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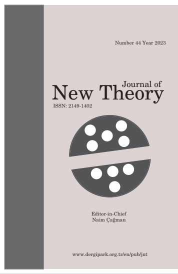

An Investigation into LRS Bianchi I Universe in Brans-Dicke Theory

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Abstract — In this study, the homogenous and anisotropic locally rotationally symmetric (LRS) Bianchi type-I universe filled with the bulk viscous and the string cloud matter has been investigated in the Brans-Dicke (BD) scalar-tensor theory. The modified Einstein field equations of the constructed model have been solved by using the relation of the scale factors $A = B^m$ and considering the deceleration parameter to be $q = m - 1$. It is found that the string does not survive for the model due to the obtained zero string energy density ($\rho_s = 0$) and agrees with some studies in recent years. Moreover, it is possible to say that string matter may be turned into phantom energy over time, depending on the obtained negative rest energy density of the matter. When BD scalar field Φ is assumed constant, the attained solutions are reduced to General Relativity (GR) solutions for the static vacuum. Thus, the constructed model has not allowed the reduction of the BD solutions to GR solutions, including the matter distributions. In addition, some expansion models for choosing a value of constant m have been obtained and provided. All the results have been analyzed in detail.

Keywords *LRS Bianchi I universe, Brans-Dicke theory, bulk viscous, string cloud, deceleration parameter*

Mathematics Subject Classification (2020) 83C05, 83C15

1. Introduction

Gravitation theories are based on Newton's theory, successful in determining the orbits of planets and other celestial bodies. However, this theory was insufficient to explain some cosmic issues. On the other hand, Einstein's General Relativity (GR) theory [1] was the first theory that succeeded in explaining gravity geometrically. But Einstein's theory has been suggested for a static universe. After studies of Friedman [2], Lemaitre [3], and Hubble [4], when considering the expanding universe model, a function determining the expansion of the universe has been needed to add to the field equations in Einstein's GR theory. Einstein has suggested the cosmological constant Λ as the added function to GR theory. Hence, the GR theory has become the most valid gravitational theory, which provides tests, such as the perihelion progression of planets, the gravitational deflection, and the redshift of light. Firstly, scientists have discussed decelerating expanding universe until some observation, such as Supernova IA observation [5], Cosmic Microwave Background radiation (CMB) [6], Wilkinson Microwave Anisotropy Probe (WMAP) [7], etc. Such studies have proven the expanding universe model by accelerating, and then researchers have been focused on studies on this expansion model. Especially alternative gravitation theories, such as Lyra theory [8], Barber's Theory [9], Creation Field

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Theory [10], Yılmaz Theory [11], and Brans-Dicke theory [12] have been investigated with some cosmic matter distribution to explain accelerating expansion of the universe.

In addition, cosmic strings formed during the phase transition have been suggested to cause variations in the density of the particles, which cause the formation of the galaxies [13, 14]. Because of this assumption, it is believed that cosmic strings are important matter structures to investigate and determine the universe's evolution from early to late. Scientists have recently studied the Brans-Dicke (BD) theory with cosmic string for various space-time models. Delice [15] has investigated BD theory for the generalized cylindrical symmetric universe with cosmic string. Reddy and Rao [16] have researched the axially symmetric metric in the presence of a string dust cloud for BD theory. Besides, in BD theory, Rao and Neelima [17] have analyzed the axially symmetric universe model with bulk string matter distribution. Vidya et al. [18] have solved equations of BD theory for the Bianchi type-III universe with bulk string matters. Sharma and Singh [19] have gotten the solutions of Bianchi II universe for the massive string in BD alternative theory. Trivedi and Bhabor [20] have investigated five-dimensional Bianchi III space-time with dark energy fluid and cosmic string in BD theory.

Moreover, Mahanta et al. [21] have calculated plane-symmetric metric with strange quark matter by attaching to string cloud and bulk viscous matter in BD theory. Chakraborty et al. [22] have analyzed BD theory for the Bianchi type-III universe in the presence of perfect fluid matter. Adhav et al. [23] have gotten exact solutions of the Bianchi III cosmological model for vacuum in BD gravity. Çağlar et al. [24] have investigated the locally rotationally symmetric (LRS) Bianchi type I universe in BD theory by assuming strange quark matter (SQM) coupled with string cloud as matter distributions. Çağlar and Aygün [25] have gotten the exact solutions of higher dimensional Friedman–Walker Friedmann-Robertson-Walker (FRW) universe in the presence of SQM with bulk viscous and cosmic string matter in Lyra theory. Furthermore, higher dimensional FRW universe filled with SQM attached to cosmic string matter for Self Creation Cosmology and BD theory, respectively, have been examined by Çağlar and Aygün [26, 27].

