ESKİŞEHİR TEKNİK ÜNİVERSİTESİ BİLİM VE TEKNOLOJİ DERGİSİ B- TEORİK BİLİMLER

Eskişehir Technical University Journal of Science and Technology B- Theoretical Sciences

2024, 12(1), pp. 32-54, DOI: 10.20290/estubtdb.1356881

RESEARCH ARTICLE

A COMPREHENSIVE STUDY ON SOFT BINARY PIECEWISE DIFFERENCE OPERATION

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ABSTRACT

Soft set theory, developed by Molodtsov, has been applied both theoretically and practically in many fields. It is a useful mathematical tool for handling uncertainty. Numerous variations of soft set operations, the crucial concept for the theory, have been described and used since its introduction. In this paper, we explore more about soft binary piecewise difference operation (defined first as “difference of soft sets”) and its whole properties are examined especially in comparison with the basic properties of difference operation in classical set theory. Several striking properties of soft binary piecewise operations are obtained as analogous to the characteristic of difference operation in classical set theory. Also, we show that the collection of all soft sets with a fixed parameter set together with the soft binary piecewise difference operation is a bounded BCK-algebra.

Keywords: Soft sets, Soft set operations, Soft binary piecewise difference operation

1. INTRODUCTION

Due to the existence of some types of uncertainty, we are unable to employ traditional ways effectively to address issues in many domains, including engineering, environmental and health sciences, and economics. Molodtsov [1], in 1999, proposed Soft Set Theory as a mathematical method to deal with these uncertainties. Since then, this theory has been applied to a variety of fields, including information systems, decision-making, optimization theory, game theory, operations research, measurement theory, and some algebraic structures. The initial contributions to soft set operations were stated by Maji et al. [2] and Pei and Miao [3]. Following this, Ali et al. [4] introduced and discussed several soft set operations, including restricted and extended soft set operations. Sezgin and Atagün [5] discussed the basic properties of soft set operations and the connections between them. They also investigated and defined the idea of restricted symmetric difference of soft sets. A brand-new soft set operation called “extended difference of soft sets” was presented by Sezgin et al. [6]. Stojanovic [7] introduced the term "extended symmetric difference of soft sets" and its characteristics were investigated. The two main categories into which the operations of soft set theory fall, according to the soft set literature, are restricted soft set operations and extended soft set operations. Soft binary piecewise operations were defined by Yavuz [8], who also carefully analyzed their core characteristics. Since the creation of new soft set operations and derivation of their algebraic properties as well as the introduction of new soft set operations and their implemantations offer new perspectives for solving parametric data problems, the operations of soft sets are the fundemantal concepts of soft set theory, and thus soft set operations have been extensively studied since 2003. For more details, we refer to [9-36].

There is a lot of algebra related to logic. Boolean algebra is related to traditional two valued Aristotelean logic. MV algebra is suitable for multi-valued logic. BCI/BCK algebra generalizes the concept of set algebra of sets with the set subtraction as the only non-nullary operation, while these algebras generalize the algebra of implication. The concept of BCI/BCK algebra was introduced by Imai and Iseki [37] to

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Received: 12.09.2023 Published: 27.02.2024

study non-classical propositional logic. Although soft sets over algebraic structures have been studied extensively with the initial study of soft groups, the soft set algebras themselves have been studied extensively by [10,18,38,39].

There are five different types of difference operation defined in the soft set literature; one of which is restricted difference of soft sets defined in [4], the other is extended difference of soft sets defined in [6], the other is complementary extended difference of soft sets defined in [9], the other is complementary soft binary piecewise difference operation defined in [28]. Eren [17], in 2019, introduced a brand-new class of soft difference operations, which we call here as "soft binary piecewise difference" to avoid confusion, and the core characteristics of the operation together with the distribution laws were analyzed. This paper is a following study of [17] and we aim to enrich the paper [17] by investigating more on soft binary piecewise difference operation.

In this study, we especially examine the full algebaric properties of this soft set operation comparatively with the basic properties of difference operation existing in classical set theory and we obtain many interesting similarities. Moreover, we prove that the set of all the soft sets with a fixed parameter set together with the soft binary piecewise difference operation is a BCK-algebra. This paper is arranged in the following manner. In Section 2, we recall the preliminary concepts in soft set theory together with BCK-algebras. In Section 3, definition and the example of soft binary piecewise difference operation defined in [17] are reminded. The full analysis of the algebraic properties of the new operation, including closure, associativity, unit, inverse element, and abelian property, are then examined. Besides, the properties of this soft set opertaion are handled comparatively with the difference operation in classical set theory and we obtain stunning analogies. In the same section, it is proved that the set of all the soft sets with a fixed parameter set with respect to soft binary piecewise difference operation is a BCK algebra. In the conclusion section, we put into focus the meaning of the study's findings and its potential influence on the field.

2. PRELIMINARIES

Definition 2.1. [1] Let U be the universal set, E be the parameter set, $\mathrm{P(U)}$ be the power set of U and $\mathrm{A} \subseteq \mathrm{E}$. A pair (F, A) is called a soft set over U where F is a set-valued function such that $\mathrm{F}: \mathrm{A} \rightarrow \mathrm{P(U)}$.

Throughout this paper, the collections of all the soft sets defined over U is designated by $\mathrm{S_E(U)}$. Let A be a fixed subset of E and $\mathrm{S_A(U)}$ be the collection of all those soft sets over U with the fixed parameter set A. Clearly, $\mathrm{S_A(U)}$ is a subset of $\mathrm{S_E(U)}$.

Definition 2.2. [4] (K, W) is called a relative null soft set (with regard to W), denoted by $\emptyset_{\mathrm{W}}$, if $\mathrm{K}(\vartheta) = \emptyset$ for all $\vartheta \in \mathrm{W}$ and (K, W) is called a relative whole soft set (with regard to W), denoted by $\mathrm{U_W}$ if $\mathrm{K}(\vartheta) = \mathrm{U}$ for all $\vartheta \in \mathrm{W}$. The relative whole soft set $\mathrm{U_E}$ with regard to E is called the absolute soft set over U. We shall denote by $\emptyset_\emptyset$ the unique soft set over U with an empty parameter set, which is called the empty soft set over U. Note that by $\emptyset_\emptyset$ and by $\emptyset_{\mathrm{A}}$ are different soft sets over U [10].

Definition 2.3. [3] For two soft sets (K, W) and $(\mathrm{T}, \c{S})$, (K, W) is a soft subset of $(\mathrm{T}, \c{S})$ and it is denoted by $(\mathrm{K}, \mathrm{W}) \mathrel{\tilde{\subseteq}} (\mathrm{T}, \c{S})$, if $\mathrm{W} \subseteq \c{S}$ and $\mathrm{K}(\vartheta) \subseteq \mathrm{T}(\vartheta)$, $\forall \vartheta \in \mathrm{W}$. Two soft sets (K, W) and $(\mathrm{T}, \c{S})$ are said to be soft equal if (K, W) is a soft subset of $(\mathrm{T}, \c{S})$ and $(\mathrm{T}, \c{S})$ is a soft subset of (K, W).

Definition 2.4. [4] The relative complement of a soft set (K, W), denoted by $(\mathrm{K}, \mathrm{W})^{\mathrm{r}}$, is defined by $(\mathrm{K}, \mathrm{W})^{\mathrm{r}} = (\mathrm{K}^{\mathrm{r}}, \mathrm{W})$, where $\mathrm{K}^{\mathrm{r}}: \mathrm{W} \rightarrow \mathrm{P(U)}$ is a mapping given by $(\mathrm{K}, \mathrm{W})^{\mathrm{r}} = \mathrm{U} \backslash \mathrm{W}(\vartheta)$ for all $\vartheta \in \mathrm{W}$. From now on, $\mathrm{U} \backslash \mathrm{K}(\vartheta) = [\mathrm{K}(\vartheta)]'$ will be designated by $\mathrm{K}'(\vartheta)$ for the sake of ease.

Soft set operations can be grouped into the following categories as a summary: If " Θ " is used to denote the set operations (Namely, Θ here can be $\cap$, $\cup$, $\backslash$, Δ), then the following soft set operations are defined as follows:

Definition 2.5. [4,5] Let (K,W) and $(\mathrm{T},\mathrm{Ş})$ be soft sets over U. The restricted Θ operation of (K,W) and $(\mathrm{T},\mathrm{Ş})$ is the soft set (B,X), denoted by, $(\mathrm{K},\mathrm{W})\Theta_{\mathrm{R}}(\mathrm{T},\mathrm{Ş}) = (\mathrm{B},\mathrm{X})$, where $\mathrm{X} = \mathrm{W} \cap \mathrm{Ş} \neq \emptyset$ and $\forall\vartheta \in \mathrm{X}$, $\mathrm{B}(\vartheta) = \mathrm{K}(\vartheta)\ \Theta\ \mathrm{T}(\vartheta)$. Here note that if $\mathrm{W} \cap \mathrm{Ş} = \emptyset$, then $(\mathrm{K},\mathrm{W})\Theta_{\mathrm{R}}(\mathrm{T},\mathrm{Ş}) = \emptyset_{\emptyset}$ [10].

