
The Eurasia Proceedings of Educational & Social Sciences (EPESS), 2014

Volume 1, Pages 286-291

ICEMST 2014: International Conference on Education in Mathematics, Science & Technology

SOLVING A NUMBER PLACEMENT GAME USING RECURSIVE BACKTRACKING ALGORITHM ON THE GRAPH MODEL

Sema BODUR
Ege University

Sevcan EMEK
Ege University

ABSTRACT: In this study, a number placement game has been developed. This application is designed on a graph model. Recursive backtracking algorithm was used in the solution of this game. Numbers on a board of $n \times n$ will be placed in a certain order under specified rules in finite time. With backtracking algorithm based on depth first recursive search method, finite number of possible solutions has been revealed. In this application that was inspired by 8-Queen problem, Knight's Tour, coloring a map, Knapsack problem and other search problems, advantages and disadvantages of this method have been discussed. The larger the board size, the much more the number of placement complexity. In further studies, the solution of this problem can be possible with the use of heuristic or informative search techniques.

Key words: Backtracking, graph, depth-first search algorithm

INTRODUCTION

Nowadays, in every sector, millions and billions of data is processed by computers. Weather forecasts from statistical analysis, patients' records sampling and finding suitable marrow, image processing and face recognition systems, training data from test drives, finding the correct way from different road routes, learning through entertainments etc. in almost every field of knowledge in computer processing have made important contributions to our daily life as well as to our future. Numerous data processed electronically in the fastest way using the least memory and few resources enabling to be accessible at a lower cost. Therefore, depending on the field of data used in the data structure to achieve the objective, mathematical model, applied algorithm and methods used are important.

In the real world, computer science contributions are very important in solving complex mathematical problems. In solving a problem, mathematical structure that are included in models must exist. Discrete structures such as trees, graphs, permutation and probabilities, equations and finite state machines are used in mathematical modeling. After modeling the problem, it is necessary to determine the best algorithm that will lead to a solution.

In this study, we are expected to place a sequence of all data set in an area of $n \times n$ in such a way that no gap is accommodated according to the specified rules. All possible solutions are searched by graph structures using backtracking algorithm. This application; maze problems, 8 queens are aligned on the chessboard while the knights are placed in L position in such a way that they don't eat each other [1] and likewise sudokupuzzle has been an inspiration in the development of such applications. Methods in application are used in detection of right path from the possible paths, the results and recommendations are discussed in details in the relevant sections.

Graph Theory

One of the branches of Mathematics known as Graph Theory, is commonly encountered in day to day life whereby in many cases the creation of a mathematical model that enables us to solve easily using different techniques from the common known methods. Expressing in a mathematical way, a G graph is formed by V; a set of elements of vertices and E; a set of elements of edges that connects any two non-binary vertices. Each

- This is an Open Access article distributed under the terms of the Creative Commons Attribution-Noncommercial 4.0 Unported License, permitting all non-commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

- Selection and peer-review under responsibility of the Organizing Committee of the conference

*Corresponding author: Sema BODUR- e-mail: sema.bodur@ege.edu.tr

elements of E is referred to as an edge(West, 2001). Directed graph is expressed by respective pairs of the vertices [2].

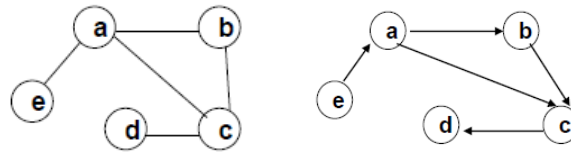


Figure 1. Graph and Directed Graph

In this study the matrix cells are formed by vertices of the graph while in a case where one cell is directed to another cell, the formation is by edges of the graph.

Backtracking Algorithm

In problems of achieving the goals, it is important to choose the right path. Backtracking algorithm tends to find a right path from the followed paths to reach the goal. All possible paths of the problem within the boundaries are tested. When unsuccessful path is followed or a path fails to reach a solution, it is abandoned and a previous step is followed to return back and that path is eliminated. Figure 2 shows the paths from the point of source to the goal.

