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A review on the current applications of genetic algorithms in mean-variance portfolio optimization

Ortalama-varyans portföy optimizasyonunda genetik algoritma uygulamaları üzerine bir literatür araştırması

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Abstract

Mean-variance portfolio optimization model, introduced by Markowitz, provides a fundamental answer to the problem of portfolio management. This model seeks an efficient frontier with the best trade-offs between two conflicting objectives of maximizing return and minimizing risk. The problem of determining an efficient frontier is known to be NP-hard. Due to the complexity of the problem, genetic algorithms have been widely employed by a growing number of researchers to solve this problem. In this study, a literature review of genetic algorithms implementations on mean-variance portfolio optimization is examined from the recent published literature. Main specifications of the problems studied and the specifications of suggested genetic algorithms have been summarized.

Keywords: Portfolio management and optimization, Mean-variance model, Evolutionary algorithms, Genetic algorithm

Öz

Markowitz'in ortaya koymuş olduğu ortalama-varyans portföy optimizasyonu, portföy yönetimi problemine temel bir cevap vermiştir. Bu model, getirinin en büyüklenmesi ve riskin en küçüklenmesi gibi iki çakışan amaç arasındaki en iyi ödünleşimi ile bir etkin sınır aramaktadır. Bir etkin sınır belirleme probleminin NP-Zor olduğu bilinmektedir. Problemin karmaşıklığı nedeniyle, giderek artan sayıda araştırmacı bu problemi çözmek için genetik algoritmaları kullanmışlardır. Bu çalışmada, mevcut literatürdeki genetik algoritmaların ortalama-varyans portföy optimizasyonu uygulamaları incelenmiştir. Çalışılmış olan problemlerin ana özellikleri ve önerilen genetik algoritma karakteristikleri özetlenmiştir.

Anahtar kelimeler: Portföy yönetimi ve optimizasyonu, Ortalama-varyans modeli, Evrimsel algoritmalar, Genetik algoritma

1 Introduction: Portfolio optimization and mean-variance model

Investors' desire is to have a non-decreasing fund even if the market is losing value. It is not always possible to achieve this by investing on only one security. In a financial market, it is rare that all securities gain or lose value at the same time. Therefore, an investor should use a diversification strategy, such as forming a portfolio, to spread the risk among assets. The main question in portfolio management is to decide on the assets and weights for a better investment. A fundamental answer to the problem of portfolio management was given by the mean–variance model [1],[2]. Mathematical formulation of unconstrained portfolio optimization problem (UCPO) according to Markowitz's standard mean-variance approach is given as follows where parameter N represents the number of available assets, μ_i represents the expected return of asset i, σ_{ij} represents the covariance between asset and asset i and asset j, R^* represents the expected return at the desired level and variable w_i represent the proportion of asset i:

$$min \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{1}$$

Subject to:

$$\sum_{i=1}^{N} w_i \mu_i = R^* \tag{2}$$

$$\sum_{i=1}^{N} w_i = 1 \tag{3}$$

$$0 \leq w_i \leq 1, \qquad i = 1, \dots, N \tag{4}$$

Equation (1) minimizes risk of the portfolio while equation (2) ensures that expected return (R^*) is at the desired level. Equation (3) guarantees that proportions add to one while the proportion of an asset neither can be less than zero nor can be greater than one (Equation (4)). In practice, it is possible to calculate an optimal solution for a particular data set with this formulation. Solving this formulation by varying values of the expected return, an efficient frontier can be found as a non-decreasing curve. This frontier represents the balance of expected return corresponding to risk that must be accepted. Figure 1 demonstrates a standard efficient frontier.