Definition 2.6. [2,4,6,7,11] Let (K,W) and $(\mathrm{T},\mathrm{Ş})$ be soft sets over U. The extended Θ operation of (K,W) and $(\mathrm{T},\mathrm{Ş})$ is the soft set (B,X), denoted by, $(\mathrm{K},\mathrm{W})\Theta_{\varepsilon}(\mathrm{T},\mathrm{Ş}) = (\mathrm{B},\mathrm{X})$, where $\mathrm{X} = \mathrm{W} \cup \mathrm{Ş}$ and $\forall\vartheta \in \mathrm{X}$,

$$\mathrm{B}(\vartheta) = \begin{cases} \mathrm{K}(\vartheta), & \vartheta \in \mathrm{W}\setminus\mathrm{Ş}, \\ \mathrm{T}(\vartheta), & \vartheta \in \mathrm{Ş}\setminus\mathrm{W}, \\ \mathrm{K}(\vartheta)\Theta\mathrm{T}(\vartheta), & \vartheta \in \mathrm{W} \cap \mathrm{Ş}. \end{cases}$$

Definition 2.7. [9,12,23] Let (K,W) and $(\mathrm{T},\mathrm{Ş})$ be soft sets over U. The complementary extended Θ operation of (K,W) and $(\mathrm{T},\mathrm{Ş})$ is the soft set (B,X), denoted by, $(\mathrm{K},\mathrm{W})\Theta_{\varepsilon}(\mathrm{T},\mathrm{Ş}) = (\mathrm{B},\mathrm{X})$, where $\mathrm{X} = \mathrm{W} \cup \mathrm{Ş}$ and $\forall\vartheta \in \mathrm{X}$,

$$\mathrm{B}(\vartheta) = \begin{cases} \mathrm{K}'(\vartheta), & \vartheta \in \mathrm{W}\setminus\mathrm{Ş}, \\ \mathrm{T}'(\vartheta), & \vartheta \in \mathrm{Ş}\setminus\mathrm{W}, \\ \mathrm{K}(\vartheta)\Theta\mathrm{T}(\vartheta), & \vartheta \in \mathrm{W} \cap \mathrm{Ş}. \end{cases}$$

Definition 2.8. [8,17] Let (K,W) and $(\mathrm{T},\mathrm{Ş})$ be soft sets over U. The soft binary piecewise Θ operation of (K,W) and $(\mathrm{T},\mathrm{Ş})$ is the soft set, (B,W), denoted by, $(\mathrm{K},\mathrm{W})\widetilde{\Theta}(\mathrm{T},\mathrm{Ş}) = (\mathrm{B},\mathrm{W})$, where $\forall\vartheta \in \mathrm{W}$,

$$\mathrm{B}(\vartheta) = \begin{cases} \mathrm{K}(\vartheta), & \vartheta \in \mathrm{W}\setminus\mathrm{Ş} \\ \mathrm{K}(\vartheta)\ \Theta\ \mathrm{T}(\vartheta), & \vartheta \in \mathrm{W}\cap\mathrm{Ş} \end{cases}$$

A set X containing a binary operation ζ and a constant 0 is called a BCI algebra if it satisfies

BCI-1 $((a \zeta b) \zeta (a \zeta c)) \zeta (c \zeta b) = 0$, BCI-2 $(a \zeta (a \zeta b)) \zeta b = 0$, BCI-3 $a \zeta a = 0$, BCI-4 $a \zeta b = 0$ and $b \zeta a = 0$ imply $a = b$. A BCI algebra is called a BCK algebra if it additionally satisfies: BCK-5 $0 \zeta a = 0$.A BCK algebra X is called bounded if there exists some element $1 \in X$ such that $a * \zeta 1 = 0$ for all $x \in X$. For a bounded BCK algebra X, if an element $a \in X$ satisfies $1 \zeta (1 \zeta a) = a$, then a is called an involution.

3. MORE ON THE PROPERTIES OF SOFT BINARY PIECEWISE DIFFERENCE OPERATION

Definition 3.1. [17] Let $(\mathrm{V},\aleph)$ and (Y,I) be soft sets over U. The complementary soft binary piecewise difference operation of $(\mathrm{V},\aleph)$ and (Y,I) is the soft set $(\mathrm{Q},\aleph)$, denoted by $(\mathrm{V},\aleph)\widetilde{\setminus}(\mathrm{Y},\mathrm{I}) = (\mathrm{Q},\aleph)$, where $\forall\vartheta\in\aleph$,

$$\mathrm{Q}(\vartheta) = \begin{cases} \mathrm{V}(\vartheta), & \vartheta\in\aleph\setminus\mathrm{I} \\ \mathrm{V}(\vartheta)\setminus\mathrm{Y}(\vartheta), & \vartheta\in\aleph\cap\mathrm{I} \end{cases}$$

Here note that, in [17], the above definition was given as "difference of soft sets"; however since there are five types of difference of soft sets operations in the literature, in order to avoid confusion, we prefer to use "soft binary piecewise operation" for the above definition.

Example 3.2. Let $E=\{e_1,e_2,e_3,e_4\}$ be the parameter set $Q=\{e_1, e_3\}$ and $I=\{e_2, e_3, e_4\}$ be the subsets of E and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the initial universe set. Assume that $(V,\aleph)$ and (Y,I) are the soft sets over U defined as follows:

$(V,\aleph) =\{(e_1,\{h_2,h_5\}), (e_3,\{h_1,h_2,h_5\})\}$

$(Y,I)=\{(e_2,\{h_1,h_4,h_5\}), (e_3,\{h_2,h_3,h_4\}),(e_4,\{h_3,h_5\})\}$. Let $(V,\aleph)\ \widetilde{\setminus}\ (Y,I)=(Q,\aleph)$. Then,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Since $\aleph=\{e_1, e_3\}$ and $\aleph\setminus I=\{e_1\}$, so $Q(e_1) = V(e_1)=\{ h_2,h_5\}$. And since $\aleph\cap I=\{e_3\}$, so $Q(e_3)= V(e_3)\setminus Y(e_3)=\{h_1, h_5\}$. Thus, $(V,\aleph)\ \tilde{\setminus}(Y,I)=\{(e_1,\{h_1,h_3,h_4\}), (e_3, \{h_1, h_5\})\}$.

In classical set theory, $V\setminus Y=V\cap Y'$. Now, we have the following analogy.

Theorem 3.3 $(V,\aleph)\ \tilde{\setminus}(Y,R) =(V,\aleph) \cap_\varepsilon (Y, \aleph \cap R)^r$ [17].

Theorem 3.4.$(V,\aleph)\ \tilde{\setminus}(Y,R) =(V,\aleph)\ \tilde{\cap}\ (Y, R)^r$.

Proof: Let $(V,\aleph)\ \tilde{\cap}\ (Y, R)^r=(Q,\aleph)$, where $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus R \\ V(\vartheta)\cap Y'(\vartheta), & \vartheta\in\aleph\cap R \end{cases}$$

Thus, $(Q,\aleph)= (V,\aleph)\ \tilde{\setminus}(Y,R)$.

Theroem 3.5. (Algebraic properties of the operation)

1) The set $S_E(U)$ is closed under the operation $\tilde{\setminus}$.

Proof: It is clear that $\tilde{\setminus}$ is a binary operation in $S_E(U)$. That is,

$$\tilde{\setminus}: S_E(U)\times S_E(U)\rightarrow S_E(U)$$
$$((V,\aleph) , (Y,I)) \rightarrow(V,\aleph)\ \tilde{\setminus}\ (Y,I)= (Q,\aleph)$$

In classical set theory, difference operation does not have associative property. Now, we have the following analogy:

2) $[(V,\aleph)\tilde{\setminus}(Y,\aleph)]\ \tilde{\setminus}(Q,\aleph) \neq (V,\aleph)\ \tilde{\setminus}[(Y,\aleph)\ \tilde{\setminus}(Q,\aleph)]$

Proof: Let $(V,\aleph)\ \tilde{\setminus}\ (Y,\aleph)=(T,\aleph)$, where $\forall\vartheta\in\aleph$;

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(T, \aleph)\ \tilde{\setminus}(Q, \aleph)=(M, \aleph)$, where $\forall\vartheta\in\aleph$;

$$M(\vartheta)=\begin{cases} T(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ T(\vartheta)\setminus Q(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus,

$$M(\vartheta)=\begin{cases} T(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)]\setminus Q(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(Y,\aleph)\tilde{\setminus}\ (Q,\aleph) =(L,\aleph)$, where $\forall\vartheta\in\aleph$;

$$L(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ Y(\vartheta)\setminus Q(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(V,\aleph)\ \tilde{\setminus}(L,\aleph) =(D,\aleph)$, where $\forall\vartheta\in\aleph$;

$$D(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus L(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus,

$$D(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus[Y(\vartheta)\setminus Q(\vartheta)], & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

It is seen that $(M, \aleph)$ and $(D,\aleph)$ are not the same soft sets.

