VERİMLİLİK DERGİSİ
JOURNAL OF PRODUCTIVITY

Technology Adoption, Innovation and the South Korean Miracle

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ABSTRACT

Purpose: The main purpose is to understand the microeconomic foundations of the South Korean miracle through careful analysis and a realistic equilibrium model.

Methodology: The paper constructs a simple equilibrium model where productivity growth is endogenous to (i) human capital, (ii) the country's distance to the global technology frontier, and (iii) the level of urban agglomeration. The paper identifies and calculates unobserved productivity terms using various observed variables from South Korean national accounts for the post-1960 period. The paper then presents the structural estimates of the model parameters and the results of the decomposition analysis.

Findings: While the South Korean economy was initially using a backward technology, it became an innovation economy in the early 1980s. Structural estimates show that urban agglomeration is not statistically significant in the South Korean case. Finally, a decomposition analysis shows that, in the early 1960s, human capital and distance to the frontier made similar contributions to productivity growth.

Originality: The model economy has two sectors. Technology in the modern sector exhibits Constant Returns to Scale, but traditional technology is constrained by Decreasing Returns to Scale. In addition, both the technology adoption regime and the innovation regime can be represented by the same mathematical function, and the article is therefore theoretically original.

Keywords: Endogenous Technology, Equilibrium Model, Structural Estimation, Catching Up.

***JEL Codes*:** O12, O33, O41.

Teknoloji Benimseme, İnovasyon ve Güney Kore Mucizesi

ÖZET

Amaç: Ana amaç, özenli bir analiz ve gerçekçi bir denge modeli ile Güney Kore mucizesinin mikroekonomik temellerini anlamaktır.

Yöntem: Makale, verimlilik büyümesinin, (i) beşerî sermayeye, (ii) ülkenin küresel teknoloji uç sınırına olan uzaklığına ve (iii) kentsel yığılma düzeyine içsel olduğu basit bir denge modeli inşa etmektedir. Makale, 1960 sonrası dönem için Güney Kore ulusal hesaplarından çeşitli gözlemlenen değişkenleri kullanarak, gözlemlenmemiş verimlilik terimlerini belirlemekte ve hesaplamaktadır. Makale, daha sonra, model parametrelerinin yapısal tahminlerini ve ayrıştırma analizinin sonuçlarını sunmaktadır.

Bulgular: Güney Kore ekonomisi başlangıçta geri bir teknoloji kullanırken, 1980'lerin başında yenilikçi bir ekonomi haline gelmektedir. Yapısal tahminler, kentsel yığılmanın Güney Kore örneğinde istatistiksel olarak anlamlı olmadığını göstermektedir. Son olarak, bir ayrıştırma analizi, 1960'ların başında beşerî sermayenin ve teknoloji uç sınırına olan uzaklığın verimlilik büyümesine benzer katkılar yaptığını göstermektedir.

Özgünlük: Model ekonominin iki sektörü vardır. Modern sektördeki teknoloji, Ölçeğe Göre Sabit Getiri sergilemektedir, ancak geleneksel teknoloji, Ölçeğe Göre Azalan Getirilerle kısıtlanmıştır. Ek olarak hem teknoloji benimseme rejimi hem de yenilik rejimi aynı matematiksel fonksiyonla temsil edilebilmektedir ve bu nedenle makale teorik olarak orijinaldir.

Anahtar Kelimeler: İçsel Teknoloji, Denge Modeli, Yapısal Tahmin, Yakalama.

JEL Kodları: O12, O33, O41.

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DOI: 10.51551/verimlilik.1344143
Research Article | Received: 16.08.2023 | Accepted: 20.12.2023
Cite: Attar, M.A. (2024). "Technology Adoption, Innovation, and the South Korean Miracle", *Verimlilik Dergisi,* 58(1), 103-118.

1. INTRODUCTION

The South Korean economy has exhibited rapid economic growth in per capita terms in the second half of the 20th century. The updated Maddison Project database indicates that, measured in 2011 international dollars, real GDP per capita has been around 38,000 United States' dollar (USD) in 2018 (Bolt and van Zanden, 2020). This is about 28 times larger than 1,373 USD recorded in 1954, i.e., in the immediate aftermath of the Korean War. From 1954 to 2018, the average growth rate of real GDP per capita has been around 5.4% per annum, and the miraculous growth rate has been around 7.4% per annum for the period starting in 1961 and ending with the Asian Financial Crisis in 1997.

The main purpose of this paper is to present new evidence on the microeconomic foundations of the South Korean miracle. The microeconomic foundations are simply the deep determinants of sectoral and aggregate productivity growth rates in the economy. The paper closely follows Lucas (2009) and constructs a two-sector catching up model to understand the drivers of endogenous productivity growth in South Korea. As in Lucas (2009), the sector that uses the modern (i.e., constant returns to scale) technology adopts foreign technologies from the world frontier, and the sector that uses the traditional (i.e., decreasing returns to scale) technology benefits from a spillover from the modern sector. But differently from Lucas (2009), the present framework introduces (i) physical capital in the modern technology as an essential input and (ii) human capital as a determinant of endogenous productivity growth. In these respects, the model studied in this paper is much more realistic. Furthermore, neglecting the sectoral differences and structural transformation in understanding miraculous economic growth may lead to wrong lessons about the true sources of growth and change, as previously demonstrated by Nelson and Pack (1999).

Since productivity levels are not directly observed by the econometrician, estimating structural parameters that determine productivity growth rates are typically infeasible. Whereas one can estimate production functions and, hence, residual productivity levels, it is not entirely straightforward to obtain the structural estimates of parameterized versions of nonlinear endogenous technology models. Besides, in multi-sector models that feature productivity spillovers, identification is all the more formidable. On the other hand, imposed theoretical structures may still allow us to identify some or all of the structural parameters if some model inputs are set arbitrarily at the outset. The present analysis benefits from such a possibility; the structural model has six structural parameters, and the empirical strategy identifies five of these six structural parameters using observed data after fixing one of them.[2] More specifically, this paper estimates two parameters of the traditional technology and three parameters that determine productivity growth rate of the modern technology by assigning a value to the labor share of the traditional technology. The observed data needed for this estimation to work covers real GDP per capita, physical capital stock, human capital stock, and the share of rural population.

Results indicate that the South Korean miracle has two distinct episodes or regimes. The former, from 1960 to the early 1980s, is an era of very rapid productivity growth. Decadal averages of productivity growth rates typically exceed 7% per annum in this first period. The major driver of productivity growth in the first regime is the distance to the frontier. That is, the South Korean economy is not a technology leader in this regime, but it keeps closing its distance with the world frontier by successfully adopting foreign technologies. Eventually, in the early 1980s, productivity in South Korea forges ahead the frontier productivity. In the second regime, South Korea becomes an innovator economy. In this regime, productivity growth rates are much lower, and the advantage of relative backwardness of the first regime now acts as an obstacle for higher productivity growth, as the economy already achieved high levels of productivity. Hence, the second regime after the early 1980s is similar to the experiences of other innovative high-income economies that are themselves technology leaders. Results also show that, in both regimes, human capital works as a crucial determinant of productivity growth in South Korea.