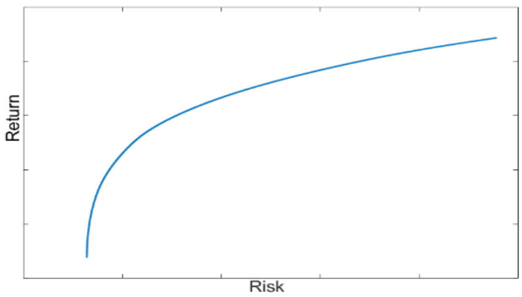

Figure 1: An example of standard efficient frontier.

The mean-variance model has been extended throughout the decade by introducing additional real-world constraints such as the cardinality constraints that impose a predetermined limit on the number of assets (K) to be held in the portfolio and the quantity constraints which restrict the proportion of each asset in the portfolio to satisfy lower (ε_i) and upper (δ_i) bounds. The mixed integer nonlinear programming formulation of cardinality constrained portfolio optimization (CCPO) problem is given as follows with additional parameters; K representing the desired number of assets to be hold in the portfolio, ε_i representing the minimum proportion of asset i, δ_i representing the maximum proportion of asset i and z_i representing a binary variable whether or not an asset i is held in the portfolio [3]:

$$min \quad \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j\, \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} w_i \mu_i\right] \tag{5}$$

Subject to:

$$\sum_{i=1}^{N} w_i = 1 \tag{6}$$

$$\sum_{i=1}^{N} z_i = K \tag{7}$$

$$\varepsilon_i z_i \le w_i \le \delta_i z_i \quad i = 1, \dots, N \tag{8}$$

$$z_i \in (0,1) \quad i = 1, \dots, N \tag{9}$$

$$0 \le w_i \le 1, \quad i = 1, \dots, N \tag{10}$$

$$0 \le \varepsilon_i \le \delta_i \le 1, \quad i = 1, \dots, N \tag{11}$$

Equation (6) and equation (10) are inherited from the original Markowitz formulation. Equation (7) guarantees that exactly K assets are hold in the portfolio while equation (8) restricts the proportion of an asset to be between predetermined values of minimum and maximum limits with decision variable defined in Equation (9). Equation (11) defines variable domains. The quadratic objective function given in equation (5) seeks the best trade-offs between two conflicting objectives, maximizing return and minimizing risk. In the equation (5), a λ parameter is used to trace the efficient frontier by gradually increasing the value of λ from 0 to 1. Thus, a weighted sum of two objectives is obtained. The resulting single objective aims to construct the cardinality constrained efficient frontier that represents the best balance of expected return and the risk that must be accepted since both objectives cannot be simultaneously achieved. Transaction costs are also considered as additional real life constraints in the literature [4]-[7]. It is not convenient to find an optimal efficient frontier in practice when real life constraints are taken into account. In fact, calculating an optimal portfolio for the standard mean-variance model is known to be NP-hard [8] since a classical quadratic optimization problem becomes NP-hard if a single cardinality constraint is added to the formulation [9]. Therefore, in the literature, several computationally efficient solution approaches have been developed in order to calculate the efficient frontier. Among those approaches, genetic algorithms (GA) is one of the most preferred algorithm for solving the problem. Metaxiotis and Liagkouras [10] presented a review of multi-objective evolutionary algorithms applied to portfolio management problem in a broad problem perspective. In this study, however, a comprehensive review of GA applications including single and multi-objective implementations specifically in mean-variance portfolio optimization is conducted. This aim of this review is to reveal the problem specifications considered and the key strategies of GA utilized to solve mean-variance portfolio optimization problem types.

Section 2 presents the genetic algorithm implementations for mean-variance portfolio optimization while Section 3 concludes the paper with a discussion of future research directions.

2 Genetic algorithms for mean-variance portfolio optimization

GA, firstly introduced by Holland [11], is a search method that can be modified to solve complex optimization problems. In GA, a set of iterative search procedures based on biological natural selection and genetic inheritance principals is executed. A population of solutions is updated over generations using selection, crossover and mutation strategies. Each individual that is evaluated in the population represents a potential solution to the problem in hand. Individuals form new individuals a stochastic transformation of individuals is achieved by genetic operators such as crossover and mutation. Crossover provides better solutions to be constructed from good solutions by a random, yet structured change of genetic materials. The role of mutation is to obtain lost or unexplored genetic materials, thereby preventing premature convergence and stuck in local optima. After several iterations, the algorithm converges a (near) optimal solution. Basic steps of the GA are given in Table 1.