That is, on the soft sets whose parameter set are the same, the operation $\tilde{\setminus}$ does not have associativity property. Moreover, we have the following:

3) $[(V,\aleph)\,\tilde{\setminus}\,(Y,I)]\,\tilde{\setminus}(Q,Z)\neq(V,\aleph)\,\tilde{\setminus}[(Y,I)\,\tilde{\setminus}(Q,Z)]$

Proof: Let $(V,\aleph)\,\tilde{\setminus}\,(Y,I)=(T,\aleph)$, where $\forall\vartheta\in\aleph$;

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Let $(T,\aleph)\,\tilde{\setminus}(Q,Z)=(M,\aleph)$, where $\forall\vartheta\in\aleph$;

$$M(\vartheta)=\begin{cases} T(\vartheta), & \vartheta\in\aleph\setminus Z \\ T(\vartheta)\setminus Q(\vartheta), & \vartheta\in\aleph\cap Z \end{cases}$$

Thus,

$$M(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in(\aleph\setminus I)\setminus Z=\aleph\cap I'\cap Z' \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in(\aleph\cap I)\setminus Z=\aleph\cap I\cap Z' \\ V(\vartheta)\setminus Q(\vartheta), & \vartheta\in(\aleph\setminus I)\cap Z=\aleph\cap I'\cap Z \\ [V(\vartheta)\setminus Y(\vartheta)]\setminus Q(\vartheta), & \vartheta\in(\aleph\cap I)\cap Z=\aleph\cap I\cap Z \end{cases}$$

Let $(Y,I)\,\tilde{\setminus}\,(Q,Z)=(K,I)$, where $\forall\vartheta\in I$;

$$K(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in I\setminus Z \\ Y(\vartheta)\setminus Q(\vartheta), & \vartheta\in I\cap Z \end{cases}$$

Let $(V,\aleph)\,\tilde{\setminus}\,(K,I)=(S,\aleph)$, where $\forall\vartheta\in\aleph$;

$$S(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus K(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus,

$$S(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap(I\setminus Z)=\aleph\cap I\cap Z' \\ V(\vartheta)\setminus[Y(\vartheta)\setminus Q(\vartheta)], & \vartheta\in\aleph\cap(I\cap Z)=\aleph\cap I\cap Z \end{cases}$$

Here let handle $\vartheta\in\aleph\setminus I$ in the first line of $S(\vartheta)$. Since $\aleph\setminus I=\aleph\cap I'$, if $\vartheta\in I'$, then $\vartheta\in Z\setminus I$ or $\vartheta\in(I\cup Z)'$. Hence, if $\vartheta\in\aleph\setminus I$, then $\vartheta\in\aleph\cap I'\cap Z'$ or $\vartheta\in\aleph\cap I'\cap Z$. Thus, it is seen that $(M, \aleph)$ and $(S,\aleph)$ are not the same soft set. That is, for the soft sets whose parameter set are not the same, the operation $\tilde{\setminus}$ does not have associativity property in the set $S_E(U)$.

In classical set theory, difference operation does not have commutative property. Now, we have the following analogy:

4) $(V,\aleph)\,\tilde{\setminus}\,(Y,I) \neq (Y,I)\,\tilde{\setminus}\,(V,\aleph)$

Proof: Let $(V,\aleph)\,\tilde{\setminus}(Y,I)=(Q,\aleph)$. Then, $\forall\vartheta\in\aleph$;

,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Let $(Y,I)\,\tilde{\setminus}\,(V,\aleph)=(T,I)$. Then $\forall\vartheta\in I$;

$$T(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in I\setminus\aleph \\ Y(\vartheta)\setminus\aleph(\vartheta), & \vartheta\in I\cap\aleph \end{cases}$$

Here, while the parameter set of the soft set of the left hand side is $\aleph$; the parameter set of the soft set of the right hand side is I. Thus, by the definition of soft equality

$$(V,\aleph)\,\tilde{\setminus}\,(Y,I) \neq (Y,I)\,\tilde{\setminus}(V,\aleph).$$

Hence, the operation $\tilde{\setminus}$ does not have commutative property in the set $S_E(U)$, where the parameter sets of the soft sets are different. Moreover, it is easy to see that $(V,\aleph)\,\tilde{\setminus}\,(Y,\aleph)\neq(Y,\aleph)\,\tilde{\setminus}(V,\aleph)$ since $V(\vartheta)\setminus Y(\vartheta) \neq Y(\vartheta)\setminus\aleph(\vartheta)$. That is, the operation $\tilde{\setminus}$ does not have commutative property when the parameter sets of the soft sets are the same.

5) $(V,\aleph)\tilde{\setminus}\,(V,\aleph)=\emptyset_\aleph$.

Proof: Let $(V,\aleph)\,\tilde{\setminus}\,(V,\aleph)=(Q,\aleph)$, where $\forall\vartheta\in\aleph$;

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus V(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Here $\forall\vartheta\in\aleph$; $Q(\vartheta)=V(\vartheta)\setminus V(\vartheta)=\emptyset$, thus $(Q,\aleph)=\emptyset_\aleph$.

That is, the operation $\tilde{\setminus}$ does not have idempotency property in the set $S_E(U)$.

6) $(V,\aleph)\,\tilde{\setminus}\,\emptyset_K=(V,\aleph)$ [17].

Here note that, for the soft sets (no matter what the parameter set is), null soft sets with respect to any parameter set (Here, K may be E, $\aleph$, $\emptyset$, or any set) is the right identity element for the operation $\tilde{\setminus}$ in the set $S_E(U)$.

7) $\emptyset_K\tilde{\setminus}\,(V,\aleph)=\emptyset_K$ [17].

Here note that, for the soft sets (no matter what the parameter set is), null soft sets with respect to any parameter (Here, K may be E, $\aleph$, $\emptyset$, or any set) is the left-absorbing element for the operation $\tilde{\setminus}$ in the set $S_E(U)$.

8) $(V,\aleph)\,\tilde{\setminus}\,U_\aleph=(V,\aleph)\,\tilde{\setminus}U_E=\emptyset_\aleph$ [17].

9) $U_\aleph\tilde{\setminus}(V,\aleph)=(V,\aleph)^r$ and $U_E\tilde{\setminus}(V,\aleph)\neq(V,\aleph)^r$ [17].

10) $(V,\aleph)\,\tilde{\setminus}(V,\aleph)^r=(V,\aleph)$.

Proof: Let $(V,\aleph)^r=(Q,\aleph)$. Hence, $\forall\vartheta\in\aleph$; $Q(\vartheta)=V'(\vartheta)$. Let $(V,\aleph)\,\tilde{\setminus}(Q,\aleph)=(T,\aleph)$, where $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\cap Q'(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Hence, $\forall\vartheta\in\aleph$; $T(\vartheta)= V(\vartheta)\cap Q'(\vartheta)=V(\vartheta)\cap V(\vartheta)= V(\vartheta)$, thus $(T,\aleph)=(V,\aleph)$.

Note that, relative complement of every soft set is its own right identity element for the operation $\tilde{\setminus}$ in the set $S_E(U)$.

11) $(V,\aleph)^r\,\tilde{\setminus}(V,\aleph) = (V,\aleph)^r$.

Proof: Let $(V,\aleph)^r=(Q,\aleph)$. Hence, $\forall\vartheta\in\aleph$; $Q(\vartheta)=V'(\vartheta)$. Let $(Q,\aleph)\tilde{\setminus}(V,\aleph)=(T,\aleph)$, where $\forall\vartheta\in\aleph$;

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ Q(\vartheta)\cap V'(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Hence, $\forall\vartheta\in\aleph$; $T(\vartheta)= Q(\vartheta)\cap V'(\vartheta)= V'(\vartheta)\cap V'(\vartheta)= V'(\vartheta)$, thus $(T,\aleph) = (V,\aleph)^r$.

Note that, relative complement of a soft set is its own left-absorbing element for the operation $\tilde{\setminus}$ in the set $S_E(U)$.

12) $[(V,\aleph)\tilde{\setminus}(Y,I)]^r=(V,\aleph)\overset{*}{\underset{+}{\sim}}(Y,I)$.

Proof: Let $(V,\aleph)\tilde{\setminus}(Y,I)=(Q,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\cap Y'(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Let $(Q,\aleph)^r=(T,\aleph)$, so $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} V'(\vartheta), & \vartheta\in\aleph\setminus I \\ V'(\vartheta)\cup Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus, $(T,\aleph)=(V,\aleph)\overset{*}{\underset{+}{\sim}}(Y,I)$.

In classical set theory, $V\cap Y = U \Leftrightarrow V = U$ and $Y = U$. Now, we have the following:

13) $(V,\aleph)\tilde{\setminus}(Y,\aleph)= U_\aleph \Leftrightarrow (V,\aleph) = U_\aleph$ and $(Y,\aleph)=\emptyset_\aleph$.

