Turk. J. Math. Comput. Sci.
16(1)(2024) 162–168

DOI : 10.47000/tjmcs.1338657

# Coefficient Bounds for the General Subclasses of Close-to-Convex Functions of Complex Order

Serap Bulut

*Kocaeli University, Faculty of Aviation and Space Sciences, Arslanbey Campus, 41285 Kartepe-Kocaeli, Türkiye.*

Received: 06-08-2023 • Accepted: 28-03-2024

Abstract. In this study, we introduce two new subclasses of close-to-convex functions of complex order, which are introduced here by means of a certain non-homogenous Cauchy-Euler-type differential equation of order $m$, and determine the coefficient bounds for functions belonging to these new classes.

*2020 AMS Classification:* 30C45, 30C50

**Keywords:** Analytic function, close-to-convex function, non-homogenous Cauchy-Euler differential equation.

## 1. Introduction, Definitions and Preliminaries

Let $\mathbb{R} = (-\infty, \infty)$ be the set of real numbers, $\mathbb{C} := \mathbb{C}^* \cup \{0\}$ be the set of complex numbers,

$$\mathbb{N} := \{1, 2, 3, \ldots\} = \mathbb{N}_0 \setminus \{0\}$$

be the set of positive integers and

$$\mathbb{N}^* := \mathbb{N} \setminus \{1\} = \{2, 3, 4, \ldots\}.$$

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{i=2}^{\infty} a_i z^i \tag{1.1}$$

which are analytic in the open unit disc

$$\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

Faisal and Darus [5] defined the following differential operator:

$$\begin{aligned}
D^0 f(z) &= f(z), \\
D_\lambda^1(\alpha,\beta,\mu) f(z) &= \left(\frac{\alpha-\mu+\beta-\lambda}{\alpha+\beta}\right) f(z) + \left(\frac{\mu+\lambda}{\alpha+\beta}\right) z f'(z), \\
D_\lambda^2(\alpha,\beta,\mu) f(z) &= D\left(D_\lambda^1(\alpha,\beta,\mu) f(z)\right) \\
&\vdots \\
D_\lambda^n(\alpha,\beta,\mu) f(z) &= D\left(D_\lambda^{n-1}(\alpha,\beta,\mu) f(z)\right).
\end{aligned} \tag{1.2}$$

Email address: serap.bulut@kocaeli.edu.tr (S. Bulut)

If $f$ is given by (1.1), then it is easily seen from (1.2) that

$$D_\lambda^n(\alpha,\beta,\mu)\,f(z) = z + \sum_{i=2}^{\infty}\left(\frac{\alpha+(\mu+\lambda)(i-1)+\beta}{\alpha+\beta}\right)^n a_i z^i$$

$$(f\in\mathcal{A};\ \alpha,\beta,\mu,\lambda\geq 0;\ \alpha+\beta\neq 0;\ n\in\mathbb{N}_0).$$

In the light of the work of Xu *et al.* [16], Bulut [3] introduced the subclasses

$$\mathcal{M}_g(n,\alpha,\beta,\mu,\lambda,\zeta,\xi) \quad\text{and}\quad \mathcal{M}_g(n,\alpha,\beta,\mu,\lambda,\zeta,\xi;m,\tau)$$

of analytic functions of complex order $\xi\in\mathbb{C}^*$, and obtained the coefficient bounds for the Taylor-Maclaurin coefficients for functions in each of the above subclasses, which is given by Definition 1.1 and Definition 1.2.

**Definition 1.1** ( [3]). Let $\varphi:\mathbb{U}\to\mathbb{C}$ be a convex function such that

$$\varphi(0)=1 \qquad\text{and}\qquad \Re\{\varphi(z)\}>0 \quad (z\in\mathbb{U}).$$

We denote by $\mathcal{M}_\varphi(n,\alpha,\beta,\mu,\lambda,\zeta,\xi)$ the class of functions $f\in\mathcal{A}$ satisfying

$$1+\frac{1}{\xi}\left(\frac{z\left[\zeta D_\lambda^{n+1}(\alpha,\beta,\mu)\,f(z)+(1-\zeta)\,D_\lambda^n(\alpha,\beta,\mu)\,f(z)\right]'}{\zeta D_\lambda^{n+1}(\alpha,\beta,\mu)\,f(z)+(1-\zeta)\,D_\lambda^n(\alpha,\beta,\mu)\,f(z)}-1\right)\in\varphi(\mathbb{U}),$$

where $z\in\mathbb{U};\ 0\leq\zeta\leq 1;\ \xi\in\mathbb{C}^*$.

