Pamukkale Univ Muh Bilim Derg,29(4), 370-376, 2023

Pamukkale Üniversitesi Mühendislik Bilimleri Dergisi

PamukkaleUniversity Journal of Engineering Sciences

Large amplitude free vibrations analysis of prismatic and non-prismatic different tapered cantilever beams

Prizmatik ve prizmatik olmayan farklı konik konsol kirişlerin büyük genlikli serbest titreşim analizi

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Received/Geliş Tarihi: 09.01.2022 Revision/Düzeltme Tarihi: 17.07.2022 doi: 10.5505/pajes.2022.02489
Accepted/Kabul Tarihi: 22.09.2022 Research Article/Araştırma Makalesi

Abstract

This article presents large deflection data for prismatic (unvarying cross section across length) and non-prismatic (tapered, i.e. varying cross section across length) cantilever beams subjected to concentrated tip loads using the finite element method for different taper ratios. The approximate nonlinear solution is derived from the perspective of a polynomial function. The tapered beams that correlate to the non-prismatic cantilever beams have different widths, depths, and diameters. Using the aforementioned large displacement data that have been analysed, a very simple approach is used to evaluate the large amplitude first mode frequency for the cantilever beam with (non-prismatic) tapered and without (prismatic) tapered. The current approach can be used effectively to find accurate results with far less computer capacity as compared to other methods available in the literature. The difference between the current findings and the bibliographic data is shown. The major goal of this work is to contribute to the simple description of polynomial functions for large amplitude first mode free vibration frequency problems with and without tapered beams in terms load parameter(λ) versus tip slope (α)and tip amplitude(a/L). Large amplitude first mode frequency (Ω) increases with tip slope(α). This indicates that the prismatic and non-prismatic cantilever beams exhibit hardening type nonlinearity. At a particular tip slope(α), the diameter taper shows higher frequency than other tapered beams and uniform beams. According to current studies, it can be restricted to a lower range of tip slope(α)or amplitude(a/L).

Keywords: Large amplitude, uniform (prismatic), Tapered (non-prismatic), Cantilever beam, Tip slope, Free vibration.

Öz

Bu makale, farklı koniklik oranları için sonlu elemanlar yöntemini kullanarak konsantre uç yüklerine maruz kalan prizmatik (uzunluk boyunca değişmeyen enine kesit) ve prizmatik olmayan (konik, yani uzunluk boyunca değişen enine kesit) konsol kirişler için büyük sapma verileri sunmaktadır. Yaklaşık doğrusal olmayan çözüm, bir polinom fonksiyonunun perspektifinden türetilir. Prizmatik olmayan konsol kirişlerle ilişkili olan konik kirişlerin farklı genişlikleri, derinlikleri ve çapları vardır. Yukarıda bahsedilen ve analiz edilen büyük yer değiştirme verileri kullanılarak, konik (prizmatik) ve konik olmayan (prizmatik) konik kiriş için büyük genlikli birinci mod frekansını değerlendirmek için çok basit bir yaklaşım kullanılır. Mevcut yaklaşım etkin bir şekilde kullanılabilir. literatürde mevcut olan diğer yöntemlere kıyasla çok daha az bilgisayar kapasitesi ile doğru sonuçları bulmaktır. Mevcut bulgular ile bibliyografik veriler arasındaki fark gösterilmektedir. Bu çalışmanın ana amacı, konik kirişli olan ve olmayan büyük genlikli birinci mod serbest titreşim frekansı problemleri için yük parametresi (λ)ile uç eğimi (α) ve uç genliği (a/L) cinsinden polinom fonksiyonlarının basit tanımına katkıda bulunmaktır. Büyük genlikli birinci mod frekansı (Ω) uç eğimi (α) ile artar. Bu, prizmatik ve prizmatik olmayan konsol kirişlerin sertleşme tipi doğrusal olmayanlık sergilediğini gösterir. Belirli bir uç eğiminde (α), çap konikliği, diğer konik kirişlerden ve düzgün kirişlerden daha yüksek frekans gösterir.Mevcut çalışmalara göre, daha düşük bir uç eğimi (α)veya genlik (a/L) aralığı ile sınırlandırılabilir.

Anahtar kelimeler: Büyük genlik, Üniforma (prizmatik), Konik (prizmatik olmayan), Konsol kiriş, Uç eğimi, Serbest titreşim.