Proof: Let $(V,\aleph)\tilde{\setminus}(Y,\aleph) = (T,\aleph)$. Hence, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph-\aleph=\emptyset \\ V(\vartheta)\cap Y'(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Since $(T,\aleph) = U_\aleph$, $\forall\vartheta\in\aleph$, $T(\vartheta)=U$. Hence, $\forall\vartheta\in\aleph$, $T(\vartheta)= V(\vartheta)\cap Y'(\vartheta)=U \Leftrightarrow \forall\vartheta\in\aleph$, $V(\vartheta)=U$ and $Y'(\vartheta)=U \Leftrightarrow \forall\vartheta\in\aleph$, $V(\vartheta)=U$ and $Y(\vartheta)=\emptyset \Leftrightarrow (V,\aleph) = U_\aleph$ and $(Y,\aleph) = \emptyset_\aleph$.

In classical set theory, for all A, $\emptyset\subseteq A$. Now, we have the following:

14) $\emptyset_\aleph \,\tilde{\subseteq}(V,\aleph)\tilde{\setminus}(Y,I)$ and $\emptyset_I \,\tilde{\subseteq}(Y,I)\tilde{\setminus}(V,\aleph)$.

In classical set theory, for all A, $A \subseteq U$. Now, we have the following:

15) $(V,\aleph)\,\tilde{\setminus}\,(Y,I) \,\tilde{\subseteq}\, U_{\aleph}$ and $(Y,I)\,\tilde{\setminus}\,(V,\aleph) \,\tilde{\subseteq}\, U_{I}$.

In classical set theory, $V\setminus Y \subseteq V$ and $Y\setminus V \subseteq Y$. Moreover, $V\setminus Y \subseteq Y'$ and $Y\setminus V \subseteq V'$ Now, we have the following analogy:

16) $(V,\aleph)\tilde{\setminus}(Y,I) \,\tilde{\subseteq}(V,\aleph)$ and $(Y,I)\,\tilde{\setminus}(V,\aleph) \,\tilde{\subseteq}(Y,I)$. Moreover, $(V,\aleph)\,\tilde{\setminus}(Y,\aleph) \,\tilde{\subseteq}(Y,\aleph)^r$ and $(Y,\aleph)\,\tilde{\setminus}(V,\aleph) \,\tilde{\subseteq}(V,\aleph)^r$.

Proof: Let $(V,\aleph)\,\tilde{\setminus}(Y,I) =(Q,\aleph)$. First of all, $\aleph \subseteq \aleph$. Moreover, $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\cap Y'(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Since $\forall\vartheta\in\aleph\setminus I$; $Q(\vartheta)\subseteq V(\vartheta)$ and $\forall\vartheta\in\aleph\cap I$; $Q(\vartheta)=V(\vartheta)\cap Y'(\vartheta) \subseteq V(\vartheta)$, thus $\forall\vartheta\in\aleph$; $Q(\vartheta)\subseteq V(\vartheta)$. This shows that $(Q,\aleph) = (V,\aleph)\tilde{\setminus}(Y,I)\tilde{\subseteq}(V,\aleph)$. $(Y,I)\tilde{\setminus}(V,\aleph)\tilde{\subseteq}(Y,I)$ can be shown similarly.

Let $(V,\aleph)\,\tilde{\setminus}(Y,\aleph) =(K,\aleph)$. First of all, $\aleph \subseteq \aleph$. Moreover, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\cap Y'(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Since $\forall\vartheta\in\aleph$; $K(\vartheta)=V(\vartheta)\cap Y'(\vartheta)\subseteq Y'(\vartheta)$, this shows that $(K,\aleph) = (V,\aleph)\tilde{\setminus}(Y,\aleph)\tilde{\subseteq}(Y,\aleph)^r$. $(Y,\aleph)\tilde{\setminus}(V,\aleph) \tilde{\subseteq}(V,\aleph)^r$ can be shown similarly.

In classial set theory; $V=(V\setminus Y) \cup (V\cap Y)$ and $Y=(Y\setminus V) \cup (Y\cap V)$. Now, we have the following analogy:

17) $(V,\aleph) =[(V,\aleph)\,\tilde{\setminus}(Y,I)\,]\,\tilde{\cup}\,[(V,\aleph)\,\tilde{\cap}\,(Y,I)\,]$ and $(Y,I) =[(Y,I)\,\tilde{\setminus}(V,\aleph)\,]\,\tilde{\cup}\,[(Y,I)\,\tilde{\cap}\,(V,\aleph)]$.

Proof: Let $(V,\aleph)\,\tilde{\setminus}(Y,I) =(Q,\aleph)$, where $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

and $(V,\aleph)\,\tilde{\cap}\,(Y,I) =(K,\aleph)$, where $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\cap Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Let $(Q,\aleph)\,\tilde{\cup}\,(K,\aleph) =(T,\aleph)$, where $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ Q(\vartheta)\cup K(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus,

$$T(\vartheta)=\begin{cases} V(\vartheta)\cup V(\vartheta), & \vartheta\in(\aleph\setminus I)\cap(\aleph\setminus I)=(\aleph\setminus I) \\ V(\vartheta)\cup[V(\vartheta)\cap Y(\vartheta)], & \vartheta\in(\aleph\setminus I)\cap(\aleph\cap I)=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)]\cup V(\vartheta), & \vartheta\in(\aleph\cap I)\cap(\aleph\setminus I)=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)]\cup[V(\vartheta)\cap Y(\vartheta)], & \vartheta\in(\aleph\cap I)\cap(\aleph\cap I)=(\aleph\cap I) \end{cases}$$

Hence,

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ [V(\vartheta)\setminus Y(\vartheta)] \cup [V(\vartheta)\cap Y(\vartheta)], & \vartheta\in\aleph\cap I \end{cases}$$

Since, $\forall\vartheta\in\aleph$, $[V(\vartheta)\setminus Y(\vartheta)] \cup [V(\vartheta)\cap Y(\vartheta)]=V(\vartheta)$, $T(\vartheta)= V(\vartheta)$. Thus, $(T,\aleph) =(V,\aleph)$. The other can be shown similarly.

In classical set theory, $V \cup Y= (V\setminus Y) \cup Y$ and $Y \cup V = (Y\setminus V) \cup V$. Now, we have the following analogy.

18) $(V,\aleph)\ \tilde{\cup}\ (Y,I) =[(V,\aleph)\ \tilde{\setminus}(Y,I)\ ]\ \tilde{\cup}(Y,I)$ and $(Y,I)\ \tilde{\cup}\ (V,\aleph) =[(Y,I)\ \tilde{\setminus}\ (V,\aleph)\ ]\ \tilde{\cup}(V,\aleph)$.

Proof: Let $(V,\aleph)\ \tilde{\setminus}(Y,I) =(Q,\aleph)$, where $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

and $(Q,\aleph)\ \tilde{\cup}(Y,I) =(K,\aleph)$, where $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus I \\ Q(\vartheta)\cup Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in(\aleph\setminus I)\setminus I=\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in(\aleph\cap I)\setminus I=\emptyset \\ V(\vartheta) \cup Y(\vartheta), & \vartheta\in(\aleph\setminus I)\cap I=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)] \cup Y(\vartheta) & \vartheta\in(\aleph\cap I)\cap(\aleph\cap I)=(\aleph\cap I) \end{cases}$$

Since $[V(\vartheta)\setminus Y(\vartheta)] \cup Y(\vartheta)= V(\vartheta)\cup Y(\vartheta)$, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta) \cup Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus, $(K,\aleph) = (V,\aleph)\ \tilde{\cup}(Y,\aleph)$. The other can be shown similarly.

In classical set theory, $V\subseteq Y \Leftrightarrow V\setminus Y = \emptyset$. In [17], it was shown that if $(V,\aleph)\ \tilde{\subseteq}(Y,I)$, then $(V,\aleph)\ \tilde{\setminus}(Y,I) =\emptyset_{\aleph}$. For satisfying also the necessity, we have the following:

19) $(V,\aleph)\ \tilde{\subseteq}(Y,\aleph) \Leftrightarrow (V,\aleph)\ \tilde{\setminus}(Y,\aleph) =\emptyset_{\aleph}$.

Proof: Let $(V,\aleph)\ \tilde{\subseteq}(Y,\aleph)$. Then, $\forall\vartheta\in\aleph, V(\vartheta) \subseteq Y(\vartheta)$. And let $(V,\aleph)\ \tilde{\setminus}(Y,\aleph) =(Q,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Since $\forall\vartheta\in\aleph$, $V(\vartheta) \subseteq Y(\vartheta)$, then $V(\vartheta)\setminus Y(\vartheta)= \emptyset$, and hence $(Q,\aleph) = (V,\aleph)\ \tilde{\setminus}(Y,\aleph) =\emptyset_{\aleph}$, For the converse, we need to show that when $(V,\aleph)\tilde{\setminus}(Y,\aleph) =\emptyset_{\aleph}$, then $(V,\aleph)\ \tilde{\subseteq}(Y,\aleph)$. In order to show this, let $(V,\aleph)\ \tilde{\setminus}(Y,\aleph) = (T,\aleph)$. Then,

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Since, $(T,\aleph)=\emptyset_{\aleph}$, $\forall\vartheta\in\aleph$, $V(\vartheta)\setminus Y(\vartheta)=\emptyset$. Thus, $\forall\vartheta\in\aleph$, $V(\vartheta)\subseteq Y(\vartheta)$. Hence, $(V,\aleph)\tilde{\subseteq}(Y,\aleph)$.