The remainder of the paper is organized as follows: Section 2 presents a discussion of the related literature and the contributions of the present paper. Section 3 introduces the model economy. Section 4 explains the methodological approach and describes the dataset. Section 5 presents the main results of the paper. Section 6 demonstrates that the main results are not much sensitive to the arbitrary parameter values. Section 7 concludes the paper with some remarks. Detailed mathematical derivations are presented in the appendices.

2. RELATED WORKS and CONTRIBUTIONS

By identifying unobserved productivity terms for South Korea using a two-sector catching-up model and by providing structural econometric estimates of the relevant microeconomic foundations, this paper makes several empirical contributions to the related literature.

[2] The qualitative nature of results is not sensitive to this arbitrary parameter value; see Section 6.

The earliest thoughts on catching up and technology adoption can be found in Veblen (1915: 1) where he discusses how technology diffuses from early-industrialized countries such as Britain to a late-industrialized country such as Germany. The advantage of relative backwardness hypothesis of Gerschenkron (1962: 1) builds upon the experience of other follower countries such as Japan and Russia. The earliest formulations of how education (or human capital) affects technology adoption can be found in Nelson and Phelps (1966) and Gomulka (1971). In the present paper, human capital is shown to play a major role in positively affecting technology adoption during the miraculous transformation of the South Korean economy.

Early empirical assessments of whether initially poorer economies grow faster has been presented by Kormendi and Meguire (1985), Baumol (1986), DeLong (1988), and Barro (1991). These works have initiated the so-called Convergence Controversy, and various methodologies have been exploited to study the world income distribution and convergence clubs.[3] The results presented here demonstrate that the South Korean economy completed its convergence to the world frontier in the early 1980s.

The seminal works of Abramovitz (1986) and Cohen and Levinthal (1989, 1990) focus on how successfully a laggard country adopts foreign technologies and what are the determinants of a country's absorptive capacity in technological catching up. Empirical results generally support the notion that human capital and education have a significant role in positively affecting the absorptive capacity in technology adoption (Rogers, 2004; Benhabib and Spiegel, 2005; Kneller and Stevens, 2006). Some of the theoretical works on catching up and falling behind, i.e., Acemoglu et al. (2006) and Stokey (2015), clarify how extended endogenous technology models may lead to no-growth or low-growth equilibria. Here, the empirical results substantiate the view that, exactly as in Acemoglu et al. (2006), the transition of South Korea from a technology adopter to an innovator is an optimal response to the changing fundamentals of relative productivity. Once the relative gain from technology adoption becomes sufficiently small, the economy spends more of its scarce resources to innovation.

Attar's (2018) is the most directly related work, and similarities and differences with that paper deserve some attention here. Also following Lucas (2009) very closely, Attar (2018) aims at understanding the comparative development differences between Türkiye and South Korea. Türkiye in the early 1960s has better development prospects, but South Korea forges ahead, leaving only missed opportunities to her fellow. Attar (2018) studies two extensions of the baseline economy in Lucas (2009) to understand this divergence. Contrary to the structural estimation work presented here, he focuses on quantitative experiments that build upon a rigorous calibration of structural parameters.

The present paper contributes to the related literature from another, theoretical perspective. Structural models of catching up and technology diffusion typically focuses on a laggard economy's distance to the frontier by assuming that the laggard country does not forge ahead the world frontier. In reality, a follower country may exhibit miraculous productivity growth and eventually become an innovative country that contributes to the world frontier. In such a case, the country would experience a reversal of fortune in productivity growth because the advantage of relative backwardness disappears once the distance to the frontier closes down. Then, faster productivity growth that moves the economy further up on the technology ladder creates its own mean-reverting force that limits productivity growth rates. In models of endogenous technology, such an effect is typically called the fishing-out effect or the low-hanging-fruit effect. In the present paper, there is a single mathematical formulation of the technology that creates productivity growth. When the economy is behind the frontier (i.e., when it is absolutely less productive), the distance to the frontier contributes positively to productivity growth. When the economy is at the frontier (once the economy forges ahead the frontier productivity level), the very same term's contribution becomes negative.

3. MODEL

The purpose here is to develop the simplest model economy that is most informative about the fundamental determinants of productivity growth in South Korea. For simplification, we assume away the household's decisions, the role of government, and international trade flows as in Lucas (2009). We also presume that physical capital and human capital grow exogenously. All of these restrictions allow us to isolate the technological microeconomic foundations within a multi-sector endogenous technology model, and the only decision problem to be solved is the sectoral allocation of resources. As in Lucas (2009), the present formulation leaves this choice to the market, and resource allocation achieves the maximization of total output in real terms.

3.1. Overview

Time in the model is discrete with an infinite horizon: $t \in \{0, 1, \dots\}$. The length of a period is a calendar year. There is a single, all-purpose good, and there are two sectors/technologies that produce this good. The

[3] See Durlauf and Quah (1999) and Islam (2003) for two extensive reviews.

modern technology exhibits Constant Returns to Scale (CRS), but the traditional technology is subject to Decreasing Returns to Scale (DRS) because of a fixed input such as land. Productivity change is endogenous in both sectors. In the modern sector, there is either technology adoption from the frontier economy in the world or domestic innovation, depending on whether the economy's aggregate modern sector productivity is at the frontier. In the traditional sector, productivity changes as a result of spillovers from the modern sector. Only the modern sector uses physical capital. Table 1 summarizes the model parameters.

