - i}{1 + i} & ; \quad i < g \end{cases} \quad (15)$$

Then, the annual energy cost per unit area for heating and cooling (C_E) over building lifetime is:

$$C_E = \frac{F}{R_{wt} + \frac{x_{ins}}{k_{ins}}} \quad (16)$$

where F is a new factor (K $/W), introduced in this paper, that has two different values for heating and cooling loads. For heating,

$$F = \frac{86400 \, HDD \, C_F \, PWF}{\eta \, Hv} \quad (17)$$

and for cooling,

$$F = \frac{0.024 \, CDD \, C_e \, PWF}{COP} \quad (18)$$

The total annual cost ($/m^2 year) of heating and cooling (C_T) an insulated building is:

$$C_T = C_E + C_{Tins} = \frac{F}{R_{wt} + \frac{x_{ins}}{k_{ins}}} + x_{ins} \, C_{ins} \quad (19)$$

Optimum insulation thickness

The OIT (x_{opt}, m) is obtained by taking:

$$\frac{d(C_T)}{d(x_{ins})} = 0 \quad (20)$$

Hence, the OIT minimizing the total heating or cooling cost is calculated as:

$$x_{opt} = \sqrt{\frac{k_{ins}\, F}{C_{ins}}} - k_{ins}\, R_{wt} \qquad (21)$$

Net energy saving and payback period

The annual total net energy saving (E_S, \$/m²) for heating and cooling loads are calculated, as the difference between the annual energy cost of un-insulated and insulated building, as:

$$E_S = F\left(\frac{1}{R_{wt}} - \frac{1}{R_{wt} + \frac{x_{ins}}{k_{ins}}}\right) - x_{ins}\, C_{ins} \qquad (22)$$

The payback period (PP, year) for heating and cooling is calculated, as the ratio between the annual energy cost of an un-insulated building and the annual total net energy saving, as:

$$PP = \frac{F}{R_{wt}\, E_S} \qquad (23)$$

Thermal insulation performance curves (TIPCs)

The OIT curves for heating and cooling loads are prepared by grouping the terms related to energy and insulation (i.e. C_F, C_e, PWF, C_{ins}, η, COP, H_v) into one factor (f) as follows:

$$x_{opt} = \sqrt{f}\, \sqrt{k_{ins}} - k_{ins}\, R_{wt} \qquad (24)$$

For heating, the f-factor (K m³/W) is:

$$f = \frac{F}{C_{ins}} = \frac{86400\ \text{HDD}\ C_F\ \text{PWF}}{C_{ins}\ \eta\ Hv} \qquad (25)$$

and for cooling, the f-factor (K m³/W) is:

$$f = \frac{F}{C_{ins}} = \frac{0.024\ \text{CDD}\ C_e\ \text{PWF}}{C_{ins}\ \text{COP}} \qquad (26)$$

Then, the OIT is plotted versus $\sqrt{f}$ at different R_{wt} and k_{ins}. Similar to the OIT curve, energy savings curves are prepared by dividing E_S by C_{ins} and grouping the same previously mentioned terms into the f-factor as:

$$e_S = \frac{E_S}{C_{ins}} = f\left(\frac{1}{R_{wt}} - \frac{1}{R_{wt} + \frac{x_{ins}}{k_{ins}}}\right) - x_{ins} \qquad (27)$$

The specific annual total net energy savings (e_S, m) is plotted versus OIT at different R_{wt} and k_{ins}. Using this curve, e_S can be found at any OIT. To find E_S (\$/m²), e_S should be multiplied by C_{ins}.

Using the f-factor, the PP is:

$$PP = \frac{f}{R_{wt}\, e_S} \qquad (28)$$

Then, the OIT is plotted versus PP (year) at different R_{wt} and k_{ins}. The use of f-factor in Eqs. (24, 27-28) eliminates the spatial and temporal variations present in Eqs. (21-23), making the curves generated based on this factor valid anytime and everywhere; taking into account the variation in fuel and insulation costs and the variation in climate conditions.

RESULTS AND DISCUSSION

Thermal insulation is done in order to minimize the heat losses caused by building elements such as walls. Regarding thermal insulation, the purpose of the developed model is to introduce a new approach in the calculation of OIT, total cost, cost savings and payback period by considering many factors. These factors are: other variables related to economic parameters and regulations such as climate conditions (degree-days), thermal conductivity and price of the insulation material, average temperature in the region, fuel price for heating, interest and inflation rates. Thus, this will provide an effective and simple guide for people working in the field to better design, analyze and operate wall thermal insulation anywhere in the world.

TIPCs

Using Eq. (24), the OIT is plotted versus $\sqrt{f}$ at different k_{ins} (0.01-0.055 W/m K) and R_{wt} (0.4-0.8 m² K/W), as shown in Fig. 1. Prior to use the TIPCs, the user should calculate the square root of the f-factor, using Eqs. (25) and (26) for heating and cooling loads, respectively. For any other plotted values of k_{ins} and R_{wt}, linear interpolation could be applied and exact results would be obtained, as shown in Table 1. For example, for the case with R_{wt}, k_{ins} and $\sqrt{f}$ of 0.4 m² K/W, 0.03 W/m K and 0.65, respectively, from Fig. 1(a), the OIT is 0.1 m. If k_{ins} is 0.033 instead of 0.03 W/m K, then applying linear interpolation between OITs at k_{ins} of 0.03 and 0.035 W/m K, as shown in Table 1, to get the OIT of 0.1048 m at k_{ins} of 0.033 W/m K, compared to the true value of 0.1049 m. Again, if R_{wt} is 0.45 instead of 0.4 m² K/W, then applying linear interpolation between OITs at R_{wt} of 0.4 and 0.5 m² K/W, as shown in Table 1, to get the OIT of 0.106 m at R_{wt} of 0.45 m² K/W, compared to the true value of 0.10585 m. From Table 1, it can be noticed that the TICC reading values are very close to true values (i.e. calculated) with percent error of less than 0.6 %. When k_{ins} falls between any two plotted curves in Fig. 1, the user can

simply read the closest curve to his value with a maximum error in OIT of less than 5 mm.

Using Eqs. (27 and 28), the PP and e_S were plotted versus OITs at different f-factors of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7 K m³/W and 0.8, k_{ins} of 0.02, 0.03, 0.04 and 0.05 W/m K, and R_{wt} of 0.4, 0.5, 0.6, 0.7 and 0.8 m² K/W, as shown in Figs. (2-6). Using these figures, e_S and PP can be only found after the OIT has been determined from Fig. (1). The e_S value determined using Figs. (2-6) must be multiplied by the current insulation material cost (C_{ins}, $/m³) to obtain the annual total net energy saving, E_S, in units of $/m². An example of TIPCs use and interpolation is shown in Table 2. This table represents the case in which the OIT is 0.1 m with an f-factor of 0.5 K m³/W and R_{wt} of 0.4 m² K/W. The PP and e_S determined from Fig. (2) at k_{ins} of 0.02, 0.03, 0.04 and 0.05 W/m K were compared with calculated (i.e. true) values. To check the validity of using linear interpolations,

three interpolations were made at k_{ins} of 0.025, 0.035 and 0.045 W/m K and compared with true values. Results show that liner interpolation is valid with percent error of less than 0.2%, which can be referred to human error in reading the curve values.

TIPCs Validation

In this section, the TIPCs are validated against published articles for both heating and cooling loads. We have selected those articles in which complete input values are provided and that the output results were represented in tabulated format; making the comparison and thus validation is possible. Using EPS as an insulation material for cooling of cold storage space in Amman city with k_{ins} of 0.034 W/m K and R_{wt} of 0.4862 m² K/W, Batiha et al. [9] found that the OIT is 0.147 m with annual energy savings of 111.503 $/m² and PP of 1.237 year. Based on the input values given by
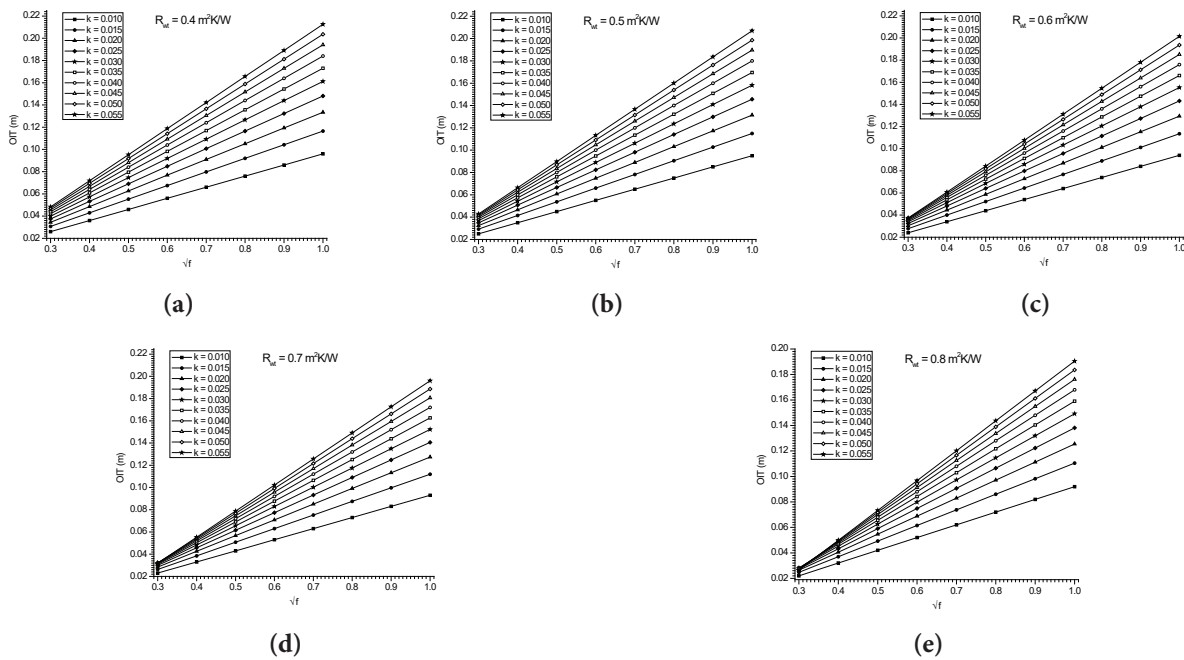
Figure 1. OIT vs. $\sqrt{f}$ at different insulation thermal conductivities: (a) R_{wt} = 0.4 m² K/W, (b) R_{wt} = 0.5 m² K/W, (c) R_{wt} = 0.6 m² K/W, (d) R_{wt} = 0.7 m² K/W and (e) R_{wt} = 0.8 m² K/W.