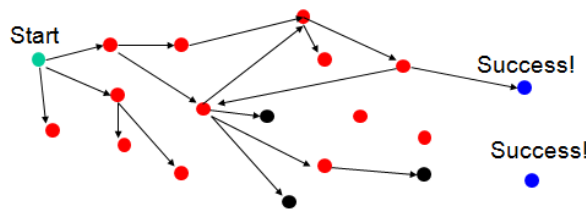


Figure 2. Backtracking Example [3]

Depth-first search algorithm is a very important part of Backtracking. Depth-first search (DFS) is an algorithm for traversing or searching tree or graph data structures. In DFS, a starting node is selected. Within the constraints of the problem, the nodes that are adjacent to neighboring nodes are added to form tree or a graph structure [4]. A right path from the source to the goal is found from all possible paths using backtracking depth-first search algorithm. Figure 3 shows part of the nodes from the point of source to the goal.

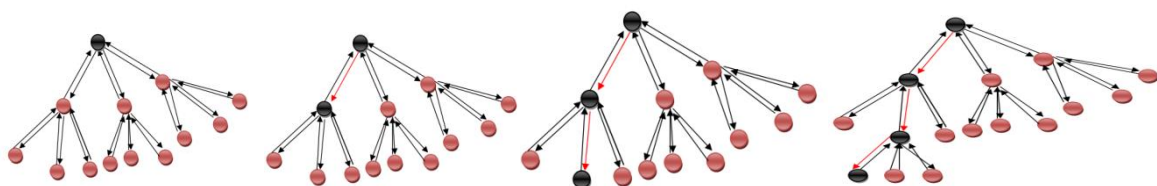


Figure 3. Application Example

In this study all possible paths that lead to the goal are taken into consideration. Backtracking depth-first search algorithm is one of the best methods for this application [5].

DEVELOPMENT OF APPLICATION

Problem Definition

In developing this application, 5x5 board model is transformed to 5x5 matrix form. The numbers are placed in a matrix form in accordance with specified rules.



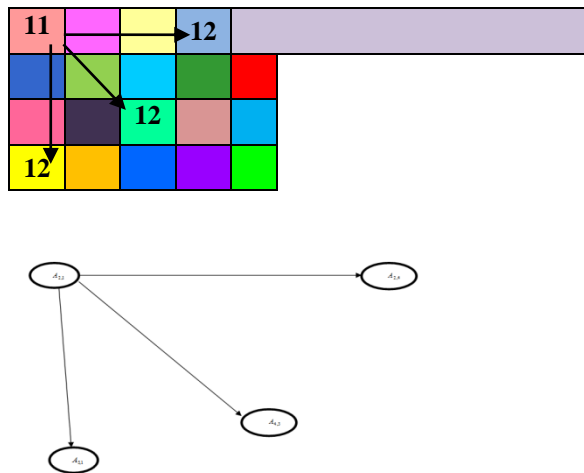


Figure 4. Number Placement Rules

In the figure $A_{2,1}$ above, the number placed on the vertex and the vertex that the next number can be placed on are shown. Our rules;

- 1- Numbers must be placed in sequential order starting from 0 or 1.
- 2- After placing a number, the next number to be placed can be placed in the same column or row after 2 empty gaps or in a diagonal way if there is free space.
- 3- For the game to end successfully, all numbers need to be placed.

Solution Algorithm

In our studies, the graph model of 25 vertices and 38 edges is shown in Figure 5.