1 Introduction

A particularly prevalent class of structural element that has been the focus of extensive study is the beam.It can be classified as prismatic (having a uniform or non-changing cross-section along the length) or non-prismatic (having a tapering or variable cross-section along the length), thin or thick, based on its geometry. Numerous beams are used in many engineering applications, and numerous studies on the transverse vibration of uniform isotropic beams have been published in the literature. However, when analysing usage and service, a non-uniform tapered (non-prismatic) beam[1] may have a superior mass and load intensity distribution than a uniform prismatic beam. This enables the needs of various engineering disciplines, such as aeronautical engineering, civil engineering, and other innovative engineering applications, to be met while taking into account economic and other factors. The natural vibration frequencies of such structures must be known in order to design them to withstand dynamic forces.

Linear vibration theory predicts that the frequency of natural vibration is amplitude-independent [1]-[3]. Such conclusions are often not valid when the amplitude of vibration is large, as one or another is accompanied by an absenceof linearity effect. A helpful overview of the literature on nonlinear beam vibrations has been provided by [4]. Large-amplitude oscillations of uniform beams are conducted with different support boundary conditions[5]. The geometric nonlinearity was approximated using a truncated series and the solution obtained by solving the mode equation by the averaging method using the assumed modal shape and Langrange equations[6]. The large-amplitude response of beams with varying cross-sectional areas was performed using a fourth-order iterative Runge-Kutta numerical scheme [7]. An

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alternative analytical approach has been proposed to assess the non-linear vibrational performance of tapered(non-prismatic) cantilever[8]. Fully constrained tapered (non-prismatic) beams experiencing large-amplitude free vibrations caused by nonlinear elastic foundations have been reported [9].

A simple relationship for the linear taper cantilever beam to determine its first mode linear natural frequency is presented. [10] depicts the relationship between a beam's linear frequency expressed as a function of beam stiffness, beam mass, and mass distribution parameters. A study was successfully proposed to obtain the large amplitude first mode of the uniform (Figure1) prismatic cantilever [11] and spring hinge cantilever [12]. In both works [11],[12] the linear stiffness of [10] was replaced by nonlinear stiffness.

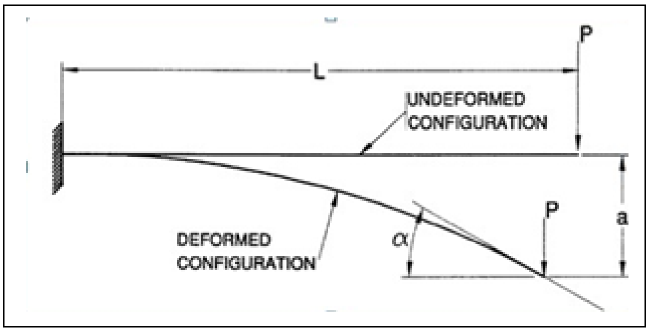
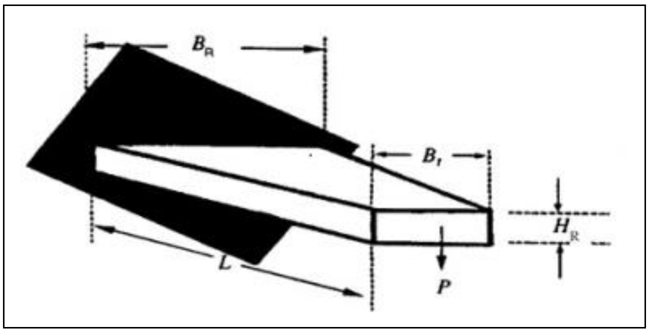


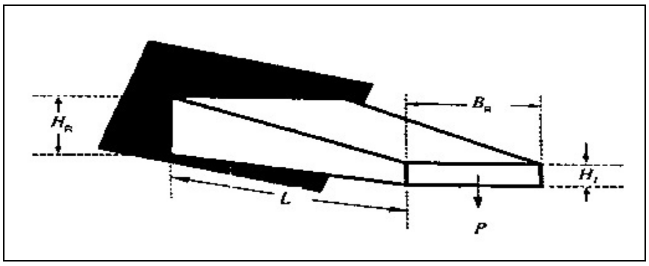
Figure 1. Prismatic (unvarying cross section along length) uniform cantilever beam experiencing large amplitudes [11]. L is length of beam; P is vertical tip load; a is tip lateral deflection; α is tip slope.