Table 1. Structural parameters of the model economy

Parameter	*Support*	*Source, identification, or estimation*
Capital share in the modern sector λ	$(0,1)$	Arbitrarily preset at $\lambda = 0.4$, see Lucas (2009)
Labor share in the traditional sector α	$(0,1)$	Identification via $\alpha = 1 - \lambda$
Productivity spillover parameter ξ	$(0,1)$	Structural estimation, joint estimates for (ξ, μ)
Traditional sector fixed productivity μ	$(0, +\infty)$	Structural estimation, joint estimates for (ξ, μ)
Productivity growth, fixed component ϕ	$(0, +\infty)$	Structural estimation, joint estimates for (ϕ, ζ, θ)
Urban agglomeration parameter ζ	$(0, +\infty)$	Structural estimation, joint estimates for (ϕ, ζ, θ)
Relative productivity, elasticity θ	$(0,1)$	Structural estimation, joint estimates for (ϕ, ζ, θ)
Frontier economy growth rate γ	$(0, +\infty)$	Estimated for the United States, see Lucas (2009)

3.2. Production Technologies

In Equations 1 and 2, let $Y_t(M)$ and $Y_t(T)$ denote output flows in the modern sector and in the traditional sector, respectively.

$$Y_t(M) = K_t^{\lambda}[A_t h_t L_t(M)]^{1-\lambda} \tag{1}$$

$$Y_t(T) = \mu A_t^{\xi}[h_t L_t(T)]^{\alpha} \tag{2}$$

where $K_t > 0$ is the stock of physical capital, $A_t > 0$ is an unobserved productivity term, $h_t > 0$ is average human capital, $L_t(M)$ and $L_t(T)$ are levels of raw labor employed, $\mu > 0$ is a fixed and exogenous productivity term (a shift parameter), $\xi \in (0,1)$ is the modern-to-traditional productivity spillover parameter, $\lambda \in (0,1)$ is the elasticity of the modern sector output with respect to physical capital, and $\alpha \in (0,1)$ is the elasticity of traditional sector output with respect to labor. For simplicity (and without loss of too much significance), we assume that h_t and K_t are exogenously given for all t.

3.3. GDP and the Static Equilibrium

Real GDP in year t, denoted by Y_t, is defined simply as in $Y_t = Y_t(M) + Y_t(T)$. As in Lucas (2009), define the static equilibrium as a resource allocation problem that maximizes Y_t by choosing $L_t(M)$ and $L_t(T)$ under the resource constraint as in Equation 3.

$$L_t(T) + L_t(M) = L_t \tag{3}$$

where L_t is also exogenous and given for all t. Formally, Equation 4 represents the optimization problem:

$$\max_{L_t(M)} \quad \mu A_t^{\xi} h_t^{\alpha}[L_t - L_t(M)]^{\alpha} + K_t^{\lambda}[A_t h_t L_t(M)]^{1-\lambda} \tag{4}$$

In general, there does not exist a closed-form solution to this problem. However, a unique, closed-form solution exists if $\alpha = 1 - \lambda$. As demonstrated in Appendix A, this unique solution implies Equation 5

$$\frac{L_t(T)}{L_t} = \frac{\tilde{A}_t}{K_t + \tilde{A}_t} = \ell_t \tag{5}$$

where $\tilde{A}_t$ is some unobserved productivity term defined in Equation 6 below

$$\tilde{A}_t = \left(\mu A_t^{\xi+\lambda-1}\right)^{\frac{1}{\lambda}} \tag{6}$$

This solution tells us that the share of labor employed in the traditional sector depends on a composite term $K_t/\tilde{A}_t$ of relative productivity. If $K_t/\tilde{A}_t$ goes to positive infinity because the spillover effect is not sufficiently strong (low enough ξ *ceteris paribus*), then the modern sector becomes increasingly more productive and labor is allocated more intensively in the modern sector; the traditional sector disappears ($\ell_t \to 0$). Conversely, if $K_t/\tilde{A}_t$ goes to zero with a sufficiently strong spillover effect (high enough ξ *ceteris paribus*), then the share of labor employed in the traditional sector goes to unity; the modern sector disappears ($\ell_t \to 1$).

3.4. Dynamics and the Growth of Productivity

Since we take the sequences $\{K_t, h_t, L_t\}_t$ as given model inputs for the entire history, the dynamics of the economy are determined by how A_t evolves in time.

Let $G_t = 1 + g_t = A_{t+1}/A_t$ denote the gross growth rate of the productivity term A_t. Equation 7 shows this growth rate that is endogenously determined:

$$G_t = \frac{\phi h_t (1-\ell_t)^{\zeta}}{a_t^{\theta}} \tag{7}$$

Here, parameters satisfy ϕ, $\zeta > 0$ and $\theta \in (0,1)$, where a_t denotes the modern sector productivity relative to the frontier economy, defined formally in Equation 8:

$$a_t = \frac{A_t}{\bar{A}_t} \tag{8}$$

Here, then, three distinct mechanisms described below govern the dynamics of productivity growth.

- *Distance to the frontier*: With relative productivity a_t defined above, the economy's distance to the frontier economy is inversely related with a_t. Whether it is additive ($d = 1 - a$) or multiplicative ($d = 1/a$), assume that productivity growth rate G_t is larger if a_t is lower, *ceteris paribus*. This is the typical catching up effect associated with relative backwardness of the follower economy (Gerschenkron, 1962; Abramowitz, 1986). In the present setup, the elasticity with respect to a_t, denoted by θ, is between 0 and 1 as in Lucas (2009).
- *Human capital*: Productivity growth rate increases with average human capital in the economy to reflect the notion that human capital of an economy's workforce is a crucial determinant of the economy's absorptive capacity for technology adoption. This is the mechanism proposed by Nelson and Phelps (1966) and Gomulka (1971).
- *Urban agglomeration*: Productivity growth rate also increases with the relative size $(1 - \ell_t)$ of the modern sector. As in Lucas (1988, 2009), cities where the modern technology firms operate are the locations where more productive and more creative people generate positive externalities for each other.

3.5. Productivity Growth in the Frontier Economy

To complete the formal characterization of the model economy, we need to specify how $\bar{A}_t$ changes in time. Equation 9 below shows the law of motion for $\bar{A}_t$:

$$\bar{A}_{t+1} = (1 + \gamma)\bar{A}_t \tag{9}$$

$\gamma > 0$ and $\bar{A}_0 > 0$ are exogenously given. Hence, the frontier economy in the world has perpetual productivity growth taking place at a fixed rate.

4. METHODOLOGY and DATA

Our fundamental task is to understand the evolution of productivity in South Korea using the above model. By this is meant (i) how the South Korean productivity relative to the frontier economy evolved in time, (ii) how the human capital, urban agglomeration, and the distance to the frontier affected the growth rate of South Korean productivity, and (iii) how much of the observed growth can be attributed to these potential sources.

For the first dimension, we need to identify and compute both the frontier productivity $\bar{A}_t$ and the South Korean productivity term A_t, as the relative productivity a_t is defined as the ratio $A_t/\bar{A}_t$. For the second dimension, we use the equilibrium definitions to formulate regression models and obtain structural estimates of several parameters. Finally, after estimating the structural parameters, we simply decompose productivity growth into its structural sources.

4.1. Identifying the Unobserved Relative Productivity a_t

Identifying the unobserved relative productivity is of central interest since it is this variable that allows us to make an inference about the date at which the South Korean economy closes its gap with the frontier. Recall from Equation 8 that this productivity term, denoted by a_t, is defined simply as the ratio $A_t/\bar{A}_t$. Therefore, one needs to identify both the numerator and denominator to identify a_t.