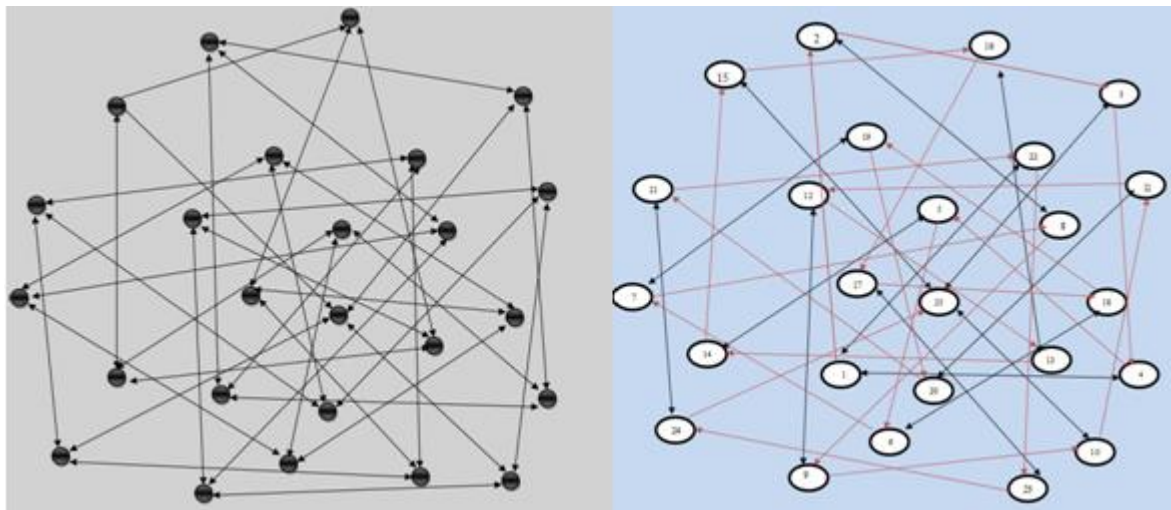


Figure 5. Number Placement Game Graph Model.

DFS pseudocode is as follows;[6]
public static void DFS (Graph g, Object vertex) {
 g.visit(vertex);
 Iterator itr = g.neighbors(vertex);
 while (itr.hasNext()) {
 Object v = itr.next();
 if (!g.isVisited(v))

```

        DFS(g, v);
    }
}
    
```

The most prominent feature of the problem is the placement pattern of the numbers on the matrix. In a square matrix being 5 x5, to go from the starting point or adjacent nodes, number of nodes is maximum 3 or 4. This condition is shown in Figure 6.

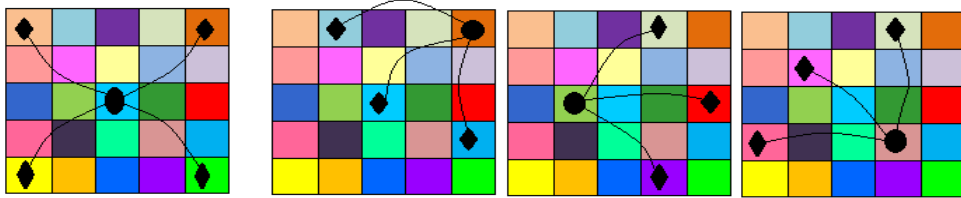


Figure 6. Destination Number of Nodes

In Figure 6, if the node position on the left is $A_{2,2}$, then the maximum number of 4 alternative nodes can be placed after itself. In the right table 3, maximum number of 3 alternative nodes can be placed.

In the process of programming our game, a sequence of rules is formed by defining the board size in x-y axis movement.

```

move_x = { 2, 2, -2, -2, 3, 0, -3, 0 };
move_y = { 2, -2, 2, -2, 0, 3, 0, -3 };
    
```

According to this x-y motion, all of the numbers are placed step by step in appropriate regions. Number of "0" is start point. Start node can be selected randomly. After number of "0" is placed in $A_{3,3}$, placement of number 1 is decided. Number 1 can be placed (-2,+2) units after $A_{3,3}$ point. Then, number of "2" is placed in $A_{1,4}$. As long as the correct spacing exists the remaining numbers can be placed.