This article presents a study to provide the large-amplitude first mode frequency for the non-prismatic(tapered beam) cantilever (Figure 2(a), (b), (c) in a simple approach, following the previously published [11],[12] approach with modification(nonliner stiffness in place of linear stiffness) of the methodology of [10]. Nonlinear stiffness is related to tip amplitude (a/L), tip slope (α), and load parameter (λ). In the literature, the nonlinear stiffness of uniform beams can be found using the relationship between tip amplitude, tip slope, and load parameters available in article [11],[13]. However,for tapered beams, these above relationships are not available in the literature. To evaluate nonlinear stiffness, nonlinear(geometric)static parametric studies through FEM[14]modeling have been carried out to derive a set of polynomial functions with different taper ratios for the different tapered types cantilever beams in terms of the load parameters $(\lambda = PL^2/EI)$and tip amplitudes(a/L), and load parameters and tip slopes(α). These polynomial functions are obtained for the prismatic and non-prismatic (tapered) cantilever beam with tip concentrated load. Based on the outcomes of the load-deflection data, the first mode nonlinear frequency is estimated by the use of the easy relation modified [11] of the tip slope in a range. Compared to the result in [7], this result shows the effectiveness of the proposed simple method. The aforementioned approach is the original aspect of the present study and contribution to the literature.

The present study is of practical interest as it applies to an ideal aeroplane wing with a tip tank or engine, gas turbine fans, and wind turbine blades, etc.



(a)



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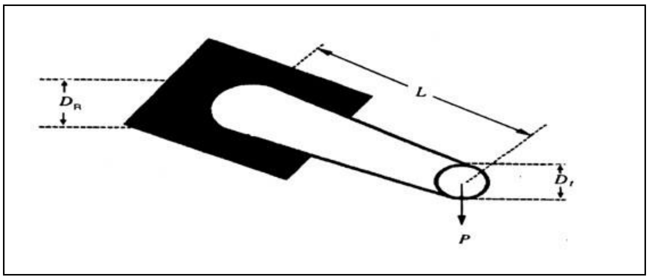
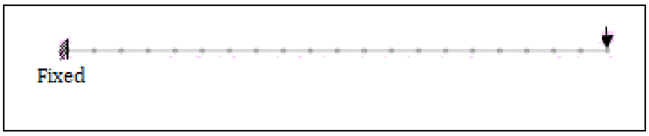


(c)

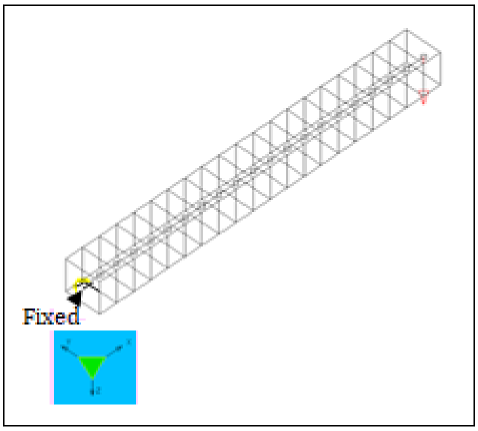
Figure 2. Non-prismatic (tapered) cantilever beam [10] experiencing large amplitudes: (a): Tapered in breadth, constant in depth. (b):Tapered in depth, constant in breadth (c):Tapered in diameter.

2 Polynomial functions for different taper ratios for large displacement under tip load of a tapered (non-prismatic) cantilever using FEM

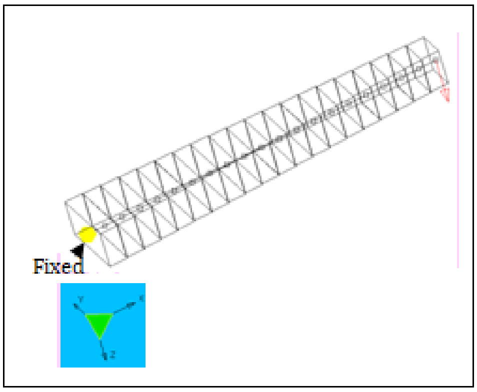
A tapered cantilever beam (linear tapered in breadth (B) taper, depth constant; linear tapered in depth (H), breadth constant and linear tapered in diameter (D)) of length L undergoing large amplitude with tip concentrated load(P) is shown in Figure 2(a), (b), and (c). The approximate nonlinear solutions in the form of polynomial functions are obtained using finite element software[14]. For analysis, 2 node(6 dof per node) prismatic 3D general beam elements (NKTP=39) are used for analysis. The formulation includes stretching and bending and accounts for geometric nonlinear(large displacement, large strain) behavior. The beam is based on the Kirchhoff assumption, so no transverse shear deflection effect is considered. Figure 3(a) shows the finite element model for a cantilever beam with elements and nodes(total elements=20 and nodes=21). Figure 3(b) shows the prismatic (uniform) cantilever beam. Figure 3(c) shows the varying cross-section B_t/B_R(breadth taper)=0.2. Figure 3(d) depicts a varying cross-section H_t/H_R(depth taper)=0.2. Figure 3(e) shows the varying cross-section D_t/D_R(diameter taper)=0.2. For clarity of modeling, the author only shows the varying cross-section for a root to tip ratio of 0.2. However, all ratios from 0.2 to 0.8 are modelled by writing a NISA programming code (BMSEC), and section properties are supplied to the software in the property card of the beam.