Table 1. TIPC reading values with examples of interpolation

k (W/m K)	R_{wt} (m² K/W)	$\sqrt{f}$	OIT (m)		
			TIPC value	True value	Error (%)
0.03	0.40	0.65	0.1000	0.1006	0.596
0.033	0.40	0.65	0.1048*	0.1049	0.095
0.035	0.40	0.65	0.1080	0.1076	0.372
0.035	0.45	0.65	0.1060*	0.10585	0.142
0.035	0.50	0.65	0.1040	0.10410	0.096

*Interpolated value
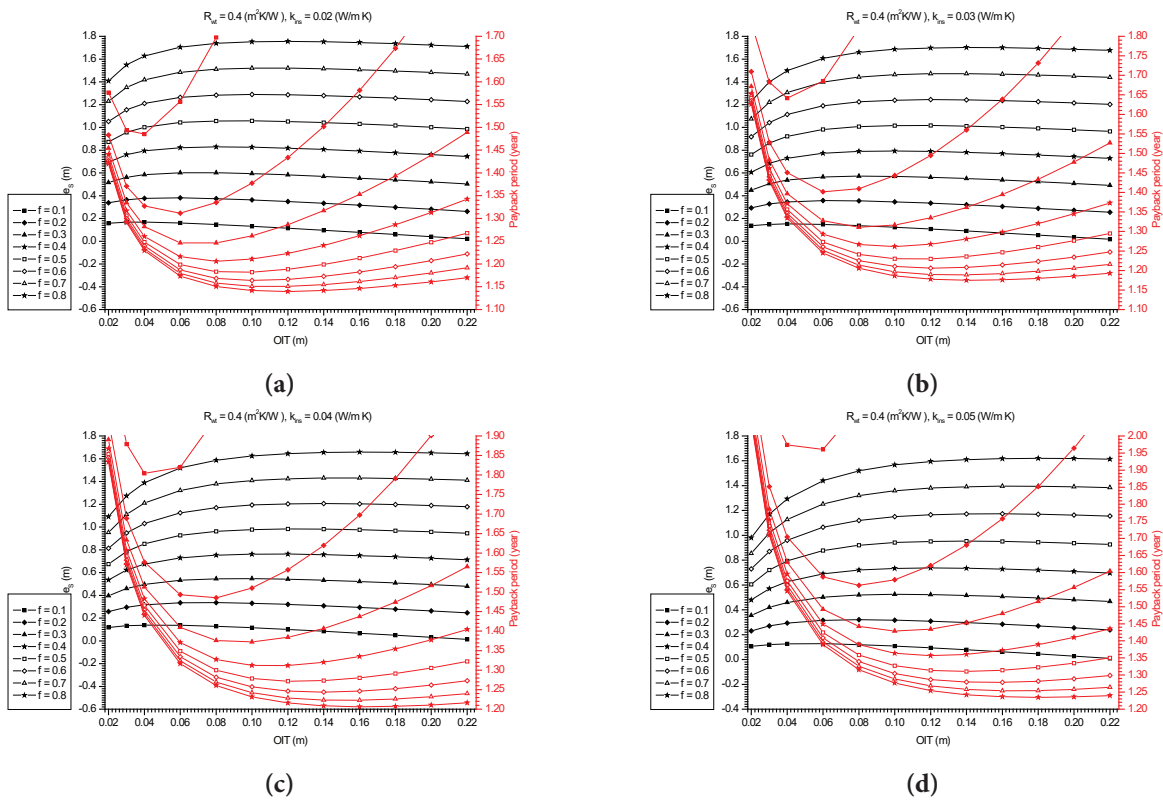
Figure 2. PP and e_S vs. OIT at R_{wt} = 0.4 m² K/W: (a) k = 0.02 W/m K, (b) k = 0.03 W/m K, (c) k = 0.04 W/m K, (d) k = 0.05 W/m K.
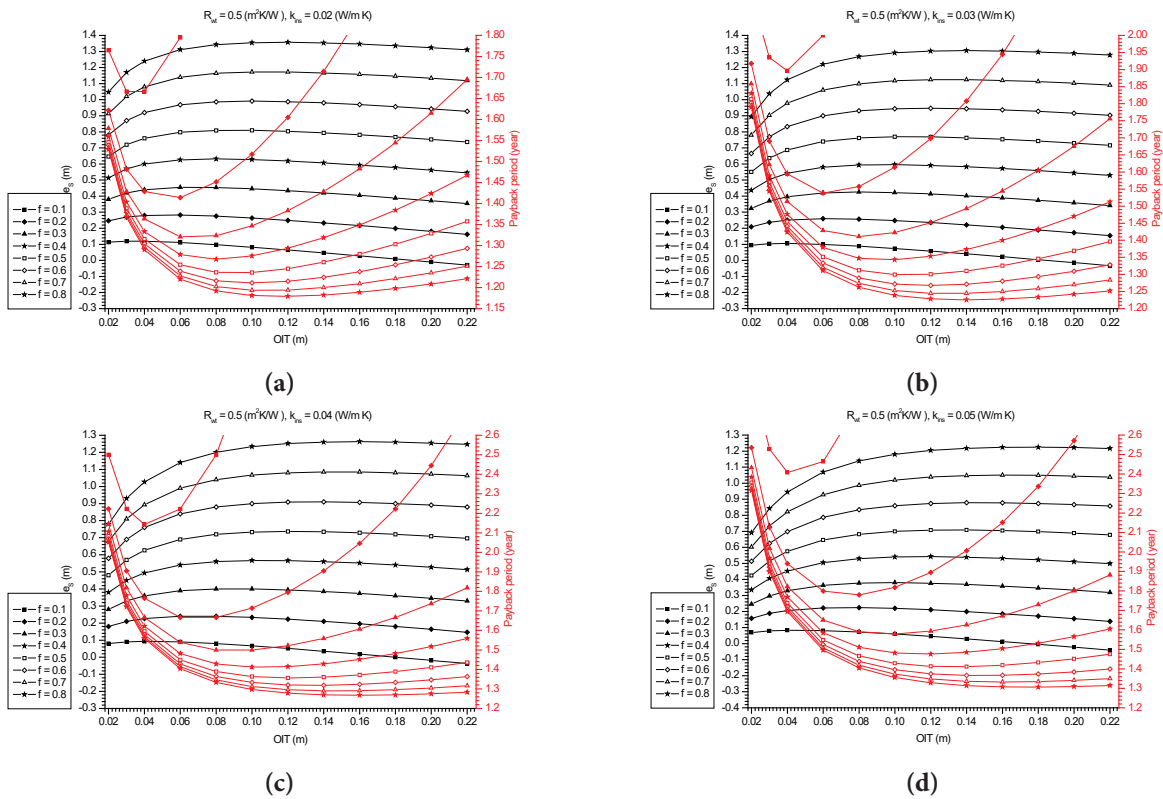
Figure 3. PP and e_S vs. OIT at R_{wt} = 0.5 m² K/W: (a) k = 0.02 W/m K, (b) k = 0.03 W/m K, (c) k = 0.04 W/m K, (d) k = 0.05 W/m K.
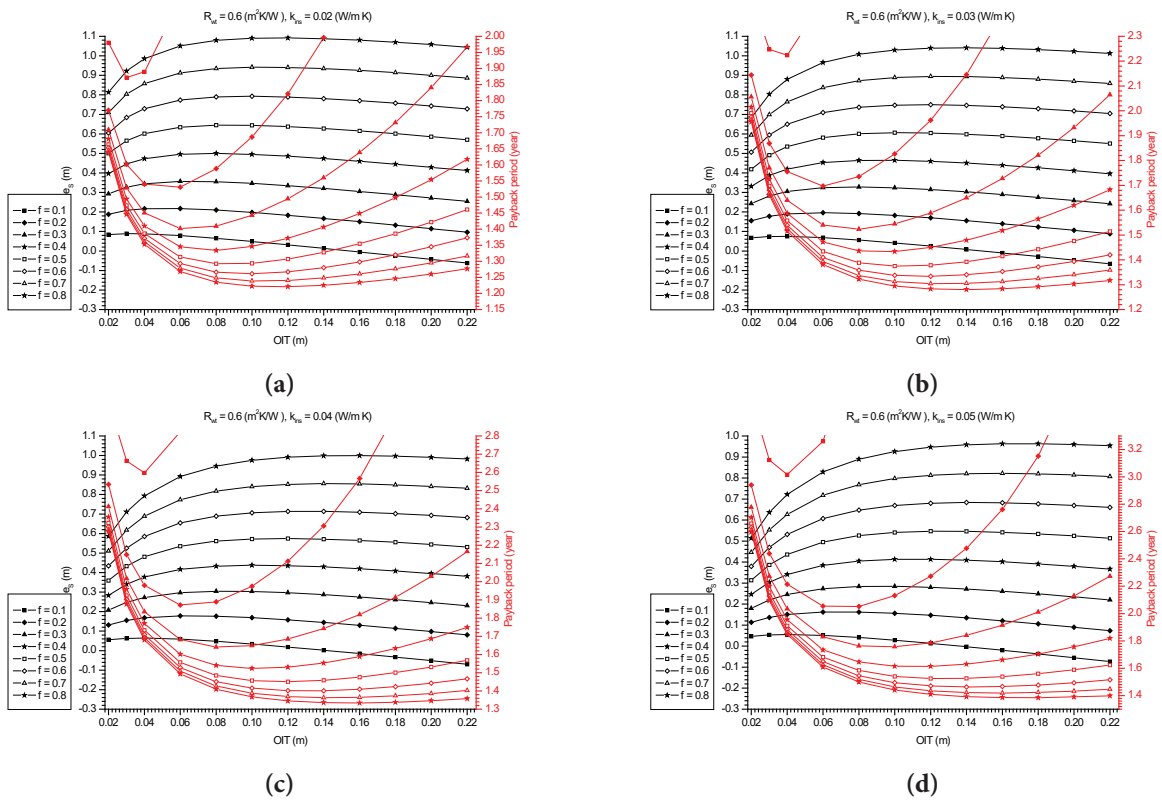
Figure 4. PP and e_S vs. OIT at R_{wt} = 0.6 m^2 K/W: (a) k = 0.02 W/m K, (b) k = 0.03 W/m K, (c) k = 0.04 W/m K, (d) k = 0.05 W/m K.
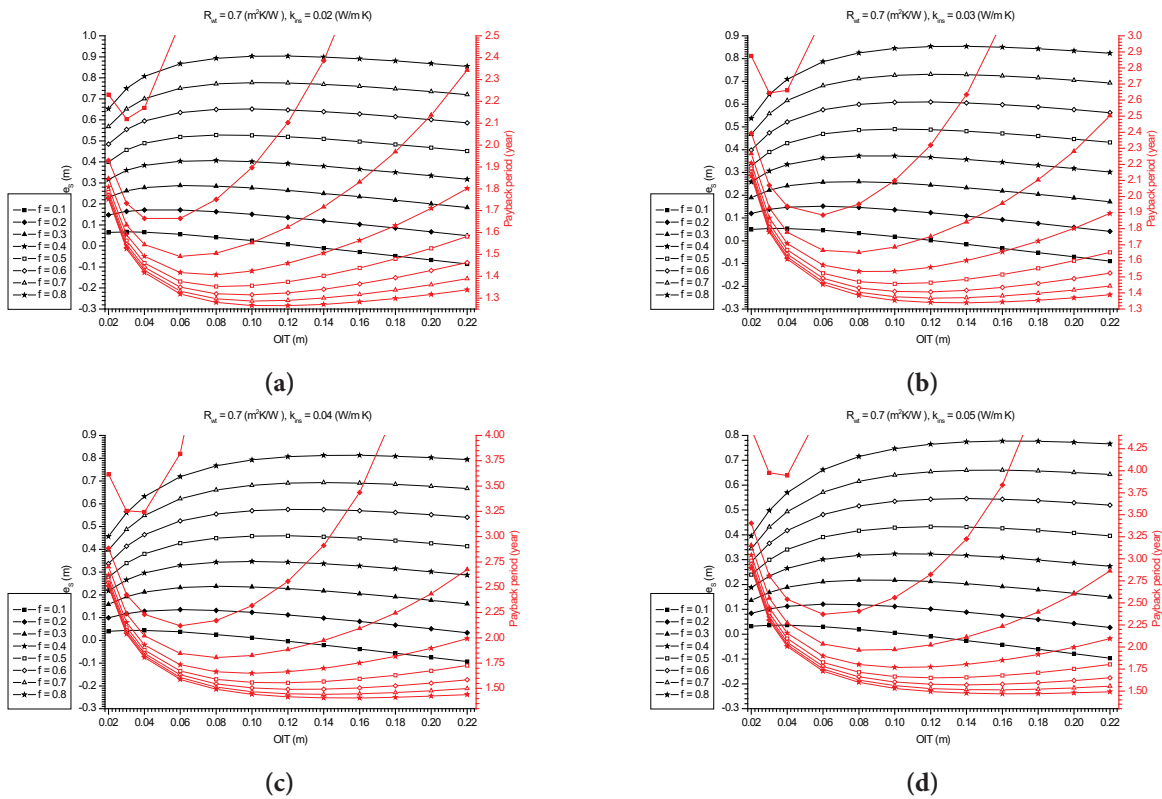
Figure 5. PP and e_S vs. OIT at R_{wt} = 0.7 m^2 K/W: (a) k = 0.02 W/m K, (b) k = 0.03 W/m K, (c) k = 0.04 W/m K, (d) k = 0.05 W/m K.
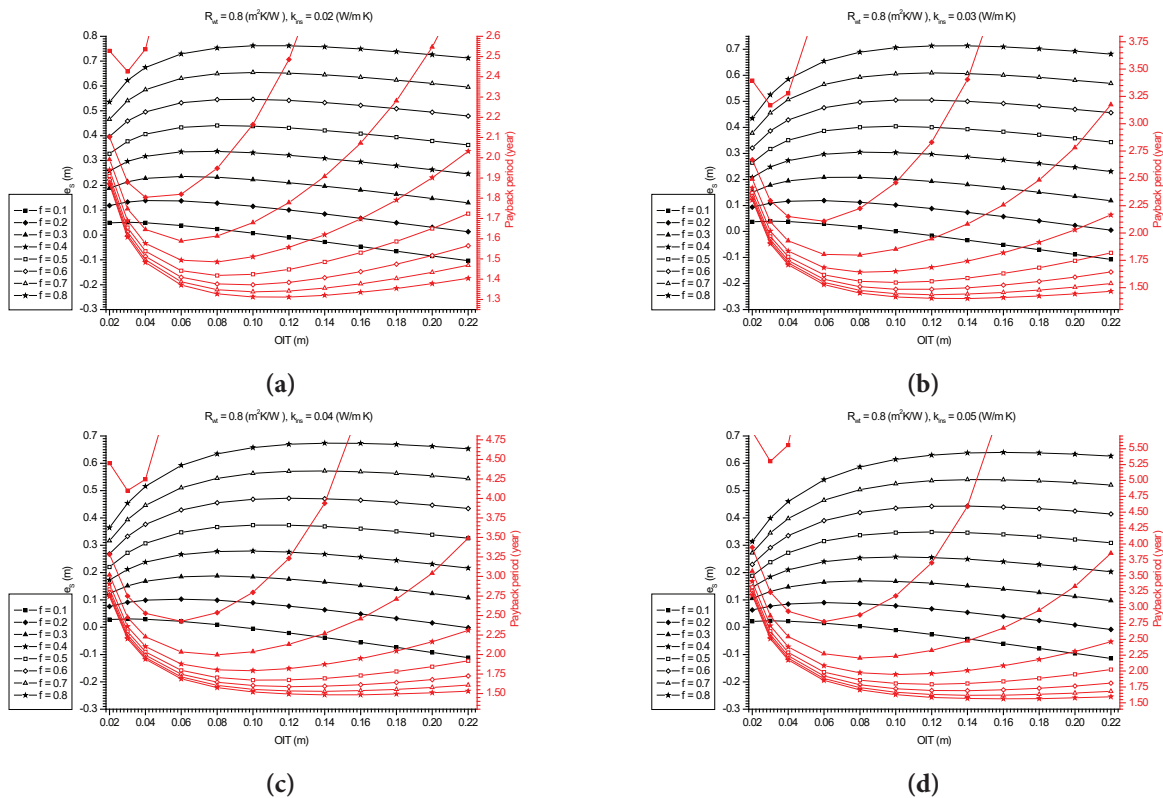
Figure 6. PP and e_S vs. OIT at R_{wt} = 0.8 m² K/W: (a) k = 0.02 W/m K, (b) k = 0.03 W/m K, (c) k = 0.04 W/m K, (d) k = 0.05 W/m K.