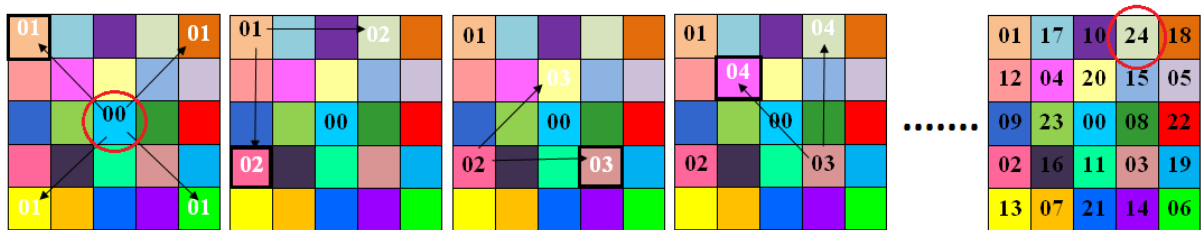


Figure 7. Step by step number placement

In solution process;

- Assigning of the movement point, in other words starting node,
- Calculation of new movement coordinates,
- Alternative nodes test or trial,
- Testing the validity of the new step coordinates,
- Not returning the previous node.

the steps above should be paid attention.

RESULTS and FINDINGS

Position of starting node can be selected randomly on the board. Number of "0" surrounded by green is start node and number of "24" surrounded by red is goal node. All of the numbers are placed according to rules. As shown below, all of the numbers from 0 to 24 have been placed successfully on the board. Figure 8 shows the successful possible paths.

| | | | | | | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 18 | 10 | 02 | 17 | 09 | 00 | 13 | 16 | 01 | 12 | 23 | 08 | 05 | 22 | 09 | 03 | 14 | 17 | 04 | 13 |
| 04 | 21 | 13 | 05 | 22 | 18 | 23 | 04 | 09 | 22 | 13 | 02 | 19 | 12 | 03 | 09 | 06 | 01 | 10 | 07 |
| 01 | 16 | 08 | 00 | 15 | 15 | 07 | 20 | 14 | 06 | 06 | 16 | 24 | 07 | 17 | 16 | 19 | 22 | 15 | 18 |
| 19 | 11 | 03 | 20 | 12 | 03 | 10 | 17 | 02 | 11 | 20 | 11 | 04 | 21 | 10 | 02 | 11 | 08 | 05 | 12 |
| 07 | 24 | 14 | 06 | 23 | 19 | 24 | 05 | 08 | 21 | 14 | 01 | 18 | 15 | 00 | 21 | 24 | 00 | 20 | 23 |
| 07 | 13 | 03 | 06 | 16 | 14 | 01 | 07 | 15 | 02 | 01 | 17 | 10 | 24 | 18 | 01 | 15 | 07 | 00 | 16 |
| 21 | 10 | 00 | 24 | 19 | 19 | 11 | 04 | 18 | 21 | 12 | 04 | 20 | 15 | 05 | 12 | 04 | 18 | 13 | 05 |
| 02 | 05 | 17 | 12 | 04 | 08 | 16 | 23 | 09 | 06 | 09 | 23 | 00 | 08 | 22 | 20 | 09 | 24 | 21 | 08 |
| 08 | 14 | 20 | 09 | 15 | 13 | 00 | 20 | 12 | 03 | 02 | 16 | 11 | 03 | 19 | 02 | 14 | 06 | 03 | 17 |
| 22 | 11 | 01 | 23 | 18 | 24 | 10 | 05 | 17 | 22 | 13 | 07 | 21 | 14 | 06 | 11 | 22 | 19 | 10 | 23 |

Figure 8. Numbers placed successfully

Figure 9 shows the steps of placing numbers according to their rules.