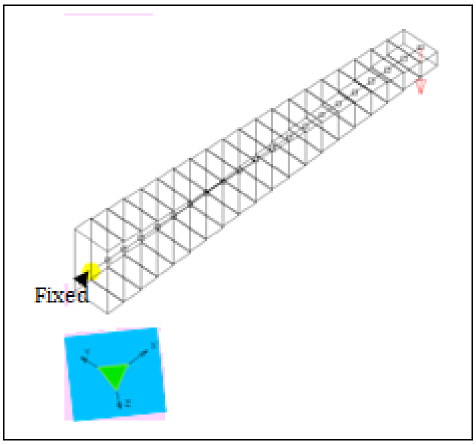
(a): Cantilever beam showing elements, nodes, boundary conditions and loads.



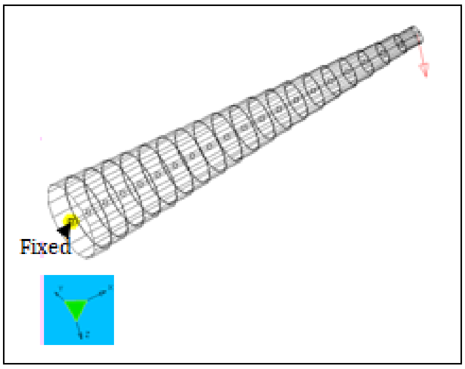
(b): Prismatic (uniform) cantilever beam.



(c): Non-prismatic cantilever beam: Breadth tapered and depth constant.



(d): Non-prismatic cantilever beam: depth tapered and breadth constant.



(e): Non-prismatic cantilever beam: diameter tapered.

Figure 3(a): Finite element model for cantilever beam showing element and node (total element= 20 and nodes=21). (b): Prismatic (uniform) cantilever beam, (c): Varying cross-section (breadth taper) B_t/B_R =0.2, (d): Varying cross-section (depth taper) H_t/H_R =0.2, (e): Varying cross-section (diameter taper)D_t/D_R =0.2.

In this work, author has followed the concept of [10] and applied it to a tapered cantilever subject to large displacement to find the frequency of the first mode. Nonlinear stiffness is used here instead of linear stiffness. Nonlinear stiffness is related to tip amplitude (a/L), tip slope (α), and load parameter (λ). In the literature, the nonlinear stiffness of uniform beams can be found using the relationship between tip amplitude, tip slope, and load parameters available in article [13],[11]. However, for tapered beams, these above relationships are not available in the literature. To evaluate nonlinear stiffness, geometric nonlinear analysis parametric studies have been performed using FEM program (NISA-II) for prismatic (uniform) and non-prismatic cantilever (different taper ratios and different taper types) beams. Based on parametric studies, the relationship between amplitude (a/L) with load parameter (λ) and tip slope (α) with load parameter (λ) is proposed in terms of polynomial functions. To find the aforementioned polynomials, first a tip load (P) is applied to a given taper ratio and taper type beam with assumed geometry (length, breadth, depth, diameter in case of circular cross section). The tip lateral deflections (a) and tip slope (α) are estimated for the applied load from the converged nonlinear solution. The load parameter ($\lambda = PL^2/EI$) versus tip amplitude (a/L) and load parameter ($\lambda = PL^2/EI$) versus tip slope (α) are plotted. The relations obtained from these plots are expressed in terms of polynomials by curve fitting the data.