4.1.1 Identifying the Unobserved Productivity Terms $\tilde{A}_t$ and A_t

In general, we do not observe the share ℓ_t of labor allocated to use traditional technologies. The closest observables for this variable are the shares of rural employment and rural population. Exactly as in Lucas (2009), we take ℓ_t as the share of rural population. But if both ℓ_t and K_t are observed, Equation 5 uniquely identifies $\tilde{A}_t$ as shown in Equation 10:

$$\tilde{A}_t = \frac{\ell_t K_t}{1-\ell_t} \tag{10}$$

After identifying $\tilde{A}_t$ using the above equation, we need another observable to identify A_t. This observable is real GDP per capita, denoted by y_t. As demonstrated in Appendix B, real GDP per capita can be written as in Equation 11:

$$y_t = A_t^{1-\lambda} h_t^{1-\lambda} \left(\frac{\tilde{A}_t}{L_t} + \frac{K_t}{L_t}\right)^{\lambda} \tag{11}$$

Hence, we only need a value of λ to identify A_t since the rest of the terms are readily available, either observed or identified. Specifically, the resulting identifying formula is shown in Equation 12:

$$A_t = \left[\frac{y_t}{h_t^{1-\lambda}\left(\frac{\tilde{A}_t}{L_t}+\frac{K_t}{L_t}\right)^{\lambda}}\right]^{\frac{1}{1-\lambda}} \tag{12}$$

The baseline results presented in the next section build upon a value of $\lambda = 0.4$ that implies $\alpha = 0.6$ as in Lucas (2009), but we also present a sensitivity analysis.

4.1.2 Identifying the Unobserved World Frontier $\bar{A}_t$

To identify the frontier productivity that is assumed to grow perpetually at a fixed rate as in Equation 9, both the initial value $\bar{A}_0$ and the growth rate γ must be specified. For both of these inputs, the US is taken as the frontier economy as in Lucas (2009).

For the initial value $\bar{A}_0$ of the US economy, a meaningful estimate is available through the relative TFP data of Isaksson (2007). In their baseline sample, the South Korean economy's TFP relative to the US is equal to $a_0 = 0.317$ in 1960. Since we have $\bar{A}_0 = A_0/a_0$ by definition and $A_0 = 179.16$ from the baseline identification, we obtain $\bar{A}_0 = 565.18$. For the growth rate γ, the value of 2.1% per annum, representing the long-term growth rate in the US, is used as a benchmark as in Lucas (2009) and Attar (2018).

4.2. Estimating the Structural Parameters

The complete model economy has five structural parameters other than $\alpha = 1-\lambda$, i.e., $(\mu, \xi, \phi, \zeta, \theta)$. The first two of these are $\mu > 0$ and $\xi \in (0,1)$ that, along with α, characterize the traditional technology of production. The other three parameters $\phi > 0$, $\zeta > 0$, and $\theta \in (0,1)$, on the other hand, describe how productivity grows from t to $t+1$; they characterize the technology of innovation and technology adoption. It turns out that, given λ, identified sequences of $\tilde{A}_t$ and A_t uniquely identify $\mu > 0$ and $\xi \in (0,1)$ through Equation 6. To see how, rewrite Equation 6 as a loglinear regression Equation 13 with an additive error term u_t that satisfies typical regulatory assumptions:

$$\ln(\tilde{A}_t) = \ln(\mu^{1/\lambda}) + \left[\frac{\xi-(1-\lambda)}{\lambda}\right]\ln(A_t) + u_t \tag{13}$$

Clearly, with uniquely identified sequences of $\tilde{A}_t$ and A_t, the ordinary least squares estimate of the intercept and the slope parameters in Equation 13 yield unique estimates of $\mu > 0$ and $\xi \in (0,1)$; see Appendix C.

For the remaining three parameters, one can easily derive an estimating equation from Equation 7 by introducing another additive error term v_t. This regression is shown in Equation 14:

$$G_t = \frac{\phi h_t (1-\ell_t)^{\zeta}}{a_t^{\theta}} + v_t \tag{14}$$

This can be estimated via nonlinear least squares, and it would return unique estimates of (ϕ, ζ, θ).

4.3. Decomposing Productivity Growth into Its Sources

Recall the interpretation of Equation 7 that specifies how productivity grows in time. If $a_t < 1$, the economy has a scope for technology adoption. In this case, productivity growth has three components associated with h_t, a_t, and $(1-\ell_t)$, respectively. Since $a_t < 1$, all of these three components have positive contributions to productivity growth.

If the economy is sufficiently advanced to have $a_t \geq 1$, technology adoption stops. From this point on, productivity growth is driven by innovation and a_t now acts as a drag that reflects the loss due to the fishing-out mechanism. Thus, a_t's contribution to growth is negative and the total contribution of h_t and $(1-\ell_t)$ is thus larger than 100%.

Since the task is to decompose the observed productivity growth into various components, we need to have an additive structure. Such a structure can be derived by taking the natural logarithm of G_t/ϕ by using Equation 7. The result is shown in Equation 15:

$$\ln\left(\frac{G_t}{\phi}\right) = \ln(h_t) + \zeta \ln(1-\ell_t) - \theta \ln(a_t) \tag{15}$$

Dividing both sides of this equation to $\ln(G_t/\phi)$ then implies the formula in Equation 16 that we can use for the decomposition:

$$\frac{\ln(h_t)}{\ln(G_t/\phi)} + \frac{\zeta \ln(1-\ell_t)}{\ln(G_t/\phi)} + \frac{-\theta \ln(a_t)}{\ln(G_t/\phi)} = 100\% \tag{16}$$

5. RESULTS

5.1. Identified Productivity Terms

Figure 1 pictures the identified sequences of absolute productivity levels A_t and $\bar{A}_t$. The first thing to note from this figure is the date at which absolute productivity A_t of the modern sector surpasses the world frontier $\bar{A}_t$. This event happens in the year 1983.[4] Within the narrative of the present framework, this means that the South Korean miracle of productivity growth made the follower South Korea an innovator economy sometime in the early 1980s. The second implication of these results is that the growth rate of absolute productivity A_t is not fixed across decades. In fact, it exhibits an early acceleration and a late slowdown, and both the acceleration and the slowdown are sizable. The average growth of rate of A_t is equal to 7.6%, 7.2%, and 8.4% per annum for the 1960-1969, 1970-1979, and 1980-1989 periods, respectively.[5] However, productivity growth rates are much lower than these impressive rates for the 1990-1999, 2000-2009, and 2010-2019 decades, being equal to 3.8%, 3.1%, and 1.1% per annum, respectively. The acceleration-slowdown pattern of productivity growth is clearly in line with the logic of convergence and catching up. After 1983, once technology adoption stops, relative productivity enters a regime at which it fluctuates around a fixed value. This is the third noteworthy implication of Figure 1.