Table 1: Basic steps of GA.

Step	Procedure
1	Generate initial population.
2	Evaluate fitness of each individual in the population.
3	Select the set of individuals for applying genetic operators.
4	Apply genetic operators and evaluate new fitness values.
5	Form new generation according to fitness values.
6	Go to step 3 if termination criteria are not satisfied.
7	End evolution and report the results.

Components of a typical GA are summarized below:

- Genetic representation (Encoding strategy): The solution of the problem that is formed by binary, integer or real numbers,
- Chromosome: A solution of encoding,
- Population: A set of chromosomes,
- Fitness: A function that evaluates how good a solution is,
- Genetic operators: Procedures such as crossover and mutation that provide to obtain new population from the current population,
- Control parameters: Input parameters such as population size, crossover and mutation rates.

Goldberg [12] pointed out search and optimization applications of GA in different areas. Efficient portfolio selection is one of the main concerns of researchers who practice in financial optimization domain. One of the most preferred solution approach for portfolio optimization is GA. Several researchers applied GA variants for solving portfolio optimization problems since 1998 [13]. In this study, 44 articles published in conferences and refereed journals between 1998-2016 are examined. Figure 2 shows the number of papers published in recent years for GA implementation on mean-variance portfolio optimization with respect to publication years. Among these studies published in the literature, 3 types of problems; UCPO, CCPO and Portfolio

Optimization with Transaction Costs (POTC) come into prominence among other types (See Figure 3).

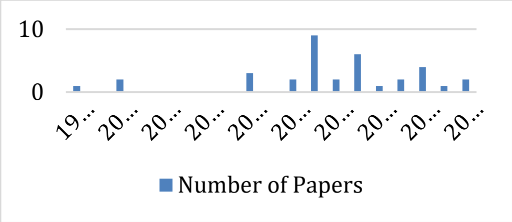

Figure 2: Number of papers published between 1998-2016 for GA implementation on mean-variance portfolio optimization.

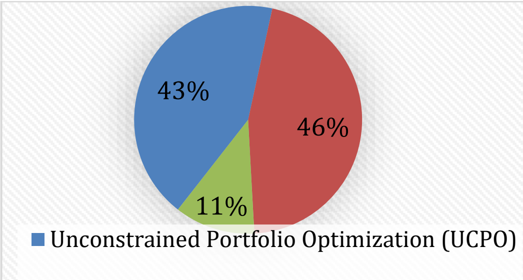

Figure 3: Problems types studied in mean-variance portfolio optimization.

Studies in the literature can be classified in many ways. In this review, two classification schemes for studies that apply GA to mean-variance portfolio optimization are used. First classification is formed according to the problem specifications such as data type, compared methods, problem type and coded programming language. Table 2 provides an up-to-date list of problem specifications. As summarized in Table 2, experimental settings are generally carried out on either real world applications or hypothetical data sets for benchmarking purposes. Methods reported in the literature are generally compared against other metaheuristics either taken from the literature or coded by the authors themselves. Literature analysis show that most of the problems types considered so far consist of UCPO, CCPO and POTC. Most of recent studies focused on CCPO and POTC while UCPO provided a basis for other types of problems.

The second classification scheme is formed according to applied algorithm specifications such as the generation methodology of initial population, size of the population, chromosome representation, crossover type and rate, mutation type and rate, type of selection mechanism, survival type, feasibility construction and termination criteria. Table 3 provides an up-to-date list of algorithm specifications. As summarized in Table 3, single objective GA are widely applied while some multi-objective GA are also suggested for mean-variance portfolio optimization. A two-stage GA is employed in [14] that firstly identifies good quality assets in terms of asset ranking and then optimizes investment allocation in the selected good quality assets. Some hybrid strategies are also suggested as in [15] that utilize quadratic programming approach with GA, in [16] that combines GA with simulated annealing approach and in [17] that utilizes a position displacement strategy of the particle swarm optimization methodology with GA.