Motivated by the studies mentioned above, the LRS Bianchi type-I universe has been studied in BD theory by assuming cosmic string with bulk viscous as matter distribution. This paper is organized as follows: In the second section, the universe model has been constructed, and the modified field equations of the model have been solved. In the last section, all the obtained solutions have been concluded in detail.

2. MEFE's of LRS Bianchi-I Universe for Brans-Dicke Theory

The modified Einstein field equations (MEFE's) in BD theory [12] have been suggested as:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{8\pi}{\Phi}T_{ij} - \frac{\omega}{\Phi^2}(\Phi_{,i}\Phi_{,j} - \frac{1}{2}g_{ij}\Phi_{,k}\Phi^{,k}) - \frac{1}{\Phi}(\Phi_{i;j} - g_{ij}\Phi^{;k}_{;k})) \tag{1}$$

Here, R, R_{ij}, and ω are Ricci scalar, Ricci tensor, and the coupling constant of BD theory, respectively. It is possible to reduce BD theory to GR theory when $\omega \to \propto$ and $\Phi = constant$ [28]. In addition, Φ symbolizes BD scalar field and it is given as follows [29]:

$$\Phi = \Phi^{;k}_{;k} = \frac{8\pi}{3+2\omega}T \tag{2}$$

Here and after, comma (,) and semicolon (;) represent partial derivative and covariant derivative, respectively. Furthermore, T_{ij} is a tensor characterized stress energy of the matter in Equation 1. In this study, it has been assumed that the string cloud matter with bulk viscous filled the universe and this matter distribution is written as

$$T_{ij} = \rho u_i u_j - \rho_s x_i x_j - \eta u^k_{;k}(u_i u_j - g_{ij}) \tag{3}$$

Here, ρ and ρ_s symbolizes the rest energy density of matter and the string energy density, respectively. Moreover, the bulk viscous coefficient has been represented as η. Besides, u_i defines 4-velocity of the particle and x_i is named unit space-like vector determined the string direction [21, 30]. Further, the LRS Bianchi type-I space-time line element can be written as

$$ds^2 = dt^2 - A(t)^2 dx^2 - B(t)^2(dy^2 + dz^2) \tag{4}$$

Here, $A(t)$ and $B(t)$ are time-depending scale factors of the universe [31, 32]. Some kinematic quantities, such as spatial volume (V), Hubble parameter (H), expansion scalar (θ), shear scalar (σ^2), and deceleration parameter (q), for LRS Bianchi type-I universe are provided as follows, respectively:

$$V = a^3 = AB^2 \tag{5}$$

$$H = \frac{a_t}{a} = \frac{A_t}{3A} + \frac{2B_t}{3B} \tag{6}$$

$$\theta = \frac{A_t}{3A} + \frac{2B_t}{B}$$

$$\sigma^2 = \frac{1}{3}\left[\frac{A_t}{A} - \frac{B_t}{B}\right]^2$$

and

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{2(AB_t - A_tB)^2 - 3AB(A_{tt}B + 2AB_{tt})}{(2AB_t + A_tB)^2} \tag{7}$$

Here and after, the lower index t represents the derivative with respect to cosmic time t. Besides, a provided in Equations 5 and 6 symbolizes the average scale factor [33]. By using Equations 1-4, the modified Einstein field equations of LRS Bianchi-I space-time for string cloud coupled with bulk viscous matter have been obtained in BD theory as follows:

$$\frac{B_t^2}{B^2} + \frac{2B_{tt}}{B} = \frac{1}{\Phi}\left[8\pi\eta\left(\frac{A_t}{A} + \frac{2B_t}{B}\right) - \frac{\omega\Phi_t^2}{2\Phi} - \frac{2B_t\Phi_t}{B} - \Phi_{tt}\right] \tag{8}$$

$$\frac{B_t^2}{B^2} + \frac{2A_tB_t}{AB} = \frac{1}{\Phi}\left[8\pi\rho + \frac{\omega\Phi_t^2}{2\Phi} - \frac{A_t\Phi_t}{A} - \frac{2B_t\Phi_t}{B}\right] \tag{9}$$

$$\frac{A_{tt}}{A} + \frac{B_{tt}}{B} + \frac{A_tB_t}{AB} = \frac{1}{\Phi}\left[8\pi\eta\left(\frac{A_t}{A} + \frac{2B_t}{B}\right) - \Phi_t\left(\frac{A_t}{A} - \frac{B_t}{B}\right) - \frac{\omega\Phi_t^2}{2\Phi} - \Phi_{tt}\right] \tag{10}$$

$$\frac{A_{tt}}{A} + \frac{B_{tt}}{B} + \frac{A_tB_t}{AB} = \frac{1}{\Phi}\left[8\pi\eta\left(\frac{A_t}{A} + \frac{2B_t}{B}\right) - \Phi_t\left(\frac{A_t}{A} - \frac{B_t}{B}\right) + 8\pi\rho_s - \frac{\omega\Phi_t^2}{2\Phi} - \Phi_{tt}\right] \tag{11}$$