**Definition 1.2** ( [3]). A function $f\in\mathcal{A}$ is said to be in the class $\mathcal{M}_\varphi(n,\alpha,\beta,\mu,\lambda,\zeta,\xi;m,\tau)$ if it satisfies the following non-homogenous Cauchy-Euler differential equation:

$$z^m\frac{d^mw}{dz^m}+\binom{m}{1}(\tau+m-1)\,z^{m-1}\frac{d^{m-1}w}{dz^{m-1}}+\cdots+\binom{m}{m}w\prod_{j=0}^{m-1}(\tau+j)=q(z)\prod_{j=0}^{m-1}(\tau+j+1)$$

$$\left(w=f(z)\in\mathcal{A};\ q\in\mathcal{M}_\varphi(n,\alpha,\beta,\mu,\lambda,\zeta,\xi);\ m\in\mathbb{N}^*;\ \tau\in(-1,\infty)\right).$$

Making use of Definition 1.1 and Definition 1.2, Bulut [3] proved the following coefficient bounds for the Taylor-Maclaurin coefficients for functions in the subclasses

$$\mathcal{M}_g(n,\alpha,\beta,\mu,\lambda,\zeta,\xi) \quad\text{and}\quad \mathcal{M}_g(n,\alpha,\beta,\mu,\lambda,\zeta,\xi;m,\tau)$$

of analytic functions of complex order $\xi\in\mathbb{C}^*$.

**Theorem 1.3** ( [3]). *Let the function $f\in\mathcal{A}$ be defined by (1.1). If*

$$f\in\mathcal{M}_\varphi(n,\alpha,\beta,\mu,\lambda,\zeta,\xi),$$

*then*

$$|a_i|\leq\frac{(\alpha+\beta)^{n+1}\prod_{j=0}^{i-2}\left[j+|\xi|\,|\varphi'(0)|\right]}{(i-1)!\,\left[\alpha+\zeta(\mu+\lambda)(i-1)+\beta\right]\left[\alpha+(\mu+\lambda)(i-1)+\beta\right]^n}\qquad(i\in\mathbb{N}^*).$$

**Theorem 1.4** ( [3]). *Let the function $f\in\mathcal{A}$ be defined by (1.1). If*

$$f\in\mathcal{M}_\varphi(n,\alpha,\beta,\mu,\lambda,\zeta,\xi;m,\tau),$$

*then*

$$|a_i|\leq\frac{(\alpha+\beta)^{n+1}\prod_{j=0}^{i-2}\left[j+|\xi|\,|\varphi'(0)|\right]\prod_{j=0}^{m-1}(\tau+j+1)}{(i-1)!\,\left[\alpha+\zeta(\mu+\lambda)(i-1)+\beta\right]\left[\alpha+(\mu+\lambda)(i-1)+\beta\right]^n\prod_{j=0}^{m-1}(\tau+j+i)}\qquad(i\in\mathbb{N}^*).$$

Here, in our present sequel to some of the aforecited work of Bulut [3], we first introduce the following subclasses of analytic functions of complex order $\xi\in\mathbb{C}^*$.

**Definition 1.5.** Let $\varphi : \mathbb{U} \to \mathbb{C}$ be a convex function such that $\varphi(0) = 1$ and $\Re\{\varphi(z)\} > 0\,(z \in \mathbb{U})$. We denote by $\mathcal{MQ}_{\varphi}^{n,\alpha,\beta,\mu,\lambda}(\zeta,\xi,\delta,\gamma)$ the class of functions $f \in \mathcal{A}$ satisfying

$$1 + \frac{1}{\xi}\left(\frac{z\left[\zeta D_{\lambda}^{n+1}(\alpha,\beta,\mu)\,f(z) + (1-\zeta)\,D_{\lambda}^{n}(\alpha,\beta,\mu)\,f(z)\right]'}{\zeta D_{\lambda}^{n+1}(\alpha,\beta,\mu)\,g(z) + (1-\zeta)\,D_{\lambda}^{n}(\alpha,\beta,\mu)\,g(z)} - 1\right) \in \varphi(\mathbb{U}) \qquad (z \in \mathbb{U}),$$

where $g \in \mathcal{M}_{\varphi}(n,\alpha,\beta,\mu,\lambda,\delta,\gamma)\,;\ 0 \le \zeta,\delta \le 1;\ \xi,\gamma \in \mathbb{C}^{*}$.