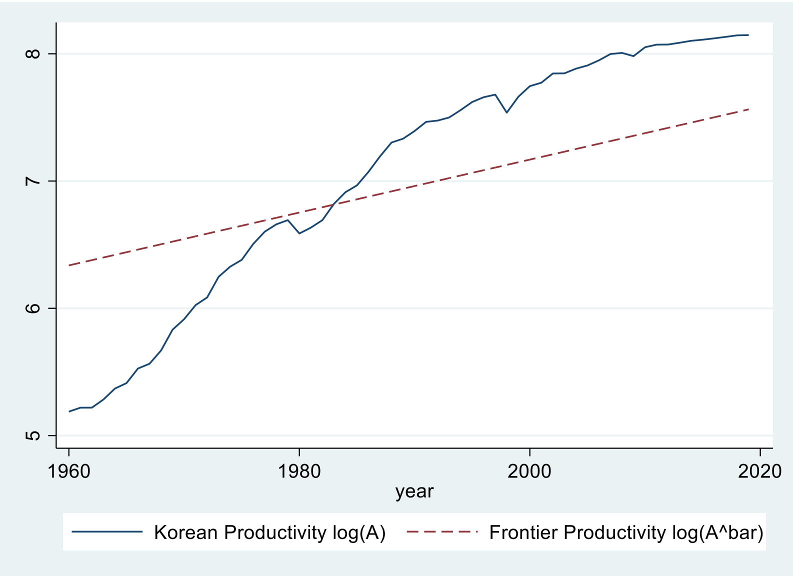

Figure 1. Identified productivity terms, South Korea versus the world frontier

[4] It is not feasible to calculate confidence intervals for this estimate since it follows from a deterministic identification.
[5] While these figures seem exceptionally high at first glance, the reader should recall that these aggregative figures originate from a two-sector model, and the aggregate production technology is not of Cobb-Douglas type.

5.2. Structural parameters

The empirical strategy estimates five structural parameters of the model. Recall that the shift parameter μ and the spillover parameter ξ of the traditional technology are estimated via the regression Equation 13. Table 2 presents the estimation results for the reduced-form parameters $(cons, b)$ and the structural parameters (μ, ξ). All of these estimates satisfy the sign expectations, and the Newey-West (robust) standard errors indicate that all of them are statistically significant at 1% level. Regarding the magnitudes, it is difficult to interpret the shift parameter μ that has an unbounded support. The magnitude of ξ, on the other hand, lies within the (0,1) interval as expected, and it is larger than the value of 0.75 that is used by Lucas (2009). Put differently, the South Korean economy has a stronger modern-to-traditional productivity spillover that characterizes the baseline economy in Lucas (2009).[6]

Note that the null hypothesis of joint residual normality can be rejected with a *p value* less than 1% while its skewness still fits a Gaussian distribution well. Results of various unit root tests (not reported here for space considerations) indicate that the residual term has a unit root. These together imply that the residual term carries a stochastic trend that the theoretical model does not capture well. A more detailed structural model would be useful in achieving efficient estimates of the structural parameters, but this is left for future research.

Table 2. Structural estimates of $\mu > 0$ and $\xi \in (0, 1)$

Reduced-form estimates of (13)		*Structural estimates*	
Intercept $(cons)$	9.1794***	Mu (μ)	39.3218***
	(0.7455)		(11.7261)
Slope (b)	0.6183***	Xi (ξ)	0.8473***
	(0.1049)		(0.0420)
		Residual Normality	
# Observations	60	Skewness *p value*	0.3902
R-squared	0.823	Kurtosis *p value*	0.0003
F stat. *p value*	0.000	Joint *p value*	0.0037

Notes: This table collects the reduced-form estimates of (13) and structural estimates of $\mu > 0$ and $\xi \in (0,1)$. These structural estimates are exactly identified. The Newey-West standard errors are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 3 collects the second set of structural parameter estimates and model diagnostics. Here, the estimating equation is Equation 14, and the estimated parameters are (ϕ, ζ, θ). The table reports estimation results for restricted models $\zeta = 0$ and $\theta = 0$, and the general model. For each of these three models, a specification with a dummy variable that controls for large contractions in productivity is estimated as well.[7] It should also be noted that all of these specifications are still restricted in the sense that the exponent of human capital h_t is equal to unity; specifications that allowing for this exponent to be different than unity do not fit the South Korean data.

Among the three specifications, the one that returns the minimum values for Akaike and Bayesian Information Criteria is the model that excludes the urban agglomeration mechanism. That is, results suggest that productivity growth is driven by human capital and relative productivity. When the agglomeration mechanism of Lucas (2009) is included through $(1 - \ell_t)^{\zeta}$, the parameter ζ is not statistically significant at 5%.

Table 4 summarizes the marginal effects of human capital and relative productivity on productivity growth rate using the preferred specification of the estimated model. As expected, both human capital and relative productivity create statistically significant marginal effects. An increase in human capital creates a very large marginal effect on the gross growth rate. Human capital h_t is equal to 1.59 and 3.77 in 1960 and 2019, respectively, and a unit increase has a marginal effect of around 1.00 on G_t whose sample average is equal to 1.053 and sample standard deviation is equal to 0.056. The marginal effect −0.43 of relative productivity a_t on gross growth rate G_t of productivity is also sizable; the sample range of relative productivity is nearly one half.

[6] Attar (2018) uses a minimum distance algorithm to calibrate ξ for South Korea within a similar setup and obtains $\xi = 0.8281$.

[7] Parameter estimates are similar in sign, in magnitude, and in statistical significance when the model excludes this dummy variable.