Table 2. TIPC reading values compared to true values with examples of interpolation at OIT of 0.1 m, f-factor of 0.5 K m³/W, and R_{wt} of 0.4 m² K/W

k_{ins} (W/m K)	e_S (m)		PP (year)	
	TIPC value (% error)	True value	TIPC value (% error)	True value
0.020	1.058 (0.09%)	1.057	1.182 (0.00%)	1.182
0.025	1.038* (0.19%)	1.036	1.206* (0.00%)	1.206
0.030	1.018 (0.20%)	1.016	1.230 (0.00%)	1.230
0.035	0.998* (0.10%)	0.997	1.255* (0.08%)	1.254
0.040	0.978 (0.00%)	0.978	1.280 (0.08%)	1.279
0.045	0.959* (0.00%)	0.959	1.304* (0.08%)	1.303
0.050	0.940 (0.21%)	0.942	1.328 (0.08%)	1.327

*Interpolated value

the authors, listed in Table 3, the calculated f-factor, using Eq. (26), for the cooling loads is 0.789 K m³/W; √f = 0.888. Hence, interpolating curve values between k_{ins} of 0.03 and 0.035 W/m K at R_{wt} of 0.4 and 0.5 m² K/W, using Fig. (1), we found that the OIT at k_{ins} of 0.034 W/m K and R_{wt} of 0.4862 m² K/W is 0.147 m, which is the same as calculated value. Instead of making interpolation, if the TICC user decided to use the closest curve to his data (e.g., k_{ins} of 0.035 W/m K and R_{wt} of 0.5 m² K/W) without making interpolations, the

OIT would be 0.148 m, which is also acceptable for decision making purposes compared to the true value of 0.147 m. Based on the OIT of 0.147 m found from Fig. (1), interpolating curve values between k_{ins} of 0.03 and 0.04 W/m K at R_{wt} of 0.4 and 0.5 m² K/W, using Figs. 2(b and c) and Figs. 3(b and c), we found that, at k_{ins} of 0.034 W/m K and R_{wt} of 0.4862 m² K/W, the PP and e_S are 1.23 year and 1.317 m, respectively. Multiplying e_S by the insulation material cost of 85 $/m³ to obtain the annual energy savings of 111.945

Table 3. Results of TIPC validation against published data

Reference		[9]	[33]	[35]	[26]	[36]
Process		Cooling	Heating	Heating	Heating	Heating
City		Amman	Antalya	Ankara	Denizli	Eskisehir
DD (°C-day)		17448	1431	2425	2055	3215
R_{wt} (m² K/W)		0.4862	0.715	0.774	0.592	0.592
Insulation material		EPS	Polystyrene	Polyurethane	Rock wool	Rock wool
k_{ins} (W/m K)		0.034	0.03	0.024	0.04	0.04
C_{ins} ($/m³)		85	75	450	107	108
Fuel type		Electricity	Coal	Natural gas	Fuel oil	Coal
C_F ($/kg, $/m³, $/kWh)		0.22	0.199	1.08	0.616	0.185
η (%)		–	65	90	80	70
H_u (J/kg, J/m³, J/kWh)		–	29.295×10⁶	34.526×10⁶	40.614×10⁶	25.54×10⁶
PWF		1.82	6.72	8.58	6.71	6.786
f-factor (K m³/W)		0.789	0.116	0.139	0.211	0.181
OIT (m)	Ref. value	0.147	0.037	0.040	0.068	0.061
	TIPC value	0.147	0.037	0.039	0.068	0.061
Savings ($/m², %)	Ref. value	111.503	40.42%	37.126	21.020	3.500%
	TIPC value	111.945	4.913	37.130	21.023	17.147
PP (year)	Ref. value	1.237	2.470	0.483	1.810	1.890
	TIPC value	1.238	2.472	2.174	1.815	1.901

$/m². It can be clearly noticed that the values found using TIPCs are very close to that calculated by Batiha et al. [9]. In a similar manner, the other data listed in Table 3 were validated. Comparing the obtained results by using the TIPCs with those calculated by authors listed in Table 3, we can conclude that TIPCs are valid and can be trusted to use with no caution.

CONCLUSION

In this paper, utilizing LCCA method, TIPCs were successfully introduced and validated, which can be considered as the first attempt toward developing more simple TIPCs. TIPCs are constructed to cover commonly used insulation materials for building walls with thermal conductivities between 0.020-0.055 W/m K. f-factor for the heating and cooling loads given first time in this paper. The findings of this paper can be summarized as follows:

1. TIPCs are timesaving and a simple tool that can be used to estimate the advantage of using thermal insulation.
2. Insulation performance, such as OIT, energy savings and PP, can be easily obtained using TIPCs based on a single factor, called the f-factor.
3. TIPCs can be used for determining the performance of insulation materials for external walls for any: (i) climate zone, (ii) loads (i.e. heating or cooling), (iii) fuel type and cost and (iv) insulation cost.
4. TIPCs can be used for the comparison and ranking of different insulation materials.

5. TIPCs are considered to be well validated and trusted tool.
6. TIPCs' user can use the curve with characteristics closest to his entry values without making interpolation; the percent error would be within acceptable margins.

NOMENCLATURE

C	Cost ($/kg, $/m³, $/kWh)
E	Energy (J)
e	Specific annual total net energy (m)
g	inflation rate (%)
h	Convection heat transfer coefficient (W/m² K)
Hv	Fuel lower heating value (J/kWh, J/kg, or J/m³)
i	Interest rate (%)
i*	Interest rate adjusted for inflation rate (%)
k	Thermal conductivity of the wall material (W/m K),
N	insulation material lifetime (year)
PP	Payback period (year)
q	Annual heating gain per unit area (W/m²)
R	Thermal resistance (K/W)
T	Temperature (°C)
U	Overall heat transfer coefficient (W/m² K)
x	Thickness (m)
η	Heating system efficiency (%)

Subscripts
A	Annual
b	Indoor base

C	Cooling
e	Electricity
E	Energy
F	Fuel
H	Heating
ins	Insulation
opt	Optimum
out	Outside
pre-ins	Pre-insulation
r	Reference
S	Saving
sa	Solar-air
T	Total
W	Wall

Abbreviations

CDD	Cooling degree-days
COP	Coefficient of performance
EPS	Expanded Polystyrene
HDD	Heating degree-days
LCCA	Life-cycle cost analysis
OIT	Optimum insulation thickness
PP	Payback period
PWF	Present worth factor
TIPCs	Thermal insulation performance curves

AUTHORSHIP CONTRIBUTIONS

Mohammad A. Batiha: conceptualization, data curation, formal analysis, investigation, methodology, project administration, resources, software, validation, visualization, writing – original draft, writing – review & editing. Saleh E. Rawadieh: data curation, methodology, software, validation, visualization, writing – original draft. Marwan M. Batiha: methodology, validation, visualization, writing – original draft. Leema A. Al-Makhadmeh: project administration, resources, writing – original draft. Abdullah A. Marachli: writing – review & editing. Muhammet Kayfeci: writing – review & editing.