GA implementations in the literature shows that initial population is widely preferred to be randomly generated while just a few studies [15],[18] employed heuristic approaches for the construction of initial population. In the studies examined, population size (PS) parameter is set to be 20 as minimum and 2000 as maximum. However, most of the researchers zoomed in the range of 100 and 300 for PS parameter. Binary and real valued chromosome representation are observed to be popular although some other representation strategies such as integer based, tree based are also utilized. Several studies differentiate from each other with the use of crossover operators such as uniform, BLX-Alpha, one point, n-point and mutation operators such as swap, one-point, Gaussian, guided, bit-flipping strategies. Tournament selection is mostly utilized while roulette wheel selection is also used. Elitism and ranking strategies are generally employed for survival of population. Feasibility of chromosomes are ensured by repair or penalty functions. It is observed that iteration number is used as the termination criterion in all of the studies examined.

Although several authors used the data as downloaded from the mentioned OR-Library to test their proposed algorithm, unfortunately, there is a limited number of papers that provide a performance comparison against other published papers in the literature.

In terms of evaluation approaches, there are two types of methodologies in the literature, namely weighted sum and pareto based approaches. As for weighted sum approach, studies make use of equation (5) given in Section 1 by combining two conflicting objectives: risk minimization and return maximization. On the other hand, pareto based methodology, especially used in multi-objective evolutionary algorithms, considers two objectives separately by systematically removing dominated solutions from the heuristic frontier during the search in solution space. Table 3 summarizes the classification.

3 Introduction

Portfolio optimization is a significant problem that intrigues investors and challenges researchers. As GA was established to be a popular technique in the optimization field, the application of GA to optimization problems related to portfolio selection has expanded since 2000 as the problem is known to be NP-Hard. Two different classifications are introduced. Firstly, the main specifications of the problems were summarized, and then implemented GAs with chromosome representations, genetic operators and the fitness functions used for performance evaluation were discussed. 44 articles were examined and grouped in chronological order.

Although, there are several implementations of GA for mean-variance portfolio optimization problem, unfortunately, the improvements and enhancements made to the algorithms' main framework is not evidently noticeable since there is a limited number of papers that provide a benchmark based comparison against other published studies in the literature.

Therefore, future studies should definitely consider such a comparison that may lead the way towards a better algorithmic design and related software implementations.

Furthermore, it would be very helpful to analyze and compare other heuristics, exact solution approaches as well as metaheuristics applied to solve this problem in a future research study. A comparison on the performance of different approaches would help researchers to move forward in the search of discovering better methodologies for solving portfolio optimization problems.

Table 2: Mean-variance portfolio optimization problem specifications.