Table 3. Structural estimates of $\phi > 0$, $\zeta > 0$, and $\theta \in (0,1)$

Dependent Variable: Gt = At+1/At Nonlinear Least Squares Estimates			
	(1)	*(2)*	*(3)*
Phi (ϕ)	0.2716***	0.4127***	0.5007***
	(0.0057)	(0.0042)	(0.0525)
Zeta (ζ)	−0.7528***		0.3496*
	(0.0311)		(0.1948)
Theta (θ)		0.4347***	0.6335***
		(0.0154)	(0.1097)
DV(G < 0.95)	Yes	Yes	Yes
# Observations	59	59	59
Adj. R-squared	0.9930	0.9945	0.9946
RMSE	0.0881	0.0781	0.0776
log(Likelihood)	61.1238	68.2458	69.1773
Akaike IC	−116.2476	−130.4916	−130.3547
Bayesian IC	−110.0150	−124.2590	−122.0445
Residual Normality			
Skewness *p value*	0.2487	0.1560	0.2568
Kurtosis *p value*	0.0344	0.3495	0.6076
Joint *p value*	0.0622	0.2199	0.4462

Notes: This table collects structural estimates of $\phi > 0$, $\zeta > 0$, and $\theta \in (0,1)$. These structural estimates are exactly identified. DV stands for a dummy variable that takes the value of unity for years in which there is a large contraction in productivity. Robust standard errors are reported in parentheses.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 4. Marginal effects of h and a on G

Specification (2)	Human capital	$\partial G/\partial h$	1.00403***
			(0.00073)
	Relative productivity	$\partial G/\partial a$	−0.43645***
			(0.01536)

Notes: This table collects estimates of marginal effects in the second specification of productivity growth estimates. Standard errors calculated via the delta method are reported in parentheses.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

5.3 Sources of Productivity Growth in South Korea

Figure 2 pictures the baseline decomposition results and 95% confidence intervals. Productivity growth is simply decomposed into its time-varying components using Equation 16 at the baseline parameter estimates reported in Table 3. The second specification with $\zeta = 0$ is adopted as the preferred model. The shares shown in the vertical axes are in percentage terms. The shaded areas show the 95% confidence intervals. The top panel shows the contribution of the fishing-out effect, the middle panel shows the contribution of relative productivity, and the bottom panel shows the contribution of human capital.

From 1960 to 1983, relative productivity satisfies $a_t < 1$, and productivity growth is mainly due to technology adoption. In the beginning of this episode, human capital and the distance to the frontier have almost equal shares, nearly 50%. In time, the contribution of the distance to the frontier decreases since the distance itself decreases as the economy converges to the frontier. The share of human capital thus increases from 1960 to 1983. In the year 1982, the share of human capital is almost 100% and the corresponding share of the distance to the frontier is almost nil.

From 1983 to the end of the sample, relative productivity satisfies $a_t > 1$, and productivity growth is mainly due to innovation. By definition, the contribution of the fishing-out effect is always negative since a larger value of a_t exceeding unity implies a lower likelihood in introducing new products and new processes. The negative contribution of the fishing-out effect to productivity growth levels out around −30%.

Results presented above are within the limits of theoretical expectations, and the overall message originating from the analysis is consistent with the established view of the catching up/falling behind literature. Human capital and the distance to the frontier jointly explain the acceleration-slowdown pattern of productivity growth. Technology adoption stops at finite time, and the initially laggard South Korean economy becomes an innovator.

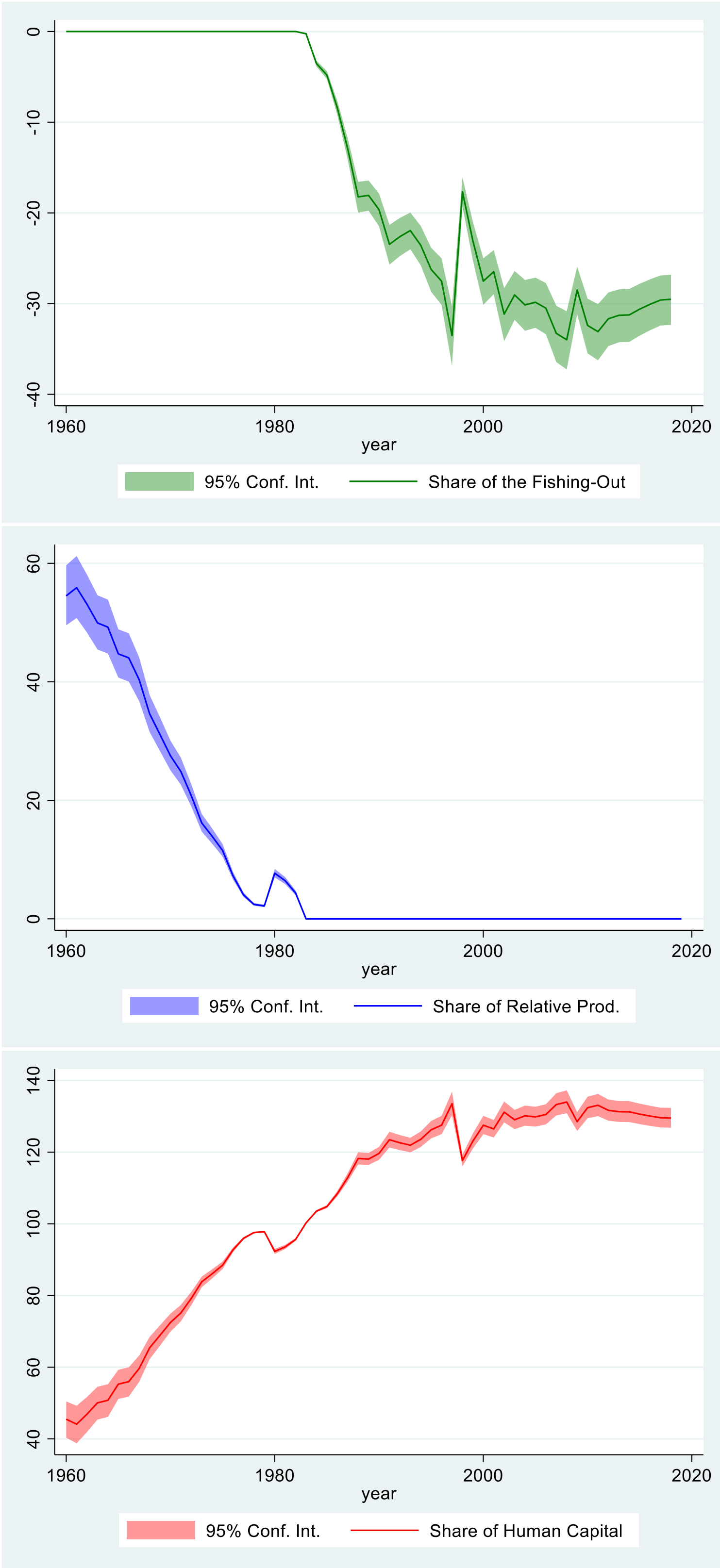

Figure 2. Decomposing productivity growth into its sources

6. ROBUSTNESS

This short section presents the results of two robustness checks.[8] It studies the effects of changes in two arbitrary model inputs. These are;

- the percentage growth rate γ of frontier productivity $\bar{A}_t$, and
- the capital share λ of the modern technology.