DATA AVAILABILITY STATEMENT

No new data were created in this study. The published publication includes all graphics collected or developed during the study.

CONFLICT OF INTEREST

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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Table S1. OIT at different values of $\sqrt{f}$, k and R_{wt}

$\sqrt{f}$	Thermal conductivity of insulation material (k, W/m K)									
	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050	0.055
OIT (m) at R_{wt} = 0.4 m² K/W										
0.2	0.01600	0.01849	0.02028	0.02162	0.02264	0.02342	0.02400	0.02443	0.02472	0.02490
0.3	0.02600	0.03074	0.03443	0.03743	0.03996	0.04212	0.04400	0.04564	0.04708	0.04836
0.4	0.03600	0.04299	0.04857	0.05325	0.05728	0.06083	0.06400	0.06685	0.06944	0.07181
0.5	0.04600	0.05524	0.06271	0.06906	0.07460	0.07954	0.08400	0.08807	0.09180	0.09526
0.6	0.05600	0.06748	0.07685	0.08487	0.09192	0.09825	0.10400	0.10928	0.11416	0.11871
0.7	0.06600	0.07973	0.09099	0.10068	0.10924	0.11696	0.12400	0.13049	0.13652	0.14216
0.8	0.07600	0.09198	0.10514	0.11649	0.12656	0.13567	0.14400	0.15171	0.15889	0.16562
0.9	0.08600	0.10423	0.11928	0.13230	0.14388	0.15437	0.16400	0.17292	0.18125	0.18907
1.0	0.09600	0.11647	0.13342	0.14811	0.16121	0.17308	0.18400	0.19413	0.20361	0.21252
OIT (m) at R_{wt} = 0.5 m² K/W										
0.2	0.01500	0.01699	0.01828	0.01912	0.01964	0.01992	0.02000	0.01993	0.01972	0.01940
0.3	0.02500	0.02924	0.03243	0.03493	0.03696	0.03862	0.04000	0.04114	0.04208	0.04286
0.4	0.03500	0.04149	0.04657	0.05075	0.05428	0.05733	0.06000	0.06235	0.06444	0.06631
0.5	0.04500	0.05374	0.06071	0.06656	0.07160	0.07604	0.08000	0.08357	0.08680	0.08976
0.6	0.05500	0.06598	0.07485	0.08237	0.08892	0.09475	0.10000	0.10478	0.10916	0.11321
0.7	0.06500	0.07823	0.08899	0.09818	0.10624	0.11346	0.12000	0.12599	0.13152	0.13666
0.8	0.07500	0.09048	0.10314	0.11399	0.12356	0.13217	0.14000	0.14721	0.15389	0.16012
0.9	0.08500	0.10273	0.11728	0.12980	0.14088	0.15087	0.16000	0.16842	0.17625	0.18357
1.0	0.09500	0.11497	0.13142	0.14561	0.15821	0.16958	0.18000	0.18963	0.19861	0.20702
OIT (m) at R_{wt} = 0.6 m² K/W										
0.2	0.01400	0.01549	0.01628	0.01662	0.01664	0.01642	0.01600	0.01543	0.01472	0.01390
0.3	0.02400	0.02774	0.03043	0.03243	0.03396	0.03512	0.03600	0.03664	0.03708	0.03736
0.4	0.03400	0.03999	0.04457	0.04825	0.05128	0.05383	0.05600	0.05785	0.05944	0.06081
0.5	0.04400	0.05224	0.05871	0.06406	0.06860	0.07254	0.07600	0.07907	0.08180	0.08426
0.6	0.05400	0.06448	0.07285	0.07987	0.08592	0.09125	0.09600	0.10028	0.10416	0.10771
0.7	0.06400	0.07673	0.08699	0.09568	0.10324	0.10996	0.11600	0.12149	0.12652	0.13116
0.8	0.07400	0.08898	0.10114	0.11149	0.12056	0.12867	0.13600	0.14271	0.14889	0.15462
0.9	0.08400	0.10123	0.11528	0.12730	0.13788	0.14737	0.15600	0.16392	0.17125	0.17807
1.0	0.09400	0.11347	0.12942	0.14311	0.15521	0.16608	0.17600	0.18513	0.19361	0.20152
OIT (m) at R_{wt} = 0.7 m² K/W										
0.2	0.01300	0.01399	0.01428	0.01412	0.01364	0.01292	0.01200	0.01093	0.00972	0.00840
0.3	0.02300	0.02624	0.02843	0.02993	0.03096	0.03162	0.03200	0.03214	0.03208	0.03186
0.4	0.03300	0.03849	0.04257	0.04575	0.04828	0.05033	0.05200	0.05335	0.05444	0.05531
0.5	0.04300	0.05074	0.05671	0.06156	0.06560	0.06904	0.07200	0.07457	0.07680	0.07876
0.6	0.05300	0.06298	0.07085	0.07737	0.08292	0.08775	0.09200	0.09578	0.09916	0.10221
0.7	0.06300	0.07523	0.08499	0.09318	0.10024	0.10646	0.11200	0.11699	0.12152	0.12566
0.8	0.07300	0.08748	0.09914	0.10899	0.11756	0.12517	0.13200	0.13821	0.14389	0.14912
0.9	0.08300	0.09973	0.11328	0.12480	0.13488	0.14387	0.15200	0.15942	0.16625	0.17257
1.0	0.09300	0.11197	0.12742	0.14061	0.15221	0.16258	0.17200	0.18063	0.18861	0.19602
OIT (m) at R_{wt} = 0.8 m² K/W										
0.2	0.01200	0.01249	0.01228	0.01162	0.01064	0.00942	0.00800	0.00643	0.00472	0.00290
0.3	0.02200	0.02474	0.02643	0.02743	0.02796	0.02812	0.02800	0.02764	0.02708	0.02636
0.4	0.03200	0.03699	0.04057	0.04325	0.04528	0.04683	0.04800	0.04885	0.04944	0.04981
0.5	0.04200	0.04924	0.05471	0.05906	0.06260	0.06554	0.06800	0.07007	0.07180	0.07326
0.6	0.05200	0.06148	0.06885	0.07487	0.07992	0.08425	0.08800	0.09128	0.09416	0.09671
0.7	0.06200	0.07373	0.08299	0.09068	0.09724	0.10296	0.10800	0.11249	0.11652	0.12016
0.8	0.07200	0.08598	0.09714	0.10649	0.11456	0.12167	0.12800	0.13371	0.13889	0.14362
0.9	0.08200	0.09823	0.11128	0.12230	0.13188	0.14037	0.14800	0.15492	0.16125	0.16707
1.0	0.09200	0.11047	0.12542	0.13811	0.14921	0.15908	0.16800	0.17613	0.18361	0.19052