Year	Researcher(s)	Performance specifications / Experimental Settings			
		Data	Benchmark	Methods compared	Problem type
1998	Shoaf and Foster [13]	Five stocks from various markets	-	-	UCPO
2000	T J Chang et al. [3]	Hang Seng, DAX100, FTSE100, S&P100, Nikkei	OR-Library	SA, TS	CCPO
2000	Xia et al. [4]	Six stocks	-	-	POTC
2006	Lai et al. [14]	One-hundred stocks from Shanghai market	-	-	UCPO
2006	ChiangLin [19]	Forty-two stocks from Taiwan market	-	-	CCPO
2006	Moral-Escudero et al. [15]	Hang Seng, DAX100, FTSE100, S&P100, Nikkei	OR-Library	Exact	CCPO
2008	W Chen et al. [5]	Fifteen stocks from China market	-	-	POTC
2008	Lin and Liu [6]	Taiwanese mutual fund data from the year 1997 to 2000	-	-	POTC
2009	Aranha and Iba [20]	NASDAQ100, NIKKEI200, S&P500	-	-	UCPO
2009	Branke et al. [21]	Hang Seng, S&P100, Nikkei	OR-Library	-	CCPO
2009	T-J Chang et al. [22]	Hang Seng, FTSE, S&P from January 2004 to December 2006	-	-	CCPO
2009	Li and Guo [23]	Ten stocks from the Shanghai 180 index	-	-	UCPO
2009	Loukeris et al. [24]	Thirty stocks in FTSE from 24 April 2001 to 29 December 2006	-	PSO, DE	UCPO
2009	Pai and Michel [25]	BSE200 and Nikkei225	-	-	CCPO
2009	Rong et al. [16]	Twenty stocks from Shanghai market	-	-	UCPO
2009	Shaikh and Abbas [26]	Twenty-three stocks form KSE30	-	-	UCPO
2009	Soleimani et al. [7]	500 and 2000 stocks randomly generated by MATLAB	-	-	POTC
2010	Anagnostopoulos and Mamanis [27]	FTSE100	OR-Library	NSGA-2, PESA, SPEA2	CCPO
2010	Ruiz-Torrubiano and Suarez [28]	Hang Seng, DAX100, FTSE100, S&P100, Nikkei	OR-Library	SA	CCPO
2011	Anagnostopoulos and Mamanis [29]	Hang Seng, DAX100, FTSE100, S&P100, Nikkei	OR-Library	SOEA	CCPO
2011	Anagnostopoulos and Mamanis [30]	DAX100	OR-Library	-	CCPO
2011	Fu et al. [31]	Four stocks from Hong Kong market	-	-	UCPO
2011	Y Chen et al. [32]	Five-hundred stock from Tokyo market	-	-	UCPO
2011	Kremmel et al. [33]	Software projects	-	SPEA2, NSGA-2	UCPO
2011	Woodside-Oriakhi et al. [34]	Hang Seng, DAX100, FTSE100, S&P100, Nikkei	OR-Library	TS, SA	CCPO
2012	Sadjadi et al. [35]	Hang Seng, DAX100, FTSE100, S&P100, Nikkei	OR-Library	-	CCPO
2013	Lu and Wang [18]	Six stocks from Chinese market	-	-	UCPO
2013	Yi and Yang [36]	Eight stocks from china market	-	-	UCPO
2014	Ackora-Prah et al. [37]	Randomly selected five stocks from Ghana market	-	-	UCPO
2014	Joglekar [38]	Seventy-two stocks from MSN Money	-	-	UCPO
2014	Liagkouras and Metaxiotis [39]	Hang Seng, DAX100, FTSE100, S&P100, Nikkei	OR-Library	MOEA-PLM	CCPO
2014	Lwin et al. [40]	Hang Seng, DAX100, FTSE100, S&P100, Nikkei, S&P500, Russel2000	OR-Library	NSGA-2, SPEA2, PEAS	CCPO
2015	Adebiyi Ayodele and Ayo Charles [41]	Hang Seng, DAX100	OR-Library	SA, TS, PSO	CCPO
2016	Hadi et al. [42]	Forty-five stocks from Egypt market	-	-	UCPO
2016	Mashayekhi and Omrani [43]	Fifty-two stocks from Iran market	-	-	CCPO

Table 3: Genetic algorithm specifications.