**Definition 1.6.** A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{KQ}_{\varphi}^{n,\alpha,\beta,\mu,\lambda}(\zeta,\xi,\delta,\gamma;m,\tau)$ if it satisfies the following non-homogenous Cauchy-Euler differential equation of order $m$ :

$$z^{m}\frac{d^{m}w}{dz^{m}} + \binom{m}{1}(\tau+m-1)\,z^{m-1}\frac{d^{m-1}w}{dz^{m-1}} + \cdots + \binom{m}{m}w\prod_{j=0}^{m-1}(\tau+j) = q(z)\prod_{j=0}^{m-1}(\tau+j+1)$$

$$\left(w = f(z) \in \mathcal{A};\ q \in \mathcal{MQ}_{\varphi}^{n,\alpha,\beta,\mu,\lambda}(\zeta,\xi,\delta,\gamma)\,;\ m \in \mathbb{N}^{*};\ \tau \in (-1,\infty)\right).$$

**Remark 1.7.** If we let $n = 0$ and $\mu + \lambda = \alpha + \beta \neq 0$ in Definition 1.5 and Definition 1.6, then we have the classes

$$\mathcal{MQ}_{\varphi}^{0,\alpha,\beta,\mu,\lambda}(\zeta,\xi,\delta,\gamma) = \mathcal{SQ}_{\varphi}(\zeta,\xi,\delta,\gamma)$$

and

$$\mathcal{KQ}_{\varphi}^{0,\alpha,\beta,\mu,\lambda}(\zeta,\xi,\delta,\gamma;m,\tau) = \mathcal{KQ}_{\varphi}(\zeta,\xi,\delta,\gamma;m,\tau)\,,$$

respectively, introduced and studied by Bulut [4].

Similar interesting results can be found into the work of Altıntaş *et al.* [1], Nasr and Aouf [7], Robertson [9], Srivastava *et al.* [11] and Ul-Haq *et al.* [13, 14], (see also [2, 6, 8, 12, 15]).

In this paper, by using the subordination principle between analytic functions, we obtain coefficient bounds for the Taylor-Maclaurin coefficients for functions in the substantially more general function classes

$$\mathcal{MQ}_{\varphi}^{n,\alpha,\beta,\mu,\lambda}(\zeta,\xi,\delta,\gamma) \quad \text{and} \quad \mathcal{KQ}_{\varphi}^{n,\alpha,\beta,\mu,\lambda}(\zeta,\xi,\delta,\gamma;m,\tau)$$

of analytic functions of complex order $\xi \in \mathbb{C}^{*}$, which we have introduced here.

## 2. Main Results and Their Demonstration

In our investigation, we shall make use of the principle of subordination between analytic functions, which is explained in Definition 2.1 below.

**Definition 2.1.** For two functions $f$ and $g$, analytic in $\mathbb{U}$, we say that the function $f$ is subordinate to $g$ in $\mathbb{U}$, and write

$$f(z) \prec g(z) \qquad (z \in \mathbb{U}),$$

if there exists a Schwarz function $\omega$, analytic in $\mathbb{U}$, with

$$\omega(0) = 0 \qquad \text{and} \qquad |\omega(z)| < 1 \qquad (z \in \mathbb{U})$$

such that

$$f(z) = g(\omega(z)) \qquad (z \in \mathbb{U}).$$

Indeed, it is known that

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \Rightarrow f(0) = g(0) \qquad \text{and} \qquad f(\mathbb{U}) \subset g(\mathbb{U}).$$

Furthermore, if the function $g$ is univalent in $\mathbb{U}$, then we have the following equivalence

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \Leftrightarrow f(0) = g(0) \qquad \text{and} \qquad f(\mathbb{U}) \subset g(\mathbb{U}).$$

In order to prove our main results (Theorems 2.3 and 2.4 below), we first recall the following lemma due to Rogosinski [10].