For a given load parameter, tip amplitude is estimated using the proposed polynomial functions for a tip slope, and different taper ratios and taper types of cantilever beam. In addition, the load parameters and amplitudes are used to calculate the nonlinear stiffness (P/a) is for a given tip slope and different taper ratios and taper types of cantilever beam. Finally, using nonlinear stiffness, beam mass and mass distribution parameter for different taper [10], first mode (fundamental) nonlinear large amplitude frequency is evaluated.

The polynomial functions in terms of tip amplitude (a/L), tip slope (α) and load parameter (λ) as illustrated in Figure1, Figure 2 and Figure3 are presented for various taper ratios (ε) as follows.

2.1 Prismatic (without taper) uniform beam

ε = 0.0;

$$\frac{a}{L} = -2.79 \text{ X } 10^4 + 0.509\lambda - 0.0396\lambda^2 \quad (1)$$

$$\alpha = -3.44 \text{ X } 10^4 + 0.341\lambda - 0.032\lambda^2 \quad (2)$$

2.2 Non-prismatic with breadth taper and depth is constant

ε = 0.2;

$$\frac{a}{L} = -8.67 \text{ X } 10^{-5} + 0.336\lambda - 0.018\lambda^2 \quad (3)$$

$$\alpha = -1.07\text{X } 10^{-4} + 0.573\lambda - 0.023\lambda^2 \quad (4)$$

ε = 0.4;

$$\frac{a}{L} = -1.06 \text{ X } 10^{-4} + 0.358\lambda - 0.023\lambda^2 \quad (5)$$

$$\alpha = -1.35\text{X } 10^{-4} + 0.559\lambda - 0.029\lambda^2 \quad (6)$$

ε = 0.6;

$$\frac{a}{L} = -1.34 \text{ X } 10^{-4} + 0.385\lambda - 0.029\lambda^2 \quad (7)$$

$$\alpha = -1.80 \text{ X } 10^{-4} + 0.621\lambda - 0.039\lambda^2 \quad (8)$$

ε = 0.8;

$$\frac{a}{L} = -1.82 \text{ X } 10^{-4} + 0.422\lambda - 0.040\lambda^2 \quad (9)$$

$$\alpha = -2.63 \text{ X } 10^{-4} + 0.718\lambda - 0.057\lambda^2 \quad (10)$$

2.3 Non-prismatic with depth taper and breadth is constant

ε = 0.2;

$$\frac{a}{L} = -4.47 \text{ X } 10^{-5} + 0.399\lambda - 0.0528\lambda^2 \quad (11)$$

$$\alpha = -6.017\text{X } 10^{-5} + 0.632\lambda - 0.070\lambda^2 \quad (12)$$

ε = 0.4;

$$\frac{a}{L} = -7.12 \text{ X } 10^{-4} + 0.488\lambda - 0.0958\lambda^2 \quad (13)$$

$$\alpha = -1.06\text{X } 10^{-3} + 0.841\lambda - 0.143\lambda^2 \quad (14)$$

ε = 0.6;

$$\frac{a}{L} = -1.173 \text{ X } 10^{-3} + 0.641\lambda - 0.198\lambda^2 \quad (15)$$

$$\alpha = -2.199\text{X } 10^{-3} + 1.266\lambda - 0.361\lambda^2 \quad (16)$$

ε = 0.8;

$$\frac{a}{L} = -2.161 \text{ X } 10^{-5} + 0.907\lambda - 0.167\lambda^2 \quad (17)$$

$$\alpha = -5.915\text{X } 10^{-5} + 2.378\lambda - 0.437\lambda^2 \quad (18)$$

2.4 Non-prismatic with diameter taper

ε = 0.2;

$$\frac{a}{L} = -1.13 \text{ X } 10^{-3} + 0.400\lambda - 0.058\lambda^2 \quad (19)$$

$$\alpha = -1.57 \text{ X } 10^{-3} + 0.647\lambda - 0.080\lambda^2 \quad (20)$$

ε = 0.4;

$$\frac{a}{L} = -2.15 \text{ X } 10^{-3} + 0.547\lambda - 0.136\lambda^2 \quad (21)$$

$$\alpha = -3.47 \text{ X } 10^{-3} + 0.996\lambda - 0.219\lambda^2 \quad (22)$$

ε = 0.6;

$$\frac{a}{L} = -2.73 \text{ X } 10^{-3} + 0.831\lambda - 0.373\lambda^2 \quad (23)$$

$$\alpha = -6.69 \text{ X } 10^{-3} + 1.874\lambda - 0.816\lambda^2 \quad (24)$$

ε = 0.8;