Field equations obtained as in Equations 8-11 have six unknown quantities such as A, B, Φ, ρ, ρ_s, and η. Therefore, two approximations have been used to solve field equations. The first one is the relationship of the scale factors came from a proportion of the shear scalar and the expansion scalar given as follows:

$$A = B^k \tag{12}$$

Here, k is an arbitrary constant [24, 34, 35]. The second assumption is a constant form of deceleration parameter by using Equation 7 given as follows:

$$q = m - 1 \tag{13}$$

where the arbitrary constant m determines the expansion model of the universe. Deceleration parameter q specifies the fate of the universe model as follows [15, 24, 36]:

i. When $m < 0$, one gets $q < -1$ named super-exponential expansion *ii.* When $m = 0$, one gets $q = -1$ named exponential expansion

iii. When $0 < m < 1$, one gets $-1 < q < 0$ named accelerating power law expansion

iv. When $m = 1$, one gets $q = 0$ named constant expansion

v. When $m > 1$, one gets $q > 0$ named decelerating expansion

By using Equation 13 with Equation 12, the scale factor B has been calculated as

$$B = (s_1t + s_2)^{\frac{3}{m(k+2)}} \tag{14}$$

Then, by using Equation 14 in Equation 12, the scale factor A has been attained as follows:

$$A = (s_1t + s_2)^{\frac{3k}{m(k+2)}} \tag{15}$$

Considering the scale factors' values in Equations 14 and 15 in field Equations 8–11, the BD scalar field Φ, the rest energy density ρ, the string energy density ρ_s, and the bulk viscous coefficient η have been obtained as follows, respectively:

$$\Phi = s_3(s_1t + s_2)^{\frac{m-3}{m}}$$

$$\rho = \frac{\left[(k+2)^2\left(6\omega + 6m - \omega m^2 - 9\omega\right) - 18k(k+2) - 54\right] s_3 s_1^2}{16\pi m^2(k+2)^2(s_1t + s_2)^{\frac{m+3}{m}}}$$

$$\rho_s = 0$$

and

$$\eta = \frac{\left[(k+2)^2\left(\omega m^2 - 6\omega - 6m + 9\omega\right) + 18k(k+2) + 54\right] s_1 s_3}{48\pi m^2(k+2)^2(s_1t + s_2)^{\frac{3}{m}}}$$

Furthermore, kinematic quantities, such as spatial volume (V), Hubble parameter (H), expansion scalar (θ), and shear scalar (σ^2) for the constructed model are calculated as follows, respectively:

$$V = (s_1t + s_2)^{\frac{1}{m}}$$

$$H = \frac{s_1}{m(s_1t + s_2)}$$

$$\theta = \frac{3s_1}{m(s_1t + s_2)}$$

and

$$\sigma^2 = \frac{3(k-1)^2 s_1^2}{m^2(k+2)^2(s_1t + s_2)^2}$$

All the obtained solutions of constructed model have been investigated in the next section.

3. Discussions

In this paper, the bulk viscous coupled with the string cloud matter distribution has been investigated in BD theory for LRS Bianchi type-I space-time. By considering the scale factor values given in Equations 14 and 15 within Equation 4, the new line element of the LRS Bianchi-I universe can be rewritten as

$$ds^2 = dt^2 - (s_1t + s_2)^{\frac{6k}{m(k+2)}} dx^2 - (s_1t + s_2)^{\frac{6}{m(k+2)}} (dy^2 + dz^2) \tag{16}$$

It is obviously seen that s_1 and k in Equation 16 are important constants to determine meaningful the universe model and must be $s_1 \neq 0$, $k \neq -2$, and $k \neq 0$. Besides, it is well know that the universe is expanding with acceleration. When $0 < m < 1$, the deceleration parameter value has given accelerating power law expansion universe. At this condition, the evolution of scale factor $A(t)$ has been shown in Figure 1. As seen in the figure, the expansion of the constructed universe model is as expected in given situations.