**Lemma 2.2.** *Let the function* $\mathfrak{g}$ *given by*

$$\mathfrak{g}(z) = \sum_{k=1}^{\infty} \mathfrak{b}_k z^k \qquad (z \in \mathbb{U})$$

*be convex in* $\mathbb{U}$. *Also let the function* $\mathfrak{f}$ *given by*

$$\mathfrak{f}(z) = \sum_{k=1}^{\infty} \mathfrak{a}_k z^k \qquad (z \in \mathbb{U})$$

*be holomorphic in* $\mathbb{U}$. *If*

$$\mathfrak{f}(z) \prec \mathfrak{g}(z) \qquad (z \in \mathbb{U}),$$

*then*

$$|\mathfrak{a}_k| \leq |\mathfrak{b}_1| \qquad (k \in \mathbb{N}).$$

We now state and prove each of our main results given by Theorems 2.3 and 2.4 below.

**Theorem 2.3.** *Let the function* $f \in \mathcal{A}$ *be defined by* (1.1). *If*

$$f \in \mathcal{MQ}_{\varphi}^{n,\alpha,\beta,\mu,\lambda}(\zeta,\xi,\delta,\gamma),$$

*then*

$$\begin{aligned} |a_i| \quad \leq \quad & \frac{(\alpha+\beta)^{n+1} \prod_{j=0}^{i-2} [j + |\gamma|\, |\varphi'(0)|]}{i!\, [\alpha + \delta(\mu+\lambda)(i-1)+\beta]\, [\alpha + (\mu+\lambda)(i-1)+\beta]^n} \\ & + \frac{(\alpha+\beta)^{n+1}\, |\xi|\, |\varphi'(0)|}{i\, [\alpha + \zeta(\mu+\lambda)(i-1)+\beta]\, [\alpha + (\mu+\lambda)(i-1)+\beta]^n} \\ & \times \left(1 + \sum_{j=1}^{i-2} \frac{[\alpha + \zeta(\mu+\lambda)(i-j-1)+\beta] \prod_{k=0}^{i-j-2} [j + |\gamma|\, |\varphi'(0)|]}{(i-j-1)!\, [\alpha + \delta(\mu+\lambda)(i-j-1)+\beta]}\right) \qquad (i \in \mathbb{N}^*), \end{aligned}$$

$$\left(g \in \mathcal{M}_{\varphi}(n,\alpha,\beta,\mu,\lambda,\delta,\gamma);\ 0 \leq \zeta,\delta \leq 1;\ \xi,\gamma \in \mathbb{C}^*\right).$$

*Proof.* Let the function $f \in \mathcal{MQ}_{\varphi}^{n,\alpha,\beta,\mu,\lambda}(\zeta,\xi,\delta,\gamma)$ be of the form (1.1). Therefore, there exists a function

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{M}_{\varphi}(n,\alpha,\beta,\mu,\lambda,\delta,\gamma)$$

so that

$$1 + \frac{1}{\xi}\left(\frac{z\left[\zeta D_{\lambda}^{n+1}(\alpha,\beta,\mu) f(z) + (1-\zeta) D_{\lambda}^{n}(\alpha,\beta,\mu) f(z)\right]'}{\zeta D_{\lambda}^{n+1}(\alpha,\beta,\mu) g(z) + (1-\zeta) D_{\lambda}^{n}(\alpha,\beta,\mu) g(z)} - 1\right) \in \varphi(\mathbb{U}). \tag{2.1}$$

Note that, by Theorem 1.3, we have

$$|b_i| \leq \frac{\prod_{j=0}^{i-2} [j + |\gamma|\, |\varphi'(0)|]}{(i-1)!\, \chi_i(\delta)} \qquad (i \in \mathbb{N}^*), \tag{2.2}$$

where

$$\chi_i(\delta) := [\alpha + \delta(\mu+\lambda)(i-1)+\beta] \frac{[\alpha + (\mu+\lambda)(i-1)+\beta]^n}{(\alpha+\beta)^{n+1}}.$$

