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INVESTIGATING TEACHING PRACTICES FOR ALGEBRAIC EXPRESSIONS WITHIN A MULTIPLE CASE STUDY

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Abstract: As effective teaching practices support students' learning; it is essential to investigate teachers' teaching practices. It can also shed light on the causes of students' difficulties. In this regard, proposing teaching practices might be important as the practices can give insight about what is going on in teaching process. Thus, the aim of this study is to extract the practices by focusing on the teachers' actions during the teaching of operations with algebraic expressions. Mathematical Knowledge for Teaching (MKT) model which is a practice-based approach to the knowledge concept is used as a framework in this study. Thus, this study focuses on teaching practices of operations with algebraic expressions (addition, subtraction, and multiplication). Data were collected from two middle school mathematics teachers' instructions. The two teachers were the cases in this qualitative research and the data were analyzed by compare and contrast method. The practices that are found in the study are: defining the concept of term, like term, constant term, and coefficient; using equal sign; using analogies to explain addition and subtraction; using algebra tiles; providing mathematical explanations for distributive property; and noticing students' misconceptions in simplification and equivalence of algebraic expression. The results suggest that the conceptual teacher knowledge influences teaching practices positively and teacher knowledge should be used in appropriate pedagogical organization to develop teaching practices.

Keywords: Algebraic expression, middle school mathematics teacher, teaching practice

Introduction

One of the factors that have an influence on students' learning is teaching practices (Saxe, Gearhart, & Seltzer, 1999) as the examination of teaching practices can shed light on the causes of students' difficulties. Practices are defined as "core activities that could and should occur regularly in the teaching of mathematics" (Franke, Kazemi & Battey, 2007, p. 249). In this regard, proposing teaching practices might also be different and significant as Charalambous (2016) stated. Teaching practices can give insight about how teachers use their knowledge in teaching process and it is meaningful to understand how students learn. Thus, the aim of this study is to extract the practices by focusing on the teachers' actions throughout the instruction of operations with algebraic expressions.

The Significance of the Study

As Wilkie (2014) indicates, the studies about teaching practice for algebra was scarce; thus this study is important for filling the void about practicing teacher algebra knowledge in classroom research. Tirosh, Even and Robinson (1998) found that the teachers who had conceptual knowledge and the knowledge of students could show effective practice with explaining like term and unlike term, and use strategies for simplification algebraic expressions appropriately. In equivalence of algebraic expression, Hallagan (2004) showed the importance of teacher's using algebra tiles appropriately underlying with area concept in learning the application of distribution.

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In algebra learning, both conceptual and procedural development of the algebraic expression is essential for forming an equation and solving it (Capraro & Joffrinon, 2006). However, manipulating algebraic expressions is one of the difficult algebra topics for students (Banerjee & Subramaniam, 2012; Livneh & Linchevski, 2007; MacGroger & Stacey, 1997; Seng, 2010). At this point, the studies about algebra teaching are scarce and there is also a need to understand the causes of students' difficulties in algebra learning. Thus, this study focuses on teaching practices of operations with algebraic expressions (addition, subtraction, and multiplication).

The Aim of the Study and Research Question

The aim of this study is to extract the practices by focusing on the teachers' actions throughout the teaching of algebraic expressions (simplification and equivalence of algebraic expressions). Thus, the following research question is framed: what is the nature of middle school mathematics teachers' teaching practices in the instruction of operations with algebraic expressions?

The Framework

Mathematical Knowledge for Teaching (MKT) model that is a practice-based approach to the knowledge concept by Ball, Thames, and Phelps (2008) is used as framework in this study (see Figure 1). MKT is defined as "the mathematical knowledge needed to carry out the work of teaching" (Ball et al., 2008, p. 395).

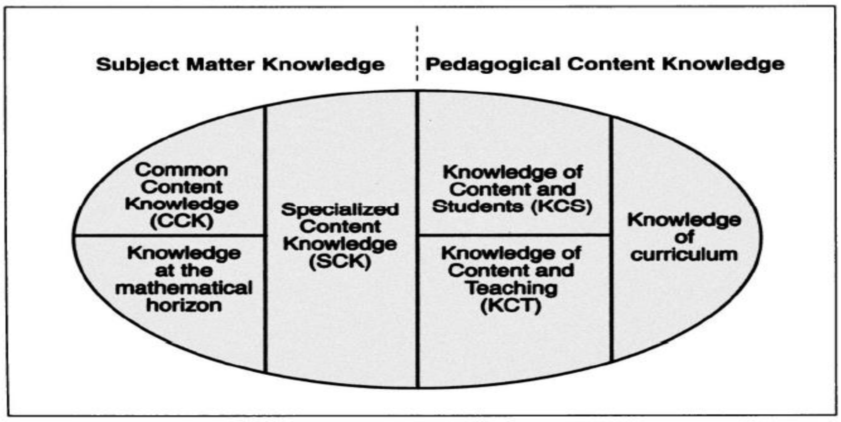

Figure 1. Mathematical knowledge for teaching model (Ball et al., 2008, p. 395)