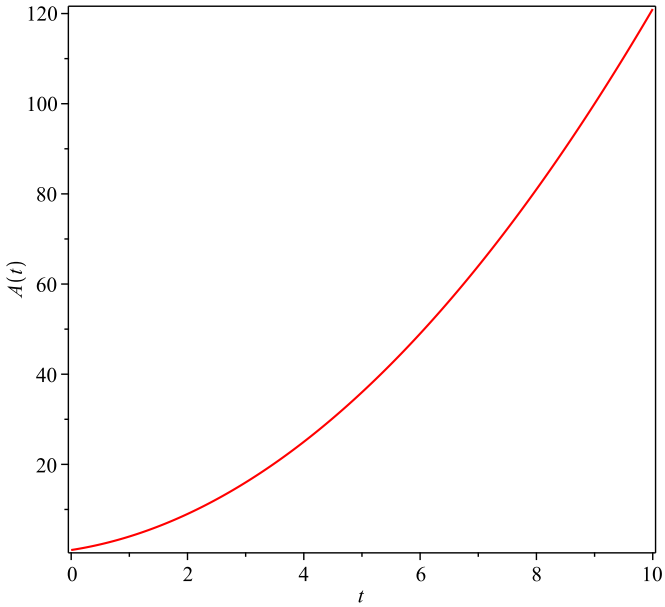

Figure 1. Evolutions of the scale factor $A(t)$ for the accelerating expanding universe ($s_1 = s_2 = k = 1$ and $m = 0.5$)

Moreover, the string energy density ρ_s vanishes for constructed model. Thus, it can be said that the string matter does not survive in the model LRS Bianchi type-I universe for BD theory with the bulk viscous and the string cloud. This finding agrees with the studies of Zel'dovich [13], Çağlar and Aygün [25–27], Sahoo and Mishra [37], Kiran and Reddy [38], etc. In addition, the obtained all the solutions give the negative energy density of particle. Hence, it is possible that the cosmic string may have disappeared by time and turned into a phantom [39]. Rest energy density and bulk viscous coefficient have been shown in Figure 2.

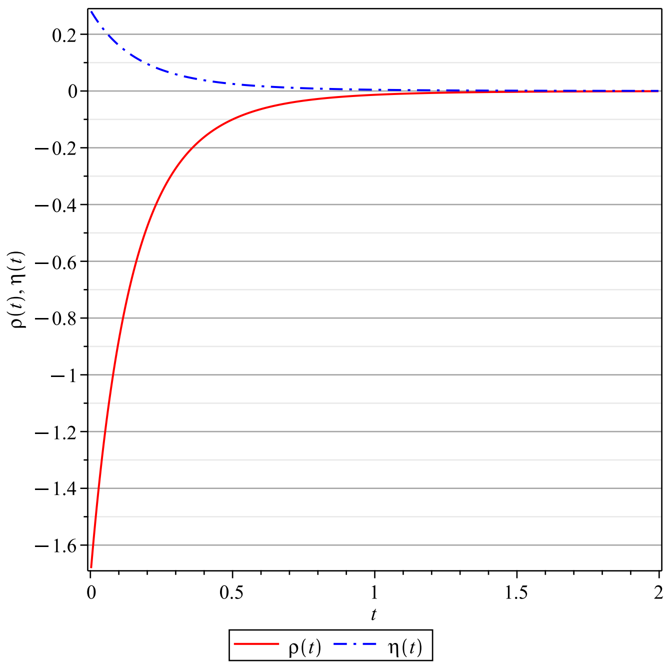

Figure 2. Evolutions of the rest energy density and the bulk viscous coefficient for the accelerating expanding universe ($s_1 = s_2 = s_3 = k = 1, \omega = 2$, and $m = 0.5$)

Furthermore, the BD scalar field Φ has been obtained time dependent and shown in Figure 3 for the accelerating expanding universe. It is observed that the scalar field of the theory is inversely proportional to time, intense at the beginning, and loses its effect over time.