Let

$$F(z) = \zeta D_{\lambda}^{n+1}(\alpha,\beta,\mu) f(z) + (1-\zeta) D_{\lambda}^{n}(\alpha,\beta,\mu) f(z) = z + \sum_{i=2}^{\infty} A_i z^i, \tag{2.3}$$

$$G(z) = \zeta D_{\lambda}^{n+1}(\alpha,\beta,\mu) g(z) + (1-\zeta) D_{\lambda}^{n}(\alpha,\beta,\mu) g(z) = z + \sum_{i=2}^{\infty} B_i z^i, \tag{2.4}$$

where

$$A_i := \chi_i(\zeta) a_i \qquad \text{and} \qquad B_i := \chi_i(\zeta) b_i,$$

with

$$\chi_i(\zeta) := [\alpha + \zeta(\mu+\lambda)(i-1)+\beta] \frac{[\alpha+(\mu+\lambda)(i-1)+\beta]^n}{(\alpha+\beta)^{n+1}}.$$

Then, (2.1) is of the form

$$1 + \frac{1}{\xi}\left(\frac{zF'(z)}{G(z)} - 1\right) \in \varphi(\mathbb{U}).$$

Let us define the function $p(z)$ by

$$p(z) = 1 + \frac{1}{\xi}\left(\frac{zF'(z)}{G(z)} - 1\right) \qquad (z \in \mathbb{U}). \tag{2.5}$$

Therefore, we deduce that

$$p(0) = \varphi(0) = 1 \qquad \text{and} \qquad p(z) \in \varphi(\mathbb{U}) \qquad (z \in \mathbb{U}).$$

So, we have

$$p(z) \prec \varphi(z) \qquad (z \in \mathbb{U}).$$

Hence, by Lemma 2.2, we obtain

$$\left|\frac{p^{(m)}(0)}{m!}\right| = |c_m| \leq |\varphi'(0)| \qquad (m \in \mathbb{N}), \tag{2.6}$$

where

$$p(z) = 1 + c_1 z + c_2 z^2 + \cdots \qquad (z \in \mathbb{U}).$$

Also from (2.5), we find

$$zF'(z) - G(z) = \xi(p(z)-1)G(z). \tag{2.7}$$

Since $A_1 = B_1 = 1$, in view of (2.7), we obtain

$$iA_i - B_i = \xi\{c_{i-1} + c_{i-2}B_2 + \cdots + c_1 B_{i-1}\} = \xi\left(c_{i-1} + \sum_{j=1}^{i-2} c_j B_{i-j}\right) \quad (i \in \mathbb{N}^*). \tag{2.8}$$

Now, we get from (2.2), (2.3), (2.4), (2.6) and (2.8),

$$|a_i| \leq \frac{\prod_{j=0}^{i-2}[j + |\gamma|\,|\varphi'(0)|]}{i!\,\chi_i(\delta)} + \frac{|\xi|\,|\varphi'(0)|}{i\,\chi_i(\zeta)}\left(1 + \sum_{j=1}^{i-2}\frac{\chi_{i-j}(\zeta)\prod_{k=0}^{i-j-2}[j+|\gamma|\,|\varphi'(0)|]}{(i-j-1)!\,\chi_{i-j}(\delta)}\right) \quad (i \in \mathbb{N}^*).$$

This evidently completes the proof of Theorem 2.3. □

**Theorem 2.4.** *Let the function $f \in \mathcal{A}$ be defined by (1.1). If*

$$f \in \mathcal{KQ}_{\varphi}^{n,\alpha,\beta,\mu,\lambda}(\zeta,\xi,\delta,\gamma;m,\tau),$$

*then*

$$|a_i| \leq \left\{ \frac{(\alpha+\beta)^{n+1} \prod_{j=0}^{i-2} [j+|\gamma|\;|\varphi'(0)|]}{i!\;[\alpha+\delta(\mu+\lambda)(i-1)+\beta]\;[\alpha+(\mu+\lambda)(i-1)+\beta]^n} \right.$$
$$+\frac{(\alpha+\beta)^{n+1}\;|\xi|\;|\varphi'(0)|}{i\;[\alpha+\zeta(\mu+\lambda)(i-1)+\beta]\,[\alpha+(\mu+\lambda)(i-1)+\beta]^n}$$
$$\left.\times\left(1+\sum_{j=1}^{i-2}\frac{[\alpha+\zeta(\mu+\lambda)(i-j-1)+\beta]\prod_{k=0}^{i-j-2}[j+|\gamma|\;|\varphi'(0)|]}{(i-j-1)!\;[\alpha+\delta(\mu+\lambda)(i-j-1)+\beta]}\right)\right\}\times\frac{\prod_{j=0}^{m-1}(\tau+j+1)}{\prod_{j=0}^{m-1}(\tau+j+i)} \quad (i\in\mathbb{N}^*), \tag{2.9}$$
$$(0\leq\zeta,\delta\leq 1;\;\xi,\gamma\in\mathbb{C}^*;\;m\in\mathbb{N}^*;\;\tau\in(-1,\infty)).$$