This model has two main domains as subject matter knowledge and pedagogical content knowledge. The domains have also sub-domains. Ball and her colleagues explained the model with the descriptors based on mathematical demands of teaching. To illustrate, "using terms and notation correctly writing on the board" (p. 399) and; "how mathematical language is used; how to choose, make, and use mathematical representations effectively" as examples for subject matter knowledge (p. 400); "anticipating or predict the mistakes or misconceptions that commonly arise during the instruction" as an example for pedagogical content knowledge (p.375). The current study investigated these mathematical practices for teaching algebraic expressions.

Methods

In this study, qualitative research design was used to investigate middle school mathematics teachers' teaching practices in the context of simplification and equivalence of algebraic expressions. In the qualitative research, a problem or issue is handled with detailed data that are got from different aspects and sources and participants' interpretations (Creswell, 2007). The two teachers were the cases in this qualitative research and the findings were presented by compare and contrast method. Particularly, multiple-case study design with single unit of analysis was used from types of case study designs (Yin, 2003).

The participants were two middle school mathematics teachers and they have been working in the same public school. They were teaching algebra topics to 7th grade students at the same time during the data collection. These teachers were selected and convenient sampling method was used because of the accessibility and their volunteering. The main data were collected from two middle school mathematics teachers' instructions. The researcher observed the lessons in two different 7th grade classes, took field notes, and video-recorded with a camera. After each class session, the researcher conducted post-observation interviews with each teacher separately.

The instructions of two teachers were analyzed and described independently first. The practices of the teachers were grouped based on the common patterns in the teacher's actions that they performed throughout the instructions. MKT model was used in order to determine the practices. The practice-based descriptions of domains were used to explain the practices.

Findings

The extracted practices in this research were: defining the concept of term, like term, constant term, and coefficient; using equal sign; using analogies to explain addition and subtraction; using algebra tiles; providing mathematical explanations for distributive property; and noticing students' misconceptions in simplification and equivalence of algebraic expression.

1. Defining the Concept of Term, Like Term, Constant Term, and Coefficient

At the beginning of the instruction, Teacher B distributed the worksheet to remind algebraic expressions that the students learnt at 6^{th} grade. She reminded what the algebraic expression, term, coefficient, constant term, and like term was. She defined term, like term, constant term, and coefficient concepts correctly by working on the worksheet which had definitions and examples for these concepts. She emphasized the importance of like terms in order to do addition and subtraction with them.

On the other hand, Teacher A had lack of knowledge about the constant term concept as she did not accept it as a term. For example; in $9x+5x^2-9x-7$, she indicated -7 as constant, but she did not accept it as a term. The lack of her knowledge caused incorrect defining of constant term. In the post interview, she admitted to the researcher that she had defined the concept of constant incorrectly when the researcher explained what the constant was.

2. Using Equal Sign

The students did not use equal sign appropriately in expressions. However, Teacher A did not correct them and make any explanations. The student answered 8m correctly, but he used the equal sign inappropriately to show the result that he found as it is shown in the Figure 2. The student added the variables and constant terms separately. He started to write the addition of variables as if it equaled the whole expression. He first added 12m + (-5m), got 7m. Then, he wrote 7m after the equal sign and added m, and got 8m. In this case, 8m was the result of the question regarding the using of equal sign. Teacher A did not intervene any of these representations. However, the student used the equal signs incorrectly. After Teacher A asked to add the constant term, the student added them (1-9) separately and found -8. Teacher A wrote the results together as 8m-8 at the bottom instead of the place of the answer.

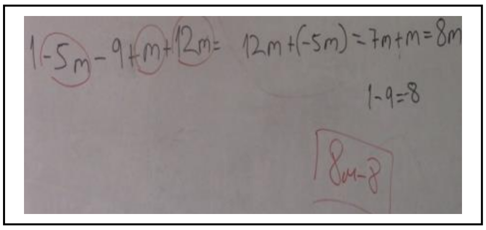

Figure 2. The use of equal sign as the result indicator

On the other hand, Teacher B was aware of students' incorrect writings and corrected them throughout the instruction. For example; she corrected the students' incorrect writings (see Figure 3) when they collected the like terms separately in another place, and they did not write as the result. Teacher B warned students against leaving the results of the addition of like terms. She suggested writing the results on the right side of the equal sign after finding the answers.