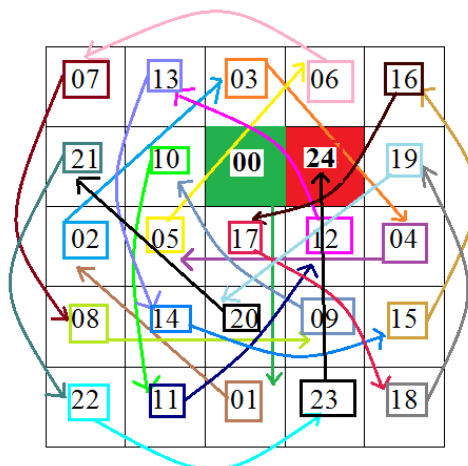


Figure 9. Example of a success path

Figure 10 shows unsuccessfully finished game with wrong paths. The numbers which are turned round by yellow colour in the cell can't move because there were no other number of steps to make a move it turns to the previous node with Backtracking. It evaluates other possible move. Thus, all possible outcomes are scanned in the search space.

| | | | | | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| | | 10 | | 01 | 05 | 11 | 16 | 06 | 06 | 03 | | | 03 | 12 | 15 | 02 | 20 | |
| 05 | 16 | 13 | 04 | 17 | 09 | 00 | 19 | 22 | 01 | 11 | | 08 | 01 | 00 | 05 | 10 | 17 | |
| 11 | 08 | 00 | | 09 | 12 | 15 | 07 | 04 | 14 | 04 | | 05 | | 14 | 08 | 19 | 13 | 07 |
| 14 | 03 | 18 | 15 | 02 | 18 | 21 | 10 | 17 | 20 | | 07 | 02 | | 04 | 11 | 16 | 01 | 21 |
| 06 | | 12 | 07 | | 08 | 03 | 13 | 23 | 02 | 10 | | 09 | 00 | | 06 | 09 | 18 | |

Figure 10. Unsuccessful paths

Probability Calculations: Although the probability condition totals to $(25 \times 24 \times 23 \dots 1) / 5!$, if we consider every possibility which doesn't contain solution, our searching space size would quite decrease. Considering our limitations;

After placing the first number on any of the 25 cells, the total number of moves sums to 23 and since at each move that can be made differently totals to 3, this makes the searching space size to be $25 \times 3^{22} \times 2$

CONCLUSION and FUTURE WORK

In this study, by placement of sequence of numbers in accordance with certain rules based on the board enabled us to develop a game. In this game, the player uses all numbers with an aim of finding possible solution paths. Thus, an individual's trial and error method of analysing different ways and decision making skills are provided and improved.

In this application, 5x5square has been tested on the board. The field can be expanded, hence possible paths are increased. Game rules defining the placement pattern of numbers can be changed. The dimensions of the playing field can be increased, 2 dimensional space can be transformed to 3 dimensional space. Exploration of our game on the possible right paths using heuristic algorithms based on learning methods may be put in trial. Thus unsuccessful paths can be avoided and the right paths can be found without selecting wrong paths. As we progress in our studies, this kind of applications can be used to compare methods based on informed and uninformed search algorithms.

REFERENCES

- Gordon, V. S., & Slocum, T. J. (2004). The Knight's Tour- Evolutionary vs. Depth-First Search, IEEE
- Tounsi, M., CS 311 Design and Algorithm Analysis Course: info.psu.edu.sa/psu/cis/mtounsi/~CS311/Chap-BKT-BB.pp
- Dündar, P. & Balçı, A.M. & Kılıç, E. (2011). Mathematical Modelling of Placement of Emergency Phone Centres in a Campus Network. *DEÜ Mühendislik Fakültesi Mühendislik Bilimleri Dergisi*, Sayı 13, Cilt 1
- Retrieved April 29, 2014 from the Wikipedia : http://en.wikipedia.org/wiki/Depth-first_search
- Problem Solving with Algorithms and Data Structures :
<http://interactivepython.org/runestone/static/pythononds/Graphs/graphdfs.html>
- Retrieved April 29, 2014 from <http://www.academic.marist.edu/~jzbv/algorithms/Backtracking.htm>