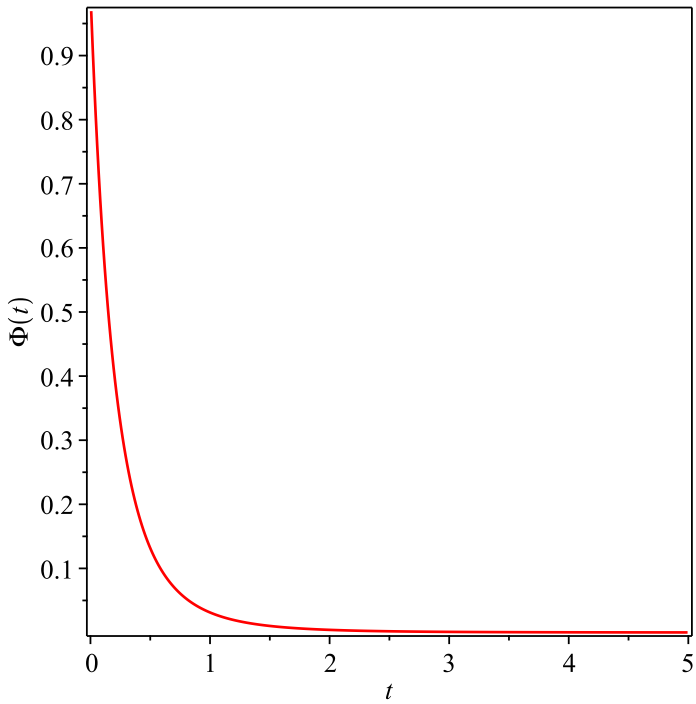

Figure 3. Evolutions of the BD scalar field for the accelerating expanding universe ($s_1 = s_2 = s_3 = 1$ and $m = 0.5$)

Considering the kinematic quantities, it is obtained that the universe has expansion with accelerating. This expansion is observed in Figures 4 and 5. Further, this result agrees with some studies [4–7].

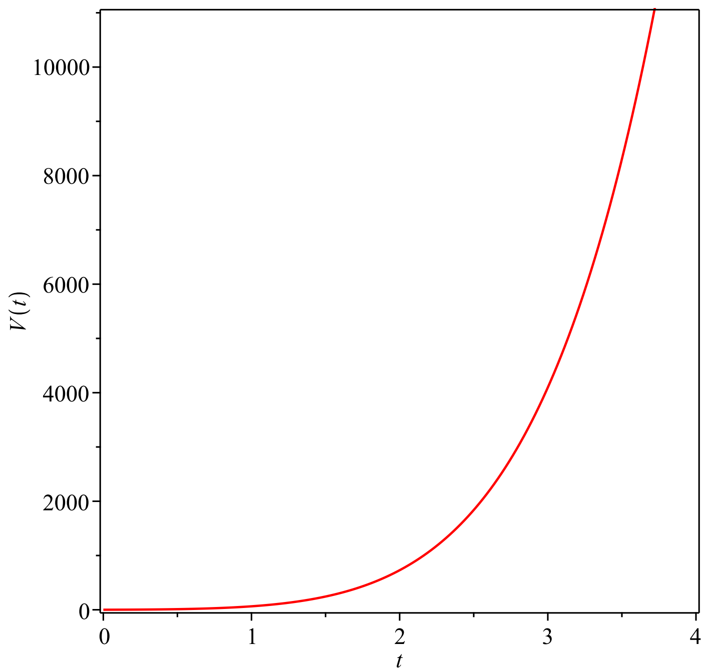

Figure 4. Evolutions of spatial volume for accelerating expanding universe ($s_1 = s_2 = s_3 = 1$ and $m = 0.5$)

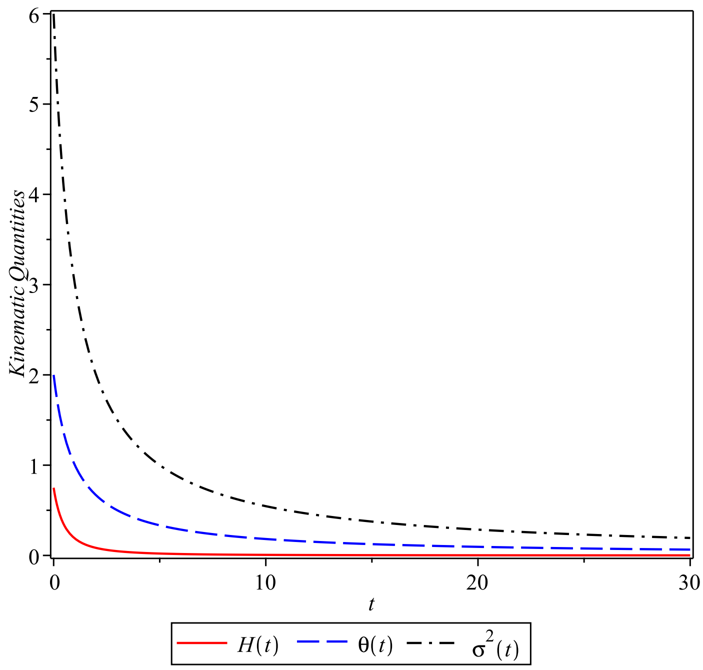

Figure 5. Evolutions of the Hubble parameter, the expansion scalar, and the shear scalar for the accelerating expanding universe ($s_1 = s_2 = s_3 = 1, k = 2$, and $m = 0.5$)

The obtained all the solutions have been calculated depending on some integer values of arbitrary constant m to investigate expansion type of the universe. By assuming $m = 2$, $m = 1$, and $m = 0$, one obtains the results of the constructed model for decelerating expansion, the constant expansion, and the exponential expansion universe, respectively. Additionally, these results have been provided in Table 1.