$$\frac{a}{L} = -1.82 \text{ X } 10^{-4} + 1.140\lambda - 1.190\lambda^2 \quad (25)$$

$$\alpha = -1.57 \text{ X } 10^{-3} + 4.308\lambda - 4.139\lambda^2 \quad (26)$$

Where, λ is the dimensionless load parameter(=PL^2/EI_R); P is the tip load; the length of the beam is L; beam moment of inertia is $I_R(=\frac{B_R H^3}{12})$, for a rectangular section beam with breadth(B) tapering linearly (depth is constant);$(=\frac{BH_R^3}{12})$for a rectangular section beam with depth (H) tapering linearly (breadth is constant); and $(=\frac{\pi D_R^4}{64})$ for a circular cross section beam of tapering diameter (D) linearly) at root; EI_R (= elastic modulus); ε is the taper ratio(= 1- TR). TR is defined as the ratio of B_t/B_R(breadth taper), H_t/H_R(depth taper), and D_t/D_R(diameter taper).The R and t mentioned as subscripts indicate the size of root and tip of a non-prismatic (tapered) beam.

3 Cantilever first mode frequency of prismatic and non-prismatic (tapered) beam

3.1 Equation for frequency (linear)

The first mode frequency equation for a tapered cantilever for a small tip deflection can be expressed as [10].

$$f_{Lin} = \bar{C}\sqrt{\frac{\bar{K}}{\bar{M}}} \quad (27)$$

Where,

f_{Lin} is the linear first mode frequency in Hertz;

$\bar{K}$ (=P/a)=stiffness (linear) in N/m;

$\bar{M} = \bar{m}\int_0^1 (1 - \varepsilon + \varepsilon\xi)^n$beam mass in Kg;

$\bar{m}$ (= mass per unit length);

ξ is non-dimensional co-ordinate;

n =1 for rectangular cross section beam with linearly breadth (B) and depth (H) tapering and

n=2 for a circular cross section beam with tapering diameter linearly.

The mass values for different taper ratio and for different taper type considered in the present investigation are presented in Table 1.

Table1. Determination of beam mass($\bar{M}$)for different taper ratios.

ε → / Taper Type ↓	0	0.2	0.4	0.6	0.8
Breadth (n = 1)	1	0.9	0.8	0.7	0.6
Depth (n = 1)	1	0.9	0.8	0.7	0.6
Diameter (n = 2)	1	0.825	0.653	0.520	0.413

$\bar{C}$ = mass distribution parameter. To determine the $\bar{C}$ value, the polynomials corresponding to the linear tapered in diameter, the linear tapered in width (depth constant), and the linear tapered in depth (width constant) are shown in [10]. The $\bar{C}$ value has been evaluated and is shown in Table 2.

Table 2. Values of $\bar{C}$ for determination of frequency corresponding to different taper ratios and taper types.

ε → / Taper Type ↓	0	0.2	0.4	0.6	0.8
Breadth	0.3233	0.3361	0.3570	0.3851	0.4415
Depth	0.3233	0.3414	0.3709	0.4147	0.5183
Diameter	0.3233	0.3454	0.411	0.524	0.680

3.2 Equation of frequency (Large amplitude)

When a cantilever is subjected to large amplitude vibrations, the first mode nonlinear frequency is estimated by making use of the nonlinear rigidity or stiffness [11]-[12] instead of the linear stiffness in equation (27).

$$f_{NL} = \bar{C}\sqrt{\frac{\bar{K}_{NL}}{\bar{M}}} \tag{28}$$

Where, $\bar{K}_{NL}$=nonlinear stiffness. The approximations is elucidated in the following section.

4 Results and discussions

For the range of tip slope(α=0.4 to 40 degree), the large amplitude first mode frequency of a prismatic (or uniform) and non-prismatic tapered (taper in width, depth constant; taper in depth, width constant; and taper in diameter) cantilever have been calculated using $\bar{C}$ (i.e. mass distribution parameter (Table 2). Here, first utilising Eqs. (1) to (26) with the tip load (P) of the prismatic and non-prismatic tapered cantilever, tip amplitude (a/L) and end slope (α) are calculated. And nonlinear stiffness (P/a) is estimated.To validate the present derived polynomial through FEA, the prismatic (or uniform) cantilever first mode nonlinear large amplitude dimensionless frequencies obtained from derived polynomial functions (Eqs.(1) to (2)) and using Eq.(28), are compared with the reference [11]. This validates the proposed polynomial approximation.