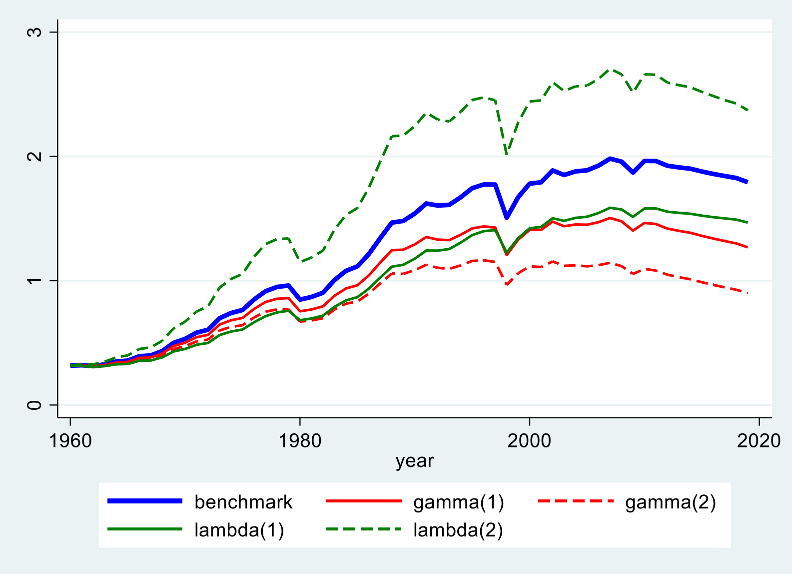

Figure 3. Alternative identifications of relative productivity in South Korea

For frontier growth rate γ, it is difficult to motivate a value other than the US long-run growth rate. But there exists a particular value for this parameter as suggested by Stokke (2004); $\gamma(1) = 0.027$. This is 0.6 percentage points larger than the benchmark value of 2.1% per annum, and the second experimented value is set to $\gamma(2) = 0.021 + 2 \times 0.6 = 0.033$. For λ, the capital share of the modern technology, the experimented values are $\lambda(1) = 0.3$ and $\lambda(2) = 0.5$.

The full set of results are pictured in Figure 3 that shows all of the experimented sequences of relative productivity as well as the benchmark sequence obtained in the baseline analysis. The main message originating from Figure 3 is that, from a qualitative perspective and except for the cases of $\lambda = \lambda(2) = 0.5$, the evolution of relative productivity is not sensitive to arbitrary model inputs. That is, it still is characterized by an initial acceleration and a later slowdown, and it still exceeds unity in the early 1980s. For the case of $\lambda = \lambda(2)$ that implies a rather large capital share in the modern sector (and a corresponding low labor share in the traditional sector), the economy passes the threshold of unity at an unrealistically early date.

The question of robustness then becomes whether the case of $\lambda = \lambda(2)$ is realistic. It should be noted that a modern sector capital share of 1/2 is perhaps too large since the aggregate labor share reported in the Penn World Tables of Feenstra et al. (2015) averages to 0.57 for the 1960-2019 period.

7. CONCLUSION

One of the most remarkable economic transformations of the postwar period was observed in South Korea. In only a few decades, the South Korean economy converged to the developed world as a result of rapid economic growth, integrated with the global value chains as a result of product diversification, and became a locus of innovation that defines the world technology frontiers in various industries.

This paper constructs a model of endogenous technology to shed new light on the South Korean miracle. The model is a two-sector catching up model where the sector that uses the modern technology adopts frontier technologies, and the sector that uses the traditional technology benefits from productivity spillovers. In time, the technology gap with the frontier closes, and the economy becomes an innovator that starts characterizing the world frontier.

[8] For space considerations, the results documented here focus only on the evolution of relative productivity. The full set of sensitivity results is available upon request.

The analysis presented in this paper devises an empirical strategy that identifies and estimates the structural parameters, and the evolution of productivity relative to the frontier. Then, the strategy allows the econometric estimations of identified structural parameters that characterize the traditional technology of production and the adoption/innovation technology of productivity growth.

Results substantiate the conventional notion that an economy that achieves fast productivity growth eventually stops technology adoption and becomes an innovation economy. Findings also reveal that, in the case of the South Korean miracle, human capital (per person) and the distance to the frontier (measured by the inverse of relative productivity) are significant drivers of productivity growth. In fact, since the South Korean economy exhibited not-so-slow productivity growth even as an innovator economy after the early 1980s, we learn that human capital accumulation was a crucial factor that trivialized the adverse fishing-out effect.

The logic of the model economy studied in this paper implies that, as an innovator economy, South Korea will remain one of the top-performing countries shaping the world technology frontiers. The observed innovation record of South Korea in the last two decades indeed shows that, among her closer peers, South Korea ranks at the top categories, especially with respect to the innovation outputs.

But does this mean that the research policy problem for South Korea has been effectively solved and should no longer be a primary policy concern? The correct answer is possibly negative. The findings presented above show that sustaining sufficiently larger relative productivity levels among the world's frontier economies requires sufficiently high human capital stocks per person. This means that a top-performing innovating economy should keep diversifying the skill content of her human capital stocks since the quantity of human capital would eventually converges to its maximum. That is, the critical task is to enlarge the innovative capacity of high-skilled researchers especially in the frontier technologies and associated, technologically-complex products. According to Harvard University's *Atlas of Economic Complexity*, South Korea has already achieved a relatively high economic complexity for its export products, and growth opportunities lie in developing and exporting new high-complexity products and processes. It is in this respect crucial for South Korea to focus on the quality of human capital and the productivity of R&D technologies. The county's projected 2.8% real GDP growth rate per annum for the next decade requires policymakers to respond to such challenges in terms of innovation inputs.

Acknowledgements
I wish to thank two anonymous reviewers and various seminar, conference, and workshop participants for their helpful comments and suggestions on earlier versions of this paper. I am responsible for all of the remaining errors and weaknesses.

Conflict of Interest
No potential conflict of interest was declared by the author.

Funding
Any specific grant has not been received from funding agencies in the public, commercial, or not-for-profit sectors.

Compliance with Ethical Standards
It was declared by the author that the tools and methods used in the study do not require the permission of the Ethics Committee.

Ethical Statement
It was declared by the author that scientific and ethical principles have been followed in this study and all the sources used have been properly cited.