Table S2. Specific savings and payback period at R_{wt} of 0.4 m² K/W

OIT (m)	f										f									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Specific savings (e_S, m)										Payback period (year)									
	k_{ins} = 0.02 W/m K																			
0.01	0.129	0.268	0.407	0.546	0.684	0.823	0.962	1.101	1.240	1.379	1.940	1.867	1.844	1.833	1.826	1.822	1.819	1.816	1.815	1.813
0.02	0.159	0.337	0.516	0.694	0.873	1.051	1.230	1.409	1.587	1.766	1.577	1.483	1.454	1.440	1.432	1.427	1.423	1.420	1.418	1.416
0.04	0.168	0.377	0.585	0.793	1.002	1.210	1.418	1.627	1.835	2.043	1.485	1.327	1.282	1.261	1.248	1.240	1.234	1.230	1.226	1.223
0.06	0.161	0.381	0.602	0.822	1.043	1.264	1.484	1.705	1.925	2.146	1.557	1.312	1.246	1.216	1.199	1.187	1.179	1.173	1.169	1.165
0.08	0.147	0.375	0.602	0.829	1.056	1.284	1.511	1.738	1.965	2.193	1.698	1.335	1.246	1.206	1.183	1.169	1.158	1.151	1.145	1.140
0.1	0.131	0.363	0.594	0.826	1.057	1.289	1.520	1.752	1.983	2.215	1.901	1.378	1.262	1.211	1.182	1.164	1.151	1.142	1.134	1.129
0.12	0.114	0.349	0.583	0.818	1.052	1.286	1.521	1.755	1.989	2.224	2.186	1.434	1.286	1.223	1.188	1.166	1.151	1.140	1.131	1.124
0.14	0.096	0.333	0.569	0.806	1.042	1.279	1.515	1.752	1.988	2.225	2.591	1.502	1.317	1.241	1.199	1.173	1.155	1.142	1.132	1.124
0.16	0.078	0.316	0.554	0.792	1.030	1.269	1.507	1.745	1.983	2.221	3.201	1.581	1.353	1.262	1.213	1.182	1.162	1.146	1.135	1.126
0.18	0.059	0.299	0.538	0.777	1.017	1.256	1.496	1.735	1.974	2.214	4.211	1.674	1.394	1.286	1.229	1.194	1.170	1.153	1.140	1.129
0.2	0.040	0.281	0.521	0.762	1.002	1.242	1.483	1.723	1.963	2.204	6.190	1.781	1.439	1.313	1.248	1.207	1.180	1.161	1.146	1.134
0.22	0.021	0.262	0.504	0.745	0.986	1.227	1.469	1.710	1.951	2.192	11.777	1.905	1.489	1.342	1.268	1.222	1.192	1.170	1.153	1.140
	k_{ins} = 0.03 W/m K																			
0.01	0.104	0.217	0.331	0.445	0.558	0.672	0.785	0.899	1.013	1.126	2.412	2.301	2.266	2.249	2.239	2.233	2.228	2.224	2.222	2.220
0.02	0.136	0.293	0.449	0.605	0.761	0.918	1.074	1.230	1.386	1.543	1.835	1.709	1.671	1.653	1.642	1.635	1.630	1.626	1.623	1.621
0.04	0.152	0.345	0.537	0.729	0.922	1.114	1.306	1.498	1.691	1.883	1.641	1.451	1.397	1.371	1.356	1.347	1.340	1.335	1.331	1.328
0.06	0.148	0.357	0.565	0.773	0.982	1.190	1.398	1.607	1.815	2.023	1.685	1.402	1.327	1.293	1.273	1.261	1.251	1.245	1.240	1.236
0.08	0.137	0.355	0.572	0.790	1.007	1.224	1.442	1.659	1.877	2.094	1.820	1.409	1.311	1.267	1.241	1.225	1.214	1.205	1.199	1.194
0.1	0.123	0.346	0.570	0.793	1.016	1.239	1.463	1.686	1.909	2.132	2.029	1.443	1.317	1.261	1.230	1.210	1.197	1.186	1.179	1.173
0.12	0.107	0.335	0.562	0.789	1.016	1.244	1.471	1.698	1.925	2.153	2.331	1.495	1.335	1.267	1.230	1.206	1.190	1.178	1.169	1.161
0.14	0.090	0.321	0.551	0.781	1.011	1.242	1.472	1.702	1.932	2.163	2.770	1.560	1.362	1.280	1.236	1.208	1.189	1.175	1.164	1.156
0.16	0.073	0.305	0.538	0.770	1.003	1.235	1.468	1.700	1.933	2.166	3.446	1.639	1.395	1.298	1.247	1.214	1.192	1.176	1.164	1.154
0.18	0.054	0.289	0.523	0.758	0.992	1.226	1.461	1.695	1.929	2.164	4.598	1.732	1.434	1.320	1.260	1.223	1.198	1.180	1.166	1.155
0.2	0.036	0.272	0.508	0.743	0.979	1.215	1.451	1.687	1.923	2.158	6.974	1.840	1.478	1.345	1.276	1.234	1.206	1.186	1.170	1.158
0.22	0.017	0.254	0.491	0.728	0.965	1.202	1.439	1.677	1.914	2.151	14.646	1.967	1.527	1.373	1.295	1.247	1.216	1.193	1.176	1.162
	k_{ins} = 0.04 W/m K																			
0.01	0.086	0.182	0.278	0.375	0.471	0.567	0.663	0.759	0.855	0.952	2.902	2.743	2.693	2.669	2.655	2.646	2.639	2.634	2.630	2.627
0.02	0.119	0.258	0.397	0.536	0.674	0.813	0.952	1.091	1.230	1.369	2.103	1.940	1.891	1.867	1.853	1.844	1.838	1.833	1.829	1.826
0.04	0.139	0.317	0.496	0.674	0.853	1.031	1.210	1.389	1.567	1.746	1.804	1.577	1.513	1.483	1.466	1.454	1.446	1.440	1.436	1.432
0.06	0.137	0.335	0.532	0.729	0.927	1.124	1.322	1.519	1.716	1.914	1.820	1.494	1.409	1.371	1.349	1.334	1.324	1.317	1.311	1.306
0.08	0.128	0.337	0.545	0.753	0.962	1.170	1.378	1.587	1.795	2.003	1.948	1.485	1.376	1.327	1.300	1.282	1.270	1.261	1.253	1.248
0.1	0.116	0.331	0.547	0.762	0.978	1.193	1.409	1.624	1.840	2.055	2.164	1.510	1.372	1.312	1.279	1.257	1.242	1.231	1.223	1.216
0.12	0.101	0.321	0.542	0.762	0.983	1.204	1.424	1.645	1.865	2.086	2.485	1.557	1.384	1.312	1.272	1.246	1.229	1.216	1.206	1.199
0.14	0.084	0.309	0.533	0.757	0.982	1.206	1.431	1.655	1.879	2.104	2.964	1.620	1.407	1.320	1.273	1.244	1.223	1.209	1.197	1.188
0.16	0.067	0.295	0.522	0.749	0.976	1.204	1.431	1.658	1.885	2.113	3.716	1.698	1.437	1.335	1.280	1.246	1.223	1.206	1.193	1.183
0.18	0.050	0.279	0.509	0.738	0.968	1.198	1.427	1.657	1.886	2.116	5.041	1.791	1.474	1.354	1.291	1.253	1.226	1.207	1.193	1.182
0.2	0.031	0.263	0.494	0.726	0.957	1.189	1.420	1.652	1.883	2.115	7.941	1.901	1.517	1.378	1.306	1.262	1.232	1.211	1.195	1.182
0.22	0.013	0.246	0.479	0.712	0.945	1.178	1.411	1.644	1.877	2.111	19.156	2.032	1.565	1.404	1.322	1.273	1.240	1.216	1.198	1.185
	k_{ins} = 0.05 W/m K																			
0.01	0.073	0.157	0.240	0.323	0.407	0.490	0.573	0.657	0.740	0.823	3.409	3.191	3.125	3.093	3.074	3.061	3.052	3.046	3.041	3.036
0.02	0.105	0.230	0.355	0.480	0.605	0.730	0.855	0.980	1.105	1.230	2.381	2.174	2.113	2.083	2.066	2.055	2.047	2.041	2.036	2.033
0.04	0.127	0.293	0.460	0.627	0.793	0.960	1.127	1.293	1.460	1.627	1.974	1.705	1.630	1.596	1.576	1.563	1.553	1.546	1.541	1.537
0.06	0.128	0.315	0.503	0.690	0.878	1.065	1.253	1.440	1.628	1.815	1.961	1.587	1.493	1.449	1.425	1.408	1.397	1.389	1.382	1.377
0.08	0.120	0.320	0.520	0.720	0.920	1.120	1.320	1.520	1.720	1.920	2.083	1.563	1.442	1.389	1.359	1.339	1.326	1.316	1.308	1.302
0.1	0.108	0.317	0.525	0.733	0.942	1.150	1.358	1.567	1.775	1.983	2.308	1.579	1.429	1.364	1.327	1.304	1.288	1.277	1.268	1.261
0.12	0.094	0.309	0.523	0.737	0.951	1.166	1.380	1.594	1.809	2.023	2.652	1.620	1.434	1.357	1.314	1.287	1.268	1.254	1.244	1.236
0.14	0.079	0.298	0.516	0.735	0.954	1.173	1.391	1.610	1.829	2.048	3.175	1.681	1.453	1.361	1.311	1.279	1.258	1.242	1.230	1.221
0.16	0.062	0.284	0.507	0.729	0.951	1.173	1.396	1.618	1.840	2.062	4.018	1.758	1.480	1.372	1.314	1.278	1.254	1.236	1.223	1.212
0.18	0.045	0.270	0.495	0.720	0.945	1.170	1.395	1.620	1.845	2.070	5.556	1.852	1.515	1.389	1.323	1.282	1.254	1.235	1.220	1.208
0.2	0.027	0.255	0.482	0.709	0.936	1.164	1.391	1.618	1.845	2.073	9.167	1.964	1.557	1.410	1.335	1.289	1.258	1.236	1.219	1.206
0.22	0.009	0.238	0.468	0.697	0.926	1.155	1.384	1.613	1.843	2.072	27.273	2.098	1.604	1.435	1.350	1.299	1.264	1.240	1.221	1.207

Table S3. Specific savings and payback period at R_{wt} of 0.5 m² K/W

OIT (m)	f																			
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Specific savings (e_S, m)										Payback period (year)									
	k_{ins} = 0.02 W/m K																			
0.01	0.090	0.190	0.290	0.390	0.490	0.590	0.690	0.790	0.890	0.990	2.222	2.105	2.069	2.051	2.041	2.034	2.029	2.025	2.022	2.020
0.02	0.113	0.247	0.380	0.513	0.647	0.780	0.913	1.047	1.180	1.313	1.765	1.622	1.579	1.558	1.546	1.538	1.533	1.529	1.525	1.523
0.04	0.120	0.280	0.440	0.600	0.760	0.920	1.080	1.240	1.400	1.560	1.667	1.429	1.364	1.333	1.316	1.304	1.296	1.290	1.286	1.282
0.06	0.111	0.283	0.454	0.626	0.797	0.969	1.140	1.311	1.483	1.654	1.795	1.414	1.321	1.279	1.254	1.239	1.228	1.220	1.214	1.209
0.08	0.098	0.276	0.453	0.631	0.809	0.987	1.164	1.342	1.520	1.698	2.045	1.452	1.324	1.268	1.236	1.216	1.202	1.192	1.184	1.178
0.1	0.082	0.264	0.445	0.627	0.809	0.991	1.173	1.355	1.536	1.718	2.444	1.517	1.347	1.275	1.236	1.211	1.194	1.181	1.172	1.164
0.12	0.065	0.249	0.434	0.618	0.803	0.988	1.172	1.357	1.542	1.726	3.095	1.605	1.383	1.294	1.245	1.215	1.194	1.179	1.168	1.159
0.14	0.047	0.233	0.420	0.607	0.793	0.980	1.167	1.353	1.540	1.727	4.286	1.714	1.429	1.319	1.261	1.224	1.200	1.182	1.169	1.158
0.16	0.028	0.216	0.405	0.593	0.781	0.969	1.158	1.346	1.534	1.722	7.083	1.848	1.483	1.349	1.280	1.238	1.209	1.189	1.173	1.161
0.18	0.009	0.199	0.388	0.578	0.767	0.957	1.146	1.336	1.525	1.715	21.111	2.011	1.545	1.384	1.303	1.254	1.221	1.198	1.180	1.166
0.2	–	0.181	0.371	0.562	0.752	0.943	1.133	1.324	1.514	1.705	–	2.211	1.615	1.424	1.329	1.273	1.235	1.209	1.189	1.173
0.22	–	0.163	0.354	0.545	0.737	0.928	1.119	1.310	1.502	1.693	–	2.460	1.695	1.467	1.358	1.293	1.251	1.221	1.199	1.181
	k_{ins} = 0.03 W/m K																			
0.01	0.070	0.150	0.230	0.310	0.390	0.470	0.550	0.630	0.710	0.790	2.857	2.667	2.609	2.581	2.564	2.553	2.545	2.540	2.535	2.532
0.02	0.094	0.209	0.323	0.437	0.551	0.666	0.780	0.894	1.009	1.123	2.121	1.918	1.858	1.830	1.813	1.803	1.795	1.789	1.785	1.781
0.04	0.105	0.251	0.396	0.542	0.687	0.833	0.978	1.124	1.269	1.415	1.897	1.594	1.514	1.477	1.455	1.441	1.431	1.424	1.418	1.414
0.06	0.100	0.260	0.420	0.580	0.740	0.900	1.060	1.220	1.380	1.540	2.000	1.538	1.429	1.379	1.351	1.333	1.321	1.311	1.304	1.299
0.08	0.088	0.257	0.425	0.594	0.762	0.931	1.099	1.267	1.436	1.604	2.262	1.557	1.411	1.348	1.312	1.290	1.274	1.262	1.254	1.247
0.1	0.074	0.248	0.422	0.596	0.770	0.943	1.117	1.291	1.465	1.639	2.706	1.614	1.423	1.343	1.299	1.272	1.253	1.239	1.228	1.220
0.12	0.058	0.236	0.413	0.591	0.769	0.947	1.124	1.302	1.480	1.658	3.462	1.698	1.452	1.353	1.301	1.268	1.245	1.229	1.216	1.206
0.14	0.041	0.221	0.402	0.583	0.763	0.944	1.125	1.305	1.486	1.666	4.921	1.808	1.493	1.373	1.310	1.271	1.245	1.226	1.211	1.200
0.16	0.023	0.206	0.389	0.571	0.754	0.937	1.120	1.303	1.486	1.669	8.750	1.944	1.544	1.400	1.326	1.280	1.250	1.228	1.212	1.199
0.18	0.005	0.189	0.374	0.558	0.743	0.928	1.112	1.297	1.482	1.666	43.333	2.114	1.605	1.433	1.346	1.294	1.259	1.234	1.215	1.200
0.2	–	0.172	0.358	0.544	0.730	0.916	1.102	1.288	1.474	1.660	–	2.324	1.675	1.470	1.369	1.310	1.270	1.242	1.221	1.204
0.22	–	0.154	0.342	0.529	0.716	0.903	1.091	1.278	1.465	1.652	–	2.590	1.756	1.512	1.396	1.328	1.284	1.252	1.229	1.210
	k_{ins} = 0.04 W/m K																			
0.01	0.057	0.123	0.190	0.257	0.323	0.390	0.457	0.523	0.590	0.657	3.529	3.243	3.158	3.117	3.093	3.077	3.066	3.057	3.051	3.046
0.02	0.080	0.180	0.280	0.380	0.480	0.580	0.680	0.780	0.880	0.980	2.500	2.222	2.143	2.105	2.083	2.069	2.059	2.051	2.045	2.041
0.04	0.093	0.227	0.360	0.493	0.627	0.760	0.893	1.027	1.160	1.293	2.143	1.765	1.667	1.622	1.596	1.579	1.567	1.558	1.552	1.546
0.06	0.090	0.240	0.390	0.540	0.690	0.840	0.990	1.140	1.290	1.440	2.222	1.667	1.538	1.481	1.449	1.429	1.414	1.404	1.395	1.389
0.08	0.080	0.240	0.400	0.560	0.720	0.880	1.040	1.200	1.360	1.520	2.500	1.667	1.500	1.429	1.389	1.364	1.346	1.333	1.324	1.316
0.1	0.067	0.233	0.400	0.567	0.733	0.900	1.067	1.233	1.400	1.567	3.000	1.714	1.500	1.412	1.364	1.333	1.313	1.297	1.286	1.277
0.12	0.051	0.223	0.394	0.566	0.737	0.909	1.080	1.251	1.423	1.594	3.889	1.795	1.522	1.414	1.357	1.321	1.296	1.279	1.265	1.254
0.14	0.035	0.210	0.385	0.560	0.735	0.910	1.085	1.260	1.435	1.610	5.714	1.905	1.558	1.429	1.361	1.319	1.290	1.270	1.254	1.242
0.16	0.018	0.196	0.373	0.551	0.729	0.907	1.084	1.262	1.440	1.618	11.250	2.045	1.607	1.452	1.372	1.324	1.291	1.268	1.250	1.236
0.18	0.000	0.180	0.360	0.540	0.720	0.900	1.080	1.260	1.440	1.620	–	2.222	1.667	1.481	1.389	1.333	1.296	1.270	1.250	1.235
0.2	–	0.164	0.345	0.527	0.709	0.891	1.073	1.255	1.436	1.618	–	2.444	1.737	1.517	1.410	1.347	1.305	1.275	1.253	1.236
0.22	–	0.147	0.330	0.513	0.697	0.880	1.063	1.247	1.430	1.613	–	2.727	1.818	1.558	1.435	1.364	1.317	1.283	1.259	1.240
	k_{ins} = 0.05 W/m K																			
0.01	0.047	0.104	0.161	0.219	0.276	0.333	0.390	0.447	0.504	0.561	4.242	3.836	3.717	3.660	3.627	3.605	3.590	3.578	3.569	3.562
0.02	0.069	0.158	0.247	0.336	0.424	0.513	0.602	0.691	0.780	0.869	2.903	2.535	2.432	2.384	2.356	2.338	2.325	2.315	2.308	2.302
0.04	0.083	0.206	0.329	0.452	0.575	0.698	0.822	0.945	1.068	1.191	2.407	1.940	1.822	1.769	1.738	1.718	1.704	1.694	1.686	1.680
0.06	0.081	0.222	0.364	0.505	0.646	0.787	0.928	1.069	1.211	1.352	2.464	1.799	1.650	1.585	1.548	1.525	1.508	1.496	1.487	1.480
0.08	0.072	0.225	0.377	0.530	0.682	0.834	0.987	1.139	1.291	1.444	2.763	1.780	1.591	1.511	1.466	1.438	1.419	1.405	1.394	1.385
0.1	0.060	0.220	0.380	0.540	0.700	0.860	1.020	1.180	1.340	1.500	3.333	1.818	1.579	1.481	1.429	1.395	1.373	1.356	1.343	1.333
0.12	0.046	0.211	0.377	0.542	0.708	0.873	1.039	1.204	1.370	1.535	4.394	1.895	1.593	1.476	1.413	1.374	1.348	1.329	1.314	1.303
0.14	0.030	0.199	0.369	0.539	0.708	0.878	1.048	1.218	1.387	1.557	6.735	2.006	1.626	1.485	1.411	1.366	1.336	1.314	1.298	1.285
0.16	0.013	0.186	0.359	0.532	0.705	0.878	1.051	1.224	1.397	1.570	15.417	2.151	1.672	1.504	1.419	1.367	1.332	1.307	1.289	1.274
0.18	–	0.171	0.347	0.522	0.698	0.874	1.049	1.225	1.400	1.576	–	2.336	1.730	1.531	1.433	1.374	1.334	1.306	1.285	1.269
0.2	–	0.156	0.333	0.511	0.689	0.867	1.044	1.222	1.400	1.578	–	2.571	1.800	1.565	1.452	1.385	1.340	1.309	1.286	1.268
0.22	–	0.139	0.319	0.498	0.678	0.858	1.037	1.217	1.396	1.576	–	2.874	1.882	1.605	1.475	1.399	1.350	1.315	1.289	1.269