The large amplitude frequency (f_{NL}) of the first mode is calculated for the varied end slope. The dimensionless frequency is$\Omega = \bar{C}\sqrt{\frac{m}{EI}}\omega L^2$. Where, ω (=2 πf_{NL}) is the radian frequency. The comparison(Table 3) shows a 1.21% difference for a 40 degree tip slope.

Table 3. First mode large amplitude frequencies for a prismatic(uniform) cantilever.

α	Ω (Present)	Ω [11]	Difference
0.4⁰	3.521	3.516	0.14
10⁰	3.541	3.537	0.16
20⁰	3.606	3.596	0.29
30⁰	3.681	3.638	1.16
40⁰	3.769	3.815	1.21

The procedure as explained above followed to find dimensionless frequency of non-prismatic(various tapered) cantilever. And the first mode non-liner frequency *Ω* of cantilever (breadth taper, depth taper and diameter taper) are compared (Tables 4, 5 and 6) with those of [7].

Table 4. Large amplitude first mode frequencies (*Ω*) for a non-prismatic cantilever (breadth taper, depth is constant) EI_R = 3364830.0 N-m², m̅=152.88 Kg/ m.

ε →	0.2		0.4		0.6		0.8	
α ↓	Ω	Diff %	Ω	Diff %	Ω	Diff %	Ω	Diff %
0.4⁰	3.838 3.763*	1.96	4.205 4.097*	2.5	4.705 4.585*	2.51	5.453 5.398*	1.0
10⁰	3.874 3.770*	2.6	4.238 4.105*	3.1	4.715 4.594*	2.56	5.585 5.407*	3.2
20⁰	3.914 3.792*	3.1	4.283 4.128*	3.6	4.769 4.619*	3.1	5.653 5.436*	3.8
30⁰	3.957 3.829*	3.2	4.332 4.168*	3.8	4.829 4.663*	3.4	5.730 5.485*	4.2
40⁰	4.023 3.883*	3.5	4.406 4.225*	4.1	4.904 4.725*	3.6	5.815 5.555*	4.4

*: Bottom values are of [7].

Table 5. Large amplitude first mode frequencies (Ω)for a non-prismatic cantilever(depth taper, breadth is constant) EI_R =12926324.N-m², m̅ =152.88 Kg / m.

ε →	0.2		0.4		0.6		0.8	
α ↓	Ω	Diff %	Ω	Diff %	Ω	Diff %	Ω	Diff %
0.4⁰	3.621 3.608*	0.35	3.871 3.737*	2.2	3.991 3.934*	1.4	4.408 4.292*	2.6
10⁰	3.653 3.615*	1.0	3.872 3.744*	3.3	4.043 3.941*	2.5	4.443 4.299*	3.2
20⁰	3.729 3.636*	2.5	3.909 3.764*	3.7	4.099 3.961*	3.3	4.477 4.317*	3.5
30⁰	3.820 3.671*	3.9	4.020 3.799*	5.5	4.225 3.994*	5.4	4.636 4.348*	6.2
40⁰	3.929 3.721*	5.3	4.161 3.848*	7.5	4.391 4.042*	7.9	4.789 4.392*	8.2

*: Bottom values are of [7].

Table 6. Large amplitude first mode frequencies (Ω) for a non-prismatic cantilever(diameter taper) EI_R = 15212946. N-m², m̅= 235.34 Kg / m.

ε →	0.2		0.4		0.6		0.8	
α ↓	Ω	Diff %	Ω	Diff %	Ω	Diff %	Ω	Diff %
0.4⁰	3.86 3.855*	0.12	4.328 4.319*	0.2	5.025 5.009*	0.3	6.229 6.196*	0.5
10⁰	3.872 3.862*	0.28	4.477 4.326*	3.37	5.225 5.017*	3.98	6.386 6.203*	3.3
20⁰	3.953 3.884*	1.77	4.574 4.349*	4.91	5.314 5.040*	5.15	6.547 6.225*	5.0
30⁰	4.056 3.921*	3.34	4.713 4.387*	6.91	5.466 5.079*	7.08	6.748 6.261*	7.2
40⁰	4.185 3.974*	5.04	4.876 4.442*	8.90	5.677 5.135*	9.54	7.012 6.314*	9.9

*: Bottom values are of[7].

Table4. shows the present dimensionless frequencies(Ω) results obtained using derived polynomial (Eqs.(3) to (10)) for different taper ratios fornon-prismatic cantilever beams (breadth taper, depth is constant).