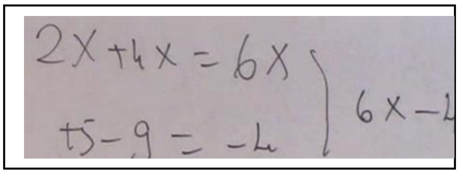

Figure 3. The simplification of 2x+5+4x-9

3. Using Analogies to Explain Addition and Subtraction

Teacher B used analogies in order to teach addition and subtraction in simplification of algebraic expression. For example, she used apple and pear analogy as well as the net worth (i.e. asset and debts) analogy to express different kind of the variables. For example, the explanations of Teacher B's for solving '2a+3-a-4' was as in the following:
B: I have 2 pears, if I give one of them, I have 1 pear. It is represented as 1a. I have 3 TL assets, 4 TL debts. Even if I get the assets, I will have 1 TL debt. Look if the algebraic expression is alone, it means 1 of it. So, we do not need to write 1. You have to know 1 before it.

Teacher B generally used asset and debt concepts and apple-pear representations for explaining the solution of these questions as indicated above. The students could understand much better when the teacher used the concept of variables with analogies. On the other hand, Teacher A did not use any analogies.

4. Using Algebra Tiles

Teacher A used algebra tiles as mathematical representations inadequately to explain multiplication since she did not explain area calculation of rectangles. Instead of this, she explained repeated addition for 2.(3x+3) as in the Figure 4. This representation can be used, but the nature of algebra tiles requires explaining area concept. First, she should have explained x and 1 which were written on the tiles were the areas of the tiles. The tiles were rectangles (for x) and squares (for 1). For x tile, the multiplication of x by 1 is 1; and for 1 tile the multiplication of 1 by 1 is 1. Then, she could have explained the sides of the modeling as rectangle in the figure. One of the sides was 6 unit, and the other side x+1, and the sum of areas was 6x+6.

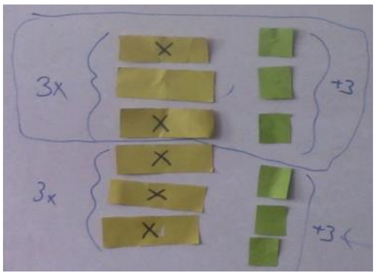

Figure 4. The representation of 2.(3x+3) with algebra tiles

On the other hand, Teacher B had stated that underlying idea of using algebra tiles was the area concept in planning process; she did not use them in teaching multiplication of algebraic expressions. Instead of modeling with tiles, she taught the multiplication procedurally by explaining the rules. Teacher B gave 4.(5-3x) to show the application of distributive property for algebraic expression. She asked the students to use the distributive property in order to do multiplication since 3x could not be subtracted from 5. She multiplied 4 by 5 and -3x respectively, and got 20-12x as the result. She stated that the result was 20-12x because of the absence of like terms that could be added or subtracted.

5. Providing Mathematical Explanations for Distributive Property

Two teachers explained and showed the application of distribution property within different strategies. Teacher A connected repeated addition with distributive property in multiplication appropriately. For example, she asked to the students to express the perimeter of the rectangle whose sides were m and n length algebraically. When the students found 2m+2n, she represented it as 2(m+n) to show distribution property as in the script:

Student: m+m+n+n=2m+2n

A: Okay, what is the common in two expressions?

Student: 2.

A: Then, we can take the parenthesis of 2, we take m and n from the expression. (She is writing 2(m+n)). It is the distribution property of multiplication to addition.

In this situation, the student added the variables separately and he did not use multiplication. Teacher A guided him to find the common number in the expression and she wrote 2(m+n) herself. She explained that it was the distribution property of multiplication to addition. This question could be useful for students for the reason that it provided to connect the repeated addition with multiplication.

On the other hand, Teacher B connected with integers to explain the application distributive property appropriately. For example; she reminded how they find the result of 3.(4-1) by distribute property, then she asked the students to compare it with 3(*x*-1). Two teachers did not use algebra tiles based on area concept to explain distribution.