Table 1. Results of Constructed Model for the Deceleration Parameters

Quantities	**Decelerating Expansion** $q = 1$	**Constant Expansion** $q = 0$	**Exponential Expansion** $q = -1$
Scale Factor A	$(s_1 t + s_2)^{\frac{3k}{2k+4}}$	$(s_1 t + s_2)^{\frac{3k}{k+2}}$	$s_4^k e^{s_5 k t}$
Scale Factor B	$(s_1 t + s_2)^{\frac{3}{2k+4}}$	$(s_1 t + s_2)^{\frac{3}{k+2}}$	$s_4 e^{s_5 t}$
BD Scalar Field Φ	$\frac{s_3}{\sqrt{s_1 t+s_2}}$	$\frac{s_3}{(s_1 t+s_2)^2}$	$\frac{s_6}{e^{(k+2)s_5 t}}$
Rest Energy Density ρ	$-\frac{\left[(\omega+6)k^2+(4\omega-12)k+4\omega+6\right]s_3 s_1^2}{96\pi(k+2)^2(s_1 t+s_2)^{\frac{3}{2}}}$	$-\frac{\left[(\omega+3)2k^2+(4\omega+3)2k+8\omega+15\right]s_3 s_1^2}{8\pi(k+2)^2(s_1 t+s_2)^4}$	$-\frac{\left[(\omega+2)k^2+4(\omega+1)k+4\omega+6)\right]s_6 s_5^2}{16\pi e^{(k+2)s_5 t}}$
Bulk Viscous Coefficient η	$\frac{\left[(\omega+6)k^2+(4\omega-12)k+4\omega+6\right]s_1 s_3}{96\pi(k+2)^2(s_1 t+s_2)^{\frac{3}{2}}}$	$\frac{\left[(\omega+3)2k^2+(4\omega+3)2k+8\omega+15\right]s_1 s_3}{24\pi(k+2)^2(s_1 t+s_2)^3}$	$\frac{\left[(\omega+2)k^2+4(\omega+1)k+4\omega+6)\right]s_5 s_6}{16\pi e^{(k+2)s_5 t}}$
Spatial Volume V	$\sqrt{s_1 t + s_2}$	$s_1 t + s_2$	$s_4 e^{\frac{(k+2)s_5 t}{3}}$
Hubble Parameter H	$\frac{s_1}{2(s_1 t+s_2)}$	$\frac{s_1}{(s_1 t+s_2)}$	$\frac{(k+2)s_5}{3}$
Expansion Scalar θ	$\frac{3s_1}{2(s_1 t+s_2)}$	$\frac{3s_1}{(s_1 t+s_2)}$	$(k+2)s_5$
Shear Scalar σ^2	$\frac{3(k-1)^2 s_1^2}{4(k+2)^2(s_1 t+s_2)^2}$	$\frac{3(k-1)^2 s_1^2}{(k+2)^2(s_1 t+s_2)^2}$	$\frac{(k-1)^2 s_5^2}{3}$

It is possible to get GR solutions of the LRS Bianchi type-I space-time with the bulk string cloud matter by assuming $\Phi = constant$ in BD theory. To get the constant BD scalar field, s_3 must be zero in the model results. In this situation, the energy density and the bulk viscous coefficient became zero, and it gives the static vacuum solutions. Thus, the obtained results do not allow GR solutions of the constructed model by reducing BD solutions with $\Phi = constant$.

4. Conclusion

This study investigated BD theory in the LRS Bianchi type-I space-time for bulk viscous and string cloud matter distribution. In the future, it is worth investigating LRS Bianchi I universe filled with bulk string matter or various cosmic matter in alternative gravitation theories such as $f(G)$, $f(Q)$, and $f(Q,T)$.

Author Contributions

The author read and approved the final version of the paper.

Conflicts of Interest

The author declares no conflict of interest.

References

[1] A. Einstein, *The Foundation of the General Theory of Relativity*, Annalen der Physik 49 (7) (1916) 769–822.

[2] A. Friedman, *Über die Krümmung des Raumes*, Zeitschrift für Physik 10 (1922) 377–386.

[3] G. Lemaitre, *Un Univers Homogène de Masse Constante et de Rayon Croissant Rendant Compte de la Vitesse Radiale des Nébuleuses Extra-galactiques*, Annales de la Société Scientifique de Bruxelles A47 (1927) 49–59.

[4] E. Hubble, *A Relation Between Distance and Radial Velocity Among Extra-Galaktik Nebulae*, Proceedings of the National Academy of Sciences of the USA 15 (3) (1929) 168–173.