*Proof.* Let the function $f \in \mathcal{A}$ be given by (1.1). Also, let

$$q(z) = z + \sum_{i=2}^{\infty} q_i\, z^i \in \mathcal{MQ}_{\varphi}^{n,\alpha,\beta,\mu,\lambda}(\zeta,\xi,\delta,\gamma).$$

We then deduce from Definition 1.6 that

$$a_i = \frac{\prod_{j=0}^{m-1}(\tau+j+1)}{\prod_{j=0}^{m-1}(\tau+j+i)} q_i \qquad (i\in\mathbb{N}^*,\;\tau\in(-1,\infty)).$$

Thus, by using Theorem 2.3 in conjunction with the above equality, we have assertion (2.9) of Theorem 2.4. This completes the proof of Theorem 2.4. □

**Remark 2.5.** If we let $n = 0$ and $\mu+\lambda = \alpha+\beta \neq 0$ in Theorem 2.3 and Theorem 2.4, then we get Theorem 1.3 and Theorem 1.4, respectively.

## 3. Conclusion

In this study, for functions of the form

$$f(z) = z + \sum_{i=2}^{\infty} a_i\, z^i \in \mathcal{A} \qquad (z\in\mathbb{U}),$$

we consider the subclass $\mathcal{M}_{\varphi}(n,\alpha,\beta,\mu,\lambda,\zeta,\xi)$ defined by means of the differential operator

$$D_{\lambda}^{n}(\alpha,\beta,\mu)\,f(z) = z + \sum_{i=2}^{\infty}\left(\frac{\alpha+(\mu+\lambda)(i-1)+\beta}{\alpha+\beta}\right)^{n} a_i\, z^i$$
$$(\alpha,\beta,\mu,\lambda\geq 0;\;\alpha+\beta\neq 0;\;n\in\mathbb{N}_0),$$

as follows:

$$\mathcal{M}_{\varphi}(n,\alpha,\beta,\mu,\lambda,\zeta,\xi) = \left\{ f\in\mathcal{A} : 1+\frac{1}{\xi}\left(\frac{z\left[\zeta D_{\lambda}^{n+1}(\alpha,\beta,\mu)\,f(z)+(1-\zeta)\,D_{\lambda}^{n}(\alpha,\beta,\mu)\,f(z)\right]'}{\zeta D_{\lambda}^{n+1}(\alpha,\beta,\mu)\,f(z)+(1-\zeta)\,D_{\lambda}^{n}(\alpha,\beta,\mu)\,f(z)}-1\right)\in\varphi(\mathbb{U})\right\},$$

where $\varphi : \mathbb{U}\to\mathbb{C}$ is a convex function such that

$$\varphi(0) = 1 \qquad \text{and} \qquad \Re\{\varphi(z)\} > 0 \quad (z\in\mathbb{U}).$$

By means of this class, we introduce following subclasses:

$$\mathcal{MQ}_{\varphi}^{n,\alpha,\beta,\mu,\lambda}(\zeta,\xi,\delta,\gamma) = \left\{ f\in\mathcal{A} : 1+\frac{1}{\xi}\left(\frac{z\left[\zeta D_{\lambda}^{n+1}(\alpha,\beta,\mu)\,f(z)+(1-\zeta)\,D_{\lambda}^{n}(\alpha,\beta,\mu)\,f(z)\right]'}{\zeta D_{\lambda}^{n+1}(\alpha,\beta,\mu)\,g(z)+(1-\zeta)\,D_{\lambda}^{n}(\alpha,\beta,\mu)\,g(z)}-1\right)\in\varphi(\mathbb{U})\right\},$$

where $z \in \mathbb{U}$; $g \in \mathcal{M}_\varphi(n, \alpha, \beta, \mu, \lambda, \zeta, \xi)$; $0 \leq \zeta, \delta \leq 1$; $\xi, \gamma \in \mathbb{C}^*$;

$$\mathcal{KQ}_\varphi^{n,\alpha,\beta,\mu,\lambda}(\zeta, \xi, \delta, \gamma; m, \tau) = \left\{ f \in \mathcal{A} : z^m \frac{d^m f(z)}{dz^m} + \cdots + \binom{m}{m} f(z) \prod_{j=0}^{m-1} (\tau + j) = q(z) \prod_{j=0}^{m-1} (\tau + j + 1) \right\},$$

where $z \in \mathbb{U}$; $q \in \mathcal{MQ}_\varphi^{n,\alpha,\beta,\mu,\lambda}(\zeta, \xi, \delta, \gamma)$; $m \in \mathbb{N}^*$; $\tau \in (-1, \infty)$.