It is found that at a taper ratio of 0.8, the difference of present dimensionless frequencies and published data[7] is 4.4% at a 40 degree tip slope. If the tip slope is less than 40 degrees, the difference is less than 4.4%.

Table 5. shows the present dimensionless frequencies(Ω) for different taper ratios for non-prismatic cantilever beam (depth taper, breadth is constant)obtained using derived polynomial(Eqs. (11) to (18)).

It is found that at a taper ratio of 0.8, the difference between present dimensionless frequencies and published data[7] is 8.2% at a 40degree tip slope. If the taper ratio and tip slope are less, the difference is less than 8.2%.

Table 6. shows the present dimensionless large amplitude frequencies(Ω) results obtained using derived polynomial (Eqs.(19) to (26)) for different taper ratios fornon-prismatic cantilever beams(diameter taper).

It is found that at a taper ratio of 0.8, the difference between present dimensionless frequencies and published data [7] is 9.9% at a 40 degree tip slope. If the taper ratio and tip slope are lower, the difference is less than 9.9%.

In the range of lower tip slopes (α) up to 40 degrees, the current results compare well with those in [7]. The difference in current results compared to [7] is due to the different analysis of the problem studied. The large amplitude first mode frequency (Ω) is observed to increase with an increase in (α). This indicates that the prismatic and non-prismatic beams with fixed at one end (cantilever) exhibit hardening type nonlinearity. In addition, this hardening effect increases with increasing taper. It has beenfound that as the taper ratio increases, the firstmode frequency of a particular tip slope also increases.

5 Conclusions

The main purpose of this work is to contribute to a simple representation of large amplitude free vibration problems of prismatic (uniform) and non-prismatic (various tapered) beams. A finite element computer program [14] was used to get approximate nonlinear solutions for polynomial functions of prismatic (uniform) and non-prismatic (width taper, depth taper, diameter taper) cantilevers. Large deflection data sets (a/L and α) with tip concentrated loads of various taper ratios are obtained.

The first mode frequency is evaluated for large-amplitude prismatic and non-prismatic cantilevers using a very simple approach (Section 3), based on the above data (discussed in the previous paragraph).

The current dimensionless frequencies of prismatic (uniform) cantilevers obtained from the polynomial functions (Eqs.(1) to (2)) are compared with the published data with a 1.21% (Table 3) difference at a tip slope of 40 degrees.

For non-prismatic cantilever beams (breadth taper, depth is constant), at a taper ratio of 0.8, the difference between present dimensionless frequencies with published data is 4.4% (Table

4) at a 40 degree tip slope. It is found that if the taper ratio and tip slope are lower, the difference is less than 4.4%.

For non-prismatic cantilever beams (depth taper, breadth is constant), at a taper ratio of 0.8, the difference in present dimensionless frequencies with published data is 8.2% (Table 5) at a 40 degree tip slope. It is observed that if the taper ratio and tip slope are lower, the difference is less than 8.2%.

For non-prismatic cantilever beams (diameter taper), it is found that at a taper ratio of 0.8, the difference between present dimensionless frequencies and published data is 9.9% (Table 6) at a 40 degree tip slope. It is seen that if the taper ratio and tip slope are lower, the difference is less than 9.9%.

It is seen that the large amplitude first mode frequency (Ω) increases with tip slope (α). This indicates that the prismatic and non-prismatic cantilever beams exhibit hardening type nonlinearity. It has been found that as taper ratio increases, the first mode frequency also increases at certain tip slopes. Therefore, this hardening effect increases with increasing taper. In addition, for a given tip slope (α), the diameter taper shows a higher frequency than other tapered beams and uniform beams.

This new simple approach provides correct computations for all practical designs with significantly less computational power and satisfactory concordance contrasted to other techniques [7],[8],thus signifying the adequacyof the simple proposed method. Therefore, all the proposed polynomials can produce a very accurate approximation to a nonlinear problem. However, current analysis may be limited to a lower range of tip slope and amplitudes.

6 Acknowledgment

The author appreciates Dr. G.V. Rao's remarks on this present study, who was once the group director for structural engineering.

7 Author contribution statements

The Chitaranjan PANY has made the literature review, formation, assessment and examining of obtained results and the spelling and checking the article in terms of content were contributed.

8 Ethics committee approval and conflict of interest statement

"Article prepared does not require approval from the Ethics Committee". "There is no conflict of interest with any person/institution in the article prepared".

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