[5] S. Perlmutter, G. Aldering, G. Goldhaber, R. Knop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, D. E. Groom, I. M. Hook, A. G. Kim, M. Y. Kim, J. C. Lee, N. J. Nunes, R. Pain, C. R. Pennypacker, R. Quimby, C. Lidman, R. S. Ellis, M. Irwin, R. G. McMahon, P. Ruiz-Lapuente, N. Walton, B. Schaefer, B. J. Boyle, A. V. Filippenko, T. Matheson, A. S. Fruchter, N. Panagia, H. J. M. Newberg, W. J. Couch, The Supernova Cosmology Project, *Measurements of Ω and Λ from 42 High-Redshift Supernovae*, The Astrophysical Journal 517 (2) (1999) 565–586.

[6] D. N. Spergel, L. Verde, H. V. Peiris, E. Komatsu, M. Nolta, C. L. Bennett, M. Halpern, G. Hinshaw, N. Jarosik, A. Kogut, M. Limon, S. S. Meyer, L. Page, G. S. Tucker, J. L. Weiland, E. Wollack, E. L. Wright, *First-Year Wilkinson Microwave Anisotropy Probe (WMAP)* Observations: Determination of Cosmological Parameters*, The Astrophysical Journal Supplement Series 148 (1) (2003) 175–194.

[7] R. A. Knop, G. Aldering, R. Amanullah, P. Astier, G. Blanc, M. S. Burns, A. Conley, S. E. Deustua, M. Doi, R. Ellis, S. Fabbro, G. Folatelli, A. S. Fruchter, G. Garavini, S. Garmond, K. Garton, R. Gibbons, G. Goldhaber, A. Goobar, D. E. Groom, D. Hardin, I. Hook, D. A. Howell, A. G. Kim, B. C. Lee, C. Lidman, J. Mendez, S. Nobili, P. E. Nugent, R. Pain, N. Panagia, C. R. Pennypacker, S. Perlmutter, R. Quimby, J. Raux, N. Regnault, P. Ruiz-Lapuente, G. Sainton, B. Schaefer, K. Schahmaneche, E. Smith, A. L. Spadafora, V. Stanishev, M. Sullivan, N. A. Walton, L. Wang, W. M. Wood-Vasey, N. Yasuda, The Supernova Cosmology Project, *New Constraints on Ω_M, Ω_Λ, and w from an Independent Set of 11 High-Redshift Supernovae Observed with the Hubble Space Telescope*, Astrophysical Journal 598 (2003) 102–137.

[8] G. Lyra, *Uber eine Modifikation der Riemannschen Geometrie*, Mathematische Zeitschrift 54 (1) (1951) 52–64.

[9] G. A. Barber, *On Two "Self-Creation" Cosmologies*, General Relativity and Gravitation 14 (1982) 117–136.

[10] F. Hoyle, J. V. Narlikar, *A Radical Departure from the 'Steady-State' Concept in Cosmology*, Proceedings of the Royal Society of London Series A, Mathematical and Physical Sciences 290 (1421) (1966) 162–176.

[11] H. Yılmaz, *New Approach to General Relativity*, Physical Review 111 (1958) 1417–1426.

[12] C. Brans, R. H. Dicke, *Mach's Principle and a Relativistic Theory of Gravitation*, Physical Review 124 (1961) 925–935.

[13] Ya. B. Zel'dovich, *Cosmological Fluctuations Produced Near a Singularity*, Monthly Notices of the Royal Astronomical Society 192 (4) (1980) 663–667.

[14] T. W. B. Kibble, *Topology of Cosmic Domains and Strings*, Journal of Physics A: Mathematical and General 9 (8) (1976) 1387–1398.

[15] Ö. Delice, *Local Cosmic Strings in Brans-Dicke Theory with a Cosmological Constant*, Physical Review D 74 (6) (2006) 067703 1–4.

[16] D. R. K. Reddy, S. M. V. Rao, *Axially Symmetric String Cosmological Model in Brans-Dicke Theory of Gravitation*, Astrophysics and Space Science 305 (2) (2006) 183–186

[17] V. U. M. Rao, D. Neelima, *Axially Symmetric Bulk Viscous String Cosmological Models in GR and Brans-Dicke Theory of Gravitation*, ISRN Astronomy and Astrophysics 2013 (2013) Article ID 543483 6 pages.

[18] T. S. Vidya, C. P. Rao, R. B. Vijaya, D. R. K. Reddy, *Bianchi Type-III Bulk Viscous String Cosmological Model in Brans-Dicke Theory of Gravitation*, Astrophysics and Space Science 349 (1) (2014) 479–483.