In classical set theory, $V\setminus(V\setminus Y)=V\cap Y$ and $Y\setminus(Y\setminus V)=Y\cap V$. Now, we have the following:

20) $(V,\aleph)\ \tilde{\setminus}\ [(V,\aleph)\ \tilde{\setminus}(Y,\aleph)]=(V,\aleph)\ \tilde{\cap}\ (Y,\aleph)$ and $(Y,\aleph)\ \tilde{\setminus}\ [(Y,\aleph)\ \tilde{\setminus}(V,\aleph)]=(Y,\aleph)\ \tilde{\cap}\ (V,\aleph)$.

Proof: Let $(V,\aleph)\ \tilde{\setminus}(Y,\aleph)=(Q,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(V,\aleph)\ \tilde{\setminus}(Q,\aleph)=(K,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus Q(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus(V(\vartheta)\setminus Y(\vartheta)), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Hence, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\cap Y(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, $\forall\vartheta\in\aleph$, $(K,\aleph)=(V,\aleph)\ \tilde{\setminus}\ (Y,\aleph)$. Moreover $(Y,\aleph)\ \tilde{\setminus}\ [(Y,\aleph)\ \tilde{\setminus}(V,\aleph)]=(Y,\aleph)\ \tilde{\cap}(V,\aleph)$ can be shown similarly.

In classical set theory, $V\setminus(Y\cap V)=V\setminus Y$ and $Y\setminus(V\cap Y)=Y\setminus V$. Now we have the following:

21) $(V,\aleph)\tilde{\setminus}\ [(Y,I)\tilde{\cap}(V,\aleph)]=(V,\aleph)\tilde{\setminus}(Y,I)$ and $(Y,I)\ \tilde{\setminus}\ [(V,\aleph)\ \tilde{\cap}\ (Y,I)]=(Y,I)\ \tilde{\setminus}(V,\aleph)$.

Proof: Let $(Y,I)\ \tilde{\cap}(V,\aleph)=(Q,\aleph)$. Then, $\forall\vartheta\in I$,

$$Q(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in I\setminus\aleph \\ Y(\vartheta)\cap V(\vartheta), & \vartheta\in I\cap\aleph \end{cases}$$

Let $(V,\aleph)\ \tilde{\setminus}(Q,I)=(K,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Q(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta) & \vartheta\in\aleph\cap(I\setminus\aleph)=\emptyset \\ V(\vartheta)\setminus[(Y(\vartheta)\cap V(\vartheta)], & \vartheta\in\aleph\cap(I\cap\aleph)=\aleph\cap I \end{cases}$$

Since $V(\vartheta)\setminus[(Y(\vartheta)\cap V(\vartheta)]=V(\vartheta)\setminus Y(\vartheta)$, hence $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus, (K,ℵ) = (V,ℵ)$\tilde{\setminus}$(Y,I). Moreover (Y,I) $\tilde{\setminus}$ [(V,ℵ)$\tilde{\cap}$ (Y,I)]= (Y,I) $\tilde{\setminus}$(V,ℵ) can be shown similarly.

In classical set theory, V\(V∩Y)=V\Y and Y\(Y∩V)=Y\V. Now, we have the following:

22) (V,ℵ)$\tilde{\setminus}$[(V,ℵ) $\tilde{\cap}$(Y,ℵ)]= (V,ℵ)$\tilde{\setminus}$(Y,ℵ) and (Y,ℵ)$\tilde{\setminus}$[(Y,ℵ)$\tilde{\cap}$(V,ℵ)]= (Y,ℵ)$\tilde{\setminus}$(V,ℵ).

Proof: Let (V,ℵ) $\tilde{\cap}$(Y,ℵ) =(Q,ℵ). Then, ∀ϑ∈ℵ,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\cap Y(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let (V,ℵ) $\tilde{\setminus}$ (Q,ℵ) =(K,ℵ). Then, ∀ϑ∈ℵ,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus Q(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, ∀ϑ∈ℵ,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus [(V(\vartheta)\cap Y(\vartheta)], & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Hence, ∀ϑ∈ℵ,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, ∀ϑ∈ℵ, (K,ℵ)=(V,ℵ)$\tilde{\setminus}$(Y,ℵ). Moreover (Y,ℵ)$\tilde{\setminus}$[(Y,ℵ)$\tilde{\cap}$(V,ℵ)]= (Y,ℵ)$\tilde{\setminus}$(V,ℵ) can be shown similarly.

NOTE: In classical set theory, V∩Y=Y∩V, hence V\(Y∩V)=V\(V∩Y)=V\Y and Y\(V∩Y)= Y\(Y∩V)=Y\V. However, since (V,ℵ) $\tilde{\cap}$ (Y,I)≠ (Y,I) $\tilde{\cap}$(V,ℵ); while Theorem 3.4 (21) is satisfied when the parameter sets of soft sets are different; Theorem 3.4. (22) is satisfied only when the parameter sets of soft sets are the same.

In classical set theory, if V∩Y= ∅, then V\Y=V. Now, we have the following analogy:

23) If (V,ℵ) $\tilde{\cap}$ (Y, I) $= \emptyset_{\aleph}$, then (V,ℵ) $\tilde{\setminus}$(Y,I) =(V,ℵ).

Proof: Let (V,ℵ) $\tilde{\cap}$(Y,I) =(Q,ℵ). Then, for all ϑ∈ℵ,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\cap Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Since, (Q,ℵ) = $\emptyset_{\aleph}$, then for all ϑ∈ℵ. Q(ϑ)= ∅. Thus, for all ϑ∈ℵ\I; Q(ϑ)=V(ϑ)= ∅, and for all ϑ∈ℵ∩I; Q(ϑ)=V(ϑ) ∩Y(ϑ)= ∅. Let (V,ℵ) $\tilde{\setminus}$(Y,I) =(S,ℵ). Then, for all ϑ∈ℵ,

$$S(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Since, for all ϑ∈ℵ∩I; V(ϑ ∩Y(ϑ)= ∅, V(ϑ) \Y(ϑ)=V(ϑ). Therefore,

$$S(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus, $(S,\aleph)=(V,\aleph)\ \tilde{\setminus}(Y,I)=(V,\aleph)$.

In classical set theory, $(V\setminus Y)\cap Y=\emptyset$ and $(Y\setminus V)\cap V=\emptyset$. Now, we have the similar analogy:

24) $[(V,\aleph)\ \tilde{\setminus}\ (Y,\aleph)\ ]\tilde{\cap}(Y,\aleph)=\emptyset_{\aleph}$ and $[(Y,\aleph)\ \tilde{\setminus}(V,\aleph)\ ]\tilde{\cap}(V,\aleph)=\emptyset_{\aleph}$.

Proof: Let $(V,\aleph)\ \tilde{\setminus}\ (Y,\aleph)=(Q,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

And let $(Q,\aleph)\tilde{\cap}(Y,\aleph)=(T,\aleph)$, where $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ Q(\vartheta)\cap Y(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)]\cap Y(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Hence, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ \emptyset, & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Since $\forall\vartheta\in\aleph$, $T(\vartheta)=\emptyset$, thus $(T,\aleph)=\emptyset_{\aleph}$. Moreover, $[(Y,\aleph)\ \tilde{\setminus}\ (V,\aleph)\ ]\ \tilde{\cap}(V,\aleph)=\emptyset_{\aleph}$ can be shown similarly.

NOTE: In classical set theory, $(V\setminus Y)\setminus Y=V\setminus Y$ (as $(V\setminus Y)\cap Y=\emptyset$) and $(Y\setminus V)\setminus V=Y\setminus V$ (as $(Y\setminus V)\cap V=\emptyset$).

As an analogy, we have the following:

25) $[(V,\aleph)\tilde{\setminus}(Y,I)\ ]\ \tilde{\setminus}(Y,I)=(V,\aleph)\tilde{\setminus}(Y,I)$ and $[(Y,I)\ \tilde{\setminus}(V,\aleph)\ ]\ \tilde{\setminus}(V,\aleph)=(Y,I)\tilde{\setminus}(V,\aleph)$.

Proof: Let $(V,\aleph)\ \tilde{\setminus}\ (Y,I)=(Q,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

And let $(Q,\aleph)\ \tilde{\setminus}\ (Y,I)=(T,\aleph)$, where $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus I \\ Q(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in(\aleph\setminus I)\setminus I=\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in(\aleph\cap I)\setminus I=\emptyset \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in(\aleph\setminus I)\cap I=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)]\setminus Y(\vartheta), & \vartheta\in(\aleph\cap I)\cap I=\aleph\cap I \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta)], & \vartheta\in\aleph\cap I \end{cases}$$

Thus, $(T,\aleph)= (V,\aleph)\tilde{\setminus}(Y,I)$. Also, $[(Y,I)\tilde{\setminus}(V,\aleph)\ ]\ \tilde{\setminus}(V,\aleph) = (Y,I)\tilde{\setminus}(V,\aleph)$ can be shown simillarly.