For functions $f$ belong to the classes

$$\mathcal{MQ}_\varphi^{n,\alpha,\beta,\mu,\lambda}(\zeta, \xi, \delta, \gamma) \quad \text{and} \quad \mathcal{KQ}_\varphi^{n,\alpha,\beta,\mu,\lambda}(\zeta, \xi, \delta, \gamma; m, \tau),$$

we investigate upper bounds for the general coefficient $|a_n|$, respectively.

## Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this article.

## Authors Contribution Statement

The author has read and agreed the published version of the manuscript.

## References


[1] Altıntaş, O., Irmak, H., Owa, S., Srivastava, H.M., *Coefficient bounds for some families of starlike and convex functions of complex order*, Appl. Math. Lett., **20**(2007), 1218–1222.

[2] Altıntaş, O., Özkan, Ö., Srivastava, H.M., *Majorization by starlike functions of complex order*, Complex Variables Theory Appl., **46**(3)(2001), 207–218.

[3] Bulut, S., *Coefficient bounds for certain subclasses of analytic functions of complex order*, Hacet. J. Math. Stat., **45**(4)(2016), 1015–1022.

[4] Bulut, S., *Coefficient bounds for certain subclasses of close-to-convex functions of complex order*, Filomat, **31**(20)(2017), 6401–6408.

[5] Faisal, I., Darus, M., *Application of nonhomogenous Cauchy-Euler differential equation for certain class of analytic functions*, Hacet. J. Math. Stat., **43**(3)(2014), 375–382.

[6] Murugusundaramoorthy, G., Srivastava, H.M., *Neighborhoods of certain classes of analytic functions of complex order*, J. Inequal. Pure Appl. Math., **5**(2)(2004), 1–8.

[7] Nasr, M.A., Aouf, M.K., *Radius of convexity for the class of starlike functions of complex order*, Bull. Fac. Sci. Assiut Univ. A, **12**(1)(1983), 153–159.

[8] Orhan, H., Răducanu, D., Çağlar, M., Bayram, M., *Coefficient estimates and other properties for a class of spirallike functions associated with a differential operator*, Abstr. Appl. Anal., (2013).

[9] Robertson, M.S., *On the theory of univalent functions*, Ann. Math. (2), **37**(2)(1936), 374–408.

[10] Rogosinski, W., *On the coefficients of subordinate functions*, Proc. London Math. Soc. (Ser. 2), **48**(1943), 48–82.

[11] Srivastava, H.M., Altıntaş, O., Kırcı Serenbay, S., *Coefficient bounds for certain subclasses of starlike functions of complex order*, Appl. Math. Lett., **24**(2011), 1359–1363.

[12] Srivastava, H.M., Xu, Q.-H., Wu, G.-P., *Coefficient estimates for certain subclasses of spiral-like functions of complex order*, Appl. Math. Lett., **23**(2010), 763–768.

[13] Ul-Haq, W., Nazneen, A., Rehman, N., *Coefficient estimates for certain subfamilies of close-to-convex functions of complex order*, Filomat, **28**(6)(2014), 1139–1142.

[14] Ul-Haq, W., Nazneen, A., Arif, M.,Rehman, N., *Coefficient bounds for certain subclasses of close-to-convex functions of Janowski type*, J. Comput. Anal. Appl., **16**(1)(2014), 133–138.

[15] Xu, Q.-X., Cai, Q.-M., Srivastava, H.M., *Sharp coefficient estimates for certain subclasses of starlike functions of complex order*, Appl. Math. Comput., **225**(2013), 43–49.

[16] Xu, Q.-H., Gui, Y.-C., Srivastava, H.M., *Coefficient estimates for certain subclasses of analytic functions of complex order*, Taiwanese J. Math., **15**(5)(2011), 2377–2386.