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APPENDIX

Derivation of Equation 5:

The allocation problem is to maximize

$$\mu A_t^{\xi} h_t^{1-\lambda}[L_t - L_t(M)]^{1-\lambda} + K_t^{\lambda}[A_t h_t L_t(M)]^{1-\lambda} \tag{A1}$$

by choosing $L_t(M)$. The first-order necessary condition (FONC) for an interior optimum is

$$-(1-\lambda)\mu A_t^{\xi} h_t^{1-\lambda}[L_t - L_t(M)]^{-\lambda} + (1-\lambda)K_t^{\lambda}(A_t h_t)^{1-\lambda}[L_t(M)]^{-\lambda} = 0 \tag{A2}$$

After some arrangements, this FONC implies

$$K_t^{\lambda} A_t^{1-\lambda} = \mu A_t^{\xi}\left[\frac{L_t - L_t(M)}{L_t(M)}\right]^{-\lambda} \tag{A3}$$

After further arrangements, we have

$$1 = \left(\frac{\mu A_t^{\xi+\lambda-1}}{K_t^{\lambda}}\right)\left[\frac{L_t(M)}{L_t - L_t(M)}\right]^{\lambda} = \left(\frac{\tilde{A}_t}{K_t}\right)\left[\frac{L_t(M)}{L_t - L_t(M)}\right] \tag{A4}$$

Defining $\ell_t(M) = L_t(M)/L_t$ and $\ell_t(T) = L_t(T)/L_t$ and recalling the definition of $\tilde{A}_t$, this equation implies

$$1 - \ell_t(T) = \left(\frac{\tilde{A}_t}{K_t}\right)\ell_t(M) \tag{A5}$$

Since we also have $\ell_t(M) + \ell_t(T) = 1$, we obtain

$$\ell_t(T) = \left(\frac{\tilde{A}_t}{K_t}\right)[1 - \ell_t(T)] \tag{A6}$$

directly implying

$$\ell_t(T) = \frac{\tilde{A}_t}{K_t + \tilde{A}_t} = \ell_t \tag{A7}$$

Derivation of Equation 11:

Notice that real GDP can be written as in

$$Y_t = \mu A_t^{\xi} h_t^{1-\lambda}(\ell_t L_t)^{1-\lambda} + K_t^{\lambda}[A_t h_t(1-\ell_t)L_t]^{1-\lambda} \tag{B1}$$

$$\frac{Y_t}{L_t} = \mu A_t^{\xi} h_t^{1-\lambda}\ell_t^{1-\lambda}L_t^{-\lambda} + K_t^{\lambda}[A_t h_t(1-\ell_t)]^{1-\lambda}L_t^{-\lambda} \tag{B2}$$

Define real GDP per capita as $y = Y/L$. Then, we have

$$y_t = h_t^{1-\lambda}\left\{\mu A_t^{\xi}\ell_t^{1-\lambda}L_t^{-\lambda} + K_t^{\lambda}[A_t(1-\ell_t)]^{1-\lambda}L_t^{-\lambda}\right\} \tag{B3}$$

Substituting ℓ_t and $1-\ell_t$, we obtain

$$y_t = h_t^{1-\lambda}\left\{\mu A_t^{\xi}\left(\frac{\tilde{A}_t}{K_t+\tilde{A}_t}\right)^{1-\lambda}L_t^{-\lambda} + K_t^{\lambda}\left[A_t\left(\frac{K_t}{K_t+\tilde{A}_t}\right)\right]^{1-\lambda}L_t^{-\lambda}\right\} \tag{B4}$$

$$y_t = h_t^{1-\lambda}\left\{\mu A_t^{\xi}\left(\frac{\tilde{A}_t}{K_t+\tilde{A}_t}\right)^{1-\lambda}L_t^{-\lambda} + A_t^{1-\lambda}K_t^{\lambda}\left(\frac{K_t}{K_t+\tilde{A}_t}\right)^{1-\lambda}L_t^{-\lambda}\right\} \tag{B5}$$

$$y_t = h_t^{1-\lambda}\left\{\mu A_t^{\xi}\left(\frac{\tilde{A}_t}{K_t+\tilde{A}_t}\right)^{1-\lambda}L_t^{-\lambda} + A_t^{1-\lambda}\left(\frac{1}{K_t+\tilde{A}_t}\right)^{1-\lambda}K_t L_t^{-\lambda}\right\} \tag{B6}$$

$$y_t = \left(\frac{h_t}{K_t+\tilde{A}_t}\right)^{1-\lambda}L_t^{-\lambda}\left\{\mu A_t^{\xi}(\tilde{A}_t)^{1-\lambda} + A_t^{1-\lambda}K_t\right\} \tag{B7}$$

After some arrangements with $(\tilde{A}_t)^{1-\lambda} = \left(\mu A_t^{\xi+\lambda-1}\right)^{\frac{1-\lambda}{\lambda}}$, we get

$$y_t = \left(\frac{h_t}{K_t+\tilde{A}_t}\right)^{1-\lambda}L_t^{-\lambda}\left\{\mu^{\frac{1}{\lambda}}A_t^{\frac{\xi}{\lambda}}A_t^{\frac{-(1-\lambda)^2}{\lambda}} + A_t^{1-\lambda}K_t\right\} \tag{B8}$$

Factoring out $A_t^{1-\lambda}$ returns

$$y_t = \left(\frac{h_t}{K_t+\tilde{A}_t}\right)^{1-\lambda} L_t^{-\lambda} \left\{ \frac{\mu^{\frac{1}{\lambda}} A_t^{\frac{\xi}{\lambda}} A_t^{\frac{-(1-\lambda)(1-\lambda)}{\lambda}}}{A_t^{1-\lambda}} + K_t \right\} A_t^{1-\lambda} \quad \text{(B9)}$$

and, given the explicit expression of $\tilde{A}_t = \mu^{\frac{1}{\lambda}} A_t^{\frac{\xi}{\lambda}} A_t^{-\frac{(1-\lambda)}{\lambda}}$, further arrangements allow us to write

$$y_t = \left(\frac{h_t}{K_t+\tilde{A}_t}\right)^{1-\lambda} L_t^{-\lambda} \left(\tilde{A}_t + K_t\right) A_t^{1-\lambda} \quad \text{(B10)}$$

$$y_t = A_t^{1-\lambda} h_t^{1-\lambda} \left(\frac{\tilde{A}_t}{L_t} + \frac{K_t}{L_t}\right)^{\lambda} \quad \text{(B11)}$$

Exact identification of μ and ξ:

Let $cons$ and b represent the intercept and slope coefficients of the ordinary least squares regression

$$\ln\left(\tilde{A}_t\right) = \ln\left(\mu^{1/\lambda}\right) + \left[\frac{\xi-(1-\lambda)}{\lambda}\right] \ln(A_t) + u_t \quad \text{(C1)}$$

Then, we have

$$\ln\left(\mu^{\frac{1}{\lambda}}\right) = cons \quad \Rightarrow \quad \mu^{\frac{1}{\lambda}} = \exp(cons) \quad \Rightarrow \quad \mu = [\exp(cons)]^{\lambda} \quad \text{(C2)}$$

and

$$\frac{\xi-(1-\lambda)}{\lambda} = b \quad \Rightarrow \quad \xi-(1-\lambda) = \lambda b \quad \Rightarrow \quad \xi = \lambda b + (1-\lambda) \quad \text{(C3)}$$