6. Noticing Students' Misconceptions in Simplification and Equivalence of Algebraic Expression

Teacher A emphasized the kind of variable without considering its power while studying on like terms. However, x and x^2 had same variable but they were unlike terms. Thus, she may cause misconceptions with this definition. Actually, most of the students had difficulty in determining like terms to simplify expressions. On the other hand, Teacher B was more careful so that she provided explanations about like term concept correctly, the procedures of addition operation, and the sign before the parenthesis adequately considering the needs of the students. Two teachers noticed some misconceptions in simplification of algebraic expressions throughout the instruction such as; thinking of 4.x as two-digit number, x.x as 2x, and adding unlike terms. For example, some students tried to add or subtract the terms that were not like such as addition of an integer and an algebraic expression. A similar error was observed in simplifying of 3n-6-3(n+2), Teacher B explained the requirement of like terms to add or subtract as in the following:

Student: I will distribute 3 to (n+2), Then I will add 3n and 3n, subtract from 6.

B: Your friend has said that he would add 3n and 3n and subtract from 6. Are there any unknowns with 6? Variable? None. We do operations only like terms.

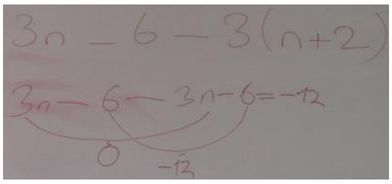

Figure 5. The simplification of the expression by the student

In this situation, Teacher B indicated that 6 was not an algebraic expression and so it could not be operated with algebraic expressions, when the student tried to do it. She noticed the misconception and addressed it by explaining like term concept.

In sum, the practices that are explained above can be presented in the following table:

Table 1. The practices in the instruction of algebraic expressions

Practices	Teacher A	Teacher B
1. Defining the concept of term, like term, constant term, and coefficient	*not accepting the constant as a term *defining of «like term» can cause misconception (having same kind of variable)	*defining the concepts correctly
2. Using equal sign	*using correctly *not correcting students' incorrect uses	*using correctly *correcting students' incorrect uses
3. Using analogies to explain addition and subtraction	*not using	* expressing the kind of the variable with apple and pear analogy *expressing the procedures with net worth concept by using assets and debts concepts

4. Using algebra tiles	*using algebra tiles as repeated addition *not explaining area calculation of rectangles in multiplication	*not using
5. Providing mathematical explanations for distributive property	*connecting with repeated addition	*connecting with integers
6. Noticing students' misconceptions in operations	(Two teachers noticed the misconceptions that arose in the instruction) *noticing the following misconceptions thinking of 4x as two-digit number, x.x as 2x, adding unlike terms	

Conclusion and Discussion

The teachers' practices of how to use mathematical representations as algebra tiles was inadequate. They did not explain the area concept that was underlying the idea of using algebra tiles. However, the instruction with connecting models or representations mathematically, particularly with algebra tiles in algebra, supports students' understanding (Çağlayan, 2013; Tchoshanov, 2011). The students had difficulty in understanding the concept of distributive property, since they did not have adequate understanding of the structure of expressions (Kieran, 1989). However, the teachers did not use algebra tiles to teach multiplication and particularly to show the application of distributive property. The equivalence of expressions using area modeling supports students' understanding of the application of distributive property (Caglayan, 2013; Hallagan, 2004).

In using mathematical language, the remediation of incorrect writings in using equal sign can provide the students an understanding of the function of the equal sign as relationship between the first algebraic expression and the simplified expression (Alibali et al., 2007). Teacher B's use of analogies to explain the procedures in the simplification of algebraic expressions is a suggested method in the literature. For example, Ojose (2015) suggested the use of real-life examples such as apple and orange as a model to teach adding unlike terms. Similarly, Filloy and Sutherland (1996) asserted that the use of concrete models such as tiles, or apple-pear analogy provides translation to abstract level of algebraic expression to facilitate students' learning.

In the aspect of the practice related to students' thinking, Teacher B had more conceptual subject matter knowledge and she could catch students' misconceptions in the instruction. Similarly, the researchers stated that when the teachers had strong subject matter knowledge, they took into account the students' thinking (Baş, Çatinkaya, & Erbaş, 2011; Depaepe et al., 2015; Jacobs, Lamb, & Brown, 2010). Thus, teacher knowledge including students' understanding and needs is essential for effective teaching practices (Anderson, White, & Sullivan, 2005).

Suggestions

Effective teaching practices require using appropriate and adequate knowledge pedagogically (Doerr, 2004; Fennema & Franke, 1992). Teachers can improve their teaching by having meeting together and share good examples and experiences from their classes. Additionally, they can ask for suggestions to their colleagues when they have difficulties in teaching. The mathematics education researchers also can present the results of the studies and suggest different type of activities or examples from the literature in in-service trainings in order to enhance teachers' practices of teaching algebra.

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