In classical set theory, $(V\setminus Y) \cap(Y\setminus V)=\emptyset$. Now, we have the following analogy.

26) $[(V,\aleph)\tilde{\setminus}(Y,\aleph)]\ \tilde{\cap}\ [(Y,\aleph)\tilde{\setminus}(V,\aleph)\ ]=\emptyset_{\aleph}$ and $[(Y,\aleph)\tilde{\setminus}(V,\aleph)]\ \tilde{\cap}\ [(V,\aleph)\tilde{\setminus}(Y,\aleph)\ ]=\emptyset_{\aleph}$.

Proof: Let $(V,\aleph)\tilde{\setminus}(Y,\aleph) =(Q,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(Y,\aleph)\tilde{\setminus}(V,\aleph) =(K,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ Y(\vartheta)\setminus V(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

And let $(Q,\aleph)\tilde{\cap}(K,\aleph) =(T,\aleph)$, where $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ Q(\vartheta)\cap K(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)]\cap[Y(\vartheta)\setminus V(\vartheta)], & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Hence, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ \emptyset, & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Since $\forall\vartheta\in\aleph$, $T(\vartheta)=\emptyset$, $(T,\aleph) =\emptyset_{\aleph}$. Moreover $[(Y,\aleph)\tilde{\setminus}(V,\aleph)\ ]\ \tilde{\cap}\ [(V,\aleph)\ \tilde{\setminus}(Y,\aleph)\ ]=\emptyset_{\aleph}$ can be shown similarly.

NOTE: In classical set theory, $(V\setminus Y)\setminus(Y\setminus V)=V\setminus Y$ (as $(V\setminus Y)\cap(Y\setminus V)=\emptyset$) and $(Y\setminus V)\setminus(V\setminus Y)=Y\setminus V$ (as $(Y\setminus V)\cap(V\setminus Y)=\emptyset$). As an analogy, we have the following:

27) $[(V,\aleph)\ \tilde{\setminus}(Y,I)]\ \tilde{\setminus}\ [(Y,I)\ \tilde{\setminus}(V,\aleph)\ ]= (V,\aleph)\ \tilde{\setminus}(Y,I)$ and $[(Y,I)\ \tilde{\setminus}(V,\aleph)\ ]\ \tilde{\setminus}\ [(V,\aleph)\tilde{\setminus}(Y,I)\ ]= (Y,I)\tilde{\setminus}(V,\aleph)$

Proof: Let $(V,\aleph)\tilde{\setminus}(Y,I) =(Q,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Let $(Y,I)\tilde{\setminus}(V,\aleph) =(K,I)$. Then, $\forall\vartheta\in I$,

$$K(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in I\setminus\aleph \\ Y(\vartheta)\setminus V(\vartheta), & \vartheta\in I\cap\aleph \end{cases}$$

And let $(Q,\aleph)\ \tilde{\setminus}\ (K,I)=(T,\aleph)$, where $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus I \\ Q(\vartheta)\setminus K(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in(\aleph\setminus I)\setminus I=\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in(\aleph\cap I)\setminus I=\emptyset \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in(\aleph\setminus I)\cap(I\setminus\aleph)=\emptyset \\ V(\vartheta)\setminus[Y(\vartheta)\setminus V(\vartheta)], & \vartheta\in(\aleph\setminus I)\cap(I\cap\aleph)=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)]\setminus Y(\vartheta), & \vartheta\in(\aleph\cap I)\cap(I\setminus\aleph)=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)]\setminus[Y(\vartheta)\setminus V(\vartheta)], & \vartheta\in(\aleph\cap I)\cap(I\cap\aleph)=I\cap\aleph \end{cases}$$

Hence, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus, $(T,\aleph)=(V,\aleph)\ \tilde{\setminus}((Y,I)$. Moreover, $[(Y,I)\tilde{\setminus}(V,\aleph)]\tilde{\setminus}[(V,\aleph)\tilde{\setminus}((Y,I)]=(Y,I)\tilde{\setminus}(V,\aleph)$ can be shown similarly.

In classical set theory, $(V\setminus Y)\cap(Y\cap V)=\emptyset$ and $(Y\setminus V)\cap(V\cap Y)=\emptyset$. Now, we have the following analogy.

28) $[(V,\aleph)\ \tilde{\setminus}\ (Y,\aleph)\ ]\ \tilde{\cap}\ [(Y,\aleph)\ \tilde{\cap}\ (V,\aleph)\ ]=\emptyset_{\aleph}$ and $[(Y,\aleph)\ \tilde{\setminus}\ (V,\aleph)\ ]\ \tilde{\cap}\ [(V,\aleph)\ \tilde{\cap}\ (Y,\aleph)\ ]=\emptyset_{\aleph}$.

Proof: Let $(V,\aleph)\ \tilde{\setminus}(Y,\aleph)=(Q,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(Y,\aleph)\tilde{\cap}\ (V,\aleph)=(K,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ Y(\vartheta)\cap V(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

And let $(Q,\aleph)\ \tilde{\cap}\ (K,\aleph)=(T,\aleph)$, where $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ Q(\vartheta)\cap K(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)]\cap[Y(\vartheta)\cap V(\vartheta)], & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Hence, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ \emptyset, & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Since $\forall\vartheta\in\aleph$, $T(\vartheta)=\emptyset$, $(T,\aleph)=\emptyset_{\aleph}$. Moreover, $[(Y,\aleph)\,\tilde{\setminus}(V,\aleph)\,]\,\tilde{\cap}\,[(V,\aleph)\ \tilde{\cap}\,(Y,\aleph)\,]=\emptyset_{\aleph}$ can be shown similarly.

NOTE: In classical set theory, $(V\setminus Y)\setminus(Y\cap V)=(V\setminus Y)$ (since $(V\setminus Y)\cap(Y\cap V)=\emptyset$) and $(Y\setminus V)\setminus(V\cap Y)=Y\setminus V$ (since $(Y\setminus V)\cap(V\cap Y)=\emptyset$). As an analogy, we have the following:

29) $[(V,\aleph)\,\tilde{\setminus}(Y,I)\,]\,\tilde{\setminus}\,[(Y,I)\,\tilde{\cap}(V,\aleph)\,]=(V,\aleph)\,\tilde{\setminus}(Y,I)$ and $[(Y,I)\,\tilde{\setminus}(V,\aleph)\,]\,\tilde{\setminus}[(V,\aleph)\,\tilde{\cap}(Y,I)\,]=(Y,I)\,\tilde{\setminus}\,(V,\aleph)$.

Proof: Let $(V,\aleph)\tilde{\setminus}(Y,I)=(Q,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Let $(Y,I)\,\tilde{\cap}\,(V,\aleph)=(K,I)$. Then, $\forall\vartheta\in I$,

$$K(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in I\setminus\aleph \\ Y(\vartheta)\cap V(\vartheta), & \vartheta\in I\cap\aleph \end{cases}$$

And let $(Q,\aleph)\,\tilde{\setminus}\,(K,I)=(T,\aleph)$, where $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus I \\ Q(\vartheta)\setminus K(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in(\aleph\setminus I)\setminus I=\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in(\aleph\cap I)\setminus I=\emptyset \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in(\aleph\setminus I)\cap(I\setminus\aleph)=\emptyset \\ V(\vartheta)\setminus[Y(\vartheta)\cap V(\vartheta)], & \vartheta\in(\aleph\setminus I)\cap(I\cap\aleph)=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)]\setminus Y(\vartheta), & \vartheta\in(\aleph\cap I)\cap(I\setminus\aleph)=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)]\setminus[Y(\vartheta)\cap V(\vartheta)], & \vartheta\in(\aleph\cap I)\cap(I\cap\aleph)=I\cap\aleph \end{cases}$$

Hence, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus, $(T,\aleph)=(V,\aleph)\ \tilde{\setminus}((Y,I)$. Moreover, $[(Y,I)\ \tilde{\setminus}(V,\aleph)]\tilde{\setminus}[(V,\aleph)\ \tilde{\cap}(Y,I)]=(Y,I)\ \tilde{\setminus}(V,\aleph)$ can be shown similarly.

In classical set theory, $V\cap(Y\setminus V)=\emptyset$ and $Y\cap(V\setminus Y)=\emptyset$. Now, we have the following analogy:

30) $(V,\aleph)\,\tilde{\cap}\,[(Y,\aleph)\tilde{\setminus}(V,\aleph)]=\emptyset_{\aleph}$ and $(Y,\aleph)\,\tilde{\cap}\,[(V,\aleph)\,\tilde{\setminus}\,(Y,\aleph)\,]=\emptyset_{\aleph}$.

Proof: Let $(Y,\aleph)\,\tilde{\setminus}(V,\aleph)=(Q,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ Y(\vartheta)\setminus V(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(V,\aleph)\tilde{\cap}\,(Q,\aleph)=(K,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\cap Q(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\cap[Y(\vartheta)\setminus V(\vartheta)], & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Hence, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ \emptyset, & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Since $\forall\vartheta\in\aleph$, $K(\vartheta)=\emptyset$, $(K,\aleph)=\emptyset_{\aleph}$. Moreover $(Y,\aleph)\ \tilde{\cap}\ [(V,\aleph)\tilde{\setminus}(Y,\aleph)]=\emptyset_{\aleph}$ can be shown similarly.