Year	Researcher(s)	Evolutionary algorithm specifications									
		Method	Evaluation approach	Initial population & size of population	Chromosome representation	Crossover type & rate (Rc)	Mutation type & rate (Rm)	Selection type	Survival type	Feasibility (Repair or Penalty)	Termination criteria
1998	Shoaf and Foster [13]	GA	Weighted sum	PS=100	Binary and real value	2-Point & Rc=60%	Rm=0.1%	Roulette wheel	-	Penalty	Iteration number
2000	T J Chang et al. [3]	GA	Pareto based	Random & PS=100	Integer and real value	Uniform & Rc=100%	One-point mutation& Rm=10%	Tournament	steady-state	Repair	Iteration number
2000	Xia et al. [4]	GA	Pareto based	Random & PS=30	Binary and real value	Uniform & Rc=30%	Heuristic & Rm=20%	Roulette wheel	-	Repair	Iteration number
2006	Lai et al. [14]	2 stage GA	Weighted sum	Random & PS=100	Integer and real value	One point crossover & Rc=50%	Rm=0.5%	Roulette wheel	-	-	Iteration number
2006	ChiangLin [19]	GA	Pareto based	Random & PS=100	Integer and real value	One point crossover & Rc=100%	Rm=3%	Roulette wheel	-	-	Iteration number
2006	Moral-Escudero et al. [15]	Hybrid(GA and quadratic programming)	Weighted sum	Heuristic & PS=100	Binary	Uniform & Rc=100%	Swap mutation & Rm=1%	Tournament	steady-state	Repair and Penalty	-
2008	W Chen et al. [5]	GA	Pareto based	Random & PS=30	Real value	One point crossover & Rc=variable	Uniform & Rm=variable	Roulette wheel	-	Repair	Iteration number
2008	Lin and Liu [6]	GA	Weighted sum	Random & PS=n	Integer and real value	One point crossover & Rc=100%	One-point mutation & Rm=5%	Roulette wheel	Replacement	Penalty	Iteration number

Table 3: Cont.