[19] N. K. Sharma, J. K. Singh, *Some Exact Solutions of Bianchi Type-II String Cosmological Models with Magnetic Field in Brans-Dicke Theory of Gravitation*, International Journal of Theoretical Physics 54 (5) (2015) 1633–1643.

[20] D. Trivedi, A. K. Bhabor, *Higher Dimensional Bianchi Type-III String Cosmological Models with Dark Energy in Brans-Dicke Scalar-Tensor Theory of Gravitation*, New Astronomy 89 (2021) Article ID 101658 9 pages.

[21] K. L. Mahanta, A. K. Biswal, P. K. Sahoo, *Bulk Viscous String Cloud with Strange Quark Matter in Brans-Dicke Theory*, European Physical Journal-Plus 129 (2014) 141–146.

[22] N. C. Chakraborty, S. Chakraborty, A. Dolgov, *Brans-Dicke Theory in Bianchi III Model and Inflationary Scenario*, International Journal of Modern Physics D 11 (3) (2002) 391–404.

[23] K. S. Adhav, M. R. Ugale, C. B. Kale, M. P. Bhende, *Plane Symmetric Vacuum Bianchi Type-III Cosmological Model in Brans-Dicke Theory*, International Journal of Theoretical Physics 48 (1) (2009) 178–182

[24] H. Çağlar, S.Aygün, C. Aktaş, D. Taşer, *LRS Binachi I Model with Strange Quark Matter Attached to String Cloud in Brans-Dicke Theory*, International Physics Conference of the Balkan Physical Union 22 (2014) 221014 129–137.

[25] H. Çağlar, S.Aygün, *Exact Solutions of Bulk Viscous with String Cloud Attached to Strange Quark Matter for Higher Dimensional FRW Universe in Lyra Geometry*, AIP Conference Proceedings 1722 (2016) 050001 1–5.

[26] H. Çağlar, S. Aygün, *Bulk Viscous String Cloud with Strange Quark Matter in Self Creation Cosmology*, IOSR Journal of Mathematics 11 (6) (2015) 53–59.

[27] H. Çağlar, S. Aygün, *Non-existence of Brans-Dicke Theory in Higher Dimensional FRW Universe*, Astrophysics and Space Science 361 (2016) 1–6.

[28] S. Weinberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, John Wiley & Sons, New York, 1972.

[29] V .U. M. Rao, D. Neelima, *Cosmological Models with Strange Quark Matter Attached to String Cloud in GR and Brans-Dicke Theory of Gravitation*, The European Physical Journal-Plus 129 (2014) 122–136.

[30] A. Dixit, R. Zia, A. Pradhan, *Anisotropic Bulk Viscous String Cosmological Models of the Universe under a Time-Dependent Deceleration Parameter*, Pramana 94 (1) (2020) Article Number 25 11 pages.

[31] G. Mohanty, S. K. Sahu, P. K. Saho, *Massive Scalar Field in the Bianchi Type I Space Time*, Astrophysics and Space Science 288 (2003) 421–427.

[32] K. S. Adhav, *LRS Bianchi Type-I Cosmological Model in $f(R,T)$ Theory of Gravity*, Astrophysics and Space Science 339 (2012) 365–369.

[33] O. Özdemir, C. Aktaş, *Anisotropic Universe Models with Magnetized Strange Quark Matter in $f(R)$ Gravity Theory*, Modern Phys. Lett. A 35 (14) (2020) 2050111 9 pages.

[34] D. Taşer, M. U. Doğru, *Scalar Fields in $f(G)$ Gravity*, International Journal of Geometric Methods in Modern Physics 17 (09) (2020) 2050132 17 pages.

[35] R. Bali, *Magnetized Cosmological Model*, International Journal of Theoretical Physics A 25 (1986) 755–761.

[36] K. S. Adhav, *LRS Bianchi Type-I Cosmological Model with Linearly Varying Deceleration Parameter*, The European Physical Journal-Plus 126 (2011) 122–128.

[37] P. K. Sahoo, B. Mishra, *String Cloud and Domain Walls with Quark Matter for Plane Symmetric Cosmological Model in Bimetric Theory*, Journal of Theoretical and Applied Physics 7 (2013) 12–16.

[38] M. Kiran, D. R. K. Reddy, *Non-existence of Bianchi Type-III Bulk Viscous String Cosmological Model in $f(R,T)$ Gravity*, Astrophysics and Space Science 346 (2013) 521–524.

[39] S. M. Carroll, M. Hoffman, *Can the Dark Energy Equation-of-State Parameter w be Less than A1?*, Physical Review D 68 (2003) 023509 1–11.