NOTE: In classical set theory, V\(Y\V)=V (since V∩(Y\V)= ∅) and Y\(V\Y)=Y (since Y∩(V\Y)= ∅).

Now, we have the followin analogy:

31) $(V,\aleph)\ \tilde{\setminus}\ [(Y,I)\tilde{\setminus}(V,\aleph)]=(V,\aleph)$ and $(Y,I)\ \tilde{\setminus}\ [(V,\aleph)\ \tilde{\setminus}(Y,I)]=(Y,I)$.

Proof: Let $(Y,I)\ \tilde{\setminus}(V,\aleph)=(Q,I)$. Then, $\forall\vartheta\in I$,

$$Q(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in I\setminus\aleph \\ Y(\vartheta)\setminus V(\vartheta), & \vartheta\in I\cap\aleph \end{cases}$$

Let $(V,\aleph)\tilde{\setminus}(Q,I)=(K,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Q(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap(I\setminus\aleph)=\emptyset \\ V(\vartheta)\setminus[Y(\vartheta)\setminus V(\vartheta)], & \vartheta\in\aleph\cap(I\cap\aleph)=\aleph\cap I \end{cases}$$

Hence, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Since $\forall\vartheta\in\aleph$, $K(\vartheta)=V(\vartheta)$, $(K,\aleph)=(V,\aleph)$. Moreover, $(Y,I)\ \tilde{\setminus}\ [(V,\aleph)\ \tilde{\setminus}(Y,I)]=(Y,I)$ can be shown similarly.

In classical set theory, V∪Y=(V\Y) ∪(Y\V) ∪(V∩Y). Now, we have the following analogy:

32) $(V,\aleph)\ \tilde{\cup}\ (Y,I)=[(V,\aleph)\ \tilde{\setminus}\ (Y,I)]\ \tilde{\cup}\ [(Y,I)\tilde{\setminus}\ (V,\aleph)]\ \tilde{\cup}\ [(V,\aleph)\ \tilde{\cap}\ (Y,I)]$.

Proof: Let $(V,\aleph)\tilde{\setminus}(Y,I)=(Q,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$Q(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Let $(Y,I)\ \tilde{\setminus}\ (V,\aleph)=(K,I)$. Then, $\forall\vartheta\in I$,

$$K(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in I\setminus\aleph \\ Y(\vartheta)\setminus V(\vartheta), & \vartheta\in I\cap\aleph \end{cases}$$

And let $(Q,\aleph)\ \tilde{\cup}\ (K,I)=(T,\aleph)$, where $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in\aleph\setminus I \\ Q(\vartheta)\cup K(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in(\aleph\setminus I)\setminus I=\aleph\setminus I \\ V(\vartheta)\setminus Y(\vartheta), & \vartheta\in(\aleph\cap I)\setminus I=\emptyset \\ V(\vartheta)\cup Y(\vartheta), & \vartheta\in(\aleph\setminus I)\cap(I\setminus\aleph)=\emptyset \\ V(\vartheta)\cup[Y(\vartheta)\setminus V(\vartheta)], & \vartheta\in(\aleph\setminus I)\cap(I\cap\aleph)=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)]\cup Y(\vartheta), & \vartheta\in(\aleph\cap I)\cap(I\setminus\aleph)=\emptyset \\ [V(\vartheta)\setminus Y(\vartheta)]\cup[Y(\vartheta)\setminus V(\vartheta)], & \vartheta\in(\aleph\cap I)\cap(I\cap\aleph)=I\cap\aleph \end{cases}$$

Hence, $\forall\vartheta\in\aleph$,

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ [V(\vartheta)\setminus Y(\vartheta)]\cup[Y(\vartheta)\setminus V(\vartheta)], & \vartheta\in\aleph\cap I \end{cases}$$

Let $(V,\aleph)\ \tilde{\cap}(Y,I)=(W,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$W(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\cap Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Let $(T,\aleph)\ \tilde{\cup}(W,\aleph)=(R,\aleph)$. Thus, for all $\vartheta\in\aleph$;

$$R(\vartheta)=\begin{cases} T(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ T(\vartheta)\cup W(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$R(\vartheta)=\begin{cases} V(\vartheta)\cup V(\vartheta), & \vartheta\in(\aleph\setminus I)\cap(\aleph\setminus I)=\aleph\setminus I \\ V(\vartheta)\cup[V(\vartheta)\cap Y(\vartheta)], & \vartheta\in(\aleph\setminus I)\cap(\aleph\cap I)=\emptyset \\ [[V(\vartheta)\setminus Y(\vartheta)]\cup[Y(\vartheta)\setminus V(\vartheta)]]\cup V(\vartheta), & \vartheta\in(\aleph\cap I)\cap(\aleph\setminus I)=\emptyset \\ [[V(\vartheta)\setminus Y(\vartheta)]\cup[Y(\vartheta)\setminus V(\vartheta)]]\cup[V(\vartheta)\cap Y(\vartheta)], & \vartheta\in(\aleph\cap I)\cap(\aleph\cap I)=\aleph\cap I \end{cases}$$

Since $[[V(\vartheta)\setminus Y(\vartheta)]\cup[Y(\vartheta)\setminus V(\vartheta)]]\cup[V(\vartheta)\cap Y(\vartheta)]=V(\vartheta)\cup Y(\vartheta)$, thus,

$$R(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\cup Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Therefore, $(R,\aleph)=(V,\aleph)\ \tilde{\cup}\ (Y,I)$.

NOTE: Since $[(V,\aleph)\ \tilde{\cup}\ (Y,I)]\ \tilde{\cup}\ (Z,K)\neq(V,\aleph)\ \tilde{\cup}\ [(Y,I))\ \tilde{\cup}\ (Z,K)]$, we should also have looked the following:

33) $(V,\aleph)\ \tilde{\cup}\ (Y,I)=[(V,\aleph)\ \tilde{\setminus}\ (Y,I)]\ \tilde{\cup}\ [[(Y,I)\tilde{\setminus}\ (V,\aleph)]\ \tilde{\cup}\ [(V,\aleph)\ \tilde{\cap}\ (Y,I)]]$.

Proof: Let $(Y,I)\tilde{\setminus}(V,\aleph)=(Q,I)$. Then, $\forall\vartheta\in I$,

$$Q(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in I\backslash \aleph \\ Y(\vartheta)\backslash V(\vartheta), & \vartheta\in I\cap \aleph \end{cases}$$

Let $(V,\aleph)\ \tilde{\backslash}\ (Y,I)=(K,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$K(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in \aleph\backslash I \\ V(\vartheta)\backslash Y(\vartheta), & \vartheta\in \aleph\cap I \end{cases}$$

And let $(Q,I)\ \tilde{\cup}\ (K,\aleph)=(T,I)$, where $\forall\vartheta\in I$,

$$T(\vartheta)=\begin{cases} Q(\vartheta), & \vartheta\in I\backslash \aleph \\ Q(\vartheta)\cup K(\vartheta), & \vartheta\in I\cap \aleph \end{cases}$$

Thus, $\forall\vartheta\in I$, hence,

$$T(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in (I\backslash \aleph)\backslash\aleph=I\backslash\aleph \\ Y(\vartheta)\backslash V(\vartheta), & \vartheta\in (I\cap \aleph)\backslash\aleph=\emptyset \\ Y(\vartheta)\cup V(\vartheta), & \vartheta\in (I\backslash \aleph)\cap(\aleph\backslash I)=\emptyset \\ Y(\vartheta)\cup[V(\vartheta)\backslash Y(\vartheta)], & \vartheta\in (I\backslash \aleph)\cap(\aleph\cap I)=\emptyset \\ [Y(\vartheta)\backslash V(\vartheta)]\cup V(\vartheta), & \vartheta\in (I\cap \aleph)\cap(\aleph\backslash I)=\emptyset \\ [Y(\vartheta)\backslash V(\vartheta)]\cup[V(\vartheta)\backslash Y(\vartheta)], & \vartheta\in (I\cap \aleph)\cap(\aleph\cap I)=\aleph\cap I \end{cases}$$

Hence, $\forall\vartheta\in I$,

$$T(\vartheta)=\begin{cases} Y(\vartheta), & \vartheta\in I\backslash \aleph \\ [Y(\vartheta)\backslash V(\vartheta)]\cup[V(\vartheta)\backslash Y(\vartheta)], & \vartheta\in I\cap \aleph \end{cases}$$

Let $(V,\aleph)\ \tilde{\backslash}(Y,I)=(W,\aleph)$. Then, $\forall\vartheta\in\aleph$,

$$W(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in \aleph\backslash I \\ V(\vartheta)\backslash Y(\vartheta), & \vartheta\in \aleph\cap I \end{cases}$$