Table S4. Specific savings and payback period at R_{wt} of 0.6 m² K/W

OIT (m)	f										f									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Specific savings (e_S, m)										Payback period (year)									
$k_{ins} = 0.02$ W/m K																				
0.01	0.066	0.142	0.217	0.293	0.369	0.445	0.520	0.596	0.672	0.748	2.535	2.355	2.301	2.275	2.260	2.249	2.242	2.237	2.233	2.229
0.02	0.084	0.188	0.293	0.397	0.501	0.605	0.709	0.813	0.918	1.022	1.980	1.770	1.709	1.681	1.664	1.653	1.645	1.639	1.635	1.631
0.04	0.088	0.216	0.345	0.473	0.601	0.729	0.857	0.986	1.114	1.242	1.890	1.540	1.451	1.410	1.387	1.371	1.361	1.353	1.347	1.342
0.06	0.079	0.218	0.357	0.496	0.634	0.773	0.912	1.051	1.190	1.329	2.113	1.531	1.402	1.345	1.313	1.293	1.279	1.268	1.261	1.254
0.08	0.065	0.210	0.355	0.500	0.645	0.790	0.934	1.079	1.224	1.369	2.567	1.588	1.409	1.334	1.293	1.267	1.248	1.235	1.225	1.217
0.1	0.049	0.198	0.346	0.495	0.644	0.793	0.942	1.090	1.239	1.388	3.415	1.687	1.443	1.346	1.294	1.261	1.239	1.223	1.210	1.201
0.12	0.032	0.183	0.335	0.486	0.638	0.789	0.941	1.092	1.244	1.395	5.288	1.821	1.495	1.372	1.307	1.267	1.240	1.221	1.206	1.195
0.14	0.014	0.167	0.321	0.474	0.628	0.781	0.935	1.088	1.242	1.395	12.338	1.996	1.560	1.406	1.328	1.280	1.248	1.225	1.208	1.195
0.16	–	0.150	0.305	0.460	0.615	0.770	0.925	1.080	1.235	1.390	–	2.221	1.639	1.449	1.355	1.298	1.261	1.234	1.214	1.199
0.18	–	0.133	0.289	0.445	0.601	0.758	0.914	1.070	1.226	1.383	–	2.516	1.732	1.498	1.386	1.320	1.277	1.246	1.223	1.206
0.2	–	0.114	0.272	0.429	0.586	0.743	0.901	1.058	1.215	1.372	–	2.912	1.840	1.554	1.422	1.345	1.295	1.260	1.234	1.214
0.22	–	0.096	0.254	0.412	0.570	0.728	0.886	1.044	1.202	1.360	–	3.469	1.967	1.617	1.461	1.373	1.316	1.277	1.247	1.225
$k_{ins} = 0.03$ W/m K																				
0.01	0.050	0.109	0.169	0.228	0.288	0.347	0.407	0.466	0.526	0.585	3.365	3.057	2.966	2.923	2.897	2.881	2.869	2.860	2.853	2.848
0.02	0.068	0.155	0.243	0.331	0.419	0.506	0.594	0.682	0.769	0.857	2.461	2.144	2.056	2.015	1.991	1.975	1.964	1.956	1.949	1.944
0.04	0.075	0.190	0.305	0.420	0.535	0.650	0.765	0.880	0.994	1.109	2.224	1.755	1.640	1.588	1.558	1.539	1.526	1.516	1.508	1.502
0.06	0.068	0.196	0.325	0.453	0.581	0.709	0.837	0.966	1.094	1.222	2.444	1.697	1.540	1.472	1.434	1.410	1.393	1.381	1.371	1.364
0.08	0.056	0.192	0.328	0.464	0.600	0.736	0.872	1.008	1.144	1.281	2.973	1.735	1.524	1.436	1.388	1.358	1.337	1.322	1.311	1.302
0.1	0.041	0.182	0.324	0.465	0.606	0.747	0.889	1.030	1.171	1.312	4.041	1.827	1.545	1.434	1.375	1.338	1.313	1.295	1.281	1.270
0.12	0.025	0.170	0.315	0.460	0.605	0.750	0.894	1.039	1.184	1.329	6.686	1.962	1.588	1.450	1.378	1.334	1.304	1.283	1.267	1.254
0.14	0.008	0.155	0.303	0.451	0.598	0.746	0.894	1.041	1.189	1.337	21.703	2.146	1.650	1.479	1.393	1.340	1.305	1.280	1.261	1.247
0.16	–	0.140	0.289	0.439	0.589	0.739	0.889	1.039	1.188	1.338	–	2.387	1.727	1.518	1.415	1.353	1.313	1.284	1.262	1.246
0.18	–	0.123	0.275	0.426	0.578	0.729	0.881	1.032	1.184	1.335	–	2.709	1.821	1.565	1.443	1.372	1.325	1.292	1.267	1.248
0.2	–	0.106	0.259	0.412	0.565	0.717	0.870	1.023	1.176	1.329	–	3.150	1.933	1.620	1.476	1.394	1.340	1.303	1.275	1.254
0.22	–	0.088	0.242	0.396	0.550	0.704	0.858	1.012	1.167	1.321	–	3.783	2.065	1.682	1.514	1.420	1.359	1.317	1.286	1.262
$k_{ins} = 0.04$ W/m K																				
0.01	0.039	0.088	0.137	0.186	0.235	0.284	0.333	0.382	0.431	0.480	4.271	3.786	3.648	3.583	3.545	3.520	3.502	3.489	3.479	3.471
0.02	0.056	0.132	0.207	0.283	0.359	0.435	0.510	0.586	0.662	0.738	2.989	2.535	2.412	2.355	2.323	2.301	2.286	2.275	2.266	2.260
0.04	0.064	0.168	0.273	0.377	0.481	0.585	0.689	0.793	0.898	1.002	2.597	1.980	1.835	1.770	1.733	1.709	1.693	1.681	1.671	1.664
0.06	0.059	0.178	0.297	0.416	0.535	0.654	0.773	0.892	1.011	1.130	2.823	1.872	1.683	1.602	1.557	1.528	1.509	1.494	1.483	1.474
0.08	0.048	0.176	0.305	0.433	0.561	0.689	0.817	0.946	1.074	1.202	3.457	1.890	1.641	1.540	1.485	1.451	1.427	1.410	1.397	1.387
0.1	0.034	0.169	0.303	0.438	0.572	0.706	0.841	0.975	1.110	1.244	4.844	1.975	1.649	1.523	1.457	1.416	1.387	1.367	1.352	1.340
0.12	0.019	0.158	0.297	0.436	0.574	0.713	0.852	0.991	1.130	1.269	8.824	2.113	1.685	1.531	1.451	1.402	1.369	1.345	1.327	1.313
0.14	0.002	0.145	0.287	0.429	0.571	0.714	0.856	0.998	1.140	1.283	73.214	2.306	1.743	1.554	1.458	1.401	1.363	1.336	1.315	1.299
0.16	–	0.130	0.275	0.420	0.565	0.710	0.854	0.999	1.144	1.289	–	2.567	1.820	1.588	1.476	1.409	1.365	1.334	1.311	1.293
0.18	–	0.114	0.261	0.408	0.555	0.702	0.849	0.996	1.144	1.291	–	2.921	1.914	1.633	1.501	1.424	1.373	1.338	1.312	1.291
0.2	–	0.098	0.246	0.395	0.544	0.693	0.842	0.990	1.139	1.288	–	3.415	2.029	1.687	1.532	1.443	1.386	1.346	1.317	1.294
0.22	–	0.081	0.231	0.381	0.531	0.682	0.832	0.982	1.132	1.283	–	4.138	2.166	1.749	1.568	1.467	1.402	1.358	1.325	1.299
$k_{ins} = 0.05$ W/m K																				
0.01	0.032	0.073	0.115	0.157	0.198	0.240	0.282	0.323	0.365	0.407	5.263	4.545	4.348	4.255	4.202	4.167	4.142	4.124	4.110	4.098
0.02	0.047	0.113	0.180	0.247	0.313	0.380	0.447	0.513	0.580	0.647	3.571	2.941	2.778	2.703	2.660	2.632	2.612	2.597	2.586	2.577
0.04	0.055	0.150	0.246	0.341	0.436	0.531	0.627	0.722	0.817	0.912	3.017	2.215	2.035	1.955	1.910	1.882	1.862	1.847	1.836	1.827
0.06	0.051	0.162	0.273	0.384	0.496	0.607	0.718	0.829	0.940	1.051	3.261	2.055	1.829	1.734	1.682	1.648	1.625	1.609	1.596	1.586
0.08	0.041	0.162	0.284	0.405	0.526	0.647	0.768	0.890	1.011	1.132	4.044	2.052	1.763	1.647	1.584	1.545	1.518	1.499	1.484	1.472
0.1	0.028	0.156	0.285	0.413	0.541	0.669	0.797	0.926	1.054	1.182	5.909	2.131	1.757	1.615	1.540	1.494	1.463	1.440	1.423	1.410
0.12	0.013	0.147	0.280	0.413	0.547	0.680	0.813	0.947	1.080	1.213	12.500	2.273	1.786	1.613	1.524	1.471	1.434	1.408	1.389	1.374
0.14	–	0.135	0.272	0.409	0.546	0.684	0.821	0.958	1.095	1.233	–	2.478	1.840	1.630	1.525	1.463	1.421	1.392	1.369	1.352
0.16	–	0.121	0.261	0.401	0.542	0.682	0.822	0.963	1.103	1.244	–	2.762	1.915	1.661	1.538	1.466	1.419	1.385	1.360	1.340
0.18	–	0.106	0.249	0.391	0.534	0.677	0.820	0.963	1.106	1.249	–	3.153	2.011	1.703	1.560	1.477	1.423	1.385	1.357	1.335
0.2	–	0.090	0.235	0.380	0.525	0.670	0.814	0.959	1.104	1.249	–	3.710	2.130	1.756	1.588	1.494	1.432	1.390	1.358	1.334
0.22	–	0.073	0.220	0.367	0.513	0.660	0.807	0.953	1.100	1.247	–	4.545	2.273	1.818	1.623	1.515	1.446	1.399	1.364	1.337