Year	Researcher(s)	Evolutionary algorithm specifications									
		Method	Evaluation approach	Initial population & size of population	Chromosome representation	Crossover type & rate (Rc)	Mutation type & rate (Rm)	Selection type	Survival type	Feasibility (Repair or Penalty)	Termination criteria
2009	Aranha and Iba [20]	Tree based GA	Weighted sum	PS=200	Tree based	Best-Worst Sub-tree Crossover & Rc=80%	Swap mutation & Rm=3%	Tournament	Ranking	-	Iteration number
2009	Branke et al. [21]	envelope-based MOEA	Pareto based	Random & PS=30	Binary and real value	Uniform &	Swap mutation	Tournament	Ranking	-	Iteration number
2009	T-J Chang et al. [22]	GA	Pareto based	Random & PS=100	Binary and real value	Uniform & Rc=100%	One-point mutation & Rm=100%	Tournament	steady-state	Repair	Iteration number
2009	Li and Guo [23]	GA	Weighted sum	-	Binary and real value	Rc=100%	Rm=70%	Roulette wheel	-	-	Iteration number
2009	Loukeris et al. [24]	GA	Pareto based	Random	Binary and real value	BLX-Alpha	Bit-flipping mutation	Ranking	Ranking	-	Iteration number
2009	Pai and Michel [25]	GA	Pareto based	Random & PS=200	Binary and real value	arithmetic variable point	Real values uniform	Roulette wheel	-	Repair	Iteration number
2009	Rong et al. [16]	Hybrid (GA and SA)	Pareto based	Random & PS=20	Binary and real value	Arithmetic & Rc=80%	Non-uniform mutation & Rm=1%	-	-	Penalty	Iteration number
2009	Shaikh and Abbas [26]	GA	Weighted sum	Random & PS=20	-	-	-	Tournament	-	-	Iteration number
2009	Soleimani et al. [7]	GA	Weighted sum	Random & PS=30	-	Random separate & Rc=100%	Rm=50%	Randomly	Ranking	-	Iteration number
2010	Anagnostopoulos and Mamanis [27]	NSGA-II, PESA, SPEA2	Pareto based	Random & PS=200-300	Binary and real value	Uniform & Rc=90%	Gaussian & Rm=100%	Tournament	Elitism	Repair	Iteration number
2010	Ruiz-Torrubiano and Suarez [28]	GA	Weighted sum	Random & PS=100	Binary and real value	RAR & Rc=100%	Bit-flipping mutation & Rm=1%	Tournament	Ranking	Repair	Iteration number
2011	Anagnostopoulos and Mamanis [29]	MOEA	Pareto based	Random & PS=250	Binary and real value	Uniform & Rc=90%	Gaussian & Rm=100%	Tournament	Elitism	Repair	Iteration number
2011	Anagnostopoulos and Mamanis [30]	MOEA	Pareto based	Heuristic & PS=500	Integer and real value	Uniform	Swap mutation & Rm=1%	Tournament	Elitism	Repair	Iteration number
2011	Fu et al. [31]	GA	Pareto based	-	-	Rc=Random	Rm=Random	-	-	-	-
2011	Y Chen et al. [32]	GRA	Weighted sum	Random & PS=300	nodes and edges	Node Swap & Rc=20%	Guided & Rm=3%	Tournament	Elitism	-	Iteration number
2011	Kremmel et al. [33]	MOEA	Weighted sum	Random & PS=500	Binary	Uniform & Rc=70%	Bit-flipping mutation & Rm=100%	Tournament	Ranking	Repair	Iteration number
2011	Woodside-Oriakhi et al. [34]	GA	Pareto based	Random & PS=100	Integer and real value	Uniform & Rc=100%	Random Swap & Rm=3%	Heuristic	Ranking	-	Iteration number
2012	Sadjadi et al. [35]	GA	Weighted sum	Random & PS=10	Integer	One point crossover & Rc=80%	Swap mutation & Rm=20%	Roulette wheel and Uniform	Ranking	Repair	-
2013	Lu and Wang [18]	GA	Pareto based	Heuristic & PS=60	Integer and real value	Rc=70%	Rm=77.8%	-	-	Penalty	Iteration number
2013	Yi and Yang [36]	FGA	Weighted sum	PS=2000	-	Rc=80%	Rm=20%	-	-	-	Iteration number
2014	Ackora-Prah et al. [37]	GA	Pareto based	PS=50	Real value	Heuristic	-	Roulette wheel	Elitism	-	Iteration number
2014	Joglekar [38]	GA	Weighted sum	Random	Real value	Random & Rc=70%	Random Swap & Rm=20%	-	Elitism	-	Iteration number
2014	Liagkouras and Metaxiotis [39]	MOEA	Weighted sum	Random & PS=variable	Binary and real value	Simulated Binary	Guided	Tournament	Ranking	Repair	-
2014	Lwin et al. [40]	MOEA	Pareto based	Random & PS=100	Binary and real value	Rc=90%	Polynomial	Tournament	Elitism	Repair	Iteration number

Table 3: Cont.

Year	Researcher(s)	Evolutionary algorithm specifications									
		Method	Evaluation approach	Initial population & size of population	Chromosome representation	Crossover type & rate (Rc)	Mutation type & rate (Rm)	Selection type	Survival type	Feasibility (Repair or Penalty)	Termination criteria
2015	Adebiyi Ayodele and Ayo Charles [41]	GDE	Weighted sum	-	-	-	-	-	-	-	-
2016	Hadi et al. [42]	GA	Pareto based	Random	Real value	-	-	-	-	-	Iteration number
2016	Mashayekhi and Omrani [43]	NSGA-II	Pareto based	Random & PS=100	Binary and real value	Rc=80%	Gaussian & Rm=10%	Tournament	-	Repair	Iteration number

n: Number of stocks; MOEA: Multi objective evolutionary algorithm, SA: Simulated annealing, NSGA-II: Non-dominated sorting genetic algorithm, PESA: Pareto envelope-based selection algorithm, SPEA2: Strength pareto evolutionary algorithm 2, GRA: Genetic relation algorithm, PSO: Particle swarm optimization, FGA: Fuzzy genetic algorithm, GDE: Generalized differential evolution.

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