Let $(W,\aleph)\tilde{\cup}(T,I)=(R,\aleph)$. Thus, for all $\vartheta\in\aleph$;

$$R(\vartheta)=\begin{cases} W(\vartheta), & \vartheta\in \aleph\backslash I \\ W(\vartheta)\cup T(\vartheta), & \vartheta\in \aleph\cap I \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$R(\vartheta)=\begin{cases} V(\vartheta) & \vartheta\in (\aleph\backslash I)\backslash I=\aleph\backslash I \\ V(\vartheta)\backslash Y(\vartheta), & \vartheta\in (\aleph\cap I)\backslash I=\emptyset \\ V(\vartheta)\cup Y(\vartheta), & \vartheta\in (\aleph\backslash I)\cap(I\backslash\aleph)=\emptyset \\ V(\vartheta)\cup[[Y(\vartheta)\backslash V(\vartheta)]\cup[V(\vartheta)\backslash Y(\vartheta)]] & \vartheta\in (\aleph\backslash I)\cap(I\cap\aleph)=\emptyset \\ [V(\vartheta)\backslash Y(\vartheta)]\cup Y(\vartheta), & \vartheta\in (\aleph\cap I)\cap(I\backslash\aleph)=\emptyset \\ [V(\vartheta)\backslash Y(\vartheta))]\cup\{[Y(\vartheta)\backslash V(\vartheta)]\cup[V(\vartheta)\backslash Y(\vartheta)]\} & \vartheta\in (\aleph\cap I)\cap(I\cap\aleph)=\aleph\cap I \end{cases}$$

Since $[V(\vartheta)\backslash Y(\vartheta)]\cup[[Y(\vartheta)\backslash V(\vartheta)]\cup[V(\vartheta)\cap Y(\vartheta)]]=V(\vartheta)\cup Y(\vartheta)$,

$$R(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus I \\ V(\vartheta)\cup Y(\vartheta), & \vartheta\in\aleph\cap I \end{cases}$$

Therefore, $(R,\aleph)=(V,\aleph)\,\widetilde{\cup}\,(Y,I)$.

In [10], it was shown that the collection of soft sets with a fixed set of parameter set associated respect to restricted difference becomes a BCK algebra. Now, we have the following:

Theorem 3.6. $(S_\aleph(U), \widetilde{\setminus}, \emptyset_\aleph)$ is a bounded BCK algebra whose every element is an involution.
Proof: Let $(V,\aleph)$, $(Q,\aleph)$,$(Z,\aleph)\in S_\aleph(U)$. Then,
BCI-1 $[((V,\aleph)\,\widetilde{\setminus}\,(Q,\aleph))\,\widetilde{\setminus}\,((V,\aleph)\,\widetilde{\setminus}\,(Z,\aleph))]\,\widetilde{\setminus}((Z,\aleph)\,\widetilde{\setminus}\,(Q,\aleph))=\emptyset_\aleph$. In fact,
Let $(V,\aleph)\,\widetilde{\setminus}\,(Q,\aleph)=(T,\aleph)$, where $\forall\vartheta\in\aleph$;

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus Q(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(V,\aleph)\,\widetilde{\setminus}(Z,\aleph)=(M,\aleph)$, where $\forall\vartheta\in\aleph$;

$$M(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus Z(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(T,\aleph)\,\widetilde{\setminus}(M,\aleph)=(W,\aleph)$, where $\forall\vartheta\in\aleph$;

$$W(\vartheta)=\begin{cases} T(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ T(\vartheta)\setminus M(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Hence, $\forall\vartheta\in\aleph$;

$$W(\vartheta)=\begin{cases} T(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ [V(\vartheta)\setminus Q(\vartheta)]\setminus[V(\vartheta)\setminus Z(\vartheta)], & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(Z,\aleph)\,\widetilde{\setminus}(Q,\aleph)=(S,\aleph)$, where $\forall\vartheta\in\aleph$;

$$S(\vartheta)=\begin{cases} Z(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ Z(\vartheta)\setminus Q(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(W,\aleph)\,\widetilde{\setminus}(S,\aleph)=(X,\aleph)$, where $\forall\vartheta\in\aleph$;

$$X(\vartheta)=\begin{cases} W(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ W(\vartheta)\setminus S(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$X(\vartheta)=\begin{cases} W(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ \{[V(\vartheta)\setminus Q(\vartheta)]\setminus[V(\vartheta)\setminus Z(\vartheta)]\}\setminus[Z(\vartheta)\setminus Q(\vartheta)], & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, $\forall\vartheta\in\aleph$;

$$X(\vartheta)=\begin{cases} W(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ \emptyset\,, & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

It is seen that $(X,\aleph)=\emptyset_\aleph$.

BCI-2 $[(V,\aleph)\,\tilde{\setminus}\,((V,\aleph)\,\tilde{\setminus}(Q,\aleph))]\,\tilde{\setminus}\,(Q,\aleph)=\emptyset_\aleph$. In fact, let $(V,\aleph)\,\tilde{\setminus}\,(Q,\aleph)=(T,\aleph)$, where $\forall\vartheta\in\aleph$;

$$T(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus Q(\vartheta)\,, & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(V,\aleph)\,\tilde{\setminus}(T,\aleph)=(M,\aleph)$, where $\forall\vartheta\in\aleph$;

$$M(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus T(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, $\forall\vartheta\in\aleph$

$$M(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\setminus[V(\vartheta)\setminus Q(\vartheta)], & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus, $\forall\vartheta\in\aleph$,

$$M(\vartheta)=\begin{cases} V(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ V(\vartheta)\cap Q(\vartheta), & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(M,\aleph)\tilde{\setminus}(Q,\aleph)=(L,\aleph)$, where $\forall\vartheta\in\aleph$;

$$L(\vartheta)=\begin{cases} M(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ M(\vartheta)\setminus Q(\vartheta)\,, & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Let $(V,\aleph)\,\tilde{\setminus}(L,\aleph)=(D,\aleph)$, where $\forall\vartheta\in\aleph$;

$$D(\vartheta)=\begin{cases} M(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ [V(\vartheta)\cap Q(\vartheta)]\setminus Q(\vartheta)\,, & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

Thus,

$$D(\vartheta)=\begin{cases} M(\vartheta), & \vartheta\in\aleph\setminus\aleph=\emptyset \\ \emptyset, & \vartheta\in\aleph\cap\aleph=\aleph \end{cases}$$

It is seen that $(D,\aleph)=\emptyset_\aleph$.

BCI-3 By Theorem 3.5. (5), $(V,\aleph)\,\tilde{\setminus}(V,\aleph)=\emptyset_\aleph$.

BCI-4 By Theorem 3.5. (19), $(V,\aleph)\tilde{\setminus}(Y,\aleph)=\emptyset_\aleph \Rightarrow (V,\aleph)\tilde{\subseteq}(Y,\aleph)$ and $(Y,\aleph)\tilde{\setminus}(V,\aleph)=\emptyset_Q \Rightarrow (Y,\aleph)\,\tilde{\subseteq}(V,\aleph)$ and thus, $(V,\aleph)=(Y,\aleph)$.

BCK-5 By Theorem 3.5. (7), $\emptyset_\aleph\tilde{\setminus}(V,\aleph)=\emptyset_\aleph$.

Thus, $(S_ℵ(U), \tilde{\setminus}, \emptyset_ℵ)$ is a BCK-algebra. Since, $(V, ℵ)\tilde{\setminus} U_ℵ=\emptyset_ℵ$ for all $(V, ℵ)\in S_ℵ(U)$ (b)y Theorem 3.5. (8), $(S_ℵ(U), \tilde{\setminus}, \emptyset_ℵ)$ is a bounded BCK-algebra, where ℵ is a fixed set of parameter set. Moreover since $U_ℵ \tilde{\setminus}[U_ℵ\tilde{\setminus}(V, ℵ)]=(V, ℵ)$ for all $(V, ℵ)\in S_ℵ(U)$, (As $U_ℵ\tilde{\setminus}(V, ℵ)= (V, ℵ)^r$ by Theorem 3.5. (9)), and $U_ℵ\tilde{\setminus}(V, ℵ)^r =[(V, ℵ)^r]^r=(V, ℵ)$), every element of $S_ℵ(U)$ is an involution.

In fact, since restricted difference soft set operation coincides with soft complementary difference operation in the collection of soft sets with a fixed parameter set, the BCK algebra in [10] and the BCK-algebra in this paper are in fact the same.

4. CONCLUSION

Since the inception of soft set theory by Molodtsov, numerous variations of soft set operations have been described and used. In this article, in order to improve the soft binary piecewise difference operation, we have explored its overall properties, especially in comparison with the fundamental properties of the difference operation in classical set theory and we have obtained very interesting analogies. Furthermore, we have proved that the set of all soft sets with a fixed parameter set is a bounded BCK-algebra together with the soft binary piecewise difference operation. Since studying the algebraic structure of soft sets from the perspective of operations provides deep insight into the algebraic structure of soft sets and its application and soft set algebra shows the potential applications of soft sets in classical and nonclassical logic, we hope this paper contibutes to the literature of soft set in this regard.

CONFLICT OF INTEREST

The authors stated that there are no conflicts of interest regarding the publication of this article.

AUTHORSHIP CONTRIBUTIONS

The authors contributed equally to this work.

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