Table S5. Specific savings and payback period at R_{wt} of 0.7 m² K/W

OIT (m)	f																			
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Specific savings (e_S, m)										Payback period (year)									
	k_{ins} = 0.02 W/m K																			
0.01	0.050	0.109	0.169	0.228	0.288	0.347	0.407	0.466	0.526	0.585	2.885	2.620	2.542	2.505	2.483	2.469	2.459	2.451	2.446	2.441
0.02	0.064	0.148	0.232	0.316	0.400	0.484	0.568	0.652	0.736	0.820	2.231	1.930	1.846	1.808	1.785	1.770	1.760	1.752	1.746	1.741
0.04	0.066	0.172	0.277	0.383	0.489	0.595	0.701	0.807	0.912	1.018	2.170	1.665	1.545	1.491	1.460	1.441	1.427	1.417	1.409	1.403
0.06	0.056	0.172	0.287	0.403	0.519	0.635	0.751	0.867	0.982	1.098	2.559	1.664	1.491	1.417	1.376	1.350	1.332	1.319	1.309	1.301
0.08	0.042	0.163	0.285	0.406	0.528	0.649	0.771	0.893	1.014	1.136	3.436	1.751	1.505	1.406	1.353	1.320	1.297	1.280	1.268	1.258
0.1	0.025	0.151	0.276	0.401	0.527	0.652	0.777	0.903	1.028	1.153	5.644	1.897	1.553	1.424	1.356	1.315	1.287	1.266	1.251	1.239
0.12	0.008	0.136	0.264	0.392	0.520	0.648	0.776	0.903	1.031	1.159	18.011	2.103	1.625	1.459	1.375	1.324	1.289	1.265	1.247	1.232
0.14	–	0.120	0.250	0.379	0.509	0.639	0.769	0.899	1.029	1.159	–	2.386	1.717	1.506	1.402	1.341	1.300	1.271	1.250	1.233
0.16	–	0.103	0.234	0.365	0.497	0.628	0.760	0.891	1.022	1.154	–	2.781	1.831	1.564	1.438	1.364	1.317	1.283	1.258	1.238
0.18	–	0.085	0.218	0.350	0.483	0.615	0.748	0.880	1.013	1.145	–	3.358	1.969	1.632	1.480	1.393	1.337	1.298	1.269	1.247
0.2	–	0.067	0.201	0.334	0.468	0.601	0.735	0.868	1.002	1.135	–	4.263	2.137	1.711	1.528	1.426	1.361	1.317	1.284	1.259
0.22	–	0.049	0.183	0.317	0.452	0.586	0.720	0.854	0.989	1.123	–	5.876	2.343	1.801	1.582	1.463	1.389	1.337	1.300	1.272
	k_{ins} = 0.03 W/m K																			
0.01	0.036	0.082	0.128	0.174	0.220	0.266	0.313	0.359	0.405	0.451	3.959	3.477	3.342	3.278	3.241	3.216	3.199	3.186	3.177	3.169
0.02	0.050	0.119	0.189	0.259	0.328	0.398	0.468	0.537	0.607	0.677	2.875	2.393	2.267	2.208	2.175	2.153	2.138	2.126	2.118	2.111
0.04	0.054	0.147	0.241	0.335	0.428	0.522	0.616	0.709	0.803	0.897	2.661	1.939	1.778	1.707	1.667	1.642	1.624	1.611	1.601	1.593
0.06	0.046	0.152	0.257	0.363	0.469	0.575	0.681	0.787	0.892	0.998	3.118	1.884	1.665	1.573	1.523	1.491	1.469	1.453	1.441	1.431
0.08	0.033	0.146	0.259	0.373	0.486	0.599	0.712	0.825	0.938	1.052	4.309	1.953	1.652	1.534	1.470	1.431	1.404	1.385	1.370	1.359
0.1	0.018	0.136	0.254	0.372	0.490	0.608	0.726	0.845	0.963	1.081	7.908	2.099	1.686	1.535	1.457	1.409	1.377	1.353	1.336	1.322
0.12	0.002	0.123	0.245	0.366	0.488	0.609	0.731	0.853	0.974	1.096	90.385	2.320	1.751	1.560	1.464	1.406	1.368	1.340	1.320	1.304
0.14	–	0.108	0.233	0.357	0.481	0.605	0.730	0.854	0.978	1.102	–	2.635	1.842	1.601	1.485	1.416	1.371	1.339	1.315	1.296
0.16	–	0.093	0.219	0.345	0.471	0.598	0.724	0.850	0.977	1.103	–	3.087	1.958	1.656	1.515	1.434	1.381	1.344	1.317	1.295
0.18	–	0.076	0.204	0.332	0.460	0.588	0.716	0.843	0.971	1.099	–	3.766	2.103	1.723	1.554	1.459	1.398	1.355	1.324	1.300
0.2	–	0.059	0.188	0.317	0.446	0.576	0.705	0.834	0.964	1.093	–	4.879	2.281	1.802	1.600	1.489	1.418	1.370	1.334	1.307
0.22	–	0.041	0.171	0.302	0.432	0.562	0.693	0.823	0.954	1.084	–	7.000	2.503	1.894	1.653	1.524	1.443	1.388	1.348	1.318
	k_{ins} = 0.04 W/m K																			
0.01	0.028	0.065	0.103	0.140	0.178	0.216	0.253	0.291	0.328	0.366	5.177	4.383	4.170	4.071	4.014	3.976	3.950	3.931	3.916	3.904
0.02	0.040	0.099	0.159	0.218	0.278	0.337	0.397	0.456	0.516	0.575	3.614	2.885	2.703	2.620	2.573	2.542	2.521	2.505	2.493	2.483
0.04	0.044	0.128	0.212	0.296	0.380	0.464	0.548	0.632	0.716	0.800	3.244	2.231	2.021	1.930	1.879	1.846	1.824	1.808	1.795	1.785
0.06	0.037	0.135	0.232	0.330	0.427	0.524	0.622	0.719	0.817	0.914	3.819	2.119	1.846	1.734	1.673	1.634	1.608	1.589	1.574	1.563
0.08	0.026	0.132	0.237	0.343	0.449	0.555	0.661	0.767	0.872	0.978	5.533	2.170	1.805	1.665	1.590	1.545	1.513	1.491	1.474	1.460
0.1	0.012	0.123	0.235	0.346	0.458	0.570	0.681	0.793	0.904	1.016	12.308	2.319	1.825	1.649	1.559	1.505	1.468	1.441	1.422	1.406
0.12	–	0.112	0.227	0.343	0.459	0.575	0.691	0.807	0.922	1.038	–	2.559	1.884	1.664	1.556	1.491	1.448	1.417	1.394	1.376
0.14	–	0.098	0.217	0.336	0.455	0.574	0.693	0.812	0.931	1.050	–	2.913	1.974	1.700	1.569	1.493	1.442	1.407	1.380	1.360
0.16	–	0.083	0.205	0.326	0.448	0.569	0.691	0.813	0.934	1.056	–	3.436	2.093	1.751	1.595	1.505	1.447	1.406	1.376	1.353
0.18	–	0.067	0.191	0.315	0.438	0.562	0.685	0.809	0.933	1.056	–	4.248	2.245	1.817	1.630	1.526	1.459	1.413	1.379	1.352
0.2	–	0.051	0.176	0.301	0.427	0.552	0.677	0.803	0.928	1.053	–	5.644	2.436	1.897	1.675	1.553	1.477	1.424	1.386	1.356
0.22	–	0.033	0.160	0.287	0.414	0.540	0.667	0.794	0.921	1.047	–	8.540	2.675	1.992	1.727	1.586	1.499	1.440	1.397	1.364
	k_{ins} = 0.05 W/m K																			
0.01	0.022	0.053	0.085	0.117	0.149	0.180	0.212	0.244	0.276	0.307	6.569	5.341	5.028	4.885	4.803	4.749	4.712	4.684	4.663	4.646
0.02	0.032	0.084	0.136	0.188	0.240	0.292	0.344	0.396	0.448	0.499	4.472	3.406	3.155	3.043	2.979	2.939	2.910	2.889	2.873	2.860
0.04	0.036	0.112	0.189	0.265	0.341	0.417	0.493	0.570	0.646	0.722	3.947	2.542	2.273	2.158	2.095	2.055	2.027	2.007	1.991	1.979
0.06	0.030	0.120	0.211	0.301	0.391	0.481	0.572	0.662	0.752	0.842	4.726	2.372	2.034	1.899	1.826	1.781	1.750	1.727	1.710	1.696
0.08	0.019	0.119	0.218	0.318	0.417	0.516	0.616	0.715	0.814	0.914	7.372	2.406	1.965	1.800	1.713	1.660	1.624	1.598	1.579	1.563
0.1	0.006	0.112	0.217	0.323	0.429	0.535	0.641	0.747	0.852	0.958	24.545	2.559	1.971	1.768	1.665	1.602	1.561	1.531	1.508	1.491
0.12	–	0.101	0.212	0.322	0.433	0.544	0.654	0.765	0.875	0.986	–	2.823	2.023	1.772	1.650	1.577	1.529	1.494	1.469	1.449
0.14	–	0.089	0.203	0.317	0.431	0.546	0.660	0.774	0.889	1.003	–	3.226	2.113	1.802	1.656	1.571	1.515	1.476	1.447	1.425
0.16	–	0.074	0.192	0.309	0.426	0.543	0.661	0.778	0.895	1.012	–	3.839	2.236	1.850	1.676	1.578	1.514	1.469	1.437	1.411
0.18	–	0.059	0.179	0.298	0.418	0.538	0.657	0.777	0.896	1.016	–	4.826	2.397	1.915	1.709	1.594	1.522	1.471	1.434	1.406
0.2	–	0.043	0.165	0.286	0.408	0.529	0.651	0.773	0.894	1.016	–	6.620	2.601	1.996	1.751	1.619	1.536	1.479	1.438	1.406
0.22	–	0.026	0.150	0.273	0.396	0.519	0.643	0.766	0.889	1.012	–	10.782	2.862	2.093	1.803	1.650	1.556	1.492	1.446	1.411

Table S6. Specific savings and payback period at R_{wt} of 0.8 m² K/W

OIT (m)	f										f									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Specific savings (e_S, m)										Payback period (year)									
	k_{ins} = 0.02 W/m K																			
0.01	0.038	0.086	0.134	0.182	0.230	0.278	0.327	0.375	0.423	0.471	3.283	2.902	2.794	2.743	2.713	2.693	2.680	2.669	2.662	2.655
0.02	0.049	0.119	0.188	0.258	0.327	0.397	0.466	0.536	0.605	0.674	2.528	2.103	1.991	1.940	1.910	1.891	1.877	1.867	1.860	1.853
0.04	0.049	0.139	0.228	0.317	0.406	0.496	0.585	0.674	0.764	0.853	2.536	1.804	1.646	1.577	1.538	1.513	1.496	1.483	1.473	1.466
0.06	0.039	0.137	0.236	0.335	0.433	0.532	0.631	0.729	0.828	0.927	3.231	1.820	1.589	1.494	1.442	1.409	1.387	1.371	1.358	1.349
0.08	0.024	0.128	0.233	0.337	0.441	0.545	0.649	0.753	0.858	0.962	5.172	1.948	1.613	1.485	1.418	1.376	1.348	1.327	1.312	1.300
0.1	0.008	0.116	0.223	0.331	0.439	0.547	0.654	0.762	0.870	0.978	16.111	2.164	1.680	1.510	1.424	1.372	1.337	1.312	1.293	1.279
0.12	–	0.101	0.211	0.321	0.431	0.542	0.652	0.762	0.873	0.983	–	2.485	1.778	1.557	1.449	1.384	1.342	1.312	1.289	1.272
0.14	–	0.084	0.197	0.309	0.421	0.533	0.645	0.757	0.870	0.982	–	2.964	1.908	1.620	1.485	1.407	1.356	1.320	1.294	1.273
0.16	–	0.067	0.181	0.295	0.408	0.522	0.635	0.749	0.863	0.976	–	3.716	2.073	1.698	1.531	1.437	1.377	1.335	1.304	1.280
0.18	–	0.050	0.164	0.279	0.394	0.509	0.624	0.738	0.853	0.968	–	5.041	2.281	1.791	1.586	1.474	1.403	1.354	1.319	1.291
0.2	–	0.031	0.147	0.263	0.379	0.494	0.610	0.726	0.842	0.957	–	7.941	2.547	1.901	1.650	1.517	1.434	1.378	1.337	1.306
0.22	–	0.013	0.130	0.246	0.363	0.479	0.596	0.712	0.829	0.945	–	19.156	2.894	2.032	1.724	1.565	1.469	1.404	1.358	1.322
	k_{ins} = 0.03 W/m K																			
0.01	0.027	0.064	0.100	0.137	0.174	0.211	0.247	0.284	0.321	0.358	4.670	3.935	3.739	3.648	3.596	3.561	3.537	3.520	3.506	3.495
0.02	0.037	0.094	0.150	0.207	0.264	0.321	0.378	0.435	0.491	0.548	3.395	2.670	2.492	2.412	2.367	2.337	2.316	2.301	2.290	2.280
0.04	0.038	0.116	0.194	0.273	0.351	0.429	0.507	0.585	0.663	0.741	3.279	2.151	1.929	1.835	1.783	1.749	1.726	1.709	1.697	1.686
0.06	0.029	0.119	0.208	0.297	0.386	0.476	0.565	0.654	0.744	0.833	4.268	2.108	1.804	1.683	1.617	1.577	1.549	1.528	1.513	1.501
0.08	0.016	0.112	0.208	0.305	0.401	0.497	0.593	0.689	0.785	0.882	7.738	2.226	1.799	1.641	1.560	1.509	1.475	1.451	1.432	1.418
0.1	0.001	0.102	0.202	0.303	0.404	0.505	0.606	0.706	0.807	0.908	155.000	2.460	1.853	1.649	1.547	1.486	1.445	1.416	1.394	1.377
0.12	–	0.088	0.193	0.297	0.401	0.505	0.609	0.713	0.818	0.922	–	2.830	1.948	1.685	1.559	1.485	1.436	1.402	1.376	1.356
0.14	–	0.073	0.180	0.287	0.394	0.500	0.607	0.714	0.820	0.927	–	3.405	2.082	1.743	1.588	1.499	1.442	1.401	1.371	1.348
0.16	–	0.057	0.166	0.275	0.383	0.492	0.601	0.710	0.818	0.927	–	4.356	2.258	1.820	1.630	1.524	1.456	1.409	1.375	1.348
0.18	–	0.041	0.151	0.261	0.371	0.482	0.592	0.702	0.813	0.923	–	6.159	2.485	1.914	1.683	1.557	1.478	1.424	1.384	1.354
0.2	–	0.023	0.135	0.246	0.358	0.470	0.581	0.693	0.804	0.916	–	10.769	2.781	2.029	1.746	1.597	1.505	1.443	1.398	1.365
0.22	–	0.005	0.118	0.231	0.344	0.456	0.569	0.682	0.794	0.907	–	46.212	3.175	2.166	1.819	1.644	1.538	1.467	1.416	1.378
	k_{ins} = 0.04 W/m K																			
0.01	0.020	0.050	0.079	0.109	0.139	0.169	0.198	0.228	0.258	0.288	6.325	5.048	4.730	4.585	4.503	4.449	4.412	4.384	4.363	4.346
0.02	0.028	0.076	0.124	0.172	0.220	0.268	0.317	0.365	0.413	0.461	4.452	3.283	3.019	2.902	2.836	2.794	2.764	2.743	2.726	2.713
0.04	0.029	0.099	0.168	0.238	0.307	0.377	0.446	0.516	0.585	0.654	4.245	2.528	2.228	2.103	2.034	1.991	1.961	1.940	1.923	1.910
0.06	0.022	0.103	0.185	0.266	0.348	0.429	0.511	0.592	0.674	0.755	5.808	2.426	2.032	1.879	1.798	1.748	1.713	1.689	1.670	1.655
0.08	0.009	0.099	0.188	0.277	0.366	0.456	0.545	0.634	0.724	0.813	13.462	2.536	1.996	1.804	1.706	1.646	1.606	1.577	1.555	1.538
0.1	–	0.089	0.184	0.279	0.373	0.468	0.563	0.658	0.752	0.847	–	2.797	2.037	1.793	1.673	1.602	1.555	1.521	1.495	1.476
0.12	–	0.077	0.176	0.275	0.373	0.472	0.571	0.669	0.768	0.867	–	3.231	2.130	1.820	1.674	1.589	1.533	1.494	1.465	1.442
0.14	–	0.063	0.165	0.267	0.369	0.470	0.572	0.674	0.776	0.877	–	3.938	2.270	1.873	1.695	1.594	1.529	1.484	1.450	1.425
0.16	–	0.048	0.153	0.257	0.361	0.465	0.569	0.673	0.778	0.882	–	5.172	2.459	1.948	1.732	1.613	1.537	1.485	1.447	1.418
0.18	–	0.032	0.138	0.245	0.351	0.457	0.563	0.669	0.775	0.881	–	7.749	2.710	2.045	1.782	1.642	1.554	1.495	1.451	1.418
0.2	–	0.016	0.123	0.231	0.339	0.447	0.554	0.662	0.770	0.878	–	16.111	3.042	2.164	1.845	1.680	1.579	1.510	1.461	1.424
0.22	–	-0.002	0.107	0.217	0.326	0.435	0.544	0.653	0.762	0.871	–	–	3.492	2.309	1.919	1.725	1.609	1.531	1.476	1.435
	k_{ins} = 0.05 W/m K																			
0.01	0.015	0.040	0.065	0.090	0.115	0.140	0.165	0.190	0.215	0.240	8.333	6.250	5.769	5.556	5.435	5.357	5.303	5.263	5.233	5.208
0.02	0.022	0.063	0.105	0.147	0.188	0.230	0.272	0.313	0.355	0.397	5.769	3.947	3.571	3.409	3.319	3.261	3.221	3.191	3.169	3.151
0.04	0.023	0.085	0.148	0.210	0.273	0.335	0.398	0.460	0.523	0.585	5.556	2.941	2.542	2.381	2.294	2.239	2.201	2.174	2.153	2.137
0.06	0.015	0.090	0.165	0.240	0.315	0.390	0.465	0.540	0.615	0.690	8.333	2.778	2.273	2.083	1.984	1.923	1.882	1.852	1.829	1.812
0.08	0.003	0.087	0.170	0.253	0.337	0.420	0.503	0.587	0.670	0.753	37.500	2.885	2.206	1.974	1.856	1.786	1.738	1.705	1.679	1.659
0.1	–	0.079	0.168	0.257	0.346	0.436	0.525	0.614	0.704	0.793	–	3.182	2.234	1.944	1.804	1.721	1.667	1.628	1.599	1.577
0.12	–	0.068	0.161	0.255	0.349	0.443	0.536	0.630	0.724	0.818	–	3.704	2.326	1.961	1.792	1.695	1.632	1.587	1.554	1.529
0.14	–	0.054	0.152	0.249	0.346	0.443	0.541	0.638	0.735	0.832	–	4.592	2.473	2.009	1.806	1.692	1.619	1.568	1.531	1.502
0.16	–	0.040	0.140	0.240	0.340	0.440	0.540	0.640	0.740	0.840	–	6.250	2.679	2.083	1.838	1.705	1.620	1.563	1.520	1.488
0.18	–	0.025	0.127	0.229	0.331	0.434	0.536	0.638	0.740	0.843	–	10.185	2.957	2.183	1.886	1.730	1.633	1.567	1.519	1.483
0.2	–	0.008	0.113	0.217	0.321	0.425	0.529	0.633	0.738	0.842	–	30.000	3.333	2.308	1.948	1.765	1.654	1.579	1.525	1.485
0.22	–	–	0.097	0.203	0.309	0.415	0.520	0.626	0.732	0.838	–	–	3.854	2.462	2.024	1.809	1.681	1.597